especially those which involve highly nonstationary environments, the difference in performance exhibited by the LS and SG lattice algorithms may be more pronounced. A comparative study of algorithm performance in these remaining applications should yield interesting and useful results.

ACKNOWLEDGMENT

The authors are grateful to L. Griffiths for suggesting that the "one-coefficient" lattice simulations be performed.

REFERENCES

To show the validity of the previous discussion in a general context, we will consider a nonparametric estimator (the periodogram) with the usual and the modified approaches.

Fig. 1(a) and (b) illustrate the difference between the two choices using a sinusoid with frequency 0.05 as the signal. Fig. 1(a) shows ten selected lags among 50 in both cases; Fig. 1(b) shows the corresponding periodogram.

The examination of this last part of the figure confirms the previous assertion: the “effective window” in the modified approach produces higher resolution than in the classical one.

Of course, the sampling involved in the lag domain will produce extra peaks for very low frequencies when the sampling violates the Nyquist criterion. Nevertheless, it seems that in many cases, it is easier to remove extra lines than to increase resolution from the initial estimate.

III. RESULTS FOR ITERATIVE ME ALGORITHMS

Recently, Lim and Malik [1] have reported an iterative ME Fourier based method in both 1-D and 2-D problems, that has the extra advantage of including (without relevant modifications) cases in which some points of the autocorrelation are not available [3]. It is also possible to introduce without modifications the proposed approach in this algorithm.

We will now show results from simulated and real experiments using Lim and Malik’s algorithm with the classical and the modified approaches.

A. 1-D Case

Using

$$r_x(n) = a^2 \delta(n) + \cos(2\pi n 0.1) + \cos(2\pi n 0.3456)$$

with $a^2 = 1$ and $r_x(n)$ known for $-9 < n < 9$, Fig. 2 displays the results after four iterations of Lim and Malik’s algorithm for the two approaches.

It can be seen that, using the 0 and four additional more significant lags (largest absolute values), the resolution is better than that exhibited by the classical procedure (lags 0–4). Some additional comments and conclusions can be obtained from this example; they can be useful not only in 2-D cases (in which we have not the easy option of using all the lags as in this 1-D problem, since a typical $64 \times 32$ data set cannot be completely used, because less than 100 constraints have to be used to keep computing time and memory under adequate limits in each iteration), but also in other nonlinear procedures of spectral estimation.

We can check that the improvement in quality is accompanied by better extrapolated (interpolated) values when the modified approach is used. This is very reasonable, since we are working with the most representative values of the autocorrelation. Table I shows the actual values of the autocorrelation (second column) and the induced ones in both approaches (third and fourth column) after the fourth iteration. Each approach approximates better the values corresponding to the selected lags; but the global result is better for the modified procedure. This last approach increases also the dynamic range of the estimated spectrum.

B. 2-D Case

In the first example, we have selected
TABLE I
LAGS SELECTED IN CLASSICAL AND MODIFIED PROCEDURES FROM
A -9 TO 9 INITIAL AUTOCORRELATION FUNCTION, ACTUAL, CONSTRAINED
(MARKED WITH ARROWS), INTERPOLATED AND EXTRAPOLATED VALUES
OF THE AUTOCORRELATION SEQUENCE FOR BOTH PROCEDURES.

<table>
<thead>
<tr>
<th>Lag</th>
<th>Actual Correlation Values</th>
<th>ME extrapolation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Classical</td>
</tr>
<tr>
<td>0</td>
<td>3.00</td>
<td>3.16</td>
</tr>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>-0.04</td>
<td>-0.07</td>
</tr>
<tr>
<td>3</td>
<td>0.66</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>-1.55</td>
<td>-1.14</td>
</tr>
<tr>
<td>5</td>
<td>-1.12</td>
<td>-0.15</td>
</tr>
<tr>
<td>6</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>7</td>
<td>-1.19</td>
<td>-0.30</td>
</tr>
<tr>
<td>8</td>
<td>0.42</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>1.56</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Selected lags:
classical approach: -4, -3, -2, -1, 0, 1, 2, 3, 4
modified approach: -9, -7, -5, -4, 0, 4, 5, 7, 9.

Fig. 3. Iterative ME algorithm. (a) Classical approach after 35 iterations. (b) Most significant lags approach after 10 iterations.

\[
r_{x}(n, m) = 1 + \cos \left( 2\pi (0.1n + 0.1m) \right) + \cos \left( 2\pi (0.2n + 0.3215m) \right)
\]

and the use of 25 (total) lags from a basic 10 \times 10 mask. The results of the classical and the modified approaches appear in Fig. 3(a) (35 iterations) and Fig. 3(b) (10 iterations), respectively. The reduction of computational burden is clear (the 70 percent obtained is a typical value; similar percentages have been obtained in other tested cases). The classical procedure does not allow the detection of the peaks; the modified version shows a well defined low-frequency peak, regardless of the number of iterations (this allows the considerable reduction of...
The advantage seems to be mainly in computational cost and computing convolutions. In this correspondence, the use of polynomial method has been described for using polynomial transforms instead of FFT in the Rader's algorithm for autocorrelation, and the number of arithmetic operations compared; it is found that the use of PT's does not result in the expected computational advantage over the FFT.

The usefulness of a particular application of the proposed idea, choosing the largest absolute values of the autocorrelation in a 2-D iterative ME method, has been tested. More work is needed to obtain a general perspective of the possible advantages in other cases.

IV. CONCLUSIONS

The idea of selecting with a certain criterion lags to be used in autocorrelation based spectral analysis has been shown to be useful in some general examples. This possibility can be very interesting in parametric methods that have to be applied under iterative algorithms, such as many objective function minimization (maximization) procedures and 2-D situations. The advantage seems to be mainly in computational cost and resolution.

The usefulness of a particular application of the proposed idea, choosing the largest absolute values of the autocorrelation in a 2-D iterative ME method, has been tested. More work is needed to obtain a general perspective of the possible advantages in other cases.

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REFERENCES

[4] Nussbaum [4] has shown that polynomial transforms (PT's) offer about 50 percent savings in computation if used for the computation of radix-2 type convolution. Hence, it is interesting to examine whether PT's prove efficient when used for computing autocorrelation functions also. First let us briefly review the Rader's method of computing autocorrelation. The autocorrelation $R_x(m)$ of a real data sequence $x(n)$, $n = 0, 1, \cdots, N - 1$, is defined by ($x(n)$ is zero for $n \geq N$)

$$R_x(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)x(n+m). \tag{1}$$

Define a series of subsequences $x_i$ and $y_i$ of length $M'$ as

$$x_i(n) = \begin{cases} x \left( n + \frac{iM'}{2} \right) & 0 \leq n < \frac{M'}{2} \\ 0 & \frac{M'}{2} \leq n < M', \ i = 0, 1, 2, \cdots \end{cases} \tag{2}$$

$$y_i(n) = \begin{cases} x \left( n + \frac{iM'}{2} \right) & 0 \leq n < \frac{M'}{2} \\ 0 & \frac{M'}{2} \leq n < M', \ i = 0, 1, 2, \cdots \end{cases} \tag{3}$$

Now consider the cyclic correlation $Z_i(m)$ of $x_i$ and $y_i$:

$$Z_i(m) = \sum_{n=0}^{M'-1} x_i(n)y_j [(n + m) \mod M'], \quad m = 0, 1, \cdots, M' - 1. \tag{4}$$

From (2) and (3)

$$Z_i(m) = \sum_{n=0}^{M'-1} x \left( n + \frac{iM'}{2} \right) \times \left( n + \frac{iM'}{2} + m \right), \quad m = 0, 1, \cdots, \frac{M'}{2}. \tag{5}$$

The relation in (5) is true for $m = 0, 1, \cdots, (M'/2)$ only because of the nature of cyclic correlation.

Now consider

$$\omega_{\lambda}(m) = \sum_{i=0}^{\lambda} Z_i(m) = \sum_{i=0}^{\lambda} M'/2 - 1 x \left( m + \frac{iM'}{2} \right) \times \left( n + \frac{iM'}{2} + m \right) = 1(\lambda+1)M'/2 - 1 x(n)x(n+m), \quad m = 0, 1, \cdots, M'/2 \tag{6}$$

on the computation of autocorrelation using polynomial transforms

S. PRAKASH AND V. V. RAO

Abstract—Polynomial transforms (PT's) are known to be efficient for computing convolutions. In this correspondence, the use of polynomial transforms for computing autocorrelation has been considered. A method has been described for using polynomial transforms instead of the FFT in the Rader’s algorithm for autocorrelation, and the number of arithmetic operations compared; it is found that the use of PT’s does not result in the expected computational advantage over the FFT method.

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