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Intermittent binary C.P.F.S.K.-0.5 signaling in a Rayleigh Fading medium

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INTERMITTENT BINARY C.P.F.S.K.-0.5

SIGNALLING IN A RAYLEIGH FADING MEDIUM

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The error rate of an intermittent continuous phase binary f.s.k. system with deviation ratio 0.5 operating in a Rayleigh fading channel is evaluated. The performance of this system is better than systems which operate continuously in the same medium. Are made comparisons with binary f.s.k. for optimum diversity.
The purpose of this letter is to illustrate the performance (average error rate) of an intermittent continuous phase binary f.s.k. (c.p.f.s.k.) operating in a Rayleigh fading medium.

The transmitted c.p.f.s.k. (with deviation ratio 0.5) signal is characterized during the "i th" bit time by

\[ S_k(t) = \frac{1}{\pi} \cos \left[ \omega_c t + \frac{\pi}{2T} \left( -1 \right)^k t + \theta_i \right] \quad \text{for} \quad iT \leq t \leq (i+1)T \]

where \( \omega_c \) is the carrier radian frequency in rad./sec, \( \theta_i \) is the carrier phase at the beginning of the "i th" interval and \( T \) is the bit period.

In the case of additive white Gaussian noise channel and with maximum likelihood coherent demodulation for an observation interval \( 0 \leq t \leq 2T \) sec, a union bound on the average bit error probability, evaluated as in Reference\(^1\), is given by

\[ P_{EB}(\gamma) = \frac{1}{8} \sum_{j \neq 0} \sum_{k \neq 0} \sum_{n \neq 0} \text{erfc} \left\{ \sqrt{\gamma (1 - \rho_{jkmn})} \right\} \quad (1) \]

where \( \gamma \) is the signal-to-noise ratio of the ideal filter output at the sampling instant which is given by

\[ \gamma = \frac{1/\pi^2}{R \cdot N_0} \]

where

- \( 1/\pi^2 \) is the received signal power
- \( R \) is the transmission rate = \( 1/T \)
- \( N_0 \) is the single-sided noise power density
The
\[
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-t^2) \, dt
\]
and the correlation coefficients $\rho_{jkmn}$ can only take on the values $0, -1$.

When we consider slow nonselective fading, the probability of error, described by (1) is also a random variable. Specifically, $\gamma$ is a random variable which is exponentially distributed as
\[
r(\gamma) = \frac{1}{\gamma_0} \exp\left(-\frac{\gamma}{\gamma_0}\right), \quad 0<\gamma<\infty
\]
(2)
where
\[
\gamma_0 = \int_{0}^{\infty} \gamma r(\gamma) \, d\gamma
\]
is the mean value of $\gamma$ averaged over the fading.

In the operation of the intermittent systems for digital transmission, when the signal-to-noise ratio $\gamma$ falls below the value $\gamma^* = 3(4.8 \text{ dB in operational systems})$ the synchronization between transmitter and receiver is deleted and the digital transmission ceases. Therefore, in order to obtain the average probability of error, it is only necessary to average expression (1) over the interval $(\gamma^*, \infty)$. The average error probability is thus given by
\[
\overline{P_{EB}} = \int_{\gamma^*}^{\infty} r(\gamma) \cdot P_{EB}(\gamma) \, d\gamma
\]
(3)

The insertion of (1) and (2) into (3), and its integrating by parts, yields the following expression for the average probability of error $^2$
\[
\overline{P_{eB}} = - \frac{1}{\sqrt{\frac{4}{4} + 4/\gamma_0}} \text{erfc} \sqrt{3(1 + 1/\gamma_0)} \\
- \frac{1}{\sqrt{\frac{4}{4} + 2/\gamma_0}} \text{erfc} \sqrt{3(1 + 0.5/\gamma_0)} \\
+ 7.5 \times 10^{-3} \left\{ \exp\left(-\frac{3}{\gamma_0}\right) + \exp\left(-\frac{1.5}{\gamma_0}\right) \right\}
\]

Equation (4) represents the average error rate of an intermittent binary c.p.f.s.k.-0.5 with ideal filtering, employing matched filter detection and operating in a Rayleigh fading medium.

In Fig. 1 the error rate curves are plotted for c.p.f. s.k. both intermittent and continuously operating systems. Also plotted are the curves for a non-fading infinite-bandwidth channel and the optimum frequency diversity\(^4\) of order \(M=1\) and \(M=2\). From these results, the main source of degradation is the continuous fading. An ideal intermittent system compares favorably with second-order diversity up to 14 dB for an average error probability of 9.7 \times 10^{-4}.
REFERENCES


FIGURE 1. Comparison of error-rate performance of both intermittent and continuously operating f.s.k. systems.

a Non-fading operating: c.p.f.s.k. -0.5
b Intermittent operating: c.p.f.s.k. -0.5
c Second order diversity: coherent f.s.k.
d Continuously operating: c.p.f.s.k. -0.5
e First-order diversity: coherent f.s.k.