LINE TRACKING PERFORMANCE OF A LEAST SQUARES ADAPTIVE LATTICE ALGORITHM

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ABSTRACT.

The work shows the performance of an algorithm for adaptive lattice structures in line tracking. The algorithm introduces some modifications in the classical LMS procedure, which theoretical background can be viewed in |1|. Basically, the algorithm uses an adaptive selection for the two basic parameters β and γ in an LMS algorithm for all-pole lattice structures. The so-called memory parameter β and weighting residual parameter γ are updated at each sample of the input signal and also they exhibit its dependence with the section number along the lattice structure. This work will show up to what degree these parameters can improve the tracking behaviour of the procedure.

1.- INTRODUCTION.

Since the all-pole lattice structure appeared, it has been succesfully applied in many problems for spectral estimation or line tracking. In both cases, the orthogonality between successive sections results in a better convergence of the algorithm than using the transversal structure |2|. Although the orthogonality is not true in an adaptive version, it can be assumed without degrading the algorithm quality |3,4|; this phenomena is more evident when the eigenvalues of the corresponding covariance matrix spread apart.

The algorithm to compute the coefficients of the lattice is least squares with object function

$$L_q(n) = \sum_{k=0}^{n} \beta^{n-k} (1-\gamma) f^2(q,n) - \gamma b^2(q,n) \quad (1)$$

where

- q denotes the section number;
- n denotes the current sample;
- $f(q,n)$ is the forward error;
- $b(q,n)$ is the backward error;
- β is the memory parameter;
- γ is the weighting parameter;
- $L_q(n)$ objective function.

Minimizing $L_q(n)$ allows us to obtain the PARCOR coefficients $K(q,n)$. It is worthwile to mention that $K(q,n)$ can be obtained recursively |5| with formula which can be viewed as a Gradient Adaptive Lattice (GAL) |3,4| algorithm with convergence parameter set to one.

2.- WEIGHTING PARAMETER γ AND MEMORY PARAMETER β.

It seems to be clear that in selecting β, a tradeoff exists between convergence rate and misadjustement error. In |1| the authors reported the following formula to compute adaptively $\beta(q,n)$ in order to get good convergence rate with adequate misadjustement noise:

$$\beta(q,n) = 1-\sigma(q,n) \quad (2)$$

where $\sigma(q,n)$ is an estimate of the mean square error between the current PARCOR $\tilde{K}(q,n)$ and the optimum one at the time n:

$$E\{[K(q,n)-\hat{K}(q,n)]^2\} = \sigma(q,n) \tag{3}$$

and

$$\sigma(q,n)=\sigma(q,n-1)\left[1-\frac{\sigma(q,n-1)b^2(q-1,n-2)}{f^2(q,n-1)+\sigma(q,n-1)b^2(q-1,n-2)}\right]+ \tag{4}$$

$$+ V(q)$$

where V(q) is a thereshold value that has to be close to zero for stationary signals and close to 0.01 for nonstationary signals (i.e. coherent sinusoids in white noise).

With respect to parameter γ, its importance has been described in speech processing |5|; and, in general, the authors in |1| relates its time evolution with the local behaviour of the signal under analysis x(n). This parameter becomes relevant in the problem of line tracking where the formula (5) allows the poles of the all-pole model to move closer to the unit circle than with the classical selection $\gamma=0.5$,

$$\gamma(n) = \frac{R(n)}{R(n)+S(n)} \tag{5}$$

where

$$S(n) = \sum_{m=0}^{Q/2-1} x^2(n-m)$$

$$R(n) = \sum_{m=0}^{Q/2-1} x^2(n-q+m)$$

In the next section we describe the experimental work carried out in the line tracking context by the authors, in order to prove the high quality of the resulting algorithm (i.e. using (3) and (5)) respect the time invariant parameters β and γ GAL procedure.

3.- EXPERIMENTAL WORK.

In order to test the algorithm the following signals were used in the experiments, which are currently used in this kind of research |2|.

a.- Two piece-wise constant frequency sinusoids with instantaneous and equal steps.

$SNR_1 = 20$ dB $\omega_1 = 7\pi/8$

$SNR_2 = 10$ dB $\omega_2 = \pi/4$

b.- The same as case (a) but opposed step sign.

c.- Two sinusoids with crossing linear variation frequency.

The used algorithm is the same proposed by Makhoul |5|, where the parameters β and γ are computed from eqs. (3) and (5); we choose the values for the time-independent parameters according to the following criteria:

1.- The parameter β is chosen to be 0,975, which is a value that allows a good compromise between convergence speed and variance of the steady-state frequency estimates. We must note that in each case a "better" β may exist. However, we do not know anything about the implied variation of the PARCOR coefficients.

2.- The parameter γ is chosen to be 0.5 which is the value that, as shown experimentally, gives a superior performance in most cases. We must remark that "a priori" the choice of γ seems not to be important since the forward and backward predictions are equivalent.

In the case of an adaptive estimation of the parameters, we must not do any previous choice excepting the magnitud V(q) or the final covariance of the PARCOR coefficients, where a value V(q)=0.01 is correct in most situations. It has been verified that his value for V(q) allows a choice of β inside the interval (0.96, 0.99). From eq. (4) we may observe that the maximum value of the β, $\beta_{max}=1-V(q)$, is not obtained since $f^2(q,n)$ does not become zero.

In fig. 1 we present the results obtained with the signal 'a' in the three cases as pointed. Let's observe that the speed of tracking of the sinusoid ω_1 with greater magnitude is slightly superior in the adaptive parameter estimation cases and the variance of the final estimates are equal. To the sinusoid ω_2 the tracking is greatly acelerated with a slight increase in the variance.

In fig. 2 we show the results obtained for the signal 'b'. It is important to remark the reduction of the tracking time of the weak sinusoid ω_2 in the adaptive parameter estimation cases. The study of the pole plot shows a faster deplacement inside the unit circle of the poles corresponding to ω_1, together with a faster use of the two extra poles (Q=6) in the tracking of the weak sinusoid.

The result obtained with signal 'c' are shown in fig. 3; we observe that the tracking of sinusoid ω_1 is slightly better and the tracking of the weak sinusoid ω_2 is much better. Likewise, a lost of one of the frequencies happens in shorter intervals.

4.- CONCLUSIONS.

From the studied experiments, we conclude that:

1.- The adaptive estimation of the parameter β gives a better frequency tracking speed in the case of sudden frequency changes, due to the assignement of β values very different from unity at the time of transition. In steady-state, the eqs. (4) and (3) give values of β near unity.

2.- The estimation of the parameter γ using eq. (5) gives rise to models with pole positions near the unit circle, since the nonstable direction of the prediction has a greater preponderance (consider the case of a decreasing exponential, where $\gamma>0.5$ and backward direction is the nonstable). This effect decreases the inertia of the algorithm and the influence of the noise that pull the poles to positions faraway from the unit circle resulting in an increase of the resolution of the algorithm.

We must note that in fact we do not need to choose "a priori" the β and γ parameters. This is a consequence of the robustness of the proposed algorithm.

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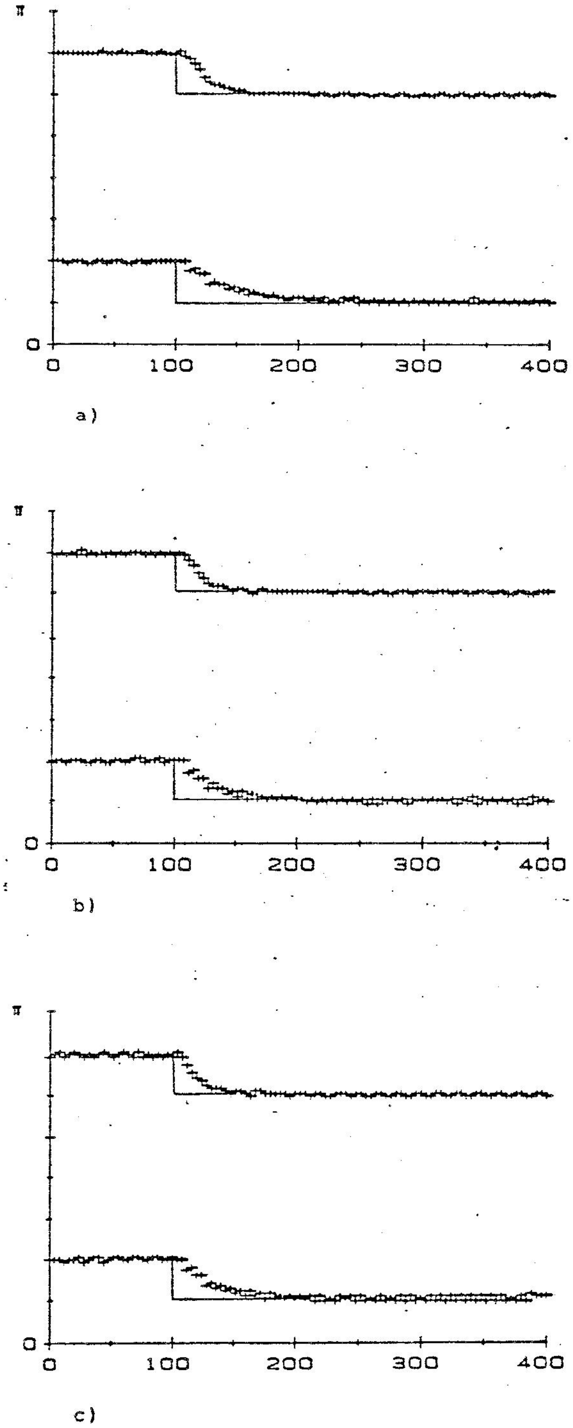
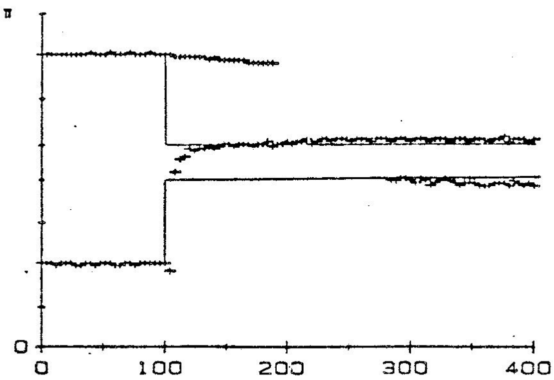


Fig. 1. Tracking performance with the signal a. Order Q=6, 256 points FFT, step magnitude: $\pi/8$.
a) $\beta=0.975$, $\gamma=0.5$. b) β adaptive, $\gamma=0.5$. c) β and γ adaptive.



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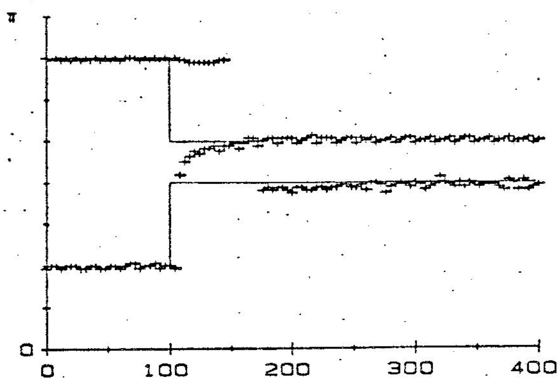

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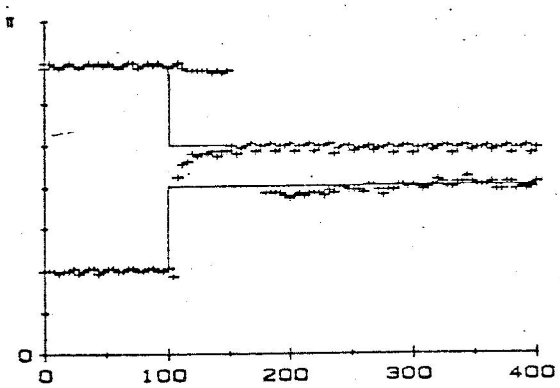
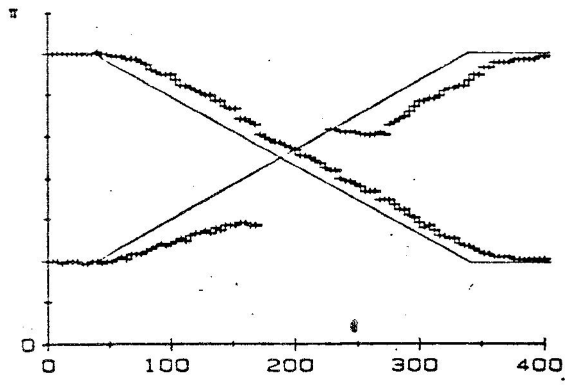


c)

Fig. 2. Tracking performance with the signal b. Order Q=6, 256 points FFT, step magnitude: π/4. a) β=0.975, γ=0.5. b) β adaptive, γ=0.5. c) β and γ adaptive.



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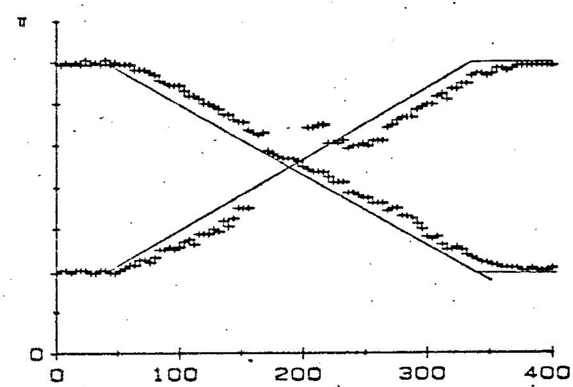

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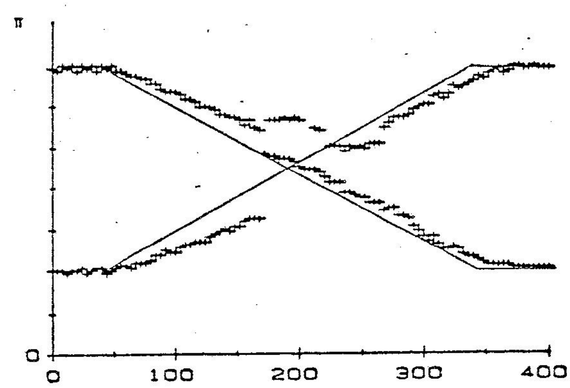

c)

Fig. 3. Tracking performance with the signal c. Order Q=6, 256 points FFT, a) β=0.975, γ=0.5. b) β adaptive, γ=0.5. c) β and γ adaptive.