State Estimation in Fuel Cell Systems: A Sliding Mode Approach

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Summary

In this paper, a nonlinear distributed parameter observer (NDPO) is developed for the estimation of the internal conditions of a proton exchange membrane fuel cell (PEMFC). In particular, the estimation of the internal concentration values for different gas species is studied. A novel high order sliding mode (HOSM) estimation technique, based on a distributed parameter model of a PEMFC is developed to reconstruct the internal variables. The observer includes the estimation of the reaction and water transport terms, which are considered as disturbances of the PEMFC estimation model. Since the approach is based on a distributed parameter model representation of the PEMFC, the observer is capable of reconstruct the full internal profile of the estimated variables. The performance and robustness of the proposed observation technique is tested through a simulation example around different operation points.

Keywords:PEMFC, distributed model, nonlinear observer, disturbance estimation, slidingmode

1 Introduction

Hydrogen as an energy vector and fuel cells are key elements in the current development of sustainable energy systems. In this context, proton exchange membrane fuel cells (PEMFC) are suitable for mass market applications such as automotive and stationary combined heat and power (CHP).

Degradation and efficient operation are directly related with the internal conditions of the PEMFC. The physical construction of these devices makes it impossible to measure internal variables. However, to develop and implement more advanced and robust control techniques that enhance the efficiency and life expectancy of the PEMFC, it is necessary that the internal variables are known by the control topology.

Full state estimation of PEMFC-based systems can be used to develop advanced control techniques as studied in [1]. Nonlinear state observation with disturbances and unknown input reconstruction is a compelling problem in modern control theory, as addressed in this paper. Model-based observers allow the use of robust control techniques in the estimation procedure. Particularly, sliding mode control (SMC) [2] offers the possibility to estimate the states with unknown input terms using equivalent output injection techniques that replace the discontinuous infinite-frequency switching terms with an equivalent smoth signal to be designed [3], as done in this paper.

In this paper a quasi-twodimensional (1+1D) distributed parameter model [4] is used to simulate the PEMFC behaviour and to develop the nonlinear distributed parameter observer (NDPO). The model is discretised to take advantage from the boundary conditions of the problem and to represent the full profile of the spatial variables. The observation model is derived from the aforesaid simulation model. Nevertheless, the observation model is simplified to improve the convergence time. The unmodelled parts are considered disturbances.

The main contribution of this paper relies on the improvement of the disturbance estimation technique presented in [5], which is applied to a PEMFC and based on a super-twisting algorithm. A novel high order sliding mode (HOSM) estimation technique based on [6] is studied and tested in a simulation environment. This proposal includes the state and disturbance estimation in the same integrated topology.

The paper is organised as follows: in Section 2 the mathematical model is presented. In Section 3 the state observer with disturbance estimation is presented. The simulation results are showed and studied in Section 4. Finally some conclusions and future work ideas are discussed in Section 5.

2 Modelling approach

2.1 Simulation model

The simulation model is a 1+1D or quasi-two dimensional parameters model [4] representation of a single channel PEMFC. The schematic representation is shown in Fig. 1 and has a z-axis or channel length (L) of 0.4 m and a y-axis or channel width (L_x) of 1×10^{-3} m. The total active area is equal to 0.4×10^{-3} m^2.

Gas channels, gas diffusion layers (GDL), catalyst layers (CL) and the electrolyte membrane are included in the simulation model.As detailed in [4], the dynamics of the gas flows are treated as spatial partial derivatives which are modelled through mass balance equations in the z-direction, coupled with the reactions and water transports through the

membrane that are described as lumped parameters perpendicular to the gas flows

$$\frac{\partial c_i(t)}{\partial t} = -\frac{\partial}{\partial z}\big(v(t)c_i(t)\big) - \frac{\dot{n}_i(t)}{\delta},$$
$$v(t) = -K\frac{\partial p(t)}{\partial z}, \tag{1}$$
$$p(t) = RT\sum_i c_i(t),$$

where subscript i stands for the reactant, being $i = H_2$ the hydrogen and $i = H_2O$ the water at the anode side. At the cathode side $i = N_2$ is the nitrogen, $i = O_2$ the oxygen and $i = H_2O$ the water.

Diffusion is modelled using Fick's second law of diffusion which states that

$$\frac{\partial c_i(t)}{\partial t} = D_i \frac{\partial^2 c_i(t)}{\partial y^2}, \tag{2}$$

with D_i being the diffusion coefficient of the i-th gas species through the GDL.

The electrochemical reaction takes place in the CLs. The molar fluxes are function of the anodic and cathodic reaction rate. For the anode side, the molar fluxes include H₂ and H₂O transports along the y-axis

$$\dot{n}_{H_2}^A = r^A, \tag{3}$$
$$\dot{n}_{H_2O}^A = \dot{n}_{H_2O}^{AM}. \tag{4}$$

Three molar fluxes are modelled in the cathode

$$\dot{n}_{H_2}^C = \frac{1}{2}r^C, \tag{5}$$
$$\dot{n}_{H_2O}^C = \dot{n}_{H_2O}^{CM} - r^C, \tag{6}$$
$$\dot{n}_{N_2}^C = 0. \tag{7}$$

A complete water transport model [4] is implemented in the membrane layer of the PEMFC. Water plays a key role in the dynamics of the proton transport; therefore it is included in the model as a variable.
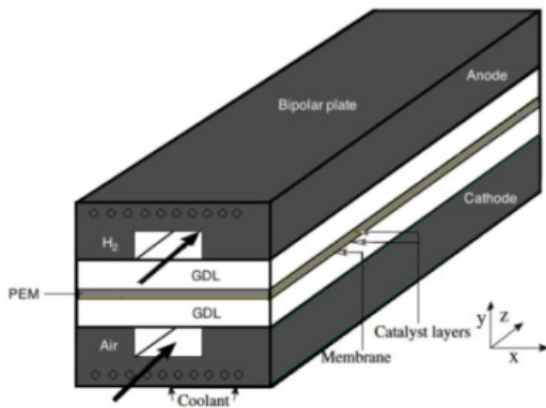
Fig. 1. Single channel PEMFC representation

The model presented in Eqs. (1)-(7) is discretised for its implementation in a simulation environment over the discretisation length Δz for a total of n_{Vol} discretisation volumes.

2.2 Observation model

As introduced in Section 1, the observation model is a simplified version of the simulation model presented in Section 2.1. Only the gas concentration values are estimated in this paper, hence it is possible to simplify the observation model and only include the mass balance gas equations shown in Eq. (1). The unmodelled reaction and water transport terms are considered as unmeasured disturbances that the observer will have to estimate alongside with the PEMFC states.

The observation model is discretised applying forward and backward spatial discretisation procedures on Eq. (1):

$$\dot{x}_{i,1}^k(t) = \frac{\dot{n}_{i,in}^k(t)}{\Delta z} - \zeta_1^k \psi^k(j) - \frac{\dot{n}_{i,j}^k(t)}{\delta^k},$$
$$\dot{x}_{i,j}^k(t) = \zeta_1^k\left(\psi^k(j-1) - \psi^k(j)\right) - \frac{\dot{n}_{i,j}^k(t)}{\delta^k}, \tag{8}$$
$$\dot{x}_{i,n_{Vol}}^k(t) = \zeta_1^k\psi^k(j-1) + \Gamma^k(j+1) - \frac{\dot{n}_{i,j}^k(t)}{\delta^k},$$

being $j < n_{Vol}$. And with

$$\psi^k(j) = x_{i,j}^k(t)\big(\sum_i x_{i,j}^k(t) - \sum_i x_{i,j+1}^k(t)\big),$$
$$\psi^k(j-1) = x_{i,j-1}^k(t)\big(\sum_i x_{i,j-1}^k(t) - \sum_i x_{i,j}^k(t)\big), \tag{9}$$
$$\Gamma^k(j+1) = \zeta_1^k x_{i,j}^k(t)\sum_i x_{i,j}^k(t) + \zeta_2^k x_{i,j}^k(t),$$

being $\zeta_1^k = KRT/\Delta z^2$ and $\zeta_2^k = Kp^{amb}/\Delta z^2$. The state variables $x_{i,j}^k$ refer to the i-th gas concentrations at the j discretisation volume. Superscript k expresses the side of the PEMFC, being $k = A$ for the anode and $k = C$ for the cathode.

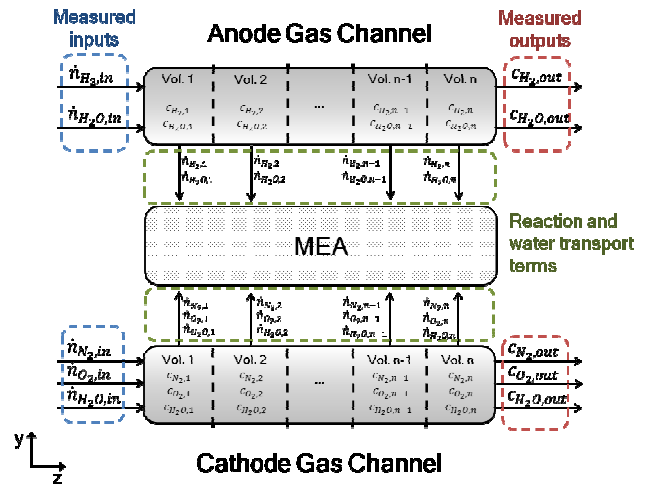The detailed representation of the discretised PEMFC is shown in Fig. 2.

Fig. 2. Discretised PEMFC

2.3 Measurements

The output measurements vector is build with the concentrations at the end of the anode and cathode gas channels [5]

$$y(t) = \begin{bmatrix} c^A_{H_2,out}(t) \\ c^A_{H_2O,out}(t) \\ c^C_{N_2,out}(t) \\ c^C_{O_2,out}(t) \\ c^C_{H_2O,out}(t) \end{bmatrix} \in \mathbb{R}^5. \qquad (10)$$

In this paper it is assumed that these measurements are feasible. For a future implementation of the observer, the output concentrations will be derived from other measurements more common in PEMFC-based systems, such as pressures, temperatures, molar flows and current.

3 HOSM observer

3.1 Observation problem

In this paper, the observation state vector $\hat{x}$ is defined as the full gas species concentrations profile vector

$$\hat{x}_i(t) \triangleq \hat{c}_i(t) \in \mathbb{R}^{5 \times n_{Vol}}. \qquad (11)$$

And the observed output vector $\hat{y}(t) \in \mathbb{R}^5$ is the estimation of Eq. (10) at the gas channels output.

The observation error e_y is defined as the difference between the estimated output $\hat{y}$ and the measured vector y

$$e_y(t) = \hat{y}(t) - y(t). \qquad (12)$$

As demonstrated in [3], if $e_y = 0$, the state estimation error goes to zero ($e_x = x - \hat{x} = 0$).

3.2 Structure of the observer

The HOSM observer of this paper is based on quasi-continuous techniques [6]. The proposed observer is faster and more accurate than previous approaches [5]. The hierarchical structure to recover the full gas concentrations state profile of the PEMFC is described Fig 3. From the value of the output states (10), a backstepping procedure is done to recover the full state profile of the concentrations. In [5] the state and disturbance estimation procedures were independent. Contrarily, the proposed HOSM observer integrates both procedures in the same topology.
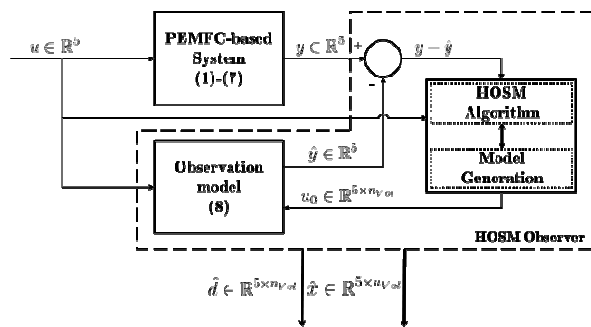

Fig. 3. NDPO with HOSM topology

3.3 HOSM backstepping algorithm

To track the output concentration value y and drive Eq. (12) to zero in a finite time a backstepping algorithm is proposed as derived from [6]. The HOSM degree is equal to n_{Vol}.

The procedure follows an r-step algorithm:

- Step $r = 1$
  $$\hat{x}_{n_{Vol}} = \phi_{n_{Vol}}(x_{n_{Vol}})$$
  $$\sigma_{n_{Vol}} = e_y(t) = \hat{y}(t) - y(t)$$

- Steps $1 < r < n_{Vol}$
  $$\hat{x}_r = \phi_r(\hat{x}_{r-1})$$
  $$\sigma_r = \hat{x}_r - \phi_{r-1}(\hat{x}_{r-1})$$

- Step $r = n_{Vol}$ (final step)
  $$\phi_1 = \hat{x}_1 - \phi_{r-1}(\hat{x}_{r-1})$$

The quasi-continuous term that has to drive the observation error (12) to zero is included in the $\phi_r(\hat{x}_r)$ terms [6], which are defined as

$$\phi_r(\hat{x}_r) = B_r(\cdot)^{-1}\{-f_r(\cdot) + u_r\} \qquad (13)$$

where $B_r(\cdot)$ is the input vector field $u(t) \triangleq \dot{n}^k_{i,in}(t) \in \mathbb{R}^5$ and $f_r(\cdot)$ the nonlinear dynamics of Eq. (8). An r-sliding homogeneus corrective action u_r [2] has to be designed for $r < n_{Vol}$

$$u_r(t) = -\alpha_r \Psi\big(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}\big), \qquad (14)$$

being α_r the tuning parameter of the sliding mode algorithm. At the final step ($r = n_{Vol}$), the corrective action is

$$u_1(t) = -\alpha_1 \, sign(\sigma_1), \qquad (15)$$

with $\sigma_1 = \hat{x}_1 - \phi(\hat{x}_1)$.

4 Simulations

4.1 Initial conditions

To test the observer, an initial state vector $x(0) \in \mathbb{R}^{5 \times n_{Vol}}$ has been considered. It is defined by the steady state corresponding to the anode and cathode stoichiometries $\lambda^A = 2.2$ and $\lambda^C = 3$, the PEMFC temperature $T = 353\,K$ and voltage $U = 0.5\,V$. In this paper $n_{Vol} = 5$.

The initial observed state vector $\hat{x}(0) = 0 \in \mathbb{R}^{5 \times n_{Vol}}$ has been considered. To test the proposed technique, ramp variations are imposed to the PEMFC inputs (Table 1) to test the dynamical behaviour of the observer.

Table 1. Step values for the dynamical analysis

Input molar flow	Time [s]	Ramp [mol m^{-2} s^{-1}]
$\dot{n}^C_{O_2,in}$	$25 - 27$	$60 \rightarrow 30$
$\dot{n}^A_{H_2,in}$	$50 - 52$	$16 \rightarrow 14$
$\dot{n}^A_{H_2O}$	$75 - 77$	$4 \rightarrow 3$

4.2 Results and discussion

The HOSM observer algorithm proposed in Section 3.3 reconstructs the full internal state profile for each of the discretised volumes of the PEMFC (8). Due to representation purposes only the state reconstruction in the middle point of the anode gas channel is shown in Fig. 4 and the cathode gas channel in Fig. 5. This point is of special significance due to the practical difficulties of sensorisingit in a real implementation environment. Nevertheless, the full gas concentration profile of the anode and cathode gas channels is estimated by the proposed HOSM observer. The use of this technique will allow the use of control techniques that improve the performance of the PEMFC without additional sensors, reducing manufacturing costs while improving efficiency.

Fig. 4 shows the observation of the gas species (dashed lines) in the anode gas channel. The state estimation error is almost negligible during all the simulation with the exception of the initial convergence time, which is common where the observer is initialised with null initial conditions as it is in the studied case.

The estimation of the cathode gas channel concentrations in the middle point of the PEMFC are shown in Fig. 5. The faster convergence time is due to the difference between the reaction dynamics at the anode and cathode sides of the PEMFC.
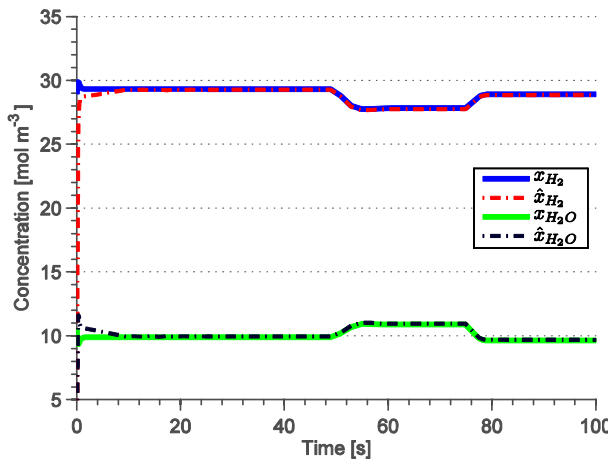

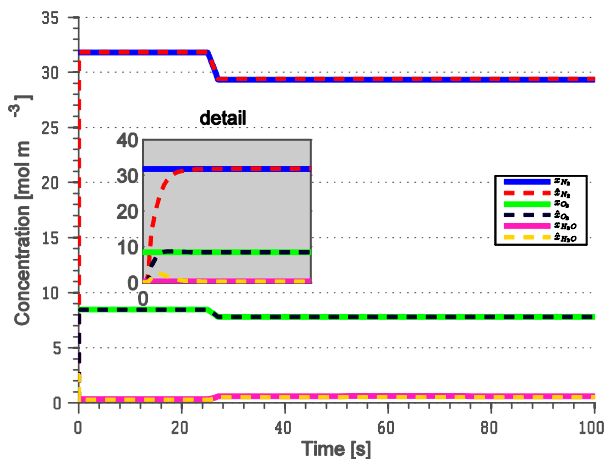**Fig. 4.** Estimation in the middle point of the anode

Fig. 5. Estimation in the middle point of the cathode

5 Conclusions

In this paper, a HOSM nonlinear observer is proposed to recover the full state profiles in a PEMFC. In addition,the observer is able to estimate the reaction and water transport terms considering them as disturbances. The performance of the observer has been tested and studied in a simulation scenario, in which satisfactory results were obtained.

The discretisation of the PEMFC model has been carried out for a total of five volumes. The complexity of the observer increases with the refinement of the discretisation, hence, a trade-off situation arises between the level of detail of the simulations and the observer mathematical complication.

The technique developed is suitable to be implemented in control topologies in order to use controllers that need full state feedback of the system without the need of installing additional sensors. Moreover, diagnosis tools to display the PEMFC health state can be designed using the HOSM observer.

6 Acknowledgements

7 References

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