An Approach to Belief in Strong Kleene Logic

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ABSTRACT

Three-valued Strong Kleene Logic [20], provides an adequate framework to deal with belief change. In this paper, *undefined* truth-value is used to denote opinionless information. Through an informative refinement, such information could become to be *true* or *false* in a step by step way. Our approach is based upon model frames [10], that are sets of informatively ordered three-valued interpretations. A relation of compatibility among frames, being a partial informative order, is defined. Belief change is accomplished by using the compatibility relation together with operations of concatenation and unification over model frames.

Underline inuitions of our belief definition are the same to those using the possible worlds semantics [18]. Syntachtical characterization for ideal reasoning and logical omniscience are given, together with the one of limited reasoning. The first two topics, undesirable when modeling *real* agents of knowledge and belief, are avoided in the proposed approach.

1 Introduction

1.1 Possible Worlds Semantics and Logical Omniscience

For knowledge and belief, the semantics of possible worlds [22] has been attractive for the intuitive way in which description of present and possible *description of the state of the things* [18] can be done. Kripke's possible worlds are sets of interpreted formulas in classical semantics. The worlds are related through an algebraical relation that, usually, satisfy either reflexivity, symmetry or transitivity. Given a world w_0, considered as the actual or present, world w is possible for w_0 whenever w_0 and w satisfy the so called possibility relation. However this relation is neihter constructive nor *causal*, and does not capture in a sutiable way, the underling intuition to be *possible* or *conceivable* for the actual world. We think this fact the fundamental weakness when applying the semantics of possible worlds to knowledge and belief: the intuitive notion of accesibility to any *description of possible state of affairs* from any world, becomes awkwarded for a nonintuitive accesibility relation.

Logics of knowledge and belief that have used the semantics of possible worlds [17, 18, 25], consider knowledge in present world w_0 as the *true* formulas in all the possible worlds of w_0. If a formula φ is knowledge and ψ is a logical consequence of φ, then ψ is *true* in all the worlds in which φ is. Thus, all logical consequences of the initial knowledge is knowledge as well. This topic, known as *logical omniscience*, is an unintuitive lateral consequence in applying the kripkean semantics of possible worlds to model knowledge and belief [17, 18, 25, 27]. In the other hand, the unrestricted application of *modus ponens* used in this approach, conveys the so called *ideal reasoning,* improper to model *real* agents of knowledge or belief. Computationally, any system that deals with logical omniscients and ideal reasoners should have unlimited time and computational resources [27]. Obviusly, this is imposible to provide.

1.2 Possible Worlds: a New Proposal

The basic idea of our approach is inspired by the metaphor of an agent who *conceives* (or imagines) worlds which are possible in (or compatible with) the existing world, by means of a well-defined iterative process. This requires the existence of a *causal* relation between the agent's existing world and the worlds that he is capable of conceiving of on the basis of that world. Furthermore, it requires that the set of possible worlds are not static, but they are the result of a dynamic process of creation and, therefore, dependent on the iterations which occur. In practice, these iterations will depend on the resources available.

In accordance with the preceeding, knowledge is treated as a concept that is relative to the worlds which have been generated. This idea seems quite natural and adequately captures the intuition that knowledge is not static: it is being continually modified on the basis of experience. In relation to the reference world, the agent's belief in the succesive iterations is constituted by those things which have been conceived of up to that iterations.

For example, an agent can construct worlds by choosing from amongst distinct options which are presented to him. He can choose between getting married (P) and not getting married ($\neg P$). If he gets married, he must choose to have children (Q) or not to have them ($\neg Q$). If he does not get married, he has the option of working (R) or not ($\neg R$), etc. In addition, he can delay its decision about having childrens or not, remaining without opinion about it until the right moment.

In a general approach, each choice creates new possible worlds from the world in which the choice is made. This procedure for constructing worlds is highly intuitive and constitutes quite a natural conceptual model of how we, as human beings, go about defining our lives by *choosing from amongst possible options,* whenever it is needed and delaying decisions if possible. This idea is in accordance with that expressed in [22], in which it is observed that statements in possible worlds must be possible in the present world or consistent with the statements of that world.

The above conceptual model can be formalised considering possible worlds as set of formulas associated with three-valued interpretations in which *undefined* truth-value is appened to classical *true* and *false* ones. *True* and *false* formulas are considered to provide *more information* than *undefined* ones. Relation among possible worlds is through an *informative refinement* such that world w is possible (compatible) from (for) w_0, if the set of more informed formulas in w_0 is subset of the ones in w. A suited motivation to informative order in modal logics has been done for Belnap [4]. He introduced it over the four-valued logic of relevance. After that, use of informative lattices to deal with knowledge representation has had an incrising appel *e. g.* [2, 10, 11, 15].

Whenever a world w is coherently more informed than a world w_0, seems to be intuitive to say that w is be possible (compatible) for (with) w_0. Using three-valued interpretations, knowledge representation can be realized more in accordance with the often real circumstance of incomplete (partial, unknown) actual information and limited resources. Whenever all the interpretation have *true* and *false* truth-values, it could be said that there is complete knowledge. But this is not the common situation. Often times we have partial knowledge describing limited situations.

In this paper we explain the case of a single agent with a modal belief (doxastic) language $\mathcal{BL}$ based on three-valued Strong Kleene Logic (SKL). The proposed procedure for generate possible worlds is iterative. Proof method is based upon analytic tableaux [13, 3, 8]. Flexibility of that method allows to define syntactical rules that captures adequately semantic and epistemic definitions [2, 10, 21, 28, 29].

This paper is organized as follows. In section 2, syntactical characterizations for perfect and limited reasoners are proposed. After that, three-valued SKL is introduced. In section 3 an informative partial order over model frames is introduced. From SKL and the informative order we outline our epistemic definitions in section 4, developing a (constructive-like) logic of belief. In section 5 correspondence between epistemic model frames and analytic tableaux is developed. It shows that tableaux proof method can be applied in our intend. In section 6 is continued our doxastic semantic. In the last section a brief comparation between AGM paradigm and *Update Theory* with respect to our approach is developed. Finally we give our concluding remarks.

2

2.1 Ideal reasoning and logical omniscience

An ideal or *perfect* reasoner is such that entails every conclusions from information and inference rules that he posses. Ideal reasoning compelle to have exhaustive and faultless reasoning capability. An agent knowing all the consequences of its knowledge must have inference mechanisms that apply in an exhaustive manner, without any fail and without time or space limitations. Natural definition of these ideas is the following (given in [27], Def. 4.1)

Definition (Ideal Reasoner)

Conceptualizing an agent β as a pair $(\Delta,\ \rho)$, where Δ is a set of statements and ρ is a set of inference rules, agent β is an **ideal reasoner** with respect to ρ if the following is satisfied:

$$\text{If } \ \Delta \vdash_{\rho} \phi \ \text{ then } \ K_{\beta}\phi,$$

where $K_{\beta}(\phi)$ is read as β knows ϕ.

In this case we say also that β is omniscient relative to ρ. When ρ is a set of complete inference rules of CP_0 (CP_1), the agent is an ideal reasoner in classical logic. In that case it is also logical omniscient in that logic. However, ideal reasoning and logical omniscience are not equivalent concepts as will be shown below.

In the previous definition, inference rules of the agent can be part of CP_1 rules, or inference rules of any kind as those of Konolige's agents [21]. That definition captures the pure meaning of ideal reasoning, pointing out that is not an absolute concept but relative to a given context, being not necessarily the classical logic. Moreover, it constitutes a general syntactical characterization of the concept, independent to the semantics of possible worlds.

Under last syntachtical characterization of perfect reasoner, it is easy to show that Levesque's agents [25] are ideal reasoners in relevance logic with its explicit belief. By other hand, Konolige's agents [21] could also be consider perfect reasoners in its particular logic [27]. (For details see there.) Now we propose a definition for the related but different concept of logic omniscience.

We can think in of realistic rational agent as that that has not obtained all conclusions from the information it posseses. It could be due because he has not *thought* about it and then he has not applied enough inference mechanisms over its information. This limitation could be considered as a kind of material limitation. It is obvious that no human being is capable to obtain all the conclusions that could be get out from possesing information. We are not perfect reasoners but very limited. At most, we can obtain conclusions from information that is focalised at a given moment. But moreover, computational agents are not always perfect reasoners. Usually they have time or resources limitations to get out conclusions.

Thus, a limited reasoner corresponds to the case that even having suitable information to obtain conclusions does not entail them. Such limitations are translated in no application of inference rules where the agent could be done. Thus, a natural way to allow a limited reasoner is in the following manner:

- restricting (the number of) performances (applications) of inference rules

If $C_\rho(\Delta) = \{\alpha \mid \Delta \vdash_\rho \alpha\}$ is the deductive closure of the ideal reasoner β, a **limited reasoner** is such that does not entail all conclusions in C_ρ. Let B_ρ be the set of conclusions obtained by the limited reasoner β'. Then

$$B_\rho(\Delta) \subset C_\rho(\Delta).$$

Example

A limited reasoner using CP_1, should apply a finite number of times the *modus ponens,* instead the infinite applications required to get out all CP_1 theorems. At this manner is not an exhaustive reasoner agent (see also the example and further details in 4.2).

Definition

An agent is **logically omniscient** with respect a formal system if and only if, it knows all the system thorems.

Thus an ideal reasoner is logically omnisciente, but not always the converse. A particular case of omniscient agents are ideal reasoners in CP_1. But there are agents that knows everything without performing any kind of reasoning. It corresponds with agents having an initial knowledge base containing all logical theorems *i. e.* with $\Delta = C_\rho$, indepently of the size of its set of inference rules. In this case ρ can be the empty set $\emptyset$.

In the present work, we deal with limited reasoners over its own beliefs (see subsection 4.2). Our formal approach is an extension of that defined by Doherty which is based upon Kleen's three-valued logic that we introduce now.

2.2 Strong Kleene Logic

In accordance with definitions of Langholm [24] for the Strong Kleene Logic, propositional language $\mathcal{L}$ is defined from a finite set of sentences Σ and the primitive connectives negation $\neg$ and disjunction $\vee$. From those connectives are defined the conjunction $\wedge$ and implication $\rightarrow$ ones in the classical way. Let $\mathcal{F}$ be the set of $\mathcal{L}(\Sigma)$-formulas. Semantical definitions for disjunction and negation in the Strong Kleene Logic are given in Table 1 and 2, respectively.

Let *true, false* and *undefined* be truth-values denoted with t, f and u.

Table 1

φ	ψ	$\varphi \vee \psi$
t	t	t
t	f	t
t	u	t
f	t	t
f	f	f
f	u	u
u	t	t
u	f	u
u	u	u

Table 2

φ	$\neg\varphi$
t	f
u	u
f	t

Definition:
A three-valued interpretation I, is a valuation function from $\mathcal{F}$ to $\{t, f, u\}$ in accordance with logical connectives definition. I is equivalent to $I^t \cup I^f \cup I^u$ in such a way that I^t is the set of *true* formulas, I^f the set of *false* formulas and I^u the set of undefined formulas. When I^u is the empty set we say that I is complete respect *true* and *false* truth-values. Let $\mathcal{I}(\mathcal{F})$ be the set of three-valued interpretations of formulas in $\mathcal{F}$.

3 Information Order and Belief

3.1 Informative Order

In this section we concern with the approach that use an informative order to deal with knowledge and belief (see *e.g* [2, 4, 10, 11, 15]), in such a way that although using semantical truth aspect of belief statements, is considered that cannot be reduced to it. Herein an informative order over interpretations of sets of beliefs is given. Underline objective is to deal with belief change in a step by step way. The intuitive point is that seems natural to consider the following:

- Having *true, false* and *undefined* statemens, *true* or *false* ones provide more information than *undefined* statements.

We propose that when undefined information turns up defined, as true or false, believes could increase.

It is well known, however, that change of belief includes also elimination, revision and updating of current beliefs. Semminal approach to these topics is the AGM paradigm [1, 14] in which in addition to expansion of belief, contraction and revision operations for it are given. In the other hand, *Belief Updating* theory [19] could be consider the classical approach to belief updating. Our aim is to show in forthcoming works that our approach provides to deal with such kind of operations in the way as we shall appoint in section 7. Now informative partial order defined for Doherty in [10] is introduce.

Definition
The **Degree of information** among t, f and u truth-values, is defined as follows: $u \leq_i t$, $u \leq_i f$ and t, f are not comparable. It constitutes a partial order over $\{t,\ f,\ u\}$ set.

Let Υ be a subset of $\mathcal{I}(\mathcal{F})$. The informative partial order $\leq_i$ is introduced over Υ in the following way: given an $I, I' \in \Upsilon$, $I \preceq_i I'$ is satisfied if and only if $I^t \subseteq I'^t$, $I^f \subseteq I'^f$ and $I^u \supseteq I'^u$. The order $\preceq_i$ is a partial one on Υ.

Intuition of that order is that given two interpretations of a set, interpretation with more *undefined* formulas is less informed than the one having more *true* or *false* formulas. Then, for $\mathcal{P} \subseteq \mathcal{F}$ such that $I(P) \preceq_i I'(P)$ for every $P \in \mathcal{P}$, I' is more informed than I with respect to $\mathcal{P}$, denoted $I \preceq_i^{\mathcal{P}} I'$. In this case is said that I' is an **informative refinement** of I with respect to $\mathcal{P}$.

Example
Let $\{P\}$ be the only sentence of language $\mathcal{L}$; $I_0 = (u)$, $I_1 = (t)$, $I_2 = (f)$ be the interpretations of $\mathcal{F}$. Then I_1, I_2 are informative refinements of I_0 and thus more informed than it.

The following formal structure providing a suitable framework for our epistemic approach is an extension of model frame defined by Doherty [10]. Modification herein introduced is that informative order in is explicitated. It is due because we are interested in a family of modal frames such that they are related by the same informative order but in a no necessarily transitive way (see 4). The underling objective is to capture the dynamic manner in which beliefs change. We consider that it deppends of *worlds being explored.* Gradual exploration is allowed by using the informative order $\preceq_i$ among model frames.

Definition (Model Frame)
A model frame is an ordered 3-tupla $\mathcal{M} = \langle \Upsilon, I_0, \preceq_i \rangle$, where $\Upsilon \subseteq \mathcal{I}(\mathcal{F})$ is a set of interpretations and $I_0 \in \Upsilon$ is called the current (actual or present) interpretation. For any $I' \in \Upsilon$, we assume that $I_0 \preceq_i I'$. Thus, any interpretation in $\mathcal{M}$ is at least as informed as I_0.

Notice that as interpretation I_0 remains fix, it constitutes an invariant and characterizes the model frame. Following definition will be used in the following comments and in belief definition.

Definition
An **interpretation** J is **maximal** in $\mathcal{M}$ if and only if there is no J' in $\mathcal{M}$ such that $J \prec_i J'$.

A model frame can be seen as a set of possible worlds that provides *a description of the state of the things together with its possibles extensions* (see fig 1). Whenever worlds corresponding with the (informatively) maximal interpretation in the model frame contain undefined statements, extension of belief could be performed as it should show below (see 4.1).

In the other hand, informative order $\preceq_i$ can be consider an accesibility relation among interpretations in Υ as follows: I' is accesible (compatible) for I whenever I' is an informative

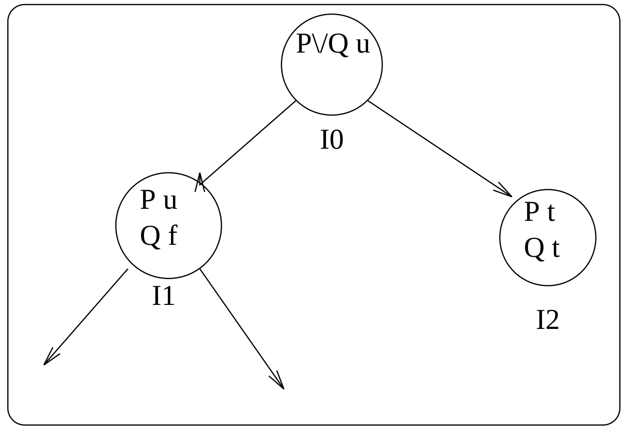

Figure 1: Model Frame

refinement of I. Moreover, if for every $I \in \Upsilon$, in a model frame $\mathcal{M}$, $I(\alpha) \in \{t, f\}$, then this last one is a Structure of Kripke[1].

3.2 Context-Dependent Belief

In [10], satisfaction and entailment definitions are relative to a given model frame $\mathcal{M}$. This relativity is mantained in our epistemic definitions: we propose that there is no *absolute* knowledge or belief, instead both are relative with respect to the actual information and context.

A related point of view has been encouraged by Konolige [21], who defines agents of belief so that the set of agent's beliefs is the deductive closure from a initial set of premises and local (particular to the agent) and efective inference rules. This approach although emphasize relativity of agent's belief to its information and inference cappabilities does not allow how to modify the agent's beliefs.

More recently, Giunchiglia et al. [16] have defined non-omniscient context-based reasoning agents. They define the non-omniscience of agents of belief depending on several kinds of incompletness: language or basic facts or axioms or inference's rules, etc. The relative agent's beliefs are defined based upon any of these kinds of incompleteness. The authors contrast the agents with respect to saturated or omniscient agents that are defined by possesing complete information and inference rules.

[1]A Structure of Kripke is a 3-tupla $< W, R, \nu >$ such that W is a set of worlds, R a possibility relation and ν a valuation function [22].

One of the recent McCarthy's proposal [26] deals with the formalization of **contexts** considered as organized sets of information, inside which sentences are declared or derived. Each context can be embedded into another one and conversely, each context is constituted by several contexts. McCarthy suggests some basic operations between contexts and provides motivations for its developement. In a cognitive sense he suggests to consider the individual mental states as *outer* context which provide the reasons (*pedegrees*) of each actual (*inner*) individual believe. McCarthy's intuition is that a set of sentences cannot describe an individual mental state. Underling each statement there are a lot of reasons that do not appear directly associated to the statement but determine it in a depper way. Thus the convenience to consider an outer context that provides the reason for any particular statement.

Our approach is tought close to this manner to understand individual mental states (see section 6). Our formalisation of context is through model frames. We define a possibility relation over them, that we find natural to call of *compatibility* because is concerned with compatible information among frames and subframes in global or local sense. Frames operations are *concatenation* and *unification*. These three elements allow us to deal with contextual belif and belief change as well.

3.3 Knowledge and Belief

In logics of knowledge and belief both concepts have been treated in a parallel way. Hinttikka considers knowledge as something *true* in every possible world including the actual (*real*) one, while belief can or cannot be *true* in the actual world [18]. It means that accesibility relation over possible worlds for knowledge statements is reflexive, while not necessarily for belief statements. This difference has been adopted in several epistemic and doxastic logics using the semantics of possible worlds (*e. g.* [17, 29]). A wider treatment of belief would be considering belief as any statement *true* in one possible world [2, 27].

Attending the common use in AI field, Levesque preferes to refer to information in the so-called *knowledge bases* as belief [25]. The reason is that this kind of information usually has a particular and not always contrastable character. The term *knowledge* seems to refer to statements having the compromise to be *true* and *provable* [18], that is not the strict manner in which is used to denote knowledge bases. Levesque's reasons suggest to use belief instead knowledge; we assume them and in the rest of this paper we will refer to belief at most. When refering to knowledge we will do it explicitely.

4 Belief

In this section a belief definition is proposed. Propositional language $\mathcal{L}$ is extended by adding a modal operator of **Belief**, B . The language extended with these modal operators is denoted $\mathcal{BL}$) by belief language. The following satisfaction definition is relative to model frame.

Definition (Satisfaction)

An interpretation $I \in \Upsilon$ in the model frame $\mathcal{M}$ satisfies a sentence $\alpha \in \mathcal{F}$, $I \models_{\mathcal{M}} \alpha$, if and only if $I(\alpha) = T$. Is said that the frame satisfies α, $\mathcal{M} \models \alpha$, if every $I \in \Upsilon$ satisfies α. For a set Γ such that any sentencence of Γ is satisfied by $\mathcal{M}$, is said that the frame $\mathcal{M}$ satisfies Γ, $\mathcal{M} \models \Gamma$.

Notice that this satisfaction definition is for no modal formulas only; for belief modal ones below is given. Relativity of model frame satisfaction is welcome in our aim to encourage the point of view considering knowledge and belief relative to actual information and context. This semantical flexibility can be syntachtical well treated using a method based on Analytic Tableaux. Now we give belief definition.

Definition
A formula α **is believed** in the interpretation I of the model frame $\mathcal{M}$, denoted $I \models_{\mathcal{M}} B(\alpha)$, if and only if, for every maximal J of $\mathcal{M}$ with $I \preceq_i J$, $J \models \alpha$ (see fig 2). Whenever for every I of $\mathcal{M}$, $I \models_{\mathcal{M}} B(\alpha)$, it is said that $\models_{\mathcal{M}} B(\alpha)$.

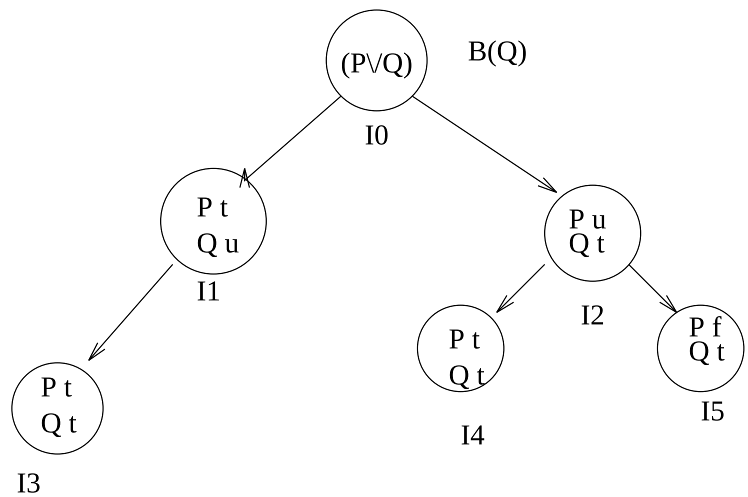

Figure 2: Belief

Then, a statement is belief in $\mathcal{M}$ if it is *true* in the interpretation I_0 or if being undefined in I_0, it becomes *true* in every maximal interpretation of the frame.

Given the model frame $\mathcal{M}$ such that the set Γ is satisfied for the current interpretation I_0 of $\mathcal{M}$, it is said that $\Gamma \models_{\mathcal{M}} B(\varphi)$ if and only if $I_0 \models_{\mathcal{M}} B(\varphi)$. Moreover, if the set $\mathcal{P}$ providing the information order of $\mathcal{M}$ is given (see 3.1), following Doherty and Lukasciewicz [10a], it is said that

$$\Gamma \models^{\mathcal{P}}_{\mathcal{M}} B(\varphi).$$

4.1 Belief Increasing

In some approaches of belief *e. g.* [25], is encoraged that since a general viewpoint the information that could become to be believed, *is initially independent* respect to logical or provability considerations. Initially an agent could be concerned with any kind of information. Thereafter, if he is rational or logical-guided proceeds to analize its belief against such criteria. It means that an arbitrary set $\mathcal{F}$ of formulas can be considered to deal with it, regardless the logical status of the formulas In this case is intuitive to assume that the agent does not believe anything.

In the proposed approach is possible to begin with any set of beliefs —perhaps contradictory— that thereafter are informatively organized in a model frame. Initial frame of belief then corresponds with the one having the single interpretation I_0 that assigns *undefined* truth-value to every formula in $\mathcal{F}$.

Definition

Initial model frame $\mathcal{M}_0 =< \Upsilon_0, I_0\ , \preceq_i' >$ is such that $\Upsilon_0 = \{I_0\}$ with $I_0(\alpha) = u\ \ \forall \alpha \in \mathcal{F}$.

By definiton of belief, no formula in $\mathcal{F}$ is believed in $\mathcal{M}_0$, because there is no interpretation assigning *true* value to any $\mathcal{F}$-formula. $\mathcal{M}_0$ is the state of no belief. In any other model frame, at least the true formulas in the initial interpretation is believed. This case is the usual one.

In [25] Levesque considers explicit belief as given information and implicit beliefs as the ones entailed from the first ones. Explicit beliefs could be contradictory. However in [25] it is not allowed neither increase nor modify explicit beliefs, except for some (obvious) logical equivalences. Our formalism provides the possibility to reason from initial (possibly not consistent) believes in a not trivial way. Each one of the contradictory sentences, if any, are considered in differents *parts* of the total frame. In fact, increasing of beliefs happens in different directions, being consistent each one of them. Below we will define compatibility relation among model frames. The intuitive idea is the following: a frame contain (is compatible with) several kinds of information, including contradictory or incoherent, but in different local contexts. Based upon a compatibility relation in addition to concatenation and unification operations over frames, change of the initial beliefs, as increasing or modification, is considered.

4.2 Compatible frames

We are interested in model frames that *change* in order to model the dynamic character of a cognitive process. A model frame defined by Doherty in [10] is static; is beforehand given and does not change at all. However, solution to our requirements can be done by introducing two operation among Doherty's model frames, concatenation and unification. The intuitive idea of concatenation is the following:

- To add information to a maximally informed world in the frame, if possible, in such a way that it allows to increase beliefs.

Let $\mathcal{M}_1 =< \Upsilon_1, I_0^1, \preceq_i^1 >$, $\mathcal{M}_2 =< \Upsilon_2, I_0^2, \preceq_i^2 >$ be two model frames.

Definition (Frames Concatenation)

Model frame $\mathcal{M} = < \Upsilon, \preceq_i, I_0^1 >$ is the concatenation of $\mathcal{M}_2$ with $\mathcal{M}_1$, denoted $\mathcal{M} = \mathcal{M}_1 \odot \mathcal{M}_2$, if and only if

1)I_0^2 is a maximal interpretation of $\mathcal{M}_1$.

2) $\Upsilon = \Upsilon_1 \cup \Upsilon_2$,

By definition, $I_0^1 \preceq I'$ for every $I' \in \Upsilon_1 \cup \Upsilon_2$. Notice that concatenation is defined over maximal interpretations only. Thus, further information is added to maximally informed worlds in the frame. It could be of interest to define concatenation for every interpretation in the model frame but this is behind the scope of this work.

Notice that frame concatenation is based upon the informative orders of model frames. Then, the possibility to concatenate $\mathcal{M}_2$ to $\mathcal{M}_1$ deppends of the fact that interpretations in $\mathcal{M}_2$ could be an informative refinaments of a maximal interpretation in $\mathcal{M}_1$ (see fig 3 and example 4.2). Concatenation is applied to increase belief from *undefined* statements by changing them to *true* or *false.* From the informative viewpoint it means to increase beliefs making refinements from informatively maximal worlds having undefined statements. Frame concatenation provides a constructive way to obtain beliefs from opinionless information.

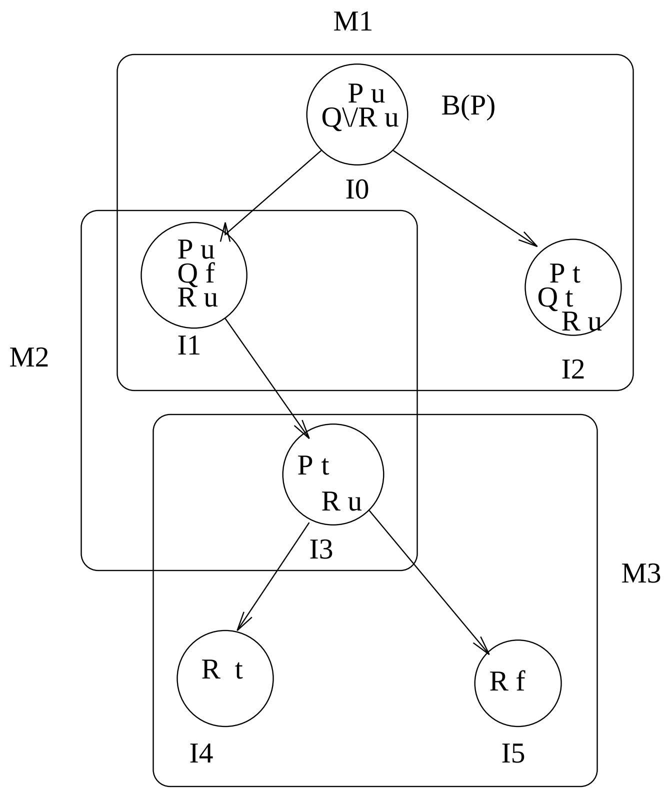

Figure 3: Model Frame Concatenation

Now we define an accesibility relation among model frames. In the introduction was mentioned that intuitions given by Kripke and Hintikka respect to accesibility relation are very suited and suggest to use semantics of possible worlds to model knowledge and belief. But the point is that we consider that formal definition of logics using that semantic does not capture adequately such intuitions. Our intend is to use concatenation to define a relation of possibility, called of compatibility, that captures the constructive and dynamic manner in which we think (hopeful close to Kripke and Hinttikka) that epistemic and doxastic alternatives (worlds) are conceived by a limited rational agent. The intuitive idea of compatibility is the following:

- Given a current frame describing a situation, any frame being coherently more informed than it, is compatible with it. (It is worth to think in the particular case when frames are defined by a single world.)

Definition (Compatibility Relation)
A model frame $\mathcal{M}_2$ is compatible with model frame $\mathcal{M}_1$ if and only if, there is a succesion of model frames $\mathcal{N}_1, \ldots \mathcal{N}_n$ such that $\mathcal{N}_1 = \mathcal{M}_1$, $\mathcal{N}_n = \mathcal{M}_2$, and $\mathcal{N}_{i+1}$ is concatenated to $\mathcal{N}_i$ for $i = 1, \ldots, n-1$.

Then, compatible model frames with $\mathcal{M}_1$ correspond to informative refinements that can be obtained from a maximal world in $\mathcal{M}_1$ (see example 1). Thus, $\mathcal{M}_2$ is compatible for $\mathcal{M}_1$ if it is more informed than $\mathcal{M}_1$ in some of the direction with undefined information that $\mathcal{M}_1$ has. By definition, any concatenated frame to $\mathcal{M}_1$ is compatible with it.

Let $\mathcal{N}_1, \ldots \mathcal{N}_l$ model frames with interpretation set $\Upsilon_1, \ldots, \Upsilon_l$, respectively. The following is a remarkable fact.

Observation 1
If each set of interpretations Υ_i for $i = 1, \ldots, l$ is a singleton, then each model frame is a world. In that case, accesibility relation is over possible worlds.

Thus, compatibility relation is a step by step possibility relation among sets of possible worlds in Kleene's three-valued logic.

Corolario 2
Partial order over Υ set of a model frame $\mathcal{M}$ corresponds to an accesibility relation inside $\mathcal{M}$.

To end this subsection we obvserve that informative order over interpretations of concatenated frames is not necessarily transitive. An example is the following: let $\mathcal{P} = \{P, Q, R\}$ with the following model frames

$\mathcal{M}_1 =< \{I_0^1 = (uuu), I_1^1 = (uut), I_2^1 = (utu)\};\ I_0^1 = (uuu);\ I_0^1 \preceq_i^1 I_1^1, I_2^1 >$,

$\mathcal{M}_2 =< \{I_0^2 = (utu), I_1^2 = (ttu), I_2^2 = (utt)\};\ I_0^2 = (utu);\ I_0^2 \preceq_i^2 I_1^2, I_2^2 >$

Let $\mathcal{M} = \mathcal{M}_1 \odot \mathcal{M}_2$. There, altough I_2^2 is more informed than I_1^1, in $\mathcal{M}$ it does not happen. This fact could be seen as an algebraical drawback, but from an epistemic viewpoint not always a rational agent has estrictly ordered all the information it posseses.

4.3 Limited reasoning and no omniscience

Before to go on, we mentione that the semantic defined so far, avoids the problem of logical omniscience and ideal reasonig. We ilustrate it with an example. Let the following set of statements, with initial interpretation I_0 such that $I_0(P) = u$ for every $P \in \mathcal{P}$.

$$\begin{aligned}
\mathcal{P} = \{&Misery \rightarrow Rebelion, && (1)\\
&Rebelion \rightarrow Negotiation \vee Represion, && (2)\\
&Rebelion \wedge Negotiation \rightarrow Solution, && (3)\\
&Rebelion \wedge Represion \rightarrow War, && (4)\\
&Solution \rightarrow \neg Misery, && (5)\\
&War \rightarrow MoreMisery && (6)\}
\end{aligned}$$

Informative order is indicated in the following model frame (fig 4). In the figure we use the initial letter of each predicate to deal with it, except to denote *Rebelion* with *Rb* and *Represion* with *Rp*. Formulas without indicated truth-value are considered undefined. It is assumed that *true* and *false* formulas in upper interpretations are present in the following ones.

There are not logical omniscience in the semantic defined due to the following reasons:

1) $\mathcal{F}$ set is an arbitrary set that rarely should contain every tautology.

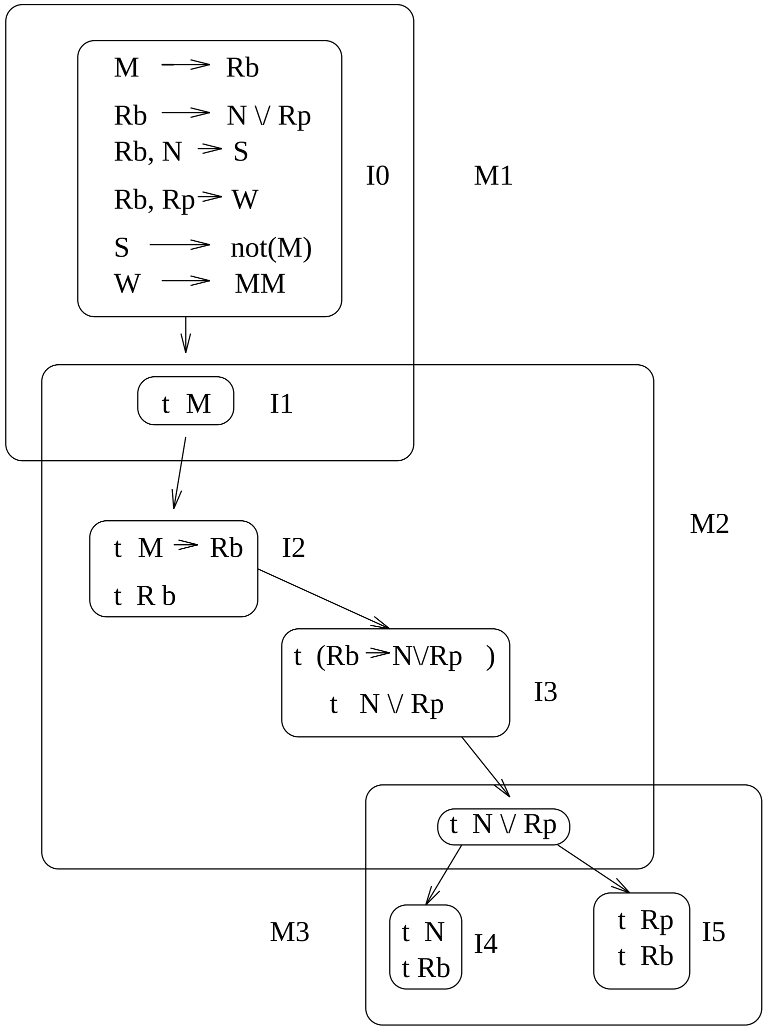

Figure 4: Frame of Belief

2) Sets of $\mathcal{F}$-formulas can begin as *undefined* and does not change to be *true* (*false*) at all. In this case, a *undefined* formula is not subject to be believed. (The case in which $\mathcal{F}$ contains all the tautologies herein is included.) At this manner limitations to reasoning about opinionless (undefined) information is allowed.

This way to avoid ideal reasoning and logical omniscience is on the direction pointed by Giunchiglia et al. [16] as well as in [27]. In both works synctactical charaterizations of limited reasoners —close between them— are given, in such a way that any of them does not deppend of the possible world semantics.

In the example we can draw the possible scenarios that could be allowed from a (unhappy) situation. It deppends of the way in which opinionless statements describing the initial situation become to be *true.* At the final steps of current frame, there is two compatible (possible) evolve frames —not resolved yet in the example— of current frame. One with a good (desired) evolution corresponds with that eventually statement (3) is *true*; the other, with unhappy evolve, corresponds with having as *true* statement (4). (It deppends of the beliefs of the active (implicit) agents.) In each case a compatible model frame can be concatenated to the current one.

Because there are sentences remaining undefined, there are not ideal reasoning nor logical omniscience over the frame.

5 Analytic Tableaux as Frames of Beliefs

We will use as proof system the analytic tableaux method [13] because its flexibility provides an adequate manner to introduce doxastic conditions over sets of formulas. Such flexibility is concerned with its widely recent use in AI theorem provers [8, 10, 21]. All those are provers based upon a two-valued logic. However, there are already characterization of Gentzen type provers for three-valued logics [2]. In fact, Avron distinguishe between the ones that are based upon a logic in which the *undefined* truth-value denote incomplet or unknown information, from those that use *undefined* to denote inconsistent information. The most known of the first ones corresponds with Kleene's and Lukasiewicz' logics[2]. Moreover, there are multivalued-logic theorem provers [5, 31].

Our proposed approach use *undefined* as incomplete or unknown, following a similar proof method as that in Doherty's work [10] that is used in the syntactical handling of epistemic model frames. We can add doxastic condition through tableaux like rules, or as belief statements in open branchs of tablaux containing current interesting information. It is easy to show from the example in 4.2 the natural manner in which a frame of beliefs can be represented through an analytic tableaux (see fig 5). The correspondence can be established under the following considerations:

- Current model frame is contained in the set of paths of the associated tableaux.
- The informative order over interpretations in the model frame allows the order in which classical tableaux rules are applied over the set of formulas.
- Whenever a formula becomes to be *true* in the frame, the closure that such change of value conveys is performed by application of classical tableaux rules.
- Classical α, β, π, ν-rules of tableaux [13] when applied over true formulas evolve in tableaux paths such that its model frames counterparts are the virtual concatenations of current model frame.
- Virtual extensions (by concatenation) of the model frame correspond with tableaux extensions through open branchs.

Incorporation of above consideration can be done appending to classical tableaux rules, the following one denoted UT:

- Whenever an *undefined* formula becomes to be *true* or *false*, then a tableux is open and all its logical consequences are deduced.

According with UT rule, whenever an *undefined* formula becomes to be *true* or *false,* a tableaux is open and classical rules are applied. In the asociated tableaux of the example, β-rule (see [13]) were applied over implication and disjunctive formulas as soon as they turn up *true*, and its logical consequences obtained. It is assumed that every tableaux satisfies all the formulas in upper tableaux. Closed tableaux are marked with X and corresponds with incompatible (impossible) situations for model frame.

[2]An example of the unconsistent view is the Paraconsistent Logics of Newton d'Acosta (see [2]).

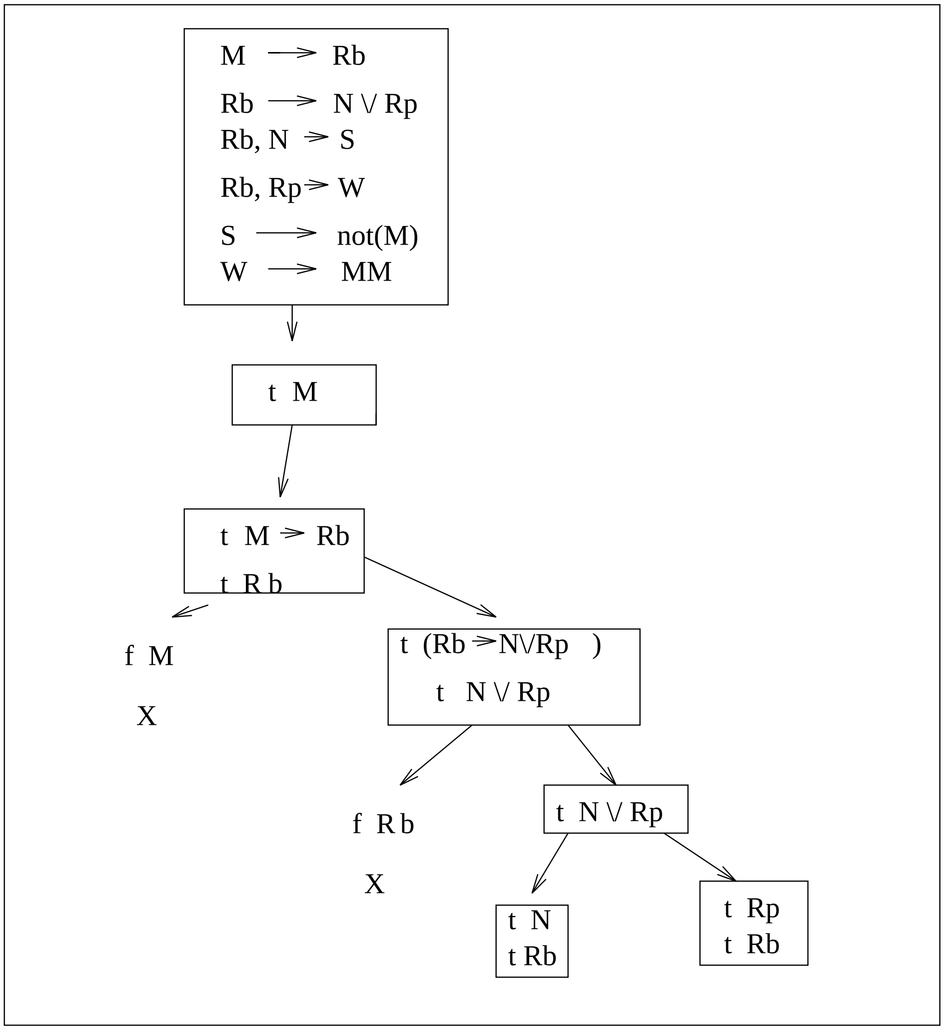

Figure 5: Analytic Tableaux as Frame of Belief

6 Epistemic States

Now we past to define epistemic states. First we need to define unification operation over model frames. Unification of frames should be used to deal with local and global information in a frame. Intuition of unification is the following:

- Given any pair of model frames there is a minimum of common information that provides global support for more precise information inside each one.

Definition (Frames Unification)
Model frame $\mathcal{M} =< \Upsilon, I_0, \preceq_i >$ is an unification of $\mathcal{M}_1$ with $\mathcal{M}_2$, denoted $\mathcal{M}_1 \sqcup \mathcal{M}_2$, if and only if, $\Upsilon_1 \cup \Upsilon_2 \subseteq \Upsilon$, and I_0 such that $I_0 \preceq_i I \quad \forall\, I \in \Upsilon_1 \cup \Upsilon_2$.

Observation
Every pair of model frames are unifiables.

Proof: Let $I_0 = I_0^t \cup I_0^f \cup I_0^u$, such that $I_0^t = (I_0^1)^t \cap (I_0^2)^t$, $I_0^f = (I_0^1)^f \cap (I_0^2)^f$ and I_0^u is given by the complement of $I_0^t \cup I_0^f$ with respect to $\mathcal{F}$. A model frame having as minimal interpretation

I_0 unifies $\mathcal{M}_1$ with $\mathcal{M}_2$. Moreover, I_0 is the informatively maximal interpretation that unifies them.

Definition (Epistemic states)

An epistemic state is defined by a model frame

Beliefs in the epistemic state are the set of beliefs in the associated frame. Let $\mathcal{M} = \bigsqcup_{i=1}^{n} \mathcal{M}_i$ be the model frame that unifies model frames $\mathcal{M}_i$, for $i = 1, \ldots, n$. $\mathcal{M}$ is an **epistemic state** that corresponds with the state given by the frame unification.

At this manner, epistemic states could correspond with McCarthy's mental states (see 3.2). The frame unification corresponds with outer context. Each concatenated frame, being a local informative refinement of other one, provides punctual reasons for its own statements. By the other hand, the unification frame provides the wider reasons (McCarthy's *pedegrees*) for the local statements.

We need the following definition.

$\mathcal{M}$' $=< \Upsilon', I_0', \preceq_i' >$, is a **submodel frame** of $\mathcal{M} =< \Upsilon, I_0, \preceq_i, \mathcal{F} >$ if and only if $\Upsilon' \subseteq \Upsilon$. (Thus, $I_0 \preceq_i I_0'$.)

Concatenation is the particular case of unification when interpretation I_0 of unification is the initial interpretation of one of the model frames. Because compatibility relation is based on concatenation, compatible model frames are unifiables.

In addition, in a model frame any pair of subframes are unifiable in the frame. And conversely, a given model frame can be seen as the unification of its subrames. The extreme case is when ecah subframe is a world. We consider that an epistemic state corresponds with a model frame. As far as more complex, as far as more complex frame unification of local frames is.

7 Related Works and Discussion

In section 3.1 was mentioned that classical approaches to belief change are those of AGM paradigm [1, 14] and Update Theory (UT) [19]. The first one define operations of *expansion*, *contraction* and *revision* over logically closed sets (called belief sets,) while the second deals with *belief updating* for arbitrary sets. In the proposed approache, using concatenation operation, it follows increasing (expansion) of belief. Now we can appoint the way in which is possible to deal with contraction, revision and update of beliefs in a nonless natural way.

Intuitive idea of revision of a beliefs set BS with new information A is to modify as little as possible BS in such a way that it entails A, preserving consistency. In our approach it conveys the modification of the frame $\mathcal{M}$ such that A becomes *true* in all maximal interpretation of $\mathcal{M}$.

If A is *true* or *undefined* in every interpretation of $\mathcal{M}$ then is an A-belief expansion on $\mathcal{M}$, while if A is false in some interpretation, it araise a contradiction in $\mathcal{M}$. In this case is proceeded in the following manner:

- It should be *erased* interpretations of the frame, by turning *undefined* formulas that being *true* conveys contradictions with formula A.

As much as needed to preserve consistency in the frame are the erased interpretations. The syntachtical operativity is performed by using the analytic tableaux associated with the model frame. Contradiction in analytic tableaux appears whenever an open branch of the tableaux becomes close. The erasure of contradictory interpretation in the frame corresponds with the erasure of the closed branches in the tableaux that were generated by doing A *true.* Unconsistency over tableaux branch is backward translated until it desappears.

Thereafter contradiction is eliminated, is possible that A becomes to be *true* in all maximal interpretation of the *revised* frame $\mathcal{M}$'.

Now we make some observations:
1.) In the case of expansion, all branches in the associated tableaux to the frame are open.
2.) The erasure of frame interpretations could be consider the contraction operation over the frame.
3.) In AGM approache, revision operation of BS by A is equivalent to contraction for $\neg A$ and expansion by A. This is the manner in which revision of a model frame is performed.

Thus, expansion, contraction and revision operation can be performed over model frames.

Respect with belief updating a similar thing should be done. Intuitive idea of UT is that the world has changed and is necessary to update beliefs. Whenever the updating formula A do not conveys contradiction with the old beliefs in BS, revision and updating are equivalents. The case in which contradictions arise, seems that UT generalizes the AGM approache [7]. There are situations in which UT allows different alternatives to belief updating that AGM does not.

However, the axiomatization of UT, given some kind of situations, results in unintuitive updatings. This is the case when A is a disjunction that is not in BS but is entailed by it. This situation corresponds with update BS with so-called *ambiguous* information [6]. Minimal change criterio of UT conveys unintuitive results when updating by such kind of information, though being so simple [6]: *e.g.* the result of flipping a coin provided by UT is always the biased result of last visible side of the coin. The same is applicable to deal with the undefinition of a person having a job, and such that it receive a new job proposal. UT choice always the old job.

Model frames approache provides to deal with different alternatives for ambiguous updating information. We have reasons to be optimistics to deal with *problematic* questions of belief change using the proposed approach. Alternative scenarios can be coherently included in a model frame without any kind of bias. While an alternative is not justified enough, it connot be a belief in the frame, but neither is excluded any of them. In this step each one is a local belief satisfied under more especific circumstances. Further information is needed to one of them becomes to be a global belief. In this case, the other sentence not necessarily will be globally rejected. It could remain as a local information. Further details provides constitutes the material of our current investigation.

Concluding Remarks

This paper emphasizes the relativity of belief from the context in a twofold manner: from current punctual information and from outer wide-context information. Our approach is tight concerned with those that use an informative order to deal with belief evolving. Formal framework is based upon model frames that are sets of informatively ordered (possible) worlds in three-valued Strong Kleene Logic. The *undefined* truth-value is used to indicate current but opinionless information. *True* and *false* information is considered more informed than the *undefined* one, and it constitutes an informative order. From that order, a compatibility relation over model frames is defined, such that in addition to concatenation and unification frames operation, belief change is allowed. The proposed formalism provides to deal, locally, with contradictory or incoherent beliefs. Syntachtical characterizations of ideal reasoning, logical omniscience and limited reasoning are given. Using the informative order in a step by step manner ideal reasoning and logical omniscience are avoided. The epistemical framework has a suited syntactical proof method using analytic tableaux.

Part of our forthcoming work is the introduction as far as possible, of strategies for tableaux calculi in modal logic that has been recently developed in [8, 9].

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