

## ON THE ADAPTIVE LATTICE ALGORITHMS WITH DATA DEPENDENT PARAMETERS

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### ABSTRACT

The classic adaptive algorithms present a behaviour greatly depending on the signal under analysis and they are sensitive to the parameter selection. In last years, several algorithms including a data dependent estimation of its convergence parameters have been published. In this work, we present a family of adaptive lattice algorithms under a common and general formulation. The estimation criterion of the parcor coefficients of the lattice is the local minimization of a generalized error function, consisting in a linear combination of the backward and forward residuals. Three different algorithms are studied in detail; two of these can be considered as the adaptive version of the Energy Weighted (AEW) [1] and the Data-Adaptive Burg (ADAB) [2] algorithms; the third algorithm, the Weighting Residuals Adaptive Lattice (WRAL), is wholly original by the authors [3]. The paper is completed with a performance comparison of these in a line tracking context.

### INTRODUCTION

Adaptive lattice algorithms have shown its utility for several applications in nonstationary environments. Speech AR modelling, line tracking and adaptive equalization are some interesting applications of these algorithms. The selection of the convergence parameters is a very important key of the adaptive filtering, and its optimum values are very dependent on the statistical of the signal. A robust data dependent estimation of these parameters should a very good solution. Some adaptive algorithms with data dependent parameters have been published in the signal processing literature in a sparse way. In this work, we present a family of adaptive lattice algorithms responding to a common formulation. The estimation criterion of the parcor coefficients is the local minimization -stage-by-stage- of the lattice- of a generalized quadratic error function. This consists in a linear combination of the forward and backward residuals. Its expression is:

$$L_p(n) = \sum_k^n w_p(n,k) [(1-\gamma_p(k)) f_p^2(k) + \gamma_p(k) b_p^2(k)] \quad p = 1, \dots, Q \quad (1)$$

where

p: index of the lattice stage

Q: lattice order

$f_p(k)$ : p<sup>th</sup> order forward residual

$b_p(k)$ : p<sup>th</sup> order backward residual

$w_p(n,k)$ : time-variant weighting window

$\gamma_p(k)$ : backward-forward weighting parameter.

The parcor expression resulting of this minimization is:

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$$K_p(n+1) = -\frac{C_p(n)}{D_p(n)} = -\frac{\sum_k^n w_p(n,k) c_p(k)}{\sum_k^n w_p(n,k) d_p(k)} \quad (2)$$

where

$$c_p(k) = f_{p-1}(k) b_{p-1}(k) \quad (3)$$

$$d_p(k) = \gamma_p(k) f_{p-1}^2(k) + (1-\gamma_p(k)) b_{p-1}^2(k-1)$$

The  $w_p(n,k)$  and  $\gamma_p(n)$  are the convergence parameters of the lattice algorithms. The  $w_p(n,k)$  parameter defines the memory of the algorithm and the relative weighting of the residuals samples. The  $\gamma_p(n)$  parameter weights the backward and forward directions of the prediction. Different estimation methods of these parameters lead to a family of adaptive lattice algorithms. In the next sections we expose three different algorithms named Weighting Residuals Adaptive Lattice (WRAL), Adaptive Energy Weighted (AEW) and Adaptive Data-Adaptive Burg (ADAB). The selection of  $w_p(n,k)=1$  and  $\gamma_p(n)=0,5$  results in the known Burg algorithm.

### THE DATA DEPENDENT ADAPTIVE LATTICE ALGORITHMS

The WRAL algorithm, wholly original by the authors, was expounded in the reference [3]. In this algorithm, the  $w_p(n,k)$  parameter is defined as:

$$w_p(n,k) = \begin{cases} \beta_p(n) w_p(n-1,k), & k \leq n-1 \\ 1, & k = n \end{cases} \quad (4)$$

and it can be interpreted as the impulse response of a time-variant first order recursive filter with a single pole of  $\beta_p(n)$  value. In this case, the expression of the parcor coefficients since formula (2) takes the following

recursive shape:

$$K_p(n+1) = - \frac{\beta p(n) C_p(n-1) + c_p(n)}{\beta p(n) D_p(n-1) + d_p(n)} \quad (5)$$

The  $\beta p(n)$  value determines the inertia of the algorithm responding to the changes of the signal statistical. It is clear that a closed unity value of  $\beta p(n)$  is good for stationary frames, and low  $\beta p(n)$  values are adequate when sudden changes of the signal statistic are presenting. Thus, a data dependent  $\beta p(n)$  is required for an optimum behaviour of the algorithm. In this algorithm we take  $\beta p(n)=1-\sigma_p(n)$ , where  $\sigma_p(n)$  is the estimation of the  $p$ th parcor error covariance. This estimation is obtained in [3] since a Kalman-Gauss formulation for the parcor evolution and the lattice equations. The resulting  $\sigma_p(n)$  expression is:

$$\sigma_p(n+1) = \sigma_p(n) \left[ \frac{f_p^2(n)}{f_p^2(n) + \sigma_p(n) b_{p-1}^2(n-1)} \right] + V_p \quad (6)$$

where  $V_p$  represents the stationary index defined in the Kalman model. Thus,  $V_p=0$  means total stationarity.

When the  $K_p(n)$  estimation is far from the optimum, the predictor errors  $f_p(n)$  are great and the  $\sigma_p(n)$  values hold high; therefore,  $\beta p(n)$  takes a low value and the relative importance of the actual data is great. On the contrary, when  $K_p(n)$  estimation is close to the optimum, the predictor residuals are small and the  $\sigma_p(n)$  value decreases quickly; therefore,  $\beta p(n) \rightarrow 1$  and the inertia or the memory of the algorithm is large. If a non-stationary environments is present, the  $V_p \neq 0$  and it prevents to the  $\sigma_p(n)$  magnitude to reach the zero value.

The  $\gamma_p(n)$  parameter is estimated as

$$\gamma_p(n) = \frac{E_{b,p}(n)}{E_{f,p}(n) + E_{b,p}(n)} \quad (7)$$

$$\text{with } E_{f,p}(n) = \sum_{i=1}^p x^2(n-i); E_{b,p}(n) = \sum_{i=1}^p x^2(n-p+i) \quad (8)$$

where  $E_{f,p}(n)$  and  $E_{b,p}(n)$  are the local energies of the signal associated to the forward and backward predictions, respectively. In a signal frame of increasing energy, the  $\gamma_p(n)$  value is smaller than 0.5 and the algorithm weights up the forward direction (see expression (1)). On the contrary, in a decreasing frame, the algorithm weights up the backward direction. Thus, the algorithm outstands the increasing direction of the signal energy evolution or unstable way, and a peaking effect in the spectral AR modelling is yielded. This fact counteracts the tendency of the standard estimation algorithms to place the poles of the related AR model in circle unit areas more inside than the true locations. The estimation (7) is based in orthogonalization principle considerations [3].

The EW algorithm was thought up initially as a block lattice algorithm [1]. In this paper we propose a time-

recursive version of this, named the AEW algorithm. The searched goal is to minimize the sum of the forward and backward residuals covariances weighted by its associated energies,  $E_{f,p}(n)$  and  $E_{b,p}(n)$ , respectively. This energy weighting minimization arises from a curve fitting problem of the covariance recursion of an AR model [1]. By dividing both adding terms by  $E_{f,p}(n) + E_{b,p}(n)$ , the object function takes a expression responding to the general formula (1), where the  $\gamma_p(n)$  parameter is

$$\gamma_p(n) = \frac{E_{f,p}(n)}{E_{f,p}(n) + E_{b,p}(n)} \quad (9)$$

and  $w_p(n,k)$  parameter is

$$w_p(n,k) = \begin{cases} \beta w_p(n-1,k) & , k \leq n-1 \\ E_{f,p}(n) + E_{b,p}(n) & , k = n \end{cases} \quad (10)$$

and the leakage parameter  $\beta$  limits the memory of the algorithm. It is adequate to work in a general nonstationary environment, and it is equivalent to a block memory length of  $1/(1-\beta)$  samples. This  $w_p(n,k)$  estimation weights up the high energy frames of the signal.

The final expression of the parcor coefficient in the AEW algorithm is:

$$K_p(n) = \frac{\beta C_p(n-1) + [E_{f,p}(n) + E_{b,p}(n)] c_p(n)}{\beta D_p(n-1) + [E_{f,p}(n) + E_{b,p}(n)] d_p(n)} \quad (11)$$

where  $c_p(n)$  and  $d_p(n)$  are defined in the expression (3).

The  $\gamma_p(n)$  expression (9) of the AEW algorithm is just the complementary of the WRAL one (7). Thus, the AEW signal algorithm outstands the stable way of the energy evolution, providing a spectral AR modelling with a low line-splitting effect [1].

The DAB algorithm, like the EW case, was thought up initially as a block lattice algorithms [2]. We propose the same time-recursion introduced in the AEW algorithm to obtain the named Adaptive DAB (ADAB) algorithm. In this case, the searched goal is to minimize the sum of the forward and backward residuals covariances weighted by the common term, named  $E_p(n)$ , of the backward  $E_{b,p}(n)$  and forward  $E_{f,p}(n)$  associated energies. This energy weighting criterion arises from the minimization of a kind of error covariances defined since orthogonalization principle considerations [2]. It resembles in part to the WRAL basis. Thus, the weighting parameter  $w_p(n,k)$  can be defined as:

$$w_p(n,k) = \begin{cases} \beta w_p(n-1,k) & , k \leq n-1 \\ E_p(n) & , k = n \end{cases}; p=2,...,Q \quad (12)$$

where the  $\beta$  parameter plays the same role like in the AEW algorithm and

$$E_p(n) = \sum_{i=1}^{p-1} x^2(n-i); p=2,...,Q \quad (13)$$

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The indetermination of the  $E_p(n)$  with  $p=1$  is solved taking  $E_1(n)=1$ . It provides a reasonable estimation for  $K_1(n)$  parcor, equivalent to the Burg case.

Also, the  $\gamma_p(n)$  parameters is constant and of 0,5 value, like in the Burg method. Thus, the ADAB algorithm does not give precedence to any predictor direction, and it weights up the high energy frames of the signal, like the AEW algorithm. The  $K_p(n+1)$  expression takes the following shape:

$$K_p(n+1) = - \frac{\beta C_p(n-1) + E_p(n)f_{p-1}(n)b_{p-1}(n-1)}{\beta D_p(n-1) + E_p(n)[f^2_{p-1}(n) + b^2_{p-1}(n-1)]} \quad (14)$$

## EMPIRICAL RESULTS

The three developed lattice algorithms and a Burg algorithm are tested in a line tracking context. It is a very important application of the adaptive processing, and also, the EW and DAB block algorithms was thought up in a line estimation context [1,2]. The signal to be considered in the test are currently used in this kind of research [4,5]. Three different signals are used:

- a. Two piece-wise constant frequency sinusoids with instantaneous and equal frequency steps.

$$\begin{aligned} SNR_1 &= 20 \text{ dB}, w_1 = 7\pi/8, \Delta w_1 = -\pi/8 \\ SNR_2 &= 10 \text{ dB}, w_1 = \pi/4, \Delta w_2 = -\pi/8 \end{aligned}$$

- b. The same as case (a) but opposed step sign

$$\Delta w_1 = -\Delta w_2 = -\pi/8$$

- c. Two sinusoids  $w_1$  and  $w_2$  with crossing linear variation frequencies.

The considered Burg algorithm minimizes the expression (1) with  $w_p(n,k) = \beta^{n-k}$  and  $\gamma_p(n) = 0.5$ . The parcor estimation is similar to expression (5) with  $\beta_p(n) = \beta$  and  $\gamma_p(n) = 0.5$ .

In the WRAL algorithm has been observed that the use of a different  $\gamma_p(n)$  parameter for each lattice stage causes an high spectral variance, specially in the first stages. The more appropriated solution is to take  $\gamma_p(n) = \gamma_Q(n)$ ,  $p=1, \dots, Q$ . It is argued that the slope of the signal energy evolution is best estimated in a  $Q$  frame. On the contrary, the use of different convergence parameters for each stage in the AEW and ADAB algorithms provides the best performance.

The convergence parameters are chosen to provide a good tradeoff between convergence speed and error variance of the steady state frequency estimates. Also, these averaged frequency error variances are taken roughly equal in all algorithms. Thus, the selection of the algorithm parameters was as follows:

Burg algorithm:  $\beta=0.98, \gamma=0.5$   
WRAL algorithm:  $V_p=0.008$

AEW algorithm:  $\beta=0.98$   
ADAB algorithm:  $\beta=0.98$ .

The order  $Q$  is 6 for all algorithms.

For the (a) and (b) signals, the convergence speed of the WRAL outperforms those of the rest of algorithms. It is due to the WRAL algorithm provides low  $\beta_p(n)$ , that is to say, low algorithm inertia, when the frequency step is presented. Also, the "unstable" property of the WRAL due to the  $\gamma_p(n)$  estimation provides a faster use of the extra poles pair in the tracking of the weak sinusoid. For the signal (c), the AEW outperforms to the rest of algorithms. The linear variation of the frequency does not require fast convergence, and the "stable" behaviour for the extra poles pair preserves from the line splitting. The line splitting is clearly present in the WRAL algorithm while stands the linear frequency variation.

Some frequency estimation evolutions are shown in the following figures in order to explain the algorithms behaviour. In the figure 1 is shown the  $\beta_Q(n)$  evolution in the WRAL algorithm for the test signal (a). It can be observed the sharp decreasing of the  $\beta_Q(n)$  value when the frequency steps are presented. Right away, the  $\beta_Q(n)$  value increases to an higher value for a best tracking of the constant frequencies. The different steady state  $\beta_Q(n)$  value after the step is due to the change of the parcor values.

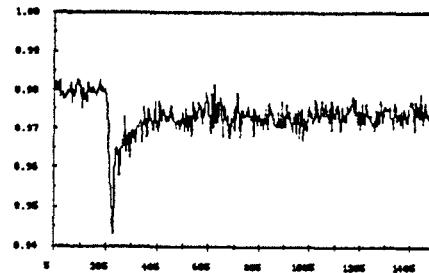


Fig. 1. WRAL  $\beta_Q(n)$  parameter evolution for the signal (a).

In the figure 2 is shown the frequency estimation evolutions of the Burg, WRAL and ADAB algorithms for signal (b). They are obtained by using a 1024 points-FFT of the related AR model. Then, the two larger maxima are tested and the corresponding over 20 realizations frequency estimation are averaged. This average estimations are shown each five samples. It is observed that WRAL algorithm outperforms the rest of algorithms.

In the figure 3 is shown the spectra evolution of the three data dependent algorithms for the signal (c). The AEW algorithm provides the best frequencies estimation due to the low line splitting. The WRAL algorithm presents line splitting in the linear frequency frame but it removes quickly the third maximum caused by the extra poles pair.

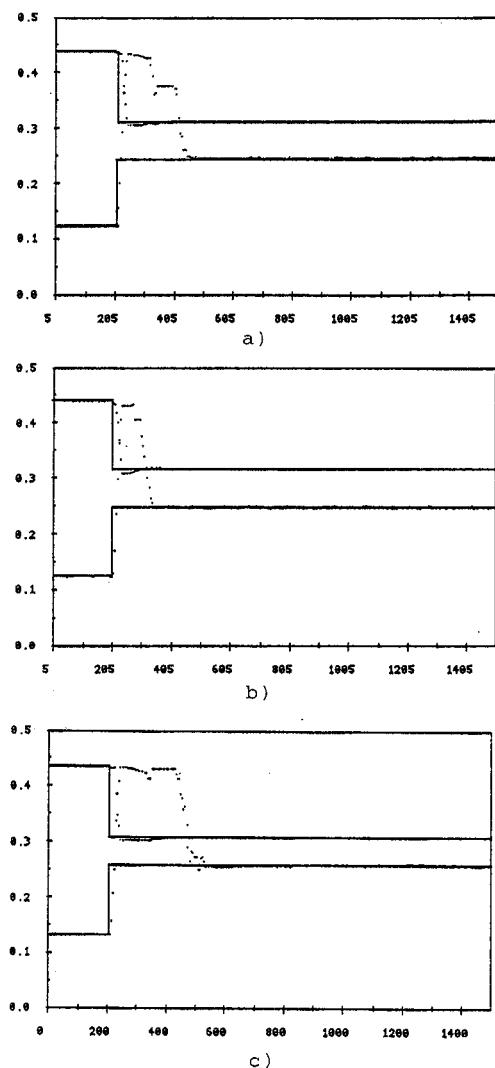


Fig. 2. Estimated frequency evolution for the signal (b) of the a) Burg, b) WRAL and c) ADAB algorithms.

## CONCLUSIONS

From a broad empirical analysis we conclude that:

- The algorithms with data dependent parameters outperform the constant parameter one in all of the tested non-stationary cases.
- The WRAL algorithm presents the highest tracking behaviour when happens a sharp step frequency. The good  $\beta_p(n)$  estimation is the basis of this optimum performance.
- The AEW algorithm behaves very well in the presence of a crossing linear frequency variation. It is due to the low line splitting effect associated to the inherent "stable" tendency.
- The WRAL algorithm present a line splitting effect due to the inherent "unstable" tendency.
- The ADAB algorithm does not outperform the rest of the algorithms in any of the studied cases.

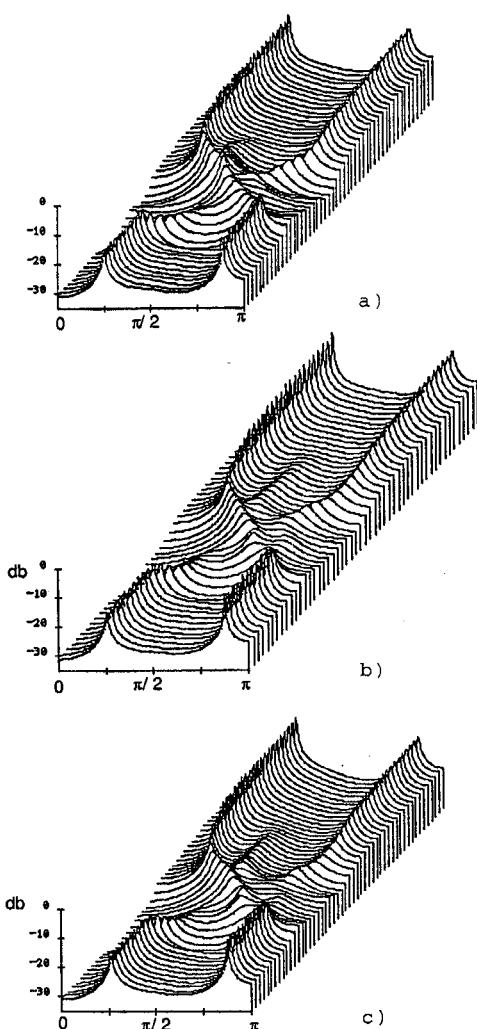


Fig. 3. Spectra evolution for the signal (c) of the a) WRAL, b) AEW and c) ADAB algorithms.

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