WIND-RETRIEVAL FROM MULTI-ANGLE
BACKSCATTER LIDAR PROFILES THROUGH
ANISOTROPIC AEROSOL STRUCTURES

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Abstract

A wind-retrieval correlation method for backscatter-lidar scanning schemes consisting of a few profiling sounding lines of sight (LOS) is mathematically formulated in matrix-solution form under general anisotropic atmospheric conditions and convective boundary layer scenarios. The method assumes the frozen atmosphere model and works with temporal cross-correlations functions that do not need to be maximized. The method also applies to the well-known case of slant scans based on the multiple-angle azimuth technique (horizontal wind retrievals). A first application of the method to a 1064-nm wavelength case example is presented for a 40-deg elevation, two-angle azimuth scan where horizontal wind speed and wind direction are retrieved.
1. Introduction

Wind lidar (light detection and ranging) remote sensing has key applications in fields such as meteorology (weather forecasting), environmental science (pollutant trajectories), and aeronautics (wind-shear early-warning systems). Lidars remotely profile the motion of atmospheric scatterers, namely molecules and aerosols, which can be applied to retrieve the mean wind vector. Equipped with a scanning system, lidars can be used to map the horizontal or even the vertical wind components, rather than just yielding the wind-radial projection on the sounding line of sight (LOS) (Rocadenbosch, 2003).

Doppler wind lidars use both coherent and direct-detection techniques (edge and fringe techniques; Comerón et al., 2010) to measure the radial velocity of the wind field along the sensor LOS, from the Doppler frequency shift of the return radiation both in ground and space-borne wind-profiling applications. Prominent missions include NASA’s LAWS (Lidar Atmospheric Wind Sounder, LAWS, 1987), ESA’s ALADIN (Atmospheric Laser Doppler Instrument, ALADIN, 1989), and NOAA’s (National Oceanic and Atmospheric Administration) Doppler lidar projects (Hall et al., 1984; Clifford et al., 1994; Huffaker and Hardesty, 1996). Backscatter lidars use pattern correlation analysis of aerosol inhomogeneities (i.e., persistent contrastable patterns having a correlated spatial structure) swept away by the wind to identify pattern similarities at specific time/range lags. In contrast to Doppler wind lidars, backscatter lidars are simple, low-cost, and yield reasonable wind speed accuracies and spatial inversion resolutions inside the atmospheric boundary layer (ABL) (Hooper and Eloranta 1986; Matsui et al. 1990; Piironen and Eloranta, 1995; Buttler et al. 2001). However,
two unwanted effects alter this straightforward “correlation” concept: spatial anisotropy of the aerosol pattern and wind turbulence. So far, two primary wind-retrieval methodologies have been developed: point-correlation methods and spatial-correlation methods.

**Point-correlation methods** antecede lidar invention, and were first applied to the study of ionospheric irregularities in downward propagating radio waves, such as horizontal drifts and time shifts, between fading curves observed from three or more ground-spaced receiving antennas (Briggs et al., 1950; Philips and Spencer, 1955; Briggs, 1968). This family of methods (see Holloway et al., 1997, for a review) requires triangulation of three sounding points on the same horizontal plane, and an analysis of the time series at these correlation points in the form of *auto- and cross-correlations*. Adaptation of these methods to lidar atmospheric sounding involves at least three non-coplanar LOS close to zenith pointing so that three points lie on each horizontal plane along the vertical direction, hence replicating the triangulation philosophy of radio-antenna measurements (Zuev et al., 1977; Clemesha et al., 1981; Kolev et al., 1988; Morley et al., 2010).

From this work (known as Full Correlation Analysis, FCA) two simplifying assumptions are applied: (i) the “frozen” pattern assumption (i.e., that the pattern details remain invariant as it moves; Stull (1988)) and (ii) the pattern *isotropy* assumption (i.e., that the range lag between two separate points yielding a given correlation value is independent of the direction chosen). Inclusion of random changes in the pattern and of anisotropic conditions has also been considered (Phillips and Spencer, 1955; Briggs, 1968).

**Spatial-correlation methods** were developed mainly for atmospheric lidar sounding. Correlation along a lidar LOS at low-elevation angles for retrieving the radial wind
component was the first example of spatial correlation analysis (Eloranta et al., 1975). Later on, additional LOS were added to derive the transversal wind component from cross-correlation function estimation in the time domain (Kunkel et al., 1980). The radial wind component was also estimated in the frequency domain (Sroga et al., 1980), including anisotropy effects for this component only (Hooper and Eloranta, 1986).

In these spatial-correlation methods, the lidar is basically configured as a transit-time wind profiler to scan back and forth at two, three, or more LOS. The LOS can be arranged in closely-spaced azimuth angles at a fixed elevation angle, so that the wind vector is decomposed into two orthogonal components: one along the LOS of the lidar (radial direction) and another perpendicular to the LOS, and, therefore, horizontal to the ground. In contrast to FCA, these spatial-correlation methods are used to retrieve the transversal-wind component by means of a correlation-function model based on advection by a random wind field, in a conceptually similar manner to previous works on solar-wind measurements (Little and Ekers, 1971). Under simplified conditions (e.g., isotropic and “frozen” atmospheres), a simple algorithm represents the maximization of the correlation function (Kunz 1996; Buttler et al., 2001), which permits identifying the time- or range-lag where aerosol pattern similarities occur.

Advanced measurement techniques since the 1990s have further developed point- and spatial-correlation methods. The vertical sounding approach was enhanced with more LOS for refined point-correlation analysis (Matsui et al., 1990; Sugimoto et al., 1998), which gave rise to a complete conical vertical scanning and the so-called kynematical analysis (Schwemmer, 1998; Wilkerson et al., 2002). Another technique combines multi-laser sounding with a
frequency-domain analysis of the correlation function (Kovalev and Eichinger, 2004). In each of these cases, a vertical profile of the wind is obtained assuming isotropy and a frozen atmosphere.

Multiple-azimuth-LOS horizontal scanning was also extended to the so-called “large-area” scanning, where an area-correlation analysis is applied. For example, Sasano et al. (1982), Schols and Eloranta (1992), and Mayor and Eloranta (2001) used a “large-area” pattern matching method (e.g., a lidar sensor scanning a circular sector at a given elevation angle) based on the two-dimensional correlation function of the aerosol pattern measured at two different times. This basically means obtaining a volume image of the horizontal wind field instead of a simple vertical profile (Schols and Eloranta, 1992; Piironen and Eloranta, 1995). The main drawback of these “large-area” scanning methods is the reduced scanning time available in practice, which usually is not fast enough to sweep the area twice before the aerosol distribution decorrelates. With a frozen atmosphere, a moving observer with the mean wind would see the aerosol pattern undistorted. As a natural evolution, further work has measured the anisotropic and turbulent properties of eddy structures (Young and Eloranta, 1995; Mayor and Eloranta, 2001; Mayor, 2010) to the point of beginning to use optical flow methods instead of correlation to retrieve high-spatial-resolution wind fields (on the order of one vector in a 10x10-m grid every 30 s; Derian et al., 2010; Mayor et al., 2013).

Wind-retrieval methods relying on temporal/spatial-lag maximization of the aerosol-pattern correlation function are not suitable for scan schemes with only a few LOS. A primary reason for this is the limited number of baselines that can be formed in the atmospheric volume of interest for velocity inversion (hereafter the “velocity-inversion volume”), where a
baseline is defined as the difference position vector between two measurement points for a common/different LOS. In the case of temporal optimization (time-lag solution maximizing the correlation function), the anisotropy of the medium introduces an apparent direction of drift that distorts the inverted wind solution with the anisotropic dominant direction, causing “false-velocity” estimates. In the case of spatial optimization, and though this method yields correct wind-velocity estimates under anisotropic conditions, a densely sampled volume is needed to solve for the baseline maximising correlation. Besides, numerical solution of the maximization problem is always cumbersome, for it involves quadratic and cross-product terms of the wind components. Further, estimation of the correlation maximum is not an easy task in scenes with moderate-to-low signal-to-noise ratios (SNR).

In this paper, we mathematically formulate a tri-dimensional wind-retrieval temporal correlation method under general anisotropic conditions, which requires at least six independent baselines from three non-coplanar LOS. Departing from this generalized formulation, the method is aimed at lidar systems with scanning capabilities limited to such multiple-azimuth scanning at low-to-mid elevation angles. The method combines both aforementioned philosophies. Thus, point correlation is applied to time series obtained from low-elevation, multiple-azimuth scanning measurements (typical of spatial-correlation methods). As a result, several advantages arise: First, and foremost, is that both wind and anisotropic information are retrieved, since spatially anisotropic aerosol structures can readily be detected by means of backscatter lidar (Mayor et al., 2003). In doing so, the only requirement is that the atmosphere is assumed to be piece-wise homogeneous in height. That is, the anisotropic properties remain the same in each vertical “layer” or atmospheric
“velocity-inversion volume”. Second is that the retrieval procedure does not rely on maximization of the temporal correlation function (as previously discussed, this may lead to false-velocity estimates under anisotropic conditions) or on “a priori” selection of a baseline aligned with the true wind direction. In contrast, the method relies on densely-sampled measurement points along the LOS of the lidar from which a subset of suitable wind-estimation baselines is selected. And third, a matrix-oriented, linear solution of the problem is presented.

The method presented is, in fact, a type of FCA. The method is intended for low turbulence situations where the mean wind velocity is much higher than the velocity standard deviation caused by shifting-wind aerosol concentration fluctuations. In this case, Taylor’s hypothesis (Taylor, 1938; Stull, 1988) holds (i.e., aerosol patterns are advected without any change in their shape as if they were “frozen”). This is a sensible approximation followed in practice even when working with advanced backscatter lidars (Sugimoto et al., 1998; Buttler et al., 2001; Wilkerson et al., 2003; Kovalev and Eichinger, 2004).

This paper is organized as follows: in Section 2, an introduction of the coordinate systems inherent to the problem and aerosol-pattern correlation fundamentals is presented. Section 3 (core section) includes the mathematical formulation of the wind and anisotropy-matrix retrieval method, and Section 4 presents a first simplified application case of the method for a two-LOS scanning scheme. Finally, concluding remarks are given in Section 5. Supplementary material (on-line accessible) shows a review of aerosol-pattern correlation models and discusses the limitations of the classic approach based on time/space maximization of the correlation function.
2. Fundamentals

2.1. Coordinate systems

Consider an aerosol pattern, \( N_{ac} (X,t) \) moving with a mean wind velocity, \( \mathbf{U} \), and the coordinate geometry of Fig.1a. The mean-wind vector defines the wind-relative coordinate system \((\mathbf{\hat{x}}_1, \mathbf{\hat{x}}_2, \mathbf{\hat{x}}_3); \text{namely, the along-wind, cross-wind and vertical coordinate vectors}\), where the mean wind at a height \( Z \) is assumed horizontally invariant and defined as

\[
\mathbf{U}(Z) = U_x \mathbf{\hat{x}}_1 + U_z \mathbf{\hat{x}}_3,
\]

with \( U_z << U_x \).

The wind-relative coordinate system is defined so that the cross-wind mean component, \( U_z \), is identically zero. Formally, the total wind is the superposition of the mean wind component (advective transport) plus the turbulent one, the latter being responsible for eddy diffusion and changes in the aerosol pattern as it moves. Because of the “frozen” atmosphere assumption (Sect. 1), the turbulent component will be neglected in what follows so that the total wind vector is the same as the mean wind vector, \( \mathbf{U} \) (\( \mathbf{U} \) in the wind-relative coordinate system, or alternatively, \( \mathbf{V} \) in the absolute coordinate system; see Fig.1b and Table 1).

In this work, a minimum of two sensing LOS are assumed for the scanning backscatter lidar. The LOS are arranged around a LOS-symmetry axis (case of multiple LOS) or bisectrix of the angle between the two LOS (case of two LOS), used as a “reference” line for the scanning scheme. Such schemes are typically the two-, three- and five-angle azimuth scan and
the four-angle square scan. The absolute coordinate system \((\hat{x}, \hat{y}, \hat{z})\), or *instrument-coordinate system* (Fig.1b), is defined relative to the lidar in such a way that \(\hat{y}\) is the horizontal projection of the reference line of the two LOS depicted (positive when \(\hat{y}\) forms an acute angle with the plane containing the two LOS); \(\hat{x}\) is defined orthogonal to \(\hat{y}\) and following a positive trihedral, and \(\hat{z} = \hat{x}_3\) is the vertical axis. \(\theta\) and \(\phi\) are, respectively, the azimuth and elevation angles of the two LOS in the absolute coordinate system \((\hat{x}, \hat{y}, \hat{z})\). The origin of both fixed coordinate systems coincide \((O(\hat{x}, \hat{y}, \hat{z}) = O'(\hat{x}_1, \hat{x}_2, \hat{x}_3))\) at the intersection point between the horizontal plane at height \(z\) and the reference line, so that the wind-relative coordinate system \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\) is the counter clock-wise rotated version of the absolute system \((\hat{x}, \hat{y}, \hat{z})\) by an azimuth angle \(\phi_w\). In other words, \(\phi_w\) is the wind direction relative to the absolute coordinate system. The rotation transform matrix is given in Table 1. For a three-angle azimuth scan, the pictorial sketch would be similar to that of Fig.1b, with the “reference” line corresponding with the central LOS.

### 2.2. Aerosol pattern correlation

#### 2.2.1 SPATIAL CORRELATION FUNCTION

The spatial correlation function is a unit-normalised statistical measurement of similarity between two functions at different spatial lags. The spatial correlation function of a static aerosol concentration field, \(N_{av}(X) \ (U = 0)\), at two separated positions \(X_0\) and \(X_1 = X_0 + \rho\) in the wind-relative coordinate system \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\) is defined as
\[
B(\mathbf{p}) = \frac{E\left[\{N_{\text{aer}}(X_0) - E[N_{\text{aer}}(X_0)]\} - E[\{N_{\text{aer}}(X_0 + \mathbf{p}) - E[N_{\text{aer}}(X_0 + \mathbf{p})]\}]\right]}{E\left[\{N_{\text{aer}}(X_0) - E[N_{\text{aer}}(X_0)]\}^2\right]},
\]

where \(E\) is the expectancy operator representing the statistical average. The terms bracketed \(\{N_{\text{aer}}(X_i) - E[N_{\text{aer}}(X_i)]\} \ i = 0,1\) stand for the aerosol concentration fluctuations around the average concentration at position \(X_i\), and the denominator is a normalization factor.

In practice, the aerosol concentration, \(N_{\text{aer}}\), is replaced by the range-corrected backscatter lidar return power \(S(R_i) = R_i^2 P(R_i), \ i = 1..N\) (\(N\) is the number of samples along the LOS). For the single-scattering elastic lidar equation (Measures, 1992), the range-corrected power is proportional to the product of the atmospheric total-backscatter coefficient (aerosol plus molecules, \(\beta = \beta_{\text{aer}} + \beta_{\text{mol}}\)) and the two-way path atmospheric transmission. Since 1) in aerosol-dominant regions such as the ABL and, particularly, towards the near infrared, \(\beta \approx \beta_{\text{aer}}\), and 2) for a well-mixed atmosphere, transmission is approximately constant in the correlation volume, the aerosol-concentration approximation above holds (Collis and Russell, 1976).

2.2.1.1 Formulation of the spatial correlation function in terms of functional models and the anisotropy matrix

\(B(\mathbf{p})\) in Eq.(2) attains a maximum value of unity at zero range lag, \(\mathbf{p} = 0\). In standard correlation analysis, it is assumed that the correlation function follows a monotonically decreasing behaviour, usually modelled by a Gaussian or exponential model (Ishimaru, 1978).
Therefore, contours of the three-dimensional spatial correlation function, $B(\rho)$, take the form of concentric ellipsoidal surfaces. Concentric ellipsoids, rather than spheres, are a consequence of pattern anisotropy, which causes stretching of the contours in one direction. This direction corresponds with the major axis of the correlation ellipsoid and does not necessarily coincide with the mean-wind direction (Fig 2). Initial large-scale concentration gradients and a spatially-variant wind field are responsible for the anisotropy of the aerosol distributions in the inertial-convective range (Elperin et al., 1996). Thus, wind shear distorts the aerosol structures directionally as well as enhancing isotropic turbulent diffusion (Stull, 1988). Atmospheric vertical stratification also gives rise to different levels of vertical correlation along the extent of the ABL, thus showing aspect ratios between the horizontal and vertical correlation lengths (Doviak et al., 1996).

From this basis, it is convenient to express the spatial correlation as

$$B(\rho) = f[q(\rho)] = (f \circ q)(\rho),$$

where $f$ is a monotonically decreasing function model (in this work we have used a simple exponential decay, see Supplementary Material) and $q(\rho)$ is a real positive quadratic form representing tri-dimensional ellipsoids centered at the origin $O'(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ of the form

$$q(\rho) = a\rho_1^2 + b\rho_2^2 + c\rho_3^2 + d\rho_1\rho_2 + e\rho_1\rho_3 + f\rho_2\rho_3.$$ (4)

It can be shown that the $d$, $e$, and $f$ terms describe ellipsoid rotation with respect to the coordinate axes $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, arising when describing the correlation ellipsoids in a fixed-observation system. The locus for which the correlation function is equal to one half ($B(\rho) = 1/2$) is called the “characteristic ellipsoid” (Briggs, 1968).
A more convenient form to express the effects of the anisotropy on the correlation pattern contours is found by rewriting the quadratic form \( q \) in terms of the anisotropy matrix, \( \mathbf{M} \), or

\[
q(\mathbf{p}) = \mathbf{p}^T \mathbf{M} \mathbf{p} = \begin{pmatrix} \rho_1 & \rho_2 & \rho_3 \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix},
\]

(5)

where superscript "T" means “transposed.” In Eq.(5), the main diagonal elements of the anisotropy matrix, \( m_i = \frac{1}{2\rho_{c,i}}, i = 1,...,3 \), are one half of the inverse of the characteristic correlation length associated to the \( \hat{x}_i \) axis, \( \rho_{c,i} \). Here, \( \rho_{c,i} \) is the length of the semi-principal axes of the correlation ellipsoid in the wind-relative coordinate system, \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\). The non-diagonal elements, \( m_{ij} \), stand for the rotations of the principal axis of the correlation ellipsoid with respect to the wind-relative coordinate system.

Scanning area measurements have shown that correlation contours are well-represented by ellipsoids whose alignment with respect to the wind may be tilted and vary in height (Mayor et al., 2003). The latter implies that the atmosphere can be modeled by horizontal layers of a given vertical extent in which the wind field is \textit{approximately} invariant in both speed and direction. This is the “piece-wise” vertical homogeneous approximation of the atmosphere outlined in Sect. 1. In each of these layers, both the wind field and the aerosol concentration field are considered time-stationary and statistically-homogeneous in space (Ferdinandov and Mitsev, 1982). Therefore, each layer, or “velocity-inversion volume”, is characterised by a unique anisotropy matrix, \( \mathbf{M} \). It is assumed that atmospheric spatial
variability does not significantly change the anisotropy matrix during the measurement time, which is consistent with the frozen-atmosphere hypothesis.

For the isotropic case, all three correlation lengths are equal, \( \rho_{c,i} = \rho_c \), \( i = 1..3 \), and \( m_y = 0 \). Therefore, \( \mathbf{M} \) is proportional to the identity matrix, and Eq.(4) above reduces to a sphere, \( q(\mathbf{p}) = \left( \rho_1^2 + \rho_2^2 + \rho_3^2 \right)/2\rho_c^2 \). The isotropic approximation has been used in practice (Kunkel et al., 1980; Sroga et al., 1980; Matsui et al., 1990) under the assumption that the average of the remotely-sensed inhomogeneities is isotropically correlated. However, this approximation cannot be taken for granted because it depends on the measuring scale and, hence, on the possibility of detecting aerosol concentration gradients.

### 2.2.2 SPACE-TIME CORRELATION FUNCTION MODEL

**Generalized model.** An observer in the wind-relative coordinate system \( \hat{x}_1, \hat{x}_2, \hat{x}_3 \) can make observations at spaced points and times to compute the space-time correlation function of the aerosol concentration field. The *generalized model* for the spatio-temporal correlation function of an atmospheric layer centered at a height, \( Z \), and time delay, \( \tau \), between these time observations is defined by Little and Ekers (1971) as

\[
R_{Z}(\mathbf{p}, \tau) = \iiint B[\mathbf{p} - \mathbf{u}(Z)\tau] p[\mathbf{u}(Z)] du_1 du_2 du_3.
\]  

(6)

Here, \( \mathbf{u}(Z) = \mathbf{U}(Z) + \mathbf{u}_r(Z) \) is the random wind field, with \( \mathbf{U}(Z) \) the mean wind and \( \mathbf{u}_r(Z) \) the turbulent random component. \( p[\mathbf{u}(Z)] \) is the probability density function (p.d.f.) of the random wind field (usually Gaussian isotropic), \( u_1, u_2, u_3 \) are the wind-speed components, and subindex \( Z \) is a reminder that Eq.(6) is computed over an atmospheric layer centered at height
Z and extending from a generic height $Z_a$ to $Z_b$ (“piece-wise” vertical homogeneous approximation).

**Frozen-atmosphere model.** The practical simplification of Eq.(6) is the frozen-atmosphere assumption (Stull, 1988). This is to say that the diffusive (i.e., turbulent) wind component is much smaller than the advective (mean wind) one (i.e., $\sigma_i^2 \ll U^2$), with $\sigma_i^2$ representing the variance of the turbulent wind, or, formally, that the wind p.d.f. tends to a Dirac’s delta distribution,

$$p(u) = \delta(u_i - U_i, u_2 - U_2, u_3 - U_3).$$

(7)

Substitution of Eq.(7) into Eq.(6) yields the physically obvious result that if the aerosol pattern is “frozen” the space-time correlation function $R_x(\rho, \tau)$ (referring to the coordinate system $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$) is just a translation of the “static” spatial correlation function of the aerosol pattern, $B(\rho)$, by an amount, $\rho_0(Z) = U(Z)\tau$. That is,

$$R_x(\rho, \tau) = B[\rho - U(Z)\tau] = f\{q[\rho - U(Z)\tau]\},$$

(8)

where we have used the corresponding form of Eq.(3).

Taylor’s hypothesis adds a valuable simplification in the atmospheric description of aerosol transport, since it relates time and spatial statistics through velocity scaling. The assumption of the frozen atmosphere (Eq.(8)) is thus required for what follows.

**2.2.3 SPACE-TIME CORRELATION FUNCTION IN ABSOLUTE COORDINATES**

From Sect. 2.1 and Table 1, the coordinate transform rotation matrix, $\Psi$, enables us to relate absolute (lidar)- and relative (wind)-coordinate baseline vectors as
\[ r = \Psi(V)\rho. \quad (9) \]

In Eq.\((9)\), it is important to note that because the coordinate system \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\) is wind relative (i.e., defined by the wind angle \(\phi_w\), see Table 1), determination of \(\Psi\) requires previous retrieval of the wind vector, \(V\).

Re-formulation of the space-time correlation model function of Sect. 2.2.1.1 in absolute coordinates can simply be done by changing the coordinates of quadratic form, \(q(\rho)\) (Eq.\((3)\)). Since rotation matrices are symmetric, \(q(\rho)\) can be expressed in the absolute coordinate system as

\[
q(\rho) = \rho^T M \rho = r^T \Psi M \Psi^T r = r^T A r, \quad (10)
\]

where

\[
A = \Psi M \Psi^T
\]

is the anisotropy matrix in absolute coordinates. \(A\) is also positive-defined and symmetric because \(\Psi\) is unitary.

By introducing the frozen-atmosphere model of Eq.\((8)\), Eq.\((10)\) takes the form

\[
q(\rho, \tau) = q(\rho - U(\tau)) = (r^T - V^T \tau) A(r - V \tau), \quad (12)
\]

where \(V = \Psi U\) is the mean wind vector referenced in the absolute coordinate system.

In the wind-retrieval method developed in Sect. 3, the velocity-inversion procedure is carried out in two steps. First, the anisotropy matrix, \(A\), is determined and, second, the wind vector, \(V\), is determined.
3. Method

3.1. Measured space-time correlation function

The space-time cross-correlation (Sect. 2.2.2) from measured lidar signals in an atmospheric aerosol layer (or velocity-inversion volume) centered at a height $Z$ (Fig. 1c) is always referenced to the “lidar” absolute-coordinate system, and it can be written as

$$\Gamma_{Z}^{ij} = \Gamma_{Z}\left(r_{pq,ij}, \tau\right) = S_{n}\left(R_{pq,ij}, t, \theta, \phi\right) \ast S_{n}\left(R_{pq,ij}, t, \theta, \phi\right),$$  \hspace{1cm} (13)

where $r_{pq,ij} = R_{q,ij} - R_{p,ij}$. Here, $\Gamma_{Z}$ denotes the measured correlation function (in contrast to $R_{Z}$, which stands for the model correlation function). The pair $(\theta_k, \phi_k)$, $k = p, q$, stands, respectively, for the elevation and azimuth angles of the $k$-th LOS, $R_{k,i}$, $k = p, q$ is the range vector to the $i$-th discrete measurement point on the $k$-th LOS, and $r_{pq,ij}$ is the baseline or range-lag vector between the $i$-th and $j$-th measurement points, respectively, on the $p$-th and $q$-th LOS (see Fig. 1c). A baseline is a vector representing the difference between a pair of range-bin positions from the available LOS of the lidar instrument in the inversion volume, which can be seen as a lattice of discrete-point lidar measurements. $\tau$ is the time delay and $\ast$ is the correlation operator.

Eq.(13) represents the temporal correlation associated with the $r_{pq,ij}$ baseline, and $r = r_{pq,ij}$ is the absolute coordinate system counterpart of $p$ (Eq.(2)) in the “wind” relative-coordinate system. $S_{n}$ is the range-corrected lidar return along a given LOS, expressed as a deviation from its time-average value and normalized by its standard deviation as
\[ S_n(R, t) = \frac{S(R, t) - S(R, t)}{\left\{ \frac{1}{M} \sum_{m=1}^{M} [S(R, t_m) - S(R, t)]^2 \right\}^{1/2}}, \quad (14) \]

where \( R_i \) is formally the distance \( R_i = |R_{k,i}| \), \( m \) stands for the \( m \)-th lidar return, \( M \) is number of lidar records, and the top bar means a time average. Consequently, \( \Gamma_z \) is normalized to \( \Gamma_z(0, 0) = 1 \).

For the sake of comparison with Eq.(8), Eq.(13) can be rewritten as

\[ \Gamma_z(r, \tau) = S_{n,p}(R_o, t) * S_{n,q}(R_0 + r, t + \tau), \quad (15) \]

where we have used \( R_{p,i} = R_o \) and \( R_{q,j} = R_{p,i} + r \) in Eq.(13). The goal of obtaining the measurement correlation function of Eq.(15), \( \Gamma_z(r, \tau) \), is to estimate the model correlation function, \( \hat{R}_z(p, \tau) \) (Sect. 2.2.2), through

\[ \Gamma_z(r, \tau) = \hat{R}_z(p, \tau)|_{p=\Psi_r}, \quad (16) \]

from which the wind components and anisotropic parameters can be derived.

Eq. (16) warrants some comments. First, a change in coordinates is involved (noted as \( \rho = \Psi^T r \); Table 1). Because \( \Gamma_z(r, \tau) \) is computed in the lidar-absolute coordinate system, \( (\hat{x}, \hat{y}, \hat{z}) \), the first step towards being able to compare both functions is to express the model correlation function, \( R_z(p, \tau) \) (Eq. (8)), in the \( (\hat{x}, \hat{y}, \hat{z}) \) absolute coordinate system, as described in Sect. 2.2.3. Second, while the model formulation of \( R_z(p, \tau) \) is 1) continuous in space, \( \rho \), and time, \( \tau \), and 2) rooted in the wind-relative coordinate system \( (\hat{x}_1, \hat{x}_2, \hat{x}_3) \),
\( \Gamma_z(\mathbf{r}, \tau) \) is discrete in space, \( r_{pq,ij} \), and time, \( \tau \). This is because of the finite number of baselines in the velocity-inversion volume and inherent time resolution of the lidar sensor.

3.2. Retrieval of the anisotropy matrix

The space-time correlation evaluated at zero-time delay is the aerosol-pattern spatial correlation function (Little and Ekers, 1971). From Eq.(8),

\[
R_z(\mathbf{p}, 0) = f\left[q(\mathbf{p})\right].
\]

In Sect. 3.1, it has been shown that the measurement correlation function, \( \Gamma_z(\mathbf{r}, \tau) \), and the model correlation function, \( R_z(\mathbf{p}, \tau) \), (the first being an estimate of the second) are analogous except for a coordinate axis rotation. This rotation relates baseline vector \( \mathbf{p} \) to \( \mathbf{r} \) (Table 1), and causes the quadratic form, \( q \), or “ellipsoid” to take the form (Eq. (10)),

\[
q(\mathbf{p}) = \mathbf{r}^T \mathbf{A} \mathbf{r}.
\]

By substituting Eq. (18) into Eq. (17), Eq. (17) leads to

\[
R_z(\mathbf{r}, 0) = f\left(\mathbf{r}^T \mathbf{A} \mathbf{r}\right).
\]

To retrieve the anisotropy matrix in Eq. (19), one is interested in the quadratic term, \( \mathbf{r}^T \mathbf{A} \mathbf{r} \), and one uses the fact that model correlation function, \( R_z(\mathbf{r}, \tau) \), is estimated from the measured one, \( \Gamma_z(\mathbf{r}, \tau) \). Thus, given the model-correlation decay function, \( f \), from the user’s side, the anisotropy matrix \( \mathbf{A} \) can formally be inverted as

\[
\mathbf{r}^T \mathbf{A} = f^{-1}[\Gamma_z(\mathbf{r}, 0)].
\]
In Eq. (20), there are six unknowns inherited from the anisotropy matrix coefficients of Eq.(5) (recall that $\mathbf{A}$ is the rotated version of $\mathbf{M}$, Tab. 1) and one single equation. In order to come up with an over-determined set of equations, we evaluate Eq.(20) above at a set of $N \geq 6$ baselines, $r_k$, $k=1..N$, from selected pairs of points in the velocity-inversion volume grid of interest (atmospheric layer at height $Z$; see Sect. 4.1 for hints on the selection criteria) so that

$$r_k^T \mathbf{A} r_k = f^{-1} [\Gamma_z(r_k,0)], \quad k = 1..N . \tag{21}$$

$\Gamma_z(r_k,0)$ represents the temporal correlation at zero-lag delay for the baselines $r_k$, $k=1..N$.

Using that,

$$\mathbf{r}^T \mathbf{A} \mathbf{r} = a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{12}xy + a_{13}xz + a_{23}yz \tag{22}$$

(counterpart of Eq.(4) in absolute coordinates, see Table 1), and Eq.(21) can be cast into the $N \times 6$ linear equation system as

$$\mathbf{P} \mathbf{a} = \mathbf{b} , \tag{23}$$

where

$$\mathbf{P} = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & 2x_1 y_1 & 2x_1 z_1 & 2y_1 z_1 \\ x_2^2 & y_2^2 & z_2^2 & 2x_2 y_2 & 2x_2 z_2 & 2y_2 z_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_N^2 & y_N^2 & z_N^2 & 2x_N y_N & 2x_N z_N & 2y_N z_N \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} a_{11} \\ a_{22} \\ \vdots \\ a_{23} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} f^{-1}[\Gamma_z(r_1,0)] \\ f^{-1}[\Gamma_z(r_2,0)] \\ \vdots \\ f^{-1}[\Gamma_z(r_N,0)] \end{bmatrix} . \tag{24}$$

In Eq.(24), $\mathbf{a}$ is the $6 \times 1$ vector reshape of the anisotropy matrix $\mathbf{A}$ (six non-redundant coefficients inherited from $\mathbf{M}$, Eq.(5)). For $N \geq 6$, and $\mathbf{P}$ a full-rank matrix, solution of Eq.(24) is obtained via the pseudo-inverse matrix of $\mathbf{P}$ as $\mathbf{a} = \mathbf{P}^+ \mathbf{b} = (\mathbf{P}^T \mathbf{P})^{-1} (\mathbf{P} \mathbf{b})$, which is
the solution that minimizes the error norm $\|a - Pb\|$ in a mean-squared sense (Barlow, 1999).

In order to have a unique solution ($P$ with rank six), at least six non-parallel baselines are needed, which in turn require at least four non-coplanar points. Once the absolute-coordinate anisotropy matrix, $A$, is solved from Eq. (24) above, retrieval of the aerosol anisotropy matrix $M$ (Eq. (5)), via Eq. (11) requires the previous retrieval of the wind vector (Sect. 3.3). This is because, as outlined in Eq. (9) and Sect. 2.1, the coordinate transformation matrix, $\Psi$, depends on the wind vector ($\Psi = \Psi(V)$, Table 1) via the wind angle, $\phi_w$. This procedure must be repeated for each atmospheric layer considered.

3.3. Retrieval of the wind vector

The limitations, and very often unsuitability, of the classic correlation-function maximization method for wind retrievals have been discussed in Sect. 1. A straightforward linear formulation is presented next based on the so-called temporal “correlation intersect method,” which does not require correlation maximization. The proposed method departs from the work of Briggs (1950) on ionospheric returns, and states that the time delay where the temporal auto- and cross-correlation functions intersect enables the wind velocity to be inverted independent of turbulence effects (assumption of frozen atmosphere). In a first step, the method is reviewed under the assumption of a one-dimensional ground for illustrative purposes (same as in the reference above). In a second step, we show its extension to the tri-dimensional case.

Correlation analysis without function maximization
In the one-dimensional case, \( \mathbf{r} = (x, y, z) \) reduces to \( x \) and the locus of constant correlation, \( R_z(x, \tau) = \text{const.} \), consists of concentric contour ellipses in the \( (x, \tau) \) domain (see Fig. 3). The space shift or “baseline”, \( x = x_{\text{opt}} \), maximizing the spatial correlation function, given lidar observations separated by a time interval, \( \tau_0 \), is also plotted and corresponds with point \( C \). The wind drift velocity is computed as \( V = x_{\text{opt}} / \tau_0 \). In the tri-dimensional case,

\[
V = \frac{r_{\text{opt}}}{\tau_0},
\]

(25)

where \( r_{\text{opt}} \) is the baseline maximizing the correlation function (spatial optimization).

In Fig. 3, the horizontal line \( \tau = \tau_0 \) intersects “contour_0” at points \( A \) and \( B \), where the correlation function attains the value \( R_z(x, \tau) = \text{const}_0 \). Contour_0 is such that point \( A \) lies on the \( \tau \) axis and, therefore, point \( A \) corresponds with the auto-correlation, \( R_z(0, \tau_0) \). Point \( B \) corresponds with cross-correlation, \( R_z(x, \tau_0) \), with baseline \( x = x_0 \). Therefore,

\[
R_z(0, \tau_0) = R_z(x, \tau_0) = \text{const}_0 \text{ at time-lag intercept, } \tau_0.
\]

According to geometrical properties of the rotated contour ellipse, the wind drift velocity is computed as \( V = x_0 / (2\tau_0) \) (see Sect. 5.iv and Eq. (17) in Briggs, 1950). In other words, given the user baseline, \( x = x_0 \), maximization of the correlation function is equivalent to finding the time lag, \( \tau_0 \), where temporal cross-correlation function \( R_z(x = x_0, \tau) \) (point \( B \)) and temporal autocorrelation function, \( R_z(0, \tau) \) (point \( A \)), intersect. Moreover, because the “correlation intersect method” relies on geometrical properties of the correlation ellipse, it also
holds for many other time intersects, including $\tau_{i,1}$ (points $A_1, B_1$; baseline $x_1$) and $\tau_{i,2}$ (points $A_2, B_2$; baseline $x_2$). In these cases, the same drift velocity is obtained as

$$V = x_0/(2\tau_0) = x_1/(2\tau_{i,1}) = x_2/(2\tau_{i,2}).$$

(26)

**Extension to the tri-dimensional case**

While in the one-dimensional case the user chosen baseline, $x$, and the wind-drift velocity, $V$, are always co-linear (because there is only one coordinate axis, the X axis), this is now no longer the usual case. In the tri-dimensional case, the user usually chooses a baseline, $r$, which, under anisotropic conditions, is in general not aligned with the velocity vector, $V$, and hence, the “natural” extension of Eq. (26) to $V = r/(2\tau)$ fails. In the following mathematical development, it will be shown that the idea behind the correlation-intersect method (Fig. 4) is that departing from a user baseline, $r = r_i$ (in general, not aligned with the wind velocity) is used to find an auxiliary vector, $V_A = r_i - 2V\tau_i$ (i.e., function of time-lag intersect $\tau_i$). Once subtracted from the user’s proposed baseline, $r_i$, we obtain a new vector, $r_i - V_A$, proportional to the maximum-correlation baseline, $r_{opt}$ (Fig. 4a). This new vector is then aligned with the true wind-drift direction, $V$. In doing so, classic spatial correlation maximization is replaced by the temporal correlation intercept method, which does not require function maximization.

**Mathematical discussion** begins with definition of the frozen-atmosphere correlation function in absolute coordinates for baseline, $r$, and time lag, $\tau$, which is obtained by substituting Eq. (12) into Eq. (8) as
From Eq. (27), the temporal auto-correlation function is given by baseline $r = 0$ as
\[
R_z(r, \tau) = f \left[ (r^T - V^T \tau) A (r - V \tau) \right].
\] (27)

The correlation-intersect method between *temporal* auto- and cross-correlation functions is formulated as
\[
R_z(r, \tau_i) = R_z(0, \tau_i), \quad (29)
\]
where subindex “i” stands for “intersect”.

Because the model decay function, $f$, is by definition the same for both auto- and cross-correlation functions (Eqs. (27)-(28) above), equality condition, Eq. (29), can be directly imposed over the respective $f$-function arguments as
\[
\left( r^T - V^T \tau \right) A (r - V \tau) = V^T A V \tau^2.
\] (30)

After simple algebraic manipulation and by using $V^T A r = r^T A V \tau$ (this term is a scalar), one obtains
\[
r^T A (r - 2V \tau) = 0.
\] (31)

Eq. (31) (to be numerically solved later) is the sought-after key equation enabling retrieval of the wind-drift velocity. This equation can physically be interpreted as the dot product between two vectors (vectors as columns); namely, $V_A = r - 2V \tau$ and $V_B = Ar$, with two possible solutions:
\[
V_A^T V_B = 0 \Rightarrow \begin{cases} V_A \perp V_B \Rightarrow (r - 2V \tau) \perp Ar \smallskip \vspace{0.1cm} V_A = 0 \Rightarrow V = \frac{r}{2\tau}, \quad \tau = \tau_i. \end{cases}
\] (32)
Given a time-lag intercept, $\tau = \tau_i$, a particular solution, $V = r/(2\tau)$, stands for the very uncommon case in which the user chosen baseline coincides with the wind-drift velocity direction. This reduces to $V = \chi/(2\tau)$ in the Briggs’ one-dimensional solution. Except for this particular case, the “correlation intercept method” must be explained from the general solution, $V_A \perp V_B$, depicted in Fig. 4b. Because the baseline, $r = r_1$, is user-defined, the anisotropy matrix, $A$, is known from Sect. 3.2. So is the vector $V_B$. From the definition of $V_A$ above, the wind-drift velocity can be expressed as

$$V = \frac{1}{2\tau_{i,1}}(r_i - V_A), \quad (33)$$

where we have substituted the user baseline, $r = r_1$, and time-lag intercept, $\tau = \tau_{i,1}$, for which $V_A \perp V_B$. From Eq. (33) it emerges that the vector $(r_i - V_A)$ is aligned with velocity vector, $V$ (Fig. 4a), which, therefore, means that $r_i - V_A$ is a vector proportional to the maximum-correlation baseline, $r_{opt}$. By comparing Eq. (33) with Eq. (25), Eq. (33) can be rewritten as

$$V = \frac{r_{opt}}{\tau_{i,1}}, \quad r_{opt} = \frac{1}{2}(r_i - V_A), \quad (34)$$

which re-encounters the well-known wind-drift velocity solution using spatial correlation maximization with baseline, $r_{opt}$, and time lag, $\tau = \tau_{i,1}$.

A numerical solution for Eq. (31) is reached by rewriting it in the form of the scalar equation as

$$r^T Ar = 2\tau_i \left(r^T A V\right), \quad (35)$$

25
where we have used $X + X^T = 2X$, with $X$ as the scalar quantity $X = r^T A V$. The left term of Eq.(35) is easily identified in Eq.(20) and can be computed as the $f$-argument of the measured correlation function, $\Gamma_z(r,0)$. Substitution of Eq.(20) into Eq.(35) yields the numerical equation for the retrieval of the wind vector as

$$r^T A V = \frac{1}{2\tau_i} f^{-1}[\Gamma_z(r,0)].$$  (36)

When Eq.(36) is evaluated for a set of baselines, $r = r_k$, $k = 1..M$ ($M \geq 3$), it is similar to Eq.(21). Therefore, the wind vector (in absolute coordinates) can be inverted from the $M \times 3$ linear equation system as

$$Q V = c,$$  (37)

where $Q = r_k^T A$ is the $M \times 3$ matrix defined by the product of the baseline vector, $r_k^T$, the anisotropy matrix, $A$, $V = (V_x, V_y, V_z)$ is the wind vector to solve, and $c = f^{-1}[\Gamma_z(r_k,0)] / 2\tau_{i,k}$ is the independent term defined by the $f$-argument of the spatial correlation function at zero time delay. $\tau_{i,k}$ is the intersecting time point from Eq.(29) for the $r_k$ baseline, which in practice is computed as $\Gamma_z(r_k, \tau_{i,k}) = \Gamma_z(0, \tau_{i,k})$.

Once $V$ is known, we compute the coordinate transform matrix, $\Psi$, from the wind azimuth angle, $\phi_w$, and Table 1. This enables recovery of the anisotropy matrix, $M$ (Eq.(5), from its rotated version, $A$, derived in Sect. 3.2.
Using 1) $\mathbf{U} = \mathbf{\Psi}^{-1}\mathbf{V}$ (Table 1), 2) $\mathbf{\Psi}^{-1} = \mathbf{\Psi}^T$ ($\mathbf{\Psi}$ is unitary) and 3) that the cross-wind component is $U_z = 0$ (by definition of the wind-relative coordinate system), it follows that the mean wind component is

$$U_1 = \sqrt{V_x^2 + V_y^2},$$ \hspace{1cm} (38)

and the wind azimuth angle $\phi_w$ is

$$\phi_w = \tan^{-1}\left(\frac{V_y}{V_x}\right).$$ \hspace{1cm} (39)

The vertical wind component $U_3 = V_z$ remains invariant. Finally, $\mathbf{M} = \mathbf{\Psi}^T \mathbf{A} \mathbf{\Psi}$ is computed from Eq.(11).

4. Simplified application case

The wind-retrieval method presented in Sect. 3 has been applied to a two-angle azimuth scan (2AAS) scheme (2 LOS, $\phi = 225$ deg and $\phi = 228$ deg in azimuth, elevation $\theta = 40$ deg) using both isotropic and anisotropic models for comparison. In contrast to the three-angle azimuth scan (3AAS) often used in the literature (Kunkel et al., 1980; Buttler et al., 2001), which offers a higher number of redundant (coplanar) baselines, the 2AAS (Fig. 1) enables faster scan rates.

Measurements were obtained 1200-1205 UTC on 19 September 2008 with the RSLAB (Remote Sensing Lab) lidar operated at 1064 nm. Because the scattering cross sections of aerosols typically follow a $\lambda^{-1}$ spectral dependency ($\lambda^{-4}$ for molecules), the measurements benefit from a comparatively higher aerosol backscatter component than the molecular one at
the operating wavelength (Collis and Russell, 1976). The system uses a Nd:YAG 20-Hz laser source with 160-mJ energy and 6-ns width pulses along with a 20-cm aperture, 2-m focal-length telescope and 3-mm avalanche-photodiode-detector (APD)-based receiving channel (7.5-m range resolution, Noise Equivalent Power ≈ 4.2×10^{-13} W·Hz^{1/2}). The lidar was aimed at 225-deg in azimuth corresponding with the southwest (SW) direction (north 0-deg, east 90-deg convention) along a 40-deg elevation slant path. The scanning period was 4 s, consisting of 1-s integration time (20 pulses) per LOS and 1-s dead time per LOS (moving time from one azimuth to the other). Coincident potential temperature and relative humidity radio-soundings at 1200 UTC, collected 600 m southeast (SE) from the lidar, are used to cross-examine the results. Wind-velocity and direction information are derived from internal GPS measurements coupled with the radiosonde measurements (Vaisala™, mod. RS92-SGP).

4.1. Practical aspects
The atmospheric vertical profile (Z) has been divided into piece-wise homogeneous velocity-inversion layers (see Sect. 2.2.1) of approximately 34 m depth (vertical inversion resolution), corresponding with seven range bins (one bin equal to 7.5 m slant path) along the LOS of the lidar instrument. In each vertical layer, Eq.(23), Sect. 3.2 and Eqs.(37)-(39), Sect. 3.3 are solved from a user-defined set of baselines formed by three “best” baselines.

The practical baseline selection procedure follows (Fig. 1c). First, in a given layer, all possible cross-correlations (\(\Gamma^{pr,ij}_z(0)\); see Eq.(13) for notation) arising from combining two-by-two the available data points in the velocity-inversion volume are computed from the normalized lidar signals, \(S_n\) (Eq.(14)). This implies that aerosol content fluctuations must
exist in the atmosphere and be intense enough to be recorded in $S_n$. Second, three correlation subsets are formed: (i) transversal correlations (i.e., correlations between points belonging to different LOS) with a baseline parallel to the $X$ axis, (ii) transversal correlations with a baseline non-parallel to the $X$ axis, and (iii) radial correlations, which by definition have the baseline following a common LOS. Finally, the three “best” baselines are those giving the absolute maximum of $\Gamma_z^{pq,ij}(0)$ in each of the subsets above. This baseline selection scheme enables choice of a baseline parallel to the $X$ axis, another one almost parallel to the $Y$ axis, and a third one following the apparent wind direction.

In the present 2AAS case example, because two LOS always lie on a plane, the wind-retrieval method cannot retrieve the three-dimensional spatial anisotropy matrix (six non-null coefficients in Eq.(5). Equivalently, the $a$ vector in Eq.(24)) nor the three wind components. Therefore, for data sufficiency, the anisotropic model of Eq.(5) must somehow be simplified. According to Mayor et al. (2003), a wind-oriented anisotropic pattern is expected to be found at the bottom of the ABL, whereas this tendency vanishes towards an isotropic one in the mixed layer. Anisotropic models can be considered for the 2AAS besides the isotropic one. A “horizontal anisotropy” model applies to low-elevation scans (as an approach to the horizontal-scan case) or, in general, to positions whose vertical distance, $z$, is small enough inside the layer to consider the vertical variation of the spatial correlation negligible. Under this assumption the anisotropy matrix, $M$ (Eq.(5)), reduces to a $2\times2$ matrix, since the coefficients related to the variation in the vertical dimension are zero ($m_3 = 0$, $m_{43} = m_{23} = 0$).
The absolute-coordinate anisotropy matrix, \( A \), is also 2×2 and has three non-null coefficients \((a_x,a_y,a_{xy})\) that can be determined (Sect. 3.2).

Computationally, once the wind vector and the anisotropy matrix, \( M \), are obtained, \( M \) is diagonalized. According to Horn and Johnson (1985) and Fig. 3, the eigenvectors of \( M \), \( e_1 \), and \( e_2 \) are the principal axes of the correlation ellipse. The major axis corresponds with the largest correlation length, and is representative of the highest correlation direction of the aerosol field. The elongation or axial ratio of the correlation ellipse is the ratio between its major and minor axes. The tilt is the deviation angle between the major axis and the wind direction.

The simplified anisotropic model above is a step beyond the usual isotropic assumption, for the degree of isotropy of the aerosol structures to be evaluated, instead of assuming it "a priori". The functional model used for the aerosol spatial correlation function, \( f \) in Eq.(3), was a unit-normalized decreasing exponential, \( f(x) = \exp(-x) \), so that \( B(p) \) in Eq.(3) was Gaussian and the space-time aerosol correlation model function, \( R_z(p,\tau) \) in Eq.(8) was a time-delayed Gaussian. The 1-s instrumental delay time between the two LOS was offset by delaying the temporal LOS cross-correlations by the same time amount.

4.2. Experimental results

Fig. 5 shows the measured range-corrected power lidar profiles as time-height images for the two LOS of the 2AAS for an observation time of 240 s (60 time records per LOS, \( M =60 \) in Eq.(14)). Three different tendencies are evident. First, between 400 and 1000 m ASL (above
sea level) aerosol patterns appear sooner for the 225-deg LOS (Fig. 5a) than for the 228-deg LOS (Fig. 5b). There is slight radial motion outwards from the lidar. This is evidenced by white solid circles at times $t_1 = 30$ s, $t_2 = 90$ s, $t_3 = 160$ s in Fig. 5a and, respectively, at times $t'_1 = 45$ s, $t'_2 = 100$ s, $t'_3 = 170$ s in Fig. 5b. Second, between approximately 1200 and 1600 m ASL, the situation reverses and the aerosol patterns appear sooner in the 228-deg LOS (Fig. 5b) than in the 225-deg LOS (Fig. 5a). This is indicated by circular shapes $B_1$, $B_2$ and $B'_1$, $B'_2$ in Fig. 5a and Fig. 5b, respectively, and reveals a wind-shear situation. Lastly, in the middle region between approximately 850-1150 m there are less contrastable patterns to visually identify the aerosol motion. Above 1600 m ASL, data are no longer represented, because of the reduced signal-to-noise ratio (Fig. 6a) and absence of significant aerosols.

Fig. 6b depicts potential temperature and water vapor mixing ratio profiles from local radiosonde. From the gradients of these profiles, the atmospheric boundary layer height is found around 650 m, which is in agreement with the approximate mean height of the contrasted structures in Fig. 5. Detection of such structures is usually limited to the ABL, because it is characterized by a vigorous mixing and patchy structures of convective origin, which give rise to relatively strong signal fluctuations. Transported aerosol layers above this level tend to be fairly homogeneous, conveying meaningless signal fluctuations (i.e., noise) once subtracted. In Fig. 5, inhomogeneities between 1200-1600 m in height are strong enough to depict two structures, which has enabled our tentatively extending the study range above the ABL up to 1600 m. The high relative humidity in the 1100-1400 m height interval can increase the atmospheric optical backscatter through deliquescence of background aerosol particles and haze activation (Tang, 1993) in Fig. 5. The large humidity gradient between 800
and 1200 m is symptomatic of wind shear, in contrast to the usual convective ABL near solar noon.

Fig. 7 a,b shows the results of the wind-retrieval method using both the isotropic and the anisotropic model. Fig. 7b shows profiles of the horizontal wind direction depicting a wind-shear phenomenon caused by two air masses with different relative humidity (80% below 800 m and up to 90% between 800-1400 m, Fig. 7b). For the lowermost height interval (< 900 m), the data in Fig. 7a,b show little variation in estimates of horizontal wind speed and direction. Both an/isotropic models retrieve similar estimates (in accordance with the radiosounding), though the isotropic estimates for wind speed are biased slightly low when compared with the radiosonde. The wind-retrieval results for the uppermost height interval (1200-1600 m) depict wind shear from 135 to 330 deg, which is identified by the anisotropic model. This is not, however, the case for the isotropic model, which oscillates over some 300-deg uncertainty, thus giving divergent wind-speed estimates. In the middle layer (850-1150 m), which is characterized by low aerosol content, performance of the anisotropic model, particularly in the retrieval of the wind direction, is again better than that of the isotropic model, and shows a progressive change in the wind direction (120 to 150 deg). But it is not as accurate as the radiosonde. One reason accounting for this discrepancy is the lack of significant aerosol content in this layer and low signal-to-noise ratio.

Fig. 7 c,d shows the horizontal anisotropy of the aerosol structures in each horizontally homogeneous layer of the wind profile (Sect. 2.2.1.1). The axial ratio is found to be between unity (1:1) and ten (10:1), progressively increasing with height (notation $\rho_{c,1} : \rho_{c,2}$, note that $m_1 = 0$, $m_{13} = m_{23} = 0$ in Eq.(5)), which resulted in an oblate ellipsoid for the quadratic function.
model, $q(p)$ (Eq.(4)). Up to 750 m, the axial ratio lies between 1 and 3. In the 800-1100 m height interval, which is characterized by less contrasted aerosol patterns, the ratios are roughly between 2 and 5, and show higher dispersion. Finally, in the transported aerosol layer aloft (1150-1400 m), the ratio is around 5. In each of these three intervals, the tilt angle is less than 30-deg clockwise. In relation to Fig. 2, the elongated aerosol structures are not perfectly (but almost) aligned with the wind direction, thus revealing a slight departure from the isotropic assumption. In the stratified aerosol layer at 1100-1400 m, the ellipse elongation increases.

5. Conclusions

Large-scale aerosol concentration gradients can effectively be sensed by multiple-lines-of-sight (LOS) scanning backscatter lidars, thus allowing for the detection of anisotropic patterns. A three-component low-troposphere wind-retrieval method under anisotropic media has been formulated for backscatter-lidar scanning schemes consisting of a reduced number of LOS. Classic spatial function maximization, which under the requirement of a densely sampled volume (LOS in many different directions) in anisotropic conditions can yield the true wind velocity, has been replaced by a temporal-correlation intersect method. The latter does not require function maximization or a densely sampled volume. In contrast, a few LOS (as in the multiple-angle azimuth scheme) with densely sampled measurements along them are needed. A further result of the proposed method is that the matrix linear solution form is obtained.
The method assumes a piece-wise “frozen” atmosphere divided into time-stationary and statistically-homogeneous vertical layers (Sect. 2.2.1.1). The algorithm is mainly intended for unstable convective boundary layer conditions and, therefore, usually limited to the atmospheric boundary layer (ABL). However, the only inherent limitation is that aerosol content fluctuations must exist in the atmosphere and be properly recorded in the mean-subtracted range-corrected signal, $S_n$. That is, $S_n$ must have acceptable signal-to-noise ratio (SNR), (an instrument requirement), and enough signal variance (an atmospheric requirement) to fairly reproduce aerosol fluctuations.

The space-time correlation function in an atmospheric aerosol layer (or velocity-inversion volume) at height $Z$ is estimated from the temporal correlation of range-corrected lidar signals measured at non-colinear baselines (difference position vector between two measurement points of a same/different LOS). Therefore, the baselines sample the velocity-inversion volume along different directions in space and with different lengths in such a way that the movement of the aerosol inhomogeneities swept away by the wind can be effectively monitored.

Because the temporal correlation function computed from a baseline, $r_k, k=1..N$ (Fig. 2), $\Gamma_z(r_k, \tau)$ depends on both the wind vector and the anisotropy of the medium, the algorithm must correct for the “false-velocity” effects caused by the anisotropy. This is done in two steps. First, the rotated anisotropy matrix, $A$, is obtained from time cross-correlations evaluated at zero-lag delay at different baselines. Second, the wind vector is obtained from the temporal auto- and cross-correlation functions (full correlation analysis) and the rotated
anisotropy matrix. Since the wind-relative coordinate system is by definition aligned with the wind direction, once the wind vector is known so is the rotation angle between both coordinate systems and the aerosol anisotropy matrix, $\mathbf{M}$, can be obtained from the rotated one, $\mathbf{A}$. This enables the anisotropy axial ratio and tilt angle of the correlation ellipse to be computed with respect to the wind direction. In the tri-dimensional case, a minimum of $N = 6$ independent baselines (equivalently, six linear equations) is required to invert the rotated anisotropy matrix, $\mathbf{A}$, and a minimum of $M = 3$ independent baselines to invert the wind vector.

A first application of the method to a simplified two-angle-azimuth-scan (2AAS), using both an/isotropic models, has shown the outperformance of the wind-retrieval anisotropic method. With SNRs ranging from 40 to 6 along the measurement range, the horizontal wind component, as well as anisotropic parameters inside the ABL and in an aerosol layer aloft, have been retrieved (axial ratios between 1 and 10 and tilt angles between 0 and -30 deg). The anisotropic ratios obtained are substantially different from the “spherical” case, so that the horizontal aerosol anisotropy was relevant enough so as to justify the application of the anisotropic model in the wind-retrieval method presented. This is corroborated by the finer and convergent results of the anisotropic model in Fig. 7. When comparing them to radiosonde data in the $[400–800] \cup [1200–1600]$-m range interval, where there is significant aerosol loading, we have:

- In the 400-to-800-m range (range interval with high SNR, typically, SNR>>10) approximate wind-speed and wind-direction $rms$ errors are 1.0 m/s and 25.0
deg, respectively, for the anisotropic model; and 1.2 m/s and 24.0 deg, for the isotropic model.

- In the 1200-to-1600-m range (range interval with low SNR, typically, SNR<10), 3.5 m/s and 49.0 deg for the anisotropic model; and, as large as 9.2 m/s and 83.0 deg, for the isotropic model.

All considered, reasonable wind velocity information can be inverted as well as on anisotropic parameters. Future research is to comprise closer study of error bounds within the system.

Acknowledgments

The data for the application case of Sect. 4 was obtained at the Remote Sensing Lab (RSLAB), UPC (http://www.tsc.upc.edu/rs) and it is available upon request from S. Tomás, tomas@ice.cat.

This work was supported by the Spanish Ministry of Economy and Competitivity (MINECO) and European Regional Development Funds (FEDER) through UPC projects TEC2012-34575 and TEC2009-09106 and ICE-CSIC project AYA2011-29183-C02-02 and, in part, by the European Union through project ITARS (Initial Training in Atmospheric Remote Sensing), GA-289923. MINECO is also thanked for S. Tomás’ predoctoral fellowship BES-2007-17047 when doing his Ph.D. at UPC.

The Astronomy and Meteorology Dep. of the Universitat de Barcelona provided the daily radio-soundings used in the experimental part of this work.
References


List of Figures

Fig. 1. Geometry of the problem and coordinate systems. (a) Sketch of the lidar scanning scheme along two generic LOS, LOS-1 ($\theta_1, \phi_1$) and LOS-2 ($\theta_2, \phi_2$) with $\theta_i$ and $\phi_i, i=1,2$ their respective elevation and azimuth angles. (b) Coordinate systems. In an atmospheric layer ($Z_a \leq Z \leq Z_b$), the wind-relative coordinate system ($\hat{x}_i, \hat{y}_i, \hat{z}_i$) is defined so that $\hat{x}_i$ is aligned with the wind direction ($\hat{x}_i$ parallel to $U_1$, Eq.(1)). The lidar-absolute coordinate system has $\hat{y}$ aligned with the horizontal projection of the bisectrix of the angle between the two LOS. $\phi_w$ is the azimuth rotation angle between both coordinate systems. (c) Detail of an atmospheric layer ($Z_a \leq Z \leq Z_b$) or velocity-inversion volume (LOS_p and LOS_q are two generic LOS). Dots represent the lattice of lidar measurement datapoints. Notation $(p,i)$ indicates LOS_p and i-th measurement point. Baseline $r = r_{pq,ij}$ is defined by measurement points $(p,i)$ and $(q,j)$. (1) and (2) are transversal baselines, (3) is a radial baseline (see Sect. 4.1).

Fig. 2. Illustration of the anisotropy ($Z$ axis exits from the paper). The Y axis and the dashed arrowed line represent two LOS. $r_{12}$, $r_{13}$, and $r_{23}$ are the baselines associated to measurement points, $P_1...P_3$. The size of the aerosol structures sensed at these measurement points is large enough to be assumed anisotropic. Anisotropy gives rise to elliptic contours of the spatial correlation function. $e_1$ and $e_2$ are the major and minor axis, respectively, of the characteristic ellipse. Note that the correlation axis $e_1$ is neither aligned with the wind direction $\hat{x}_i$ (Eq.(1))...
nor with the baselines. The orientation of the ellipse with respect to the wind direction is the tilt angle $\alpha$.

Fig. 3. Contours of constant correlation $R_z(x, \tau) = \text{const.}$ as rotated concentric ellipses under the one-dimensional case (X-axis).

Fig. 4. Vector representation of Eq. (32) general solution, $V_A \perp V_B \cdot$ (a) Concept idea of auxiliary vector, $V_A \cdot$ (b) Detail of vector $V_A$ for the time-lag intersect solution, $\tau = \tau_{i,1}$, giving $V_A = r_i - 2V \tau_{i,1} \cdot V$ is the wind-drift velocity vector and $r = r_i$ is the user’s baseline. $V_B = A r_i$ is the anisotropy-rotated baseline.

Fig. 5. Two-angle azimuth time-height images of the mean-subtracted range-corrected lidar signal, $S_a(R, \tau)$ (Eq.(14)) corresponding to September 19, 2008, 1200-1205 UTC, $\theta = 40$ deg, $\phi_i = 225$ deg, $\phi_f = 228$ deg, 1064-nm wavelength. Each figure represents a LOS time-height record.

Fig. 6. Parameters associated to September 19, 2008 measurement (Fig. 5). (a) SNR estimate (1-s time average). The shaded area (400-1600 m) indicates the wind-retrieval range. (b) Potential temperature (solid line with crosses) and water vapour mixing ratio (solid line with void circles) derived from 1200 UTC, radio-sounding data. The dashed line around 650 m indicates the estimated mixing-layer depth.
Fig. 7. Horizontal wind-velocity and horizontal-anisotropy estimates as a function of height. (a) Horizontal wind speed, (b) wind direction. (Black trace, black circles) Estimates using the horizontal anisotropic model (see Sect. 4.1). (Dashed trace, grey void circles) Estimates using the isotropic model. (Grey trace, crosses) Radio-sounding data. (c, d) Retrieved horizontal-anisotropy parameters: (c) Axial ratio, (d) tilt angle of the major axis of the correlation ellipse with respect to the wind direction ($\alpha$ angle in Fig. 2). Horizontal dashed lines indicate height intervals with similar anisotropic parameters.
### Tables

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>RELATIVE COORD. SYSTEM</th>
<th>ABSOLUTE COORD. SYSTEM</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$(\hat{x}_1, \hat{x}_2, \hat{x}_3)$</td>
<td>$(\hat{x}, \hat{y}, \hat{z})$</td>
</tr>
<tr>
<td>Range vector</td>
<td>$X = X_1\hat{x}_1 + X_2\hat{x}_2 + X_3\hat{x}_3 \rightarrow P[X_1, X_2, X_3]$</td>
<td>$R = X\hat{x} + Y\hat{y} + Z\hat{z} \rightarrow P[X, Y, Z]$</td>
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<tr>
<td>Baseline (difference) vector</td>
<td>$\rho = X_2 - X_1 = (\rho_1, \rho_2, \rho_3)$</td>
<td>$r = R_2 - R_1 = (x, y, z)$</td>
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<tr>
<td>Mean wind</td>
<td>$U = U_1\hat{x}_1 + U_3\hat{x}_3$</td>
<td>$V = V_1\hat{x} + V_3\hat{y} + V_5\hat{z}$</td>
</tr>
<tr>
<td>Coordinate transform (range vectors)</td>
<td>$r = \Psi \rho$, $\Psi = \begin{pmatrix} \cos \phi_w &amp; -\sin \phi_w &amp; 0 \ \sin \phi_w &amp; \cos \phi_w &amp; 0 \ 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>(note that $\Psi = \Psi(U)$ via the wind angle $\phi_w$)</td>
</tr>
<tr>
<td>Other relationships</td>
<td>$A = \Psi M \Psi^T$ (anisotropy matrix)</td>
<td>$V = \Psi U$ (wind vector)</td>
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Tab. 1 Variable definition for the relative and absolute coordinate systems used. Note that $X$ and $R$ (upper-case letters) stand for range vectors associated to point $P$ whereas $\rho$ and $r$ (lower case) stand for baseline vectors (i.e., difference position vectors).
\[ V = \frac{x_{\text{opt}}}{r_0} = \frac{x_0}{2r_0} = \frac{x_1}{2r_{1,1}} = \frac{x_2}{2r_{1,2}} \]