A SUBOPTIMUM RANK TEST FOR RADAR DETECTION

José L. Sanz-González*, Aníbal R. Figueiras-Vidal*,
José B. Mariño-Acebal**.

** ETSI Telecomunicación-UPB, Jorge Girona Salgado s/n, Barcelona-34 Spain.

ABSTRACT

In this communication, a new nonparametric rank scheme is introduced for Radar application and its performance briefly analyzed.

The proposed scheme has a suboptimum character, since it is introduce as a simplification of a general optimum rank detector - obtained for hypotheses defined by sample ranks in a multiple pulse situation.

We compare the characteristics of this new structure with some classical nonparametric tests used in Radar detection, and we conclude with a discussion derived from it.
1. INTRODUCTION

It is well known that the control of false alarms is very important in many situations of Radar detection; that is, the false alarm probability $P_{fa}$ has to be fixed at a specified value (say $10^{-6}$) under a wide variety of circumstances. On the other hand, the detection probability $P_d$ must be as high as possible.

This necessity of controlling the false alarms in an optimum Radar detector (designed by application of the Neyman-Pearson criterion assuming white Gaussian noise with known power level) when the power noise is unknown, has led to parametric CFAR (Constant False Alarm Rate) detectors [1],[2],[3]. These detectors make use of noise reference samples to estimate the noise power, and so to control adequately the threshold for a given $P_{fa}$.

The parametric CFAR detectors perform satisfactorily when only the noise level varies, but not when the noise sample distribution departs from the distributions initially assumed. This last situation is very frequent in Radar environments because of the usual nonstationarity of disturbances.

To maintain a constant $P_{fa}$ assuming "any" distribution of noise samples needs nonparametric CFAR detectors; in the sequel, simply nonparametric detectors.

A nonparametric detector is composed of a distribution-free statistic and a threshold, deciding target present / target absent when the statistic is greater / lower than the threshold. It is said that a statistic (or function of the received samples) is distribution-free if it is independent of the present noise distribution.

Many types of distribution-free statistics have been used in Radar and Communication applications. We have among others sign detectors [4],[5], rank detectors [6],[7],[8], and detectors of conditional threshold (based in conditional tests) [9],[10],[11],[12],[13].

In this communication, we will deal only with rank detectors - applied to Radar. We define the hypotheses, then, we will propose and analyze a suboptimum detector.

2. DEFINITION OF THE HYPOTHESES

The problem of Radar detection can be easily outlined in terms of binary statistical hypotheses: $H_0$ (absent target) and $H_1$ (present target).
Let us suppose that we employ \( N \) pulses by antenna beamwidth and that we get \( M \) noise reference samples for each range cell under test and each of these \( N \) pulses; that is, we work with

\[
\bar{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iM}, x_i), \quad i = 1, 2, \ldots, N
\]

where \( \bar{x}_i \) is a vector of \( M+1 \) components (samples taken at the output of the envelope detector), corresponding to the \( i \)-th pulse, being \( x_{ij} \) (\( j = 1, 2, \ldots, M \)) noise reference samples and \( x_i \) the samples under test. Also, we will consider that the reference samples are independent and identically distributed, the usual condition assumed when working with distribution-free tests.

The hypotheses are defined as follows

\[
H_0: P(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N/H_0) = \prod_{i=1}^{N} P_{\bar{x}_i}(\bar{x}_i/H_0)
\]

where

\[
P_{\bar{x}_i}(\bar{x}_i/H_0) = P_{\bar{x}_i}(x_i) \prod_{j=1}^{N} P_{\bar{x}_i}(x_{ij})
\]

being \( P_{\bar{x}_i}(x) \) the distribution function of a noise sample that corresponds to the \( i \)-th pulse;

\[
H_1: P(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N/H_1) = \int P(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N/\mathfrak{S}, H_1) \cdot p_{\mathfrak{S}}(\mathfrak{S}) \cdot d\mathfrak{S}
\]

where

\[
F(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N/\mathfrak{S}, H_1) = \prod_{i=1}^{N} F_{\bar{x}_i}(\bar{x}_i/\mathfrak{S}_i, H_1)
\]

\[
F_{\bar{x}_i}(\bar{x}_i/\mathfrak{S}_i, H_1) = F_{\bar{x}_i}(x_i/\mathfrak{S}_i, H_1) \prod_{j=1}^{N} F_{\bar{x}_i}(x_{ij})
\]

\( \mathfrak{S} = (\mathfrak{S}_1, \mathfrak{S}_2, \ldots, \mathfrak{S}_N) \) is the signal-to-noise ratio vector (\( \mathfrak{S}_i, i = 1, 2, \ldots, N \) is the signal-to-noise ratio corresponding to \( x_i \), the sample under test of the \( i \)-th sweep);

\( p_{\mathfrak{S}}(\mathfrak{S}) \) is the distribution function of the sample under test \( x_i \) conditioned by \( \mathfrak{S}_i \) and \( H_1 \).

\( p_{\mathfrak{S}_0}(\mathfrak{S}) \) is the probability density function that defines the target, with \( \mathfrak{S}_0 = (\mathfrak{S}_{01}, \mathfrak{S}_{02}, \ldots, \mathfrak{S}_{0N}) \) as a parameter of the distribution.
We have to realize that under the null hypothesis $H_0$, the components of $\bar{X}_i=(x_1, x_2, \ldots, x_M, x_i)$ are independent and identically distributed (IID). Under the alternative hypothesis $H_1$, the components of $\bar{X}_i$ are independent (the reference samples keep the IID character).

The ranks $r_i$ of the samples under test $x_i$ are

$$ r_i = \sum_{j=1}^{M} u(x_i-x_{ij}), \quad i=1,2,\ldots,N $$

being

$$ u(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} $$

the unit step function.

It can be proved [14] that $r_i$ ($i=1,2,\ldots,N$) are sufficient statistics to test $H_0$ against $H_1$; that is, the information of the ranks of the reference samples is statistically irrelevant.

The hypotheses defined over ranks are

$$ H_0: P(\bar{R}=\bar{r}/H_0) = \prod_{i=1}^{N} P(R_i=r_i/H_0) = \left( \frac{1}{M+1} \right)^N $$

$$ 0 \leq r_i \leq M $$

that is, $\bar{R} = (R_1, R_2, \ldots, R_N)$ is uniformly distributed and with IID components;

$$ H_1: P(\bar{R}=\bar{r}/H_1) = \int \prod_{i=1}^{N} P(R_i=r_i/S_i,H_1) \cdot p_{\bar{S}}(\bar{S}) \cdot d\bar{S} $$

being

$$ P(R_i=r_i/S_i,H_1) = \prod_{i=1}^{N} P(R_i=r_i/S_i,H_1) $$

$$ P(R_i=r_i/S_i,H_1) = \binom{M}{r_i} \cdot \int_{0}^{\infty} \left[ P_{oi}(x) \right]^{r_i} \cdot \left[ 1 - P_{oi}(x) \right]^{M-r_i} \cdot f_{X_i}(x/S_i) \cdot dx $$

where

$$ f_{X_i}(x/S_i) $$

is the probability density function of $X_i$ with signal-to-noise ratio $S_i$.

3. STRUCTURE OF THE SUBOPTIMUM RANK DETECTOR.

By applying Neyman-Pearson lemma to the hypotheses (2.2), defined in terms of ranks, we obtain the optimum rank detector:

$$ \frac{P(\bar{R}=\bar{r}/H_1)}{P(\bar{R}=\bar{r}/H_0)} = (M+1)^N \cdot \prod_{i=1}^{N} P(R_i=r_i/S_i,H_1) \cdot p_{\bar{S}}(\bar{S}) d\bar{S} \quad H_1 \quad H_0 $$

$$ N \quad \gamma $$

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We have to realize that the detector (3.1) depends on the target model and the signal-to-noise ratio \( S_o \).

With a low signal-to-noise ratio, \( S_o \rightarrow 0 \) (\( H_1 \rightarrow H_0 \)), we can obtain the locally optimum detector, with a structure (assuming Gaussian noise)

\[
\sum_{i=1}^{N} a(r_i) \left( \frac{H_i}{H_0} \right) T_{OL} \quad (3.2-a)
\]

where

\[
a(r_i) = \sum_{k=0}^{r_i} \frac{1}{M-k+1} \quad (3.2-b)
\]

that correspond to the Modified Savage detector [8].

By doing approximations in (3.1), we can obtain (for any target model and any signal-to-noise ratio) the following suboptimum detector [14]:

\[
\sum_{i=1}^{N} a(r_i) \left( \frac{H_i}{H_0} \right) T_C \quad (3.3-a)
\]

where

\[
a(r_i) = \ln \left( \frac{M+\alpha+1}{M-r_i+\alpha} \right), \quad 0 < \alpha < \infty \quad (3.3-b)
\]

If \( \alpha = 0.56 \), it can be proved easily that (3.3) is equivalent to (3.2); if \( \alpha \rightarrow 0 \), (3.3) is equivalent to the MST proposed in [4]; and if \( \alpha \rightarrow \infty \), (3.3) is equivalent to the GST proposed in [5].

The parameter \( \alpha \) is fixed to maximize \( P_d \) in an interval of practical interest (for instance, in \( 0.2 \leq P_d \leq 0.95 \)). We can anticipate that the detectability curves have low sensitivity to \( \alpha \) variations.

4. DETECTOR PERFORMANCE.

Detector (3.3) is exhaustively analyzed in [14]. We have calculated its ARE (Asymptotic Relative Efficiency), its \( P_d \) versus signal-to-noise ratio (S) curves, the effect of interfering targets, and the effect of sample quantization.

In Fig. 4.1, we present ARE versus \( M \) (number of reference samples) and versus \( \alpha \) (detector parameter). It can be observed that the ARE is maximum at \( \alpha = 0.56 \); it equals the ARE of the locally optimum detector (3.2). This fact confirm what it is previously said in the last item: if \( \alpha = 0.56 \), the detector (3.3) is equivalent to the detector (3.2).
Fig. 4.2 shows \( P_d \) vs. \( S \) curves of a single Swerling II target, assuming \( 10^{-6} \) and \( 10^{-8} \) for \( P_{fa} \) and the other parameters there indicated. From Monte-Carlo simulations showing in [14] (and also theoretically), it can be concluded that if the target model is a priori unknown, the locally optimum detector (\( \alpha = 0.56 \)) will be the adequate suboptimum detector.

In the case of interfering targets in reference cells, assuming \( P_{fa} \leq 10^{-6} \) (the equality is getting when there are not interfering targets), \( P_d \) within practical values (for instance, \( 0.2 \leq P_d \leq 0.95 \)), and with nonfluctuating and Swerling II targets, it can be concluded [14]:

a) For \( M=7 \) (number of reference cells), we need \( N \) (number of integrated pulses) greater than 16 with one interfering target, and \( N \geq 30 \) with two interfering targets.

b) For \( M=15 \), we need \( N \geq 16 \) with two interfering targets, and \( N \geq 30 \) with three ones.

In Fig. 4.3, two cases of detectability with 0, 1, 2 and 3 interfering nonfluctuating targets are presented.

Fig. 4.4 shows \( P_d \) vs. \( S \) curves with the quantization step (normalized by the noise standard deviation) as a parameter. If the quantization step is zero (\( q=0 \)), the curves will correspond to the continuous case. From the work [14] it can be concluded that if the quantization step is a half of the noise standard deviation, the detectability losses with respect to the continuous case are not significant (\( \approx 0.2 \) dB) in all practical cases.

5. CONCLUSIONS.

In this communication, we have presented a suboptimum rank Radar detector. We have proved by Monte-Carlo simulations that the locally optimum detector is the most suitable for every target model. The minimum number of integrated pulses (\( N \)) has been given in order to minimize interfering targets effects. It is also said that the quantization effect is negligible when the quantization step is lower than half of noise standard deviation (the uniform randomization of ties maintains the distribution-free character of the rank detector, independently of the quantization step).

Finally, we remember that the MST [4] and the GST [5] are particular cases of our detector structure, and then, they have been implicitly considered in our optimization process.
REFERENCES.


Fig. 4.1: ARE curves of detector (3.3):
(a) as a function of $M$, taken as a parameter;
(b) as a function of $\alpha$, taken $M$ as a parameter.
Fig. 4.2: Detection probability $P_d$ vs. signal-to-noise ratio with Swerling II target models ($SW$-$II$)
Fig. 4.3: Detectability curves of a nonfluctuating target with identical interfering targets to the detecting target (BMNF).
Fig. 4.4: Detectability curves with quantizing samples and Swerling II targets (SW-II).

q = quantization step/noise standard deviation.