Author's Reply

It is gratifying to learn of the work of Vant et al. [1], of which I was previously unaware. I had myself also originally conceived of the nonseparable two-dimensional transform domain approach to synthetic aperture radar (SAR) digital phase history processing in 1977, while at the Jet Propulsion Laboratory. At that time I proposed the approach to C. Wu, D. Held, and R. Goldstein, in relation to SEASAT and other SAR digital phase history processing applications. It was not until 1986 that I eventually was able to build a nonseparable transform domain processor. I hope I did not convey the impression that our implementation was the first; I simply had not found any references to an earlier such processor in the literature. For this reason, the abstract says that our implementation was "perhaps the first." Also, a key idea in my paper is that, regardless of when or where the nonseparable transform domain implementation first appeared, its utility relative to that of the older traditional approaches needs now to be reexamined. Thus, in [2, sect. VI], I state "We have reexamined the nonseparable approach to digital SAR strip mode processing, and conclude that it may offer a very attractive digital processing alternative for current and future SAR missions, which tend to emphasize image quality, control simplicity, flexibility, and use of the latest technology.... While the older separable approaches should also benefit from recent technology advances, the template correlation approach appears to benefit most of all, because it needs only standard signal processing chips." The key idea (of the recently improved relative utility of the nonseparable transform domain approach) is in fact the basis for the title of the paper, "A New Look at Nonseparable Synthetic Aperture Radar Processing."

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Analysis of Some Modified Ordered Statistic CFAR: OSGO and OSSO CFAR

It is necessary for automatic detection radars to be adaptive to variations in background clutter in order to maintain a constant false alarm rate (CFAR). A CFAR based on an ordered statistic technique (OS CFAR) has some advantages over the cell-averaging technique (CA CFAR), especially in clutter edges or multiple target environments; unfortunately the large processing time required by this technique limits its use. We present two new OS CFARs that require only half the processing time. One is an ordered statistic greatest of CFAR (OSGO), while the other is an ordered statistic smallest of CFAR (OSSO). The OSGO CFAR has the advantages of the OS CFAR with only a negligible increment to the CFAR loss.

I. INTRODUCTION

The behavior of cell-averaging (CA) ordered statistic (OS) constant false alarm rate (CFARs) on clutter edges and in multiple target environments have been analyzed by M. Weiss [1] and H. Rohling [2], respectively. The advantages provided by using OS CFARs in these kinds of situations are well known [2], although they require a longer processing time to obtain a representative clutter sample than a CA CFAR.

Employing two specialized processors simultaneously, one for each set of neighboring cells, it is possible to reduce by half the CA CFAR processing time, possible to reduce by half the CA CFAR processing time, without altering the estimation of the clutter statistics average, due to the associative property of the addition. On the other hand, for the OS CFAR, if the previous and following set of cells

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we can find a new random variable $Z$

$$Z = \max(Y_{k1}, Y_{k2})$$

(2)

with a pdf given by [5]

$$f_z(z) = 2f_y(y)F_y(y)_{\mu z}$$

(3)

$$f_z(z) = 2k^2 \left( \frac{M/2}{k} \right)^2 \left( 1 - e^{-\frac{z}{\mu}} \right)^{k-1} \left( 1 - e^{-\frac{z}{\mu}} \right)^{M/2-k}$$

$$\times \sum_{i=0}^{M/2-k} \left( \frac{M/2-k-i}{i} \right)^2 \left( \frac{z}{\mu} \right)^{k-1} \left( 1 - e^{-\frac{z}{\mu}} \right)^{M/2-i}.$$  

(4)

According to this, the probability of false alarm, $P_a$ becomes

$$P_a = \int_0^\infty \Pr(Y_0 \geq T \cdot Z)f_z(z)dz$$  

(5)

$$P_a \text{OSGO} = \int_0^{\infty} \Pr(Y_0 \geq T \cdot Z)f_z(z)dz$$

(6)

$$P_a \text{OSGO} = 2k^2 \left( \frac{M/2}{k} \right)^{2M/2-k} \left( \frac{M/2-k}{j} \right) \left( \frac{M-j}{i} \right) \times \Gamma(M-j-1)\Gamma(T+1) \Gamma(M-j-i+1)$$

(7)

This expression, where $\Gamma(\cdot)$ is the Gamma function [6], and $M$ th number of CFAR cells, proves that this algorithm is a CFAR procedure, because the false alarm probability depends only on the scale factor or threshold level $T$ and not on the value inferred as representative of the interference level.

Solving (7) for a given $P_a$ yields the threshold level $T$ for an OSGO CFAR as a function of $k$, the cell number taken as representative of the previous and following sets of interference estimating cells.

For an OSSO CFAR the algorithm takes in this case the smallest value of the two representative cells.

$$P_a \text{OSSO} = \frac{2k^2 \left( \frac{M/2}{k} \right)^{2M/2-k} \left( \frac{M/2-k}{j} \right) \left( \frac{M-j}{i} \right) \times \Gamma(M-j-1)\Gamma(T+1) \Gamma(M-j-i+1)}{\Gamma(M-j-i+1)}.$$
TABLE I
Scaling Factor $T$ For OS, OSGO, and OSSO CFAR. $P_{fa} = 10^{-8}$, $M = 32$ (Square Law Detector).

<table>
<thead>
<tr>
<th>$k$</th>
<th>$1 \leq k \leq M$</th>
<th>$1 \leq k \leq M/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OS CFAR</td>
<td>OSGO CFAR</td>
</tr>
<tr>
<td>32</td>
<td>5.440406</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>6.579738</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>7.609999</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>8.615433</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>9.664402</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>10.7159</td>
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<td>26</td>
<td>11.85498</td>
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<tr>
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<td>7</td>
<td>317.6604</td>
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</tr>
<tr>
<td>6</td>
<td>50.07664</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>71.00034</td>
<td></td>
</tr>
</tbody>
</table>

The same procedure used for the OSGO CFAR now gives

$$P_{fa}^{OSGO} = 2k \frac{M/2}{k} \frac{\Gamma(k)\Gamma(T + M/2 - k + 1)}{\Gamma(T + M/2 + 1)} - k \frac{M/2}{k} \sum_{j=0}^{M/2-k} \frac{M/2-k}{j} \frac{M/2-k}{i} \times \frac{(-1)^{M/2-j-1} \Gamma(M-j-1)\Gamma(T+1)}{\Gamma(M-j-i+T+1)}.$$

This expression has the same properties as (7), giving the threshold level $T$ versus order $k$, for a given $P_{fa}$. Tables I and II show representative values of $T$ for $M = 32$ and $P_{fa}$ of $10^{-8}$ and $10^{-6}$ for the OS, OSGO, and OSSO CFARs.

III. CHOICE OF THE OPTIMUM $k$TH VALUE

Since constant false alarm is achieved, having another degree of freedom leads to choose $k$ in such a way that maximizes the probability of detection ($P_d$).

To find this optimum value for $k$, the ADT criteria was used [2]. Figs. 2 and 3 show a minimum detection loss for $k = (5/6)M$ for all three OS, OSGO, and OSSO CFARs. Referring to ADT for CA CFAR, $M = 32$, $P_{fa} = 10^{-8}$. 

Fig. 2. Average decision threshold for OS, OSGO, and OSSO CFARs. Referenced to ADT for CA CFAR. $M = 32$, $P_{fa} = 10^{-8}$.
CFAR:

\[
P_d = 2k^2 \frac{M/2}{k} \sum_{j=0}^{M/2-k} \sum_{i=0}^{j} \left( \frac{M/2-k}{j} \right) \left( \frac{M/2-k}{i} \right) \left( \frac{-1)^{M-2k-i-j-i}}{M/2-i} \Gamma(M-j-i) \Gamma(T/(1+S)+1) \right)
\]

\[
P_d = 2k \left( \frac{M/2}{k} \right) \left( \frac{1}{\Gamma(T/(1+S)+M/2+1)} \Gamma(T/(1+S)+M/2-k+1) \right)
\]

\[
- k \left( \frac{M/2}{k} \right) \sum_{j=0}^{M/2-k} \sum_{i=0}^{j} \left( \frac{M/2-k}{j} \right) \left( \frac{M/2-k}{i} \right) \left( \frac{-1)^{M-2k-i-j-i}}{M/2-i} \Gamma(M-j-i) \Gamma(T/(1+S)+1) \right)
\]

where \( S \) is the signal-to-noise ratio (SNR) at the CFAR input.

The above expressions have been compared with those corresponding to CA, CAGO, OS, and the Neyman Pearson optimum detector. The results are presented in Figs. 4 and 5. The CFAR loss is shown for \( M = 32, P_b = 10^{-6} \) and \( P_b = 10^{-8} \).

It can be seen that the OSGO and OS CFARs have almost equal loss factors.

IV. TEST PROTOCOL

CFAR procedures were originally developed using a statistical model of uniform background noise, however this is not representative of real situations. It is impossible to describe all radar working conditions by a single model. We have chosen a model with clutter clouds and stationary targets in different critical cases. We have compared uniform clutter, clutter edges, and multiple targets in six different CFAR procedures.

A uniform clutter model describes the classical situation with stationary noise in the reference window. In such a model there are two interesting cases: a target over uniform background noise in the reference cell, and uniform background noise over the entire reference window. In both situations it is assumed that the cells of the reference window are independent and register the same statistics.

Clutter edges are used to describe transition areas between regions with very different noise characteristics.

Multiple target situations occur occasionally in radar signal processing when two or more targets are at a very similar range. The consequent masking of one target by the others is called suppression.

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Fig. 3. Average decision threshold for OS, OSGO, and OSSO CFARs. Referred to ADT for CA CFAR. \( M = 32, P_b = 10^{-8} \).

Fig. 4. Probability of detection for optimal detector, CA, OS, OSGO, and OSSO CFAR processors with Rayleigh statistics target. \( M = 32, P_b = 10^{-6} \). Selected values \( k \) for the OS CFAR processors are optimums in ADT sense (27, 13, 13).

Fig. 5. Probability of detection for optimal detector, CA, OS, OSGO, and OSSO CFAR processors with Rayleigh statistics target. \( M = 32, P_b = 10^{-8} \). Selected values \( k \) for OS CFAR processors are optimums in ADT sense (27, 13, 13).
Fig. 6. Behavior of CA CFAR ($M = 32$, $P_{fa} = 10^{-6}$) in 256 resolution cells with statistically independent clutter amplitudes and 4 Marcum targets.

Fig. 8. Behavior of CO CFAR ($M = 32$, $P_{fa} = 10^{-6}$) in 256 resolution cells with statistically independent exponential clutter amplitudes, as test protocol that combines 2 clutter edges and 4 Marcum targets.

Fig. 7. Behavior of CAGO CFAR ($M = 32$, $P_{fa} = 10^{-6}$) in 256 resolution cells with statistically independent exponential clutter amplitudes, as test protocol that combines 2 clutter edges and 4 Marcum targets.

Fig. 9. Behavior of OS CFAR ($M = 32$, $P_{fa} = 10^{-6}$) in 256 resolution cells with statistically independent exponential clutter amplitudes, as test protocol that combines 2 clutter edges and 4 Marcum targets.

We have studied a test system trying to unify these situations. Over a total of 256 cells we arrange a clutter edge of 30 dB with respect to background noise extending from the 30th to the 190th cell. Over this clutter cloud appear 3 targets with SNR values of 19, 54, and 19 dB located at cells 100, 105, and 110, respectively. Outside the clutter cloud at position 215 there is a fourth target with a SNR of 22 dB. The test has been run for two $P_{fa}$ ($10^{-6}$ and $10^{-8}$).

In Fig. 6 we see that CA CFAR gives three detection failures due to suppression and edge effects; the same happens with CAGO CFAR (Fig. 7) and the censoring of CFAR [7] (CO CFAR). Figs. 8, Figs. 9 and 10 show that OS CFAR and OSGO CFAR detect
V. CONCLUSIONS

Two new CFAR procedures derived from OS CFAR have been studied. OSGO CFAR has all the advantages of OS CFAR in nonhomogeneous and multiple target situations with a negligible CFAR loss. Also it requires only half of the OS processing time. Concerning the OSSO CFAR, its unique advantage is that it has the same processing speed as the OSGO CFAR, but it has a much higher loss than the OS CFAR and it behaves poorly in nonhomogeneous clutter situations.

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