Level forecasting in the Ebro River during flood episodes using adaptive predictive expert models

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Abstract – In order to minimize the catastrophic effects of floods, it is essential to have good forecasts of the flood dynamics. To carry out these forecasts, commercial computing tools use hydraulic models based on the Saint-Venant equations. Instead of these hydraulic models, this paper proposes the use of input-output adaptive predictive expert (APE) models with properly adjusted parameters. For the initial parameter setting of the APE models used in this paper, four flood episodes occurred in the Ebro river in 2001, 2003, 2007 and 2008 were analysed. In a second stage, the flood episode occurred in February 2009 was forecasted with these adjusted models, and the results were compared to the ones made with the commercial forecasting model MIKE11.

I. INTRODUCTION

The problem of water in Spain is caused by the unequal distribution of precipitation both in space and in time, which reduces its availability. As a result of that irregularity, important effects related to droughts and floods arose and their control boosted the development of important hydraulic infrastructures whose safety is vital to ensure during flood episodes. In view of this situation, and after the tragic floods that took place in the north and east of Spain at the beginning of the eighties of the last century, the need arose to deploy Automatic Hydrological Information Systems (SAIH) in the Hydrographical Confederations, allowing to have real-time hydrologic-hydraulic data and Decision Support Systems (SAD) [1]. With the data provided by the SAIH and with other complementary data (weather forecasts, snow storage, etc) and, mainly using mathematical models, these SAD systems estimate the time evolution of levels and flows along the different watershed rivers.

For the flood forecasting in the Confederación Hidrográfica del Ebro, flood management technicians count with the SAIH and the SAD systems. The essential tool for forecasting is the SAD simulator. It integrates several mathematical models and computer tools which include the NAM and ASTER hydrological models in charge of modelling the terrestrial water cycle, including snow melting and the hydraulic propagation model MIKE11 that models the transmission of the river flows along the basin.

This work is focused on improving the river level forecasting during flood episodes. Its main practical aim is to propose one way for predictions to be carried out autonomously without human intervention.

This paper is organized in eight sections. In section II, a brief review of hydraulic models is presented. Section III describes how the basin under study has been divided for the present work. Section IV describes the adaptive predictive expert models and then Section V shows, as an example, how the initial parameters for one of these models are obtained. In Sections VI and VII
the results from the flood forecasting of February 2009 are displayed, analyzed and compared. Finally, Section VIII presents the conclusions.

II. HYDRAULIC MODELS

Nowadays, for the analysis of the flood propagation effects in rivers, one-dimensional models in gradually varied regime and fixed background are still used, and they are enough for studies where the time evolution is not a factor to take into account and the flow is mainly one-dimensional. In case the process to study is clearly non-permanent, one-dimensional Saint-Venant equations must be used. The MIKE11 model used by SAD simulator implements this approach. This hydraulic model, besides solving one-dimensional Saint-Venant equations, allows different approaches to the phenomenon (permanent regime, diffusive kinematics wave, full equations, etc). The main objection to these hydraulic models is that the obtained result in the forecast may depend largely on the estimation of the parameters, the discretization of the geometry of the watercourse and the selected calculation options, so that, through his choice, the model user has a great responsibility for the validity of the obtained results [2].

This paper proposes, in replacement of the propagation hydraulic models, to use input/output (I/O) models such as the adaptive predictive expert models (APE). These models, once their parameters are adjusted, may serve for the flood forecasting in an autonomous way. There is abundant literature related to the use of input/output type models for flood forecasting. Some of the approaches found to address the problem of forecasting using non-deterministic models are: forecasting on the basis of ARIMA type models [3], forecasting using flood routing with fuzzy neural networks [4], forecasting upon rainfall-flow type models under state space formulations using Kalman filter for optimal state estimation [5], and forecasting with nonlinear models identified through data based methods [6].

III. BASINS AND SUB-BASINS OBJECT OF THIS STUDY

Figure 1 shows a general map of the most significant river gauge stations of the Ebro River Basin SAIH network. Some of them are used as inputs and outputs of different basins and sub-basins in this study. The most important floods that occur in the Ebro river basin often take place between the river gauge station located in Ebro-CASTEJÓN (A002) and the station located in Ebro-Zaragoza (A011). This is why in this work only forecast levels in these two stations will be shown. For this aim the dynamics of the most important river located upstream Ebro-Zaragoza (A011) have been analysed. These rivers have been divided into basins and sub-basins, using for each an appropriate adaptive predictive expert model.

The watershed studied in this work was divided in the following manner:
• Basin 1. It represents a 124 km long section of the Ebro River between the river gauge stations Ebro-Miranda (A001) and Ebro-Menda via (A120). This system has four inputs, stations A001, A165, A074 and A281 and an output station A120. In turn, station A001 is the output of a sub-basin with two entries, A161 and A188.

Figure 2 shows the time evolution of levels during the flood episode in February 2009 registered by the river gauge stations of Basin 1.

• Basin 2. It represents a 74 km long section of the Ebro River between the river gauge stations Ebro-Mendavia (A120) and Ebro-Castejón (A002). This system has five inputs, stations A120, A003, A004, A005 and A253 and one output, station A002. In turn, station A003 is the output of a sub-basin with an input A071, station A004 is the output of another sub-basin with two inputs A069 and A084 and station A005 is the output of a third sub-basin with two inputs A065 and A101.

Figure 3 shows the time history of levels during the flood episode in February 2009 registered by the river gauge stations of Basin 2.

• Basin 3. It represents a 137 km long section of the Ebro River between the river gauge stations Ebro-Castejón (A002) and Ebro-Zaragoza (A011). This system has two inputs, stations A002 and A260 and one output, station A011. In turn, A260 station is the output of a sub-basin with one input, station A290.

Figure 4 shows the time evolution of levels during the flood episode in February 2009 registered by the river gauge stations of Basin 3.
IV. **ADAPTIVE PREDICTIVE EXPERT MODELS**

Each one of the basins and sub-basins described in the previous section can be analysed as a non-linear input-output system, with noise associated to measurements and with non-measurable disturbances affecting the output. This kind of systems can be described with discrete-time difference equations with time-varying parameters. The parameter variation is one way of capturing the nonlinear nature of the system. For the general case of a system with one input and one output we may write:

\[
\gamma(k) = \sum_{i=1}^{n} a_i(k) \cdot \gamma(k-i) + \sum_{i=1}^{m} b_i(k) \cdot u(k-r(k)-i) + \Delta(k)
\]  

(1)

where \(\gamma(k)\) and \(u(k)\) are, respectively, the real measured output and input values at instant \(k\); \(a_i(k)\) and \(b_i(k)\) are the system parameters, generally unknown and time-varying; \(r(k)\) is the system time delay, generally time-varying; and \(\Delta(k)\) represents the joint effect at time \(k\) of the non-measurable disturbances and the measurement noise. This model considers a discretization period \(T\), so that the actual time is \(kT\), where \(k=1,2,3,...\). This equation can be written in the compact form

\[
\gamma(k) = \Theta^T(k) \cdot \Phi(k-d(k)) + \Delta(k)
\]  

(2)

where \(\Theta^T(k)\) is the transposed vector with the system parameters, \(\Phi(k-d(k))\) is the input-output vector and \(d(k)=r(k)+1\) represents the time delay associated to the system input plus the one additional delay due to the time discretization.

Using this compact notation, the following adaptive model can be defined to give an “a priori” output estimation of the output at time \(k\) using the model parameters estimated at previous instant \(k-1\), i.e. \(a_i(k-1)\) and \(b_i(k-1)\):

\[
\hat{\gamma}(k | k-1) = \hat{\Theta}^T(k-1) \cdot \Phi(k-d(k))
\]  

(3)

The parameter estimation is performed by an adaptation mechanism defined later on in this section.

In a similar way, the “a posteriori” output estimation at \(k\) with the estimated adaptive model parameters given by the adaptation mechanism at \(k\) is defined in the form

\[
\hat{\gamma}(k | k) = \hat{\Theta}^T(k) \cdot \Phi(k-d(k))
\]  

(4)
The sub-index \( r \) indicates the dimension used for the estimated parameters vectors and that of the adaptive models, being this dimension \( r \) of order below the real system dimensions \( n \) and \( m \), which are generally high. This difference in the model and real system dimensions suggest the use of a normalization of the variables as follows [7].

Define

\[
\gamma^r(k) = \frac{\gamma(k)}{n(k)}
\]

(6)

\[x(k-d(k)) = \frac{1}{n(k)} \cdot \Phi(k-d(k))
\]

(7)

\[x_r(k-d(k)) = \frac{1}{n(k)} \Phi_r(k-d(k))
\]

(8)

Replacing (6) and (7) into (2), the normalized real system equation is

\[\gamma^r(k) = \Theta^T(k) \cdot x(k-d(k)) + \frac{\Delta(k)}{n(k)}
\]

(9)

Taking into account the reduced dimension of the adaptive model we can write

\[\gamma^r(k) = \Theta_{r}^T(k) \cdot x_r(k-d(k)) + \Theta_{r}^T(k) \cdot x_{r}(k-d(k)) + \frac{\Delta(k)}{n(k)}
\]

(10)

where the parameters and input-output vectors have been split into two parts, one with the adaptive model dimension \( r \) and another of dimension \( u \) with the remaining components. You can now interpret jointly the action of the last two terms of the second member of (10) as a normalized disturbance signal acting on the normalised system output as described in [7]:

\[\gamma^r(k) = \Theta_r^T(k) \cdot x_r(k-d(k)) + \Delta^r(k)
\]

(11)

\[\Delta^r(k) = \Theta_r^T(k) \cdot x_r(k-d(k)) + \frac{\Delta(k)}{n(k)}
\]

(12)

This way, the real system and the adaptive model have the same structure.

The adaptive model is extended with an expert rule for the calculation of the system time delay, at each instant \( k \), by means of a linear function in the form

\[r(k) = f(\gamma(k))
\]

(13)

Now the a priori normalized estimation error is defined as follows:

\[e^r(k/k-1) = \gamma^r(k) - \Theta_r(k-1) \cdot x_r(k-d(k))
\]

(14)

With this error, the adaptation mechanism for the calculation of the estimated parameter vector at time \( k \) is defined. This mechanism is given in [7] has the form of a recursive linear filter with a variable gain.
where \( G(k) \) is the variable gain vector whose expression is:

\[
G(k) = \frac{B \cdot x_r(k - d(k))}{1 + x_r^T(k - d(k)) \cdot B \cdot x_r(k - d(k))}
\]

where \( B \) is a positive definite matrix.

With the adapted parameters at instant \( k \), the a posteriori normalized estimation error is defined as:

\[
e^*(k) = \gamma^*(k) - \hat{\Theta}_r(k) \cdot x_r(k - d(k))
\]

The predictive model has the same structure as the adaptive model and is used, at each instant \( k \) after the adaptation, for future forecasting of the system output as follows:

\[
\hat{y}(k + j) = \hat{\Theta}_r^j(k, j) \cdot \Phi_r(k - d(k))
\]

where \( j = 1, 2, ..., d(k) \) represents the forecast horizon at time \( k \), which is generally time variable.

In this model, the parameter vector has the following structure:

\[
\hat{\Theta}_r^j(k, j) = (\hat{A}_i(k, j); \hat{B}_i(k, j)) = (\hat{a}_i(k); \hat{A}_i(j); \hat{b}_i(k); \hat{B}_i(j))
\]

\( \hat{a}_i(k), \hat{b}_i(k) \) are the estimated parameters adapted by the adaptation mechanism at instant \( k \) and \( A_i(j) \) and \( B_i(j) \) are new parameters calculated with for each instant \( j \) within the forecast horizon. These new parameters introduce an expert component to the predictive model. In this work, they are calculated using fuzzy logic.

V. INITIAL PARAMETERS OF THE ADAPTIVE MODELS

In order to obtain the initial parameters of the adaptive models, \( \hat{a}_i \) and \( \hat{b}_i \), and the delay \( r(k) \) of each basin and sub-basin, four flood episodes occurred during the years 2001, 2003, 2007 and 2008 were analysed. As an example, Figure 5 shows the graphs of the time evolution of levels for each of these floods in the river gauge stations of basin 3.

![Figure 5. Levels in A011, A002 and A260 river gauge stations](image)

Following the analysis of these data, the following a priori estimation adaptive model resulted for basin 3:
\[
\hat{\gamma}_{ao11}(k) = \hat{a}(k-1) \cdot \gamma_{ao11}(k-1) + \hat{b}(k-1) \cdot \gamma_{ao22}(k - r_{ao22}(k-1)) + \hat{c}(k-1) \cdot \gamma_{ao26}(k - r_{ao26}(k-1))
\]  

with the initial parameters \( \hat{a}(0) = 0.8 \), \( \hat{b}(0) = 0.13 \) and \( \hat{c}(0) = 0.02 \).

For the two delays of this basin it was obtained:

\( r_{ao22}(k) = 18 \) hours for \( \gamma_{ao11} \leq 1.5 mts \)

\( r_{ao22}(k) = 65 \) hours for \( \gamma_{ao11} \geq 5.5 mts \)

\( r_{ao22}(k) = f(\gamma_{ao11}) \) for \( 1.5 mts \leq \gamma_{ao11} \leq 5.5 mts \). Calculated by cubic interpolation.

\( r_{ao26}(k) = 12 \) hours for \( \gamma_{ao11} \leq 1.5 mts \)

\( r_{ao26}(k) = 28 \) hours for \( \gamma_{ao11} \geq 3.75 mts \)

\( r_{ao26}(k) = f(\gamma_{ao11}) \) for \( 1.5 mts \leq \gamma_{ao11} \leq 3.75 mts \). Calculated by cubic interpolation.

The predictive model resulted in the form:

\[
\hat{\gamma}_{ao11}(k + j) = \hat{A}(k, j) \cdot \hat{\gamma}_{ao11}(k + j - 1) + \hat{B}(k, j) \cdot \gamma_{ao22}(k - r_{ao22}(k) + j - 1) + \hat{C}(k, j) \cdot \gamma_{ao26}(k - r_{ao26}(k) + j - 1)
\]  

where

\[
\hat{\gamma}_{ao11}(k + 1 - 1) = \gamma_{ao11}(k + 1 - 1)
\]  

is the measured output at instant \( k \), and with parameters

\[
\hat{A}(k, j) = \hat{a}(k) \cdot A(j)
\]  

\[
\hat{B}(k, j) = \hat{b}(k) \cdot B(j)
\]  

\[
\hat{C}(k, j) = \hat{c}(k) \cdot C(j)
\]

Figure 6 shows as an example the fuzzy logic structure used for the calculation of parameter \( B(j) \).

![Fuzzy logic structure for B(j)](image)

This fuzzy system has two inputs, with four and two fuzzy sets respectively, which define at each instant \( j \) the flood dynamics by means of the level increases in the river gauge stations of Ebro-
Zaragoza (A011) and Ebro - Castejón (A002), a set with eight rules and an output with four fuzzy sets for the parameter $B(j)$. Figure 7 shows the surface for output $B(j)$.

**VI. VALIDATION OF THE APE MODELS**

In this section, the validity of the adaptive predictive expert models (APE) for flood forecasting is assessed. For this purpose, the effectiveness of the adaptation mechanism and the predictive model accuracy will be analysed separately. All this is done with data from the forecasting of the flood that took place in the Ebro river in February 2009, whose plots have been shown in section III. Level data from all the river gauge stations are hourly data provided by the SAIH system and with them forecasts are launched at every hour $k$ for all the river gauge stations that are output of the basins and sub-basins described in section III. In this paper, only the forecast results for Ebro-Castejón (A002) and Ebro-Zaragoza (A011) are displayed, which are the most important two river gauge stations in this study.

The effectiveness of the adaptation mechanism can be evaluated with both the a priori and a posteriori estimation adaptive models and with their respective estimation errors. Figure 8 shows the a posteriori estimation and the error for the whole flood episode at the station Ebro-Castejón (A002).

The mean of the absolute values of the a posteriori errors for the whole flood episode is

$$
e_{mad}^{a}(k)_{A002} = \frac{1}{k} \sum_{k=1}^{k=\text{length}(A002)} |e^{a}(k/k)_{A002}| = 0.009\text{mts}$$  \hspace{1cm} (26)

The percentage of validation of the a posteriori estimation with regard to the actual values measured at each $k$, calculated according to [8] is
\[ val_{A002}(\%) = 1 - \frac{\|\hat{y}_{A002}(k) - y_{A002}^n(k)\|}{\|y_{A002}^n(k) - \text{med}(y_{A002}^n(k))\|} \cdot 100 = 90.92\% \quad (27) \]

Figure 9 shows for the whole flood episode the a posteriori estimation and error at the station Ebro-Zaragoza (A011).

\[ e_{\text{med}}^n(k/k)_{A011} = \frac{\sum_{k=1}^{k=\text{length}(A011)} |e^n(k/k)_{A011}|}{k} = 0.003\text{mts} \quad (28) \]

The percentage of validation of the a posteriori estimation with regard to the actual values measured at each \( k \), calculated according to [8] is

\[ val_{A011}(\%) = 1 - \frac{\|\hat{y}_{A011}(k) - y_{A011}^n(k)\|}{\|y_{A011}^n(k) - \text{med}(y_{A011}^n(k))\|} \cdot 100 = 94.92\% \quad (29) \]

These results revealed the excellent adaptation capability of the mechanism defined in (15) and (16).

To analyze the accuracy of the predictive model given in (18) and (19), two criteria are used: (1) the first one analyses how well the maximum flood levels are forecast; (2) the second one analyses, for all forecasts launched at time \( k \), the mean values of the absolute prediction errors at several instants \( j \) of the forecast horizon.

Figure 10 shows the peak level forecast carried out for Ebro - Castejón (A002) at \( k = 588 \), the measured real level values and the errors for a forecast horizon \( j \) of 24 hours.
Figure 10. Forecast level for Ebro-Castejón (A002) at $k=588$.

The graph shows two main issues: (1) how well the flood peak level in A002 is forecast 22 hours in advance at $k = 588$; and (2) the prediction errors are less than 20 cm during the whole prediction forecast.

Figure 11 shows the forecast peak level for Ebro-Zaragoza (A011) at $k = 605$.

The flood peak level in A011 is forecast 42 hours in advance at $k = 605$ with an error lower than 5 cm. It is also observed that, during the forecast horizon, prediction errors are lower than 10 cm.

In order to have a criterion for assessing the predictive model for the whole flood episode, the means of the prediction error absolute values made at several instants $j$ of the forecast horizons are calculated for the $k$ predictions. The results are shown in TABLE I.

<table>
<thead>
<tr>
<th></th>
<th>6 hours</th>
<th>12 hours</th>
<th>18 hours</th>
<th>24 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>A002</td>
<td>0.047</td>
<td>0.082</td>
<td>0.123</td>
<td>0.194</td>
</tr>
<tr>
<td>A011</td>
<td>0.024</td>
<td>0.044</td>
<td>0.057</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Figure 12 displays, for the station Ebro-Zaragoza (A011), the graph with the value of the predictions for $j = 18$ at each time $k$, the real value of the measured level at $k + 18$ and the prediction error.
VII. COMPARATIVE OF PREDICTIONS WITH SAD AND APE

In this section some graphs are displayed in order to compare the forecasts obtained with adaptive predictive expert models (APE) and with the forecasts made by the SAD system for the flood episode of February 2009. It should be pointed out that the SAD system does not forecast every hour but system technicians perform forecasts usually every 24 hours. This is why SAD forecasts shown here are those really available. The main feature to be demanded from a flood forecasting system is the capability to forecast the flood peak levels, both in magnitude and in their time evolution, as far in advance as possible. During the flood episode of February 2009, two flood peaks occurred reaching maximum levels in the stations A002 and A011. Below the forecasts with both systems are presented. Figure 13 shows forecasts made by the SAD for the first flood peak level at the station Ebro-Castejón (A002).

The graph shows the forecast made on the 27/01/2009 at 15 pm. It contains 30 hours of forecasts and 17 hours of level real data. This prediction of the SAD corresponds to the prediction at $k = 206$ of the adaptive predictive expert model shown in Figure 14. This graph shows 30 hours of predictions, 30 hours of level real data and prediction errors.
Figure 14. Prediction models (APE) for Ebro-Castejón (A002) in $k=206$ and prediction errors.

The graph for the first flood peak level in Ebro-Zaragoza (A011) with the SAD system is shown in Figure 15. It was carried out on the 29/01/2009 at 11 am. It contains 32 hours of predictions and 28 hours of level real data.

Figure 15. SAD prediction for Ebro-Zaragoza (A011).

This prediction of SAD corresponds to the prediction at $k = 253$ of the adaptive predictive expert models shown in Figure 16. This graph shows 32 hours of predictions, 32 hours of level real data and the prediction errors.

Figure 16. Prediction models (APE) for Ebro-Zaragoza (A011) in $k=253$ and prediction errors.
Figure 17 shows the SAD forecasts for the second flood peak level at station Ebro-Castejón (A002). It was calculated on the 12/02/2009 at 5 am. It contains 32 hours of prediction and 27 hours of level real data.

![Figure 17. SAD forecast for Ebro-Castejón (A002).](image)

The corresponding prediction with adaptive predictive expert models is at \( k = 580 \) and it is shown in Figure 18.

![Figure 18. Prediction models (APE) for Ebro-Castejón (A002) at \( k=580 \) and prediction errors.](image)

This graph shows 27 hours of predictions, 27 hours of level real data and the prediction errors.

In Figure 19, SAD forecasts are shown for the second flood peak at station Ebro-Zaragoza (A011). It was calculated on the 13/02/2009 at 9 am. It contains 32 hours of predictions and 32 hours of level real data.

![Figure 19. SAD forecast for Ebro-Zaragoza (A011).](image)
The corresponding prediction with APE models is at $k = 611$ and it is shown in Figure 20.

![Figure 20. Prediction models (APE) for Ebro-Zaragoza (A011) at $k=611$ and prediction errors.](image)

This graph shows 32 hours of predictions, 32 hours of level real data and the prediction errors.

One way to objectively evaluate the predictions of the flood peak levels with both systems is to calculate the mean of the absolute values of the errors made by both systems in the displayed predictions of Figures 12 to 19. TABLE II shows these means of the prediction errors for each prediction horizon $j$.

<table>
<thead>
<tr>
<th></th>
<th>A002 $k=206$</th>
<th>A011 $k=253$</th>
<th>A002 $k=580$</th>
<th>A011 $k=611$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j=17$ hours</td>
<td>0.332</td>
<td>0.096</td>
<td>0.423</td>
<td>0.072</td>
</tr>
<tr>
<td>$j=28$ hours</td>
<td>0.043</td>
<td>0.074</td>
<td>0.082</td>
<td>0.061</td>
</tr>
</tbody>
</table>

### VIII. CONCLUSIONS

The results shown in this work on the forecast of the flood episode of February 2009 using adaptive predictive expert (APE) models confirm the possibility of using these input-output type models for the forecast of flood propagation in replacement of the classic hydraulic models based on single-dimensional Saint-Venant equations. Moreover, this forecast methodology provides the following added benefits:

- The APE models, once the initial parameters are adjusted, do not need any additional further parameterization done by the flood manager technician.
- With these models, due to the fact that calculations are carried out only with levels and not with flows, predictions are not affected by changes occurring in river gauge level-flow tables of each river gauge station as a consequence of an important flood that could alter the geometry of the river bed and therefore modifying river gauge tables.
- During floods of major intensity, river overflows and floods happen. This flood flow coming out of the river between two river gauge stations, which is always of unknown magnitude, is source of errors in the predictions made by hydraulic models-based conventional systems. In these situations of floods, adaptive predictive expert will be able to identify the new flood dynamics caused by flooding and will give more precise forecasts.
REFERENCES


