Harvesting Management in Multiuser MIMO Systems with Simultaneous Wireless Information and Power Transfer

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Abstract—**In this paper, we focus on a broadcast multiuser multiple-input multiple-output (MIMO) system where we consider that some terminals harvest power and, thus, recharge their batteries through wireless power transfer from the transmitter, while others are simultaneously being served with data transmission. The weighted sum-rate of the terminals that are receiving information data is considered as the optimization policy where minimum energy harvesting constraints are taken into account. We propose a procedure for managing the minimum energy to be harvested by the terminals considering the effect in the target system performance.**

I. Introduction

Energy harvesting is a promising technology to provide longer connectivity to battery-powered nodes in wireless networks [1], [2]. Such technology enables to recharge the batteries of the network terminals and, thus, to enhance their lifetimes. In fact, energy harvesting is particularly useful in scenarios where the nodes are placed in positions where the replacement of the battery may incur high costs or even be impossible.

Traditionally, energy harvesting techniques have been developed based on energy sources such as, for example, wind or solar energy. Nevertheless, there are other techniques that could be applied to moving sensors (this may be the case of cellular phones) based on piezoelectric technologies. Additionally, ambient radio frequency (RF) signals can be used as a source for energy scavenging. Unfortunately, some measurements in today's urban landscape show that the actual strength of the received electric field is not high and, thus, the proximity to the transmitter is important [1]. In this sense, it is important to emphasize that the newer applications require higher data rates and that this implies that more capacity efficient network deployments must be considered. Up to now, this increase in capacity efficiency has been shown to be achieved through the deployment of networks with reduced coverage area (e.g. femtocells [3]). The use of this kind of networks allows to increase the received power levels and, consequently, to make mobile terminals be able to harvest power from the received radio signals when they are not detecting information data. This is commonly named as wireless power transfer.

The concept of simultaneous energy and data transmission was first proposed by Varshney [4]. He showed that, for the single-antenna additive white Gaussian noise (AWGN) channel, there exists a nontrivial trade-off in maximizing the data rate versus the power transmission. Later, in [5], authors extended the previous work considering frequency-selective single-antenna AWGN channels. In [6] (and its journal version [7]), authors considered a multiple-input multiple-output (MIMO) scenario with one transmitter capable of transmitting information and power simultaneously to two receivers. They proposed two receiver architectures, namely time-switching and power-splitting that were able to combine both sources (information and energy) at the same time. There is another extension that considers the case of wireless information and power transfer with imperfect channel state information (CSI) [8]. The key idea is that the users that harvest energy benefit from the radiated power intended to the information users.

However, in the previous works, the minimum energy that a given user must harvest is usually known and fixed. Only the existing trade-off between the harvested energy and the system performance has been evaluated [6]. The scope of this paper is to provide a procedure that manages and configures how much energy a given user should harvest from ambient RF signals considering the impact in terms of weighted sum-rate for the users that receive information data.

The rest of this paper is organized as follows. In Section II we present the system model. Section III summarizes the precoder design for simultaneous power and data transmission. In Section IV we develop strategies for managing the minimum energy to be harvested. Section V presents some numerical results and, finally, conclusions are drawn in Section VI.

II. System Model

A. Signal Model

We consider a wireless broadcast system consisting of one base station (BS) transmitter equipped with n_T antennas and a set of receivers, denoted as $\mathcal{U}_T = \{1, 2, \ldots, K\}$, where the k-th receiver is equipped with n_{R_k} antennas. The proposed system is depicted in Fig. 1.

We assume that $n_T > n_R - \min_k\{n_{R_k}\}$ is fulfilled. The set of users is partitioned into two subsets. One of the sets contains the users that receive information, denoted as $\mathcal{U}_I$, being $\mathcal{U}_I \subseteq \mathcal{U}_T$ and $|\mathcal{U}_I| = N$, and the other set contains users that harvest energy coming from the power radiated by the BS which is intended to the information receivers. This subset is denoted as $\mathcal{U}_E$ being $\mathcal{U}_E \subseteq \mathcal{U}_T$ and $|\mathcal{U}_E| = M$. We assume that a given user is not able to decode information and to harvest energy simultaneously, i.e., $\mathcal{U}_I \cap \mathcal{U}_E = \emptyset$, $|\mathcal{U}_I| + |\mathcal{U}_E| = N + M = K$. To simplify the notation when needed, we will assume that the indexing of the users is such that $\mathcal{U}_E = \{1, 2, \ldots, M\}$ and $\mathcal{U}_I = \{M+1, M+2, \ldots, K\}$. The baseband channel from the BS to the k-th receiver is denoted by $\mathbf{H}_k \in \mathbb{C}^{n_{R_k} \times n_T}$. We assume perfect CSI at BS and at the receivers.

It can be assumed that the total harvested RF-band power by the j-th user during the t-th frame, denoted by $\bar{Q}_j(t)$, from all receiving antennas is proportional to that of the equivalent

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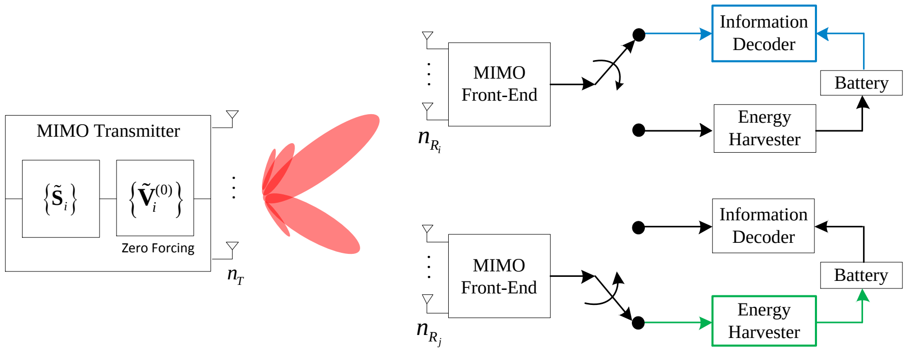

Fig. 1. Schematic representation of the downlink broadcast multiuser communication system. Note that each user can switch from an information decoder receiver to an energy harvester receiver.

baseband signal, i.e.,

$$\bar{Q}_j(t) = \zeta_j \sum_{i \in \mathcal{U}_I} \sum_{n \in t} \mathbb{E}[\|\mathbf{H}_j(t)\mathbf{B}_i(t)\mathbf{x}_i(t,n)\|^2], \tag{1}$$

where ζ_j is a constant that accounts for the loss in the energy transducer, t denotes frame, and n denotes the transmission instant within the frame. In the previous notation, $\mathbf{B}_i(t)\mathbf{x}_i(t,n)$ represents the transmitted signal for user $i \in \mathcal{U}_I$, where $\mathbf{B}_i(t) \in \mathbb{C}^{n_T \times n_{S_i}}$ is the precoder matrix and $\mathbf{x}_i(t,n) \in \mathbb{C}^{n_{S_i} \times 1}$ represents the information symbol vector. n_{S_i} denotes the number of streams assigned to user $i \in \mathcal{U}_I$ and we assume that $n_{S_i} = \min\{n_{R_i}, n_T - (n_R - n_{R_i})\} \, \forall i \in \mathcal{U}_I$ is fulfilled. The transmit covariance matrix is $\mathbf{S}_i(t) = \mathbf{B}_i(t)\mathbf{B}_i^H(t)$ if we assume w.l.o.g. that $\mathbb{E}\left[\mathbf{x}_i(t,n)\mathbf{x}_i^H(t,n)\right] = \mathbf{I}_{n_{S_i}}$. Notice that, for simplicity, in (1) we have omitted the harvested energy due to the noise term since it can be assumed negligible.

As far as the signal model is concerned, the received signal for the i-th information receiver can be modeled as

$$\begin{aligned} \mathbf{y}_i(t,n) = \;& \mathbf{H}_i(t)\mathbf{B}_i(t)\mathbf{x}_i(t,n) \\ & + \mathbf{H}_i(t) \sum_{\substack{k \in \mathcal{U}_I \\ k \neq i}} \mathbf{B}_k(t)\mathbf{x}_k(t,n) + \mathbf{n}_i(t,n), \quad \forall i \in \mathcal{U}_I, \end{aligned} \tag{2}$$

where $\mathbf{n}_i(t,n) \in \mathbb{C}^{n_{R_i} \times 1}$ denotes the receiver noise vector, that is considered Gaussian with $\mathbb{E}\left[\mathbf{n}_i(t,n)\mathbf{n}_i^H(t,n)\right] = \mathbf{I}_{n_{R_i}}$. For the sake of clarity, we will drop the frame and time dependence whenever possible.

Let $\tilde{\mathbf{x}} = \mathbf{B}\mathbf{x}$ denote the signal vector transmitted by the BS, where the joint precoding matrix is defined as $\mathbf{B} = [\mathbf{B}_1 \;\; \ldots \;\; \mathbf{B}_N] \in \mathbb{C}^{n_T \times n_S}$, being $n_S = \sum_{i=1}^N n_{S_i}$ the total number of streams of all information users, and the data vector as $\mathbf{x} = \left[\mathbf{x}_1^T \;\; \ldots \;\; \mathbf{x}_N^T\right]^T \in \mathbb{C}^{n_S \times 1}$, that must satisfy the power constraint formulated as $\mathbb{E}[\|\tilde{\mathbf{x}}\|^2] = \sum_{i=1}^N \mathrm{Tr}(\mathbf{S}_i) \leq P_{\max}$, where $P_{\max}$ is the total radiated power at the BS.

III. Review of the Sum-Rate Maximization With Individual Harvested Power Constraints

In this section, we present a brief summary of the design of the covariance matrices $\{\mathbf{S}_i\}$ based on the maximization of weighted sum-rate with individual power harvesting constraints presented in [9]. The optimization problem is as follows:

$$\begin{aligned} \underset{\{\mathbf{S}_i\}_{\forall i \in \mathcal{U}_I}}{\text{maximize}} \quad & \sum_{i \in \mathcal{U}_I} \omega_i \log\det\left(\mathbf{I} + \mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right) \\ \text{subject to} \quad & C1: \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\mathbf{H}_j\mathbf{S}_i\mathbf{H}_j^H) \geq Q_j, \quad \forall j \in \mathcal{U}_E \\ & C2: \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\mathbf{S}_i) \leq P_{\max} \\ & C3: \mathbf{H}_k\mathbf{S}_i\mathbf{H}_k^H = 0, \quad \forall k \neq i, \;\; k,i \in \mathcal{U}_I \\ & C4: \mathbf{S}_i \succeq 0, \quad \forall i \in \mathcal{U}_I, \end{aligned} \tag{3}$$

where $Q_j = \frac{\bar{Q}_j^{\min}}{\zeta_j}$, being $\{\bar{Q}_j^{\min}\}$ the set of minimum power harvesting constraints. Constraint $C1$ is associated with the minimum power to be harvested for a given user. Notice that $\{\bar{Q}_j^{\min}\}$ are considered known and fixed. The goal of the paper is to propose some strategies to configure the value of such constants. These techniques will be described in Section IV. Constraint $C3$ forces the complete cancellation of the interference inspired by block-diagonalization [11] making the problem convex since, otherwise, the objective function, i.e., the sum-rate, would not be convex. As a consequence, it can be solved efficiently with, for example, interior point methods [10]. However, in this case, it is possible to obtain the structure of the transmit covariance matrices, $\{\mathbf{S}_i\}$, and then develop a simplified and efficient iterative algorithm.

Notice that constraint $C3$ from the original problem (3) forces the precoder matrix $\mathbf{B}_i$ to lie in the right null space of matrix $\tilde{\mathbf{H}}_i = [\mathbf{H}_1^T \;\; \ldots \;\; \mathbf{H}_{i-1}^T \;\; \mathbf{H}_{i+1}^T \;\; \ldots \;\; \mathbf{H}_i^T]^T$ [11]. Computing the SVD of $\tilde{\mathbf{H}}_i$ yields $\tilde{\mathbf{H}}_i = \tilde{\mathbf{U}}_i\tilde{\mathbf{\Lambda}}_i[\tilde{\mathbf{V}}_i^{(1)} \;\; \tilde{\mathbf{V}}_i^{(0)}]^H$. Thus, $\mathbf{B}_i = \tilde{\mathbf{V}}_i^{(0)}\tilde{\mathbf{B}}_i$ and then, $\mathbf{S}_i = \tilde{\mathbf{V}}_i^{(0)}\tilde{\mathbf{S}}_i\tilde{\mathbf{V}}_i^{(0)H}$ where we define $\tilde{\mathbf{S}}_i = \tilde{\mathbf{B}}_i\tilde{\mathbf{B}}_i^H$. Now, the optimization variables are $\{\tilde{\mathbf{S}}_i\}$. Let $\hat{\mathbf{H}}_i = \mathbf{H}_i\tilde{\mathbf{V}}_i^{(0)}$ and $\hat{\mathbf{H}}_{ji} = \mathbf{H}_j\tilde{\mathbf{V}}_i^{(0)}$. Then, problem (3) is reformulated as

$$\begin{aligned} \underset{\{\tilde{\mathbf{S}}_i\}_{\forall i \in \mathcal{U}_I}}{\text{maximize}} \quad & \sum_{i \in \mathcal{U}_I} \omega_i \log\det\left(\mathbf{I} + \hat{\mathbf{H}}_i\tilde{\mathbf{S}}_i\hat{\mathbf{H}}_i^H\right) \\ \text{subject to} \quad & C1: \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\hat{\mathbf{H}}_{ji}\tilde{\mathbf{S}}_i\hat{\mathbf{H}}_{ji}^H) \geq Q_j, \quad \forall j \in \mathcal{U}_E \\ & C2: \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\tilde{\mathbf{S}}_i) + P_c^{tx} \leq P_{\max} \\ & C3: \tilde{\mathbf{S}}_i \succeq 0, \quad \forall i \in \mathcal{U}_I, \end{aligned} \tag{4}$$

The above problem can be easily checked to be convex and to satisfy Slater's conditions [10]. Hence, the duality gap is zero and the problem can be solved using tools derived from the Lagrange duality theory and the optimal structure of the transmit covariance matrices $\{\tilde{\mathbf{S}}_i\}$ can be revealed. Let $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_M)$ be the vector of dual variables associated with constraint $C1$ and μ be the dual variable associated with constraint $C2$. The optimal solution of problem (4) is given by the following theorem in terms of $\boldsymbol{\lambda}^\star$ and $\mu^\star$.

Theorem 1: The optimal solution of problem (4) has the following form:

$$\tilde{\mathbf{S}}_i^\star(\boldsymbol{\lambda}^\star, \mu^\star) = \mathbf{A}_i^{-1/2}\hat{\mathbf{V}}_i\hat{\mathbf{D}}_i\hat{\mathbf{V}}_i^H\mathbf{A}_i^{-1/2}, \tag{5}$$

where $\mathbf{A}_i = \mu^\star\mathbf{I} - \sum_{j=1}^M \lambda_j^\star\hat{\mathbf{H}}_{ji}^H\hat{\mathbf{H}}_{ji}$, $\hat{\mathbf{V}}_i \in \mathbb{C}^{(n_T - n_R + n_{R_i}) \times n_{S_i}}$ is obtained from the reduced SVD of the matrix $\hat{\mathbf{H}}_i^H\mathbf{A}_i^{-1/2} = \hat{\mathbf{U}}_i\hat{\mathbf{\Sigma}}_i^{1/2}\hat{\mathbf{V}}_i^H$, with $\hat{\mathbf{\Sigma}}_i = \mathrm{diag}(\hat{\sigma}_{1,i}, \ldots, \hat{\sigma}_{n_{S_i},i})$, $\hat{\sigma}_{1,i} \geq \hat{\sigma}_{2,i} \geq \cdots \geq \hat{\sigma}_{n_{S_i},i} > 0$, and $\hat{\mathbf{D}}_i = \mathrm{diag}(\hat{d}_{1,i}, \ldots, \hat{d}_{n_{S_i},i})$, with $\hat{d}_{k,i} =$

TABLE I
ALGORITHM FOR SOLVING PROBLEM (4)

1:	initialize $\boldsymbol{\lambda} \succeq 0$, $\mu \geq 0$ such that
2:	$\mu \mathbf{I} - \sum_{j=1}^{M} \lambda_j \hat{\mathbf{H}}_{ji}^H \hat{\mathbf{H}}_{ji} \succ 0, \forall i$
3:	**repeat**
4:	compute $\tilde{\mathbf{S}}_i(\boldsymbol{\lambda}, \mu)$ $\forall i$ using (5)
5:	compute subgradient of $g(\boldsymbol{\lambda}, \mu)$:
6:	$[\mathbf{t}]_j = Q_j - \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\hat{\mathbf{H}}_{ji} \tilde{\mathbf{S}}_i \hat{\mathbf{H}}_{ji}^H)$ for $1 \leq j \leq M$
7:	$[\mathbf{t}]_{M+1} = \mathrm{Tr}(\tilde{\mathbf{S}}_i) - P_T$
8:	update $\boldsymbol{\lambda}$, μ using ellipsoid method subject to:
9:	$\boldsymbol{\lambda} \succeq 0$, $\mu \geq 0$ and $\mu \mathbf{I} - \sum_{j=1}^{M} \lambda_j \hat{\mathbf{H}}_{ji}^H \hat{\mathbf{H}}_{ji} \succ 0, \forall i$
10:	**until** dual variables converge

$(\omega_i / \log(2) - 1/\hat{\sigma}_{k,i})^+$, $\forall i \in \mathcal{U}_I$.

Proof: See [9]. ■

Finally, the optimum data rate achieved by user i is, thus,

$$R_i^\star = \sum_{j=1}^{n_{S_i}} \omega_i \log(1 + \hat{\sigma}_{j,i} \hat{d}_{j,i}), \quad \forall i \in \mathcal{U}_I. \tag{6}$$

Now, the computation of the dual variables can be obtained by maximizing the dual function $g(\boldsymbol{\lambda}, \mu)$ subject to $\boldsymbol{\lambda} \succeq 0$, $\mu \geq 0$, and $\mathbf{A}_i \succ 0$ $\forall i$. This can be addressed by applying any subgradient-type method, such as for example the ellipsoid method [10]. It can be shown that the subgradient of $g(\boldsymbol{\lambda}, \mu)$ denoted as $\mathbf{t}$ is given by $[\mathbf{t}]_j = Q_j - \sum_{i \in \mathcal{U}_I} \mathrm{Tr}(\hat{\mathbf{H}}_{ji} \tilde{\mathbf{S}}_i \hat{\mathbf{H}}_{ji}^H)$ for $1 \leq j \leq M$ and $[\mathbf{t}]_{M+1} = \mathrm{Tr}(\tilde{\mathbf{S}}_i) - P_T$ [10], where $[\mathbf{t}]_j$ denotes the j-th entry of vector $\mathbf{t}$ and $\tilde{\mathbf{S}}_i$ is the optimal solution of problem (4) for a given $\boldsymbol{\lambda}$ and μ computed as in (5). Since the duality gap is zero, when we obtain the optimal dual variables ($\boldsymbol{\lambda}^\star$ and $\mu^\star$), the optimal solution $\tilde{\mathbf{S}}_i^\star(\boldsymbol{\lambda}^\star, \mu^\star)$ converges to the primal optimal solution of problem (4). The algorithm that solves problem (4) is described in Table I.

IV. MANAGEMENT OF THE ENERGY HARVESTED

In the previous section, we considered that the minimum energies to be harvested, i.e., $\{Q_j\}$, $\forall j \in \mathcal{U}_E$ were known and fixed. However, the particular value of such constants affects considerably the system performance, i.e., the weighted sum-rate. In [9] we presented the multidimensional trade-off that there exists between the sum-rate and the individual harvesting constraints. In this section we will develop an approach to configure (i.e., recalculate) such harvesting constants $\{Q_j\}$, $\forall j \in \mathcal{U}_E$ under a pre-established target weighted sum-rate.

In situations where the original problem is feasible but the sum-rate obtained is not enough, the system may be forced to relax (decrease) the energy harvesting constraints so that the overall sum-rate is enhanced. The idea is to identify which are the harvesting constraints that produce the largest enhancement of sum-rate when they are reduced and to apply a reduction on them. On the other hand, if the target sum-rate is below the one achieved, we could spend more resources on recharging the batteries of the harvesting users. In this case, a strategy for increasing the harvesting constants $\{Q_j\}$ is also needed. Ideally, we would like to modify the harvesting constants that accept a larger positive change and yield a small sum-rate loss.

In order to identify the constraints to be changed, we use the theory of perturbation analysis from convex optimization theory [10]. It is well-known that the Lagrange multipliers (dual variables) provide information about the sensitivity of the objective function with respect to the perturbations in the constraints. Let $\mathbf{q}^0$ be the vector of initial power harvesting constraints, i.e., $\mathbf{q}^0 = [Q_1^0, Q_2^0, \ldots, Q_M^0]^T$. Let $p^\star(\mathbf{q}^0)$ be the optimal value of problem (3), that is, $f_0(\{\mathbf{S}_i^\star\}_{\forall i}) = p^\star(\mathbf{q}^0)$, where $f_0(\cdot)$ denotes the objective function in (3) and (4). From [10] we know that the function $p^\star(\mathbf{q})$ is concave with respect to $\mathbf{q}$ where $\mathbf{q} = [Q_1, \ldots, Q_M]^T$ is the power harvesting perturbed vector defined as $\mathbf{q} = \mathbf{q}^0 + \Delta\mathbf{q}$, where $\Delta\mathbf{q} = [\Delta Q_1, \ldots, \Delta Q_M]^T$ being ΔQ_j a small change in the initial Q_j^0. Given this, we have that the optimal objective value of the relaxed problem can be upper bounded as

$$p^\star(\mathbf{q}) \leq p^\star(\mathbf{q}^0) + \nabla_{\mathbf{q}} p^\star(\mathbf{q}^0)^T (\mathbf{q} - \mathbf{q}^0). \tag{7}$$

Then, applying the following result from local sensitivity [10],

$$\frac{\partial p^\star(\mathbf{q}^0)}{\partial Q_i} = -\lambda_i^\star(\mathbf{q}^0), \quad \text{with } \lambda_i^\star(\mathbf{q}^0) \geq 0, \forall i, \tag{8}$$

it follows that

$$\begin{aligned} \nabla_{\mathbf{q}} p^\star(\mathbf{q}^0) &= \left[\frac{\partial p^\star(\mathbf{q}^0)}{\partial Q_1} \; \frac{\partial p^\star(\mathbf{q}^0)}{\partial Q_2} \; \ldots \; \frac{\partial p^\star(\mathbf{q}^0)}{\partial Q_M} \right]^T \\ &= - \left[\lambda_1^\star(\mathbf{q}^0) \; \lambda_2^\star(\mathbf{q}^0) \; \ldots \; \lambda_M^\star(\mathbf{q}^0) \right]^T = \boldsymbol{\lambda}^\star(\mathbf{q}^0), \end{aligned} \tag{9}$$

and the expression for the relaxed problem fulfills the following inequality defined by an hyperplane:

$$p^\star(\mathbf{q}) \leq p^\star(\mathbf{q}^0) - \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T (\mathbf{q} - \mathbf{q}^0). \tag{10}$$

Now let us define the target sum-rate as r_t and let us assume throughout the paper that $r_t > p^\star(\mathbf{q}^0)$[1]. We would like to find a vector $\mathbf{q}$ such that $r_t = p^\star(\mathbf{q})$, but since $p^\star(\mathbf{q})$ is not known, we force r_t to be equal to the upper bound in (10):

$$r_t = p^\star(\mathbf{q}^0) - \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T (\mathbf{q} - \mathbf{q}^0). \tag{11}$$

However, since $p(\cdot)^\star$ is a concave function, the solution obtained $p^\star(\mathbf{q})$ will be indeed below the desired sum-rate, i.e., $p^\star(\mathbf{q}) \leq r_t$. In order to get a very close solution, that is $p^\star(\mathbf{q}) \approx r_t$, we must proceed iteratively by applying successive perturbations on vector $\mathbf{q}$ in a way similar to the well-known Newton's method [10]. Before presenting the iterative algorithm, let us present different approaches (modeled as convex optimization problems) of how we can compute the new relaxed power harvesting parameters $\{Q_j\}$ since, as we are referring to a vector of variables, there exist different ways to update the vector $\mathbf{q}$ that yield the same sum-rate solution.

The first approach we propose is the simplest one. In this case, we fix the perturbed vector $\mathbf{q}$ to be a scaled version of the original vector, that is, $\mathbf{q} = \alpha\mathbf{q}^0$. In such a case, all the power harvesting constraints are reduced proportionally by the same amount. We seek to find the maximum value of α that produces the perturbed vector to yield the desired sum-rate. Let us define $\tilde{r}_t = p^\star(\mathbf{q}^0) + \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T \mathbf{q}^0 - r_t$ and assume that $\tilde{r}_t \geq 0$, otherwise we cannot find any feasible vector $\mathbf{q}$, i.e., any $\mathbf{q} \succeq 0$, where $\succeq$ refers to component-wise non-negativity. The problem is modeled as follows:

$$\begin{aligned} \underset{\alpha}{\text{maximize}} \quad & \alpha \\ \text{subject to} \quad & C1 : \alpha \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T \mathbf{q}^0 \leq \tilde{r}_t \\ & C2 : \alpha \geq 0. \end{aligned} \tag{12}$$

Lemma 1: The optimal solution of problem (12) and the optimal perturbed vector are given by

$$\alpha^\star = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T \mathbf{q}^0}, \quad \mathbf{q}^\star = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T \mathbf{q}^0} \mathbf{q}^0. \tag{13}$$

[1] In case we had $r_t < p^\star(\mathbf{q}^0)$, then we should modify slightly the optimization problems presented in the paper in order to increase the initial harvesting constraints until $r_t = p^\star(\mathbf{q})$.

TABLE II
ALGORITHM FOR ADJUSTING THE HARVESTING CONSTRAINTS

$k = 0$, $\mathbf{q}^{(k)} = (Q_1^{(k)}, \dots, Q_M^{(k)})^T$
solve problem (3) $\longrightarrow r^{(k)} = p^{\star}(\mathbf{q}^{(k)})$, $\boldsymbol{\lambda}^{\star}(\mathbf{q}^{(k)})$
while $(r^{(k)} < r_t - \epsilon)$
  obtain $\mathbf{q}^{(k+1)}$ following any strategy from (13), (15), and (17)
  solve problem (3) $\longrightarrow r^{(k+1)} = p^{\star}(\mathbf{q}^{(k+1)})$, $\boldsymbol{\lambda}^{\star}(\mathbf{q}^{(k+1)})$
  update $k \longleftarrow k+1$
end while

Proof: See Appendix A. ∎

Now, we propose a different approach to compute the perturbed vector $\mathbf{q}$. In this approach, we let the harvesting constraints have different relaxations and the objective is to minimize the sum of the harvesting reduction, i.e., $\|\Delta\mathbf{q}\|_1 = \|\mathbf{q} - \mathbf{q}^0\|_1$. The problem is modeled as follows:

$$\begin{aligned} \underset{\mathbf{q}}{\text{minimize}} \quad & \|\mathbf{q} - \mathbf{q}^0\|_1 \\ \text{subject to} \quad & C1 : \boldsymbol{\lambda}^{\star}(\mathbf{q}^0)^T \mathbf{q} \leq \tilde{r}_t \\ & C2 : \mathbf{q} \succeq \mathbf{0}. \end{aligned} \tag{14}$$

The optimal solution of previous problem is given in the following result.

Lemma 2: Let n be the index corresponding to the maximum Lagrange multiplier, i.e., $\lambda_n^{\star} > \lambda_m^{\star}$, $\forall m \neq n$.[2] The optimal solution of problem (14) is given by

$$q_n^{\star} = \frac{1}{\lambda_n^{\star}} \left(\tilde{r}_t - \sum_{i \neq n} \lambda_i Q_i^0 \right), \qquad q_m^{\star} = Q_m^0 \quad \forall m \neq n. \tag{15}$$

Proof: See Appendix B. ∎

As it can be seen, the optimal solution applies the harvesting power reduction to the user who has the largest Lagrange multiplier associated with its harvesting constraint whereas the rest of the users remain with the same harvested power.

The final proposed approach tries to be fair in terms of harvested reduction. The fairness in achieved by considering the objective function to be the maximization of the minimum q_i. The reformulated (differentiable) problem is

$$\begin{aligned} \underset{\mathbf{q},\, t}{\text{maximize}} \quad & t \\ \text{subject to} \quad & C1 : t\mathbf{1} \preceq \mathbf{q} \\ & C2 : \boldsymbol{\lambda}^{\star}(\mathbf{q}^0)^T \mathbf{q} \leq \tilde{r}_t \\ & C3 : \mathbf{q} \succeq \mathbf{0}. \end{aligned} \tag{16}$$

Lemma 3: The optimal solution of problem (16) is given by

$$\mathbf{q}^{\star} = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^{\star}(\mathbf{q}^0)^T \mathbf{1}} \mathbf{1}. \tag{17}$$

Proof: See Appendix C. ∎

As it can be seen, due to the maximin approach, when we introduce fairness in terms of harvested power reduction, all users end up with the same perturbed power constraint. As a consequence, some users could end up with more harvested energy than the initial one (i.e., $q_j^{\star} > Q_j^0$ for some j).

As it was commented before, the three previous approaches only yield a solution such that the actual rate $r_t \geq p^{\star}(\mathbf{q})$ due to the concavity of function $p^{\star}(\cdot)$. For this reason, it is not enough with just one iteration and we have to apply the previous algorithm iteratively to get a better solution (closer solution to the target sum-rate). Let us denote the obtained perturbed vector and the sum-rate at iteration k be $\mathbf{q}^{(k)}$ and $r^{(k)} = p^{\star}(\mathbf{q}^{(k)})$, respectively. Let us introduce the parameter ϵ that trades-off the speed of convergence and the solution accuracy. The idea behind the iterative algorithm, presented in Table II, is to use the previous procedures ((13), (15), and (17)) but with different iterations over Q_j, starting with Q_j^0.

[2]We have assumed that there is just one maximum Lagrange multiplier. In case there were more than just one, we would choose one randomly.

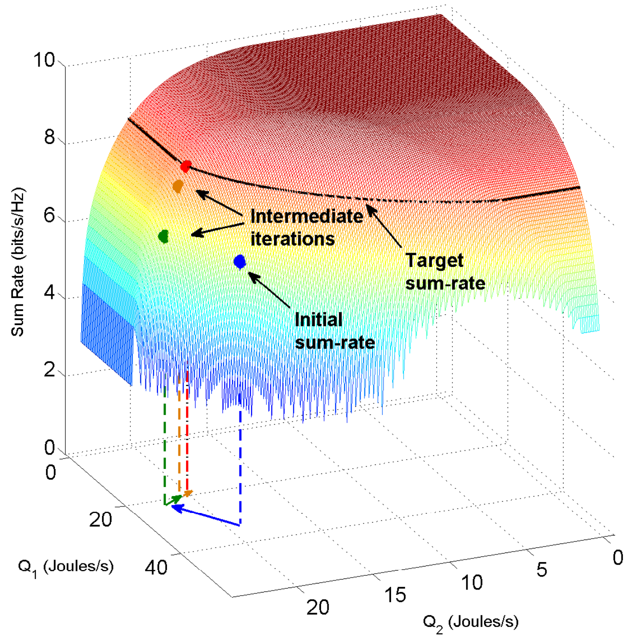

Fig. 2. Performance of the proposed algorithm with minimum energy management based on (16).

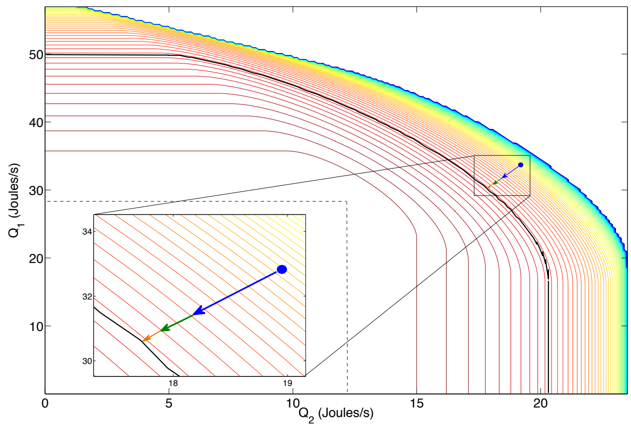

Fig. 3. Performance of the proposed algorithm based on (12).

V. Numerical Evaluation

In this section we present illustrative examples of the behavior of the different strategies developed in the paper. The set up is a BS with four transmit antennas, and two information users and two harvesting users with two antennas each. The maximum transmission power at the BS is $P_{\max} = 10$ W. The entries of the matrix channels are generated from a complex Gaussian distribution with zero mean and unit variance. The values of the initial minimum power to be harvested are $Q_1^0 = 33$ J/s and $Q_2^0 = 19$ J/s. The target weighted sum-rate is $r_t = 8.5$ bits/s/Hz where the weights are $\omega_j = 1/2, \forall j$.

Figure 2 depicts the three dimensional curve of the achieved sum-rate as a function of different values of $\{Q_j\}$ known as *Rate-Energy* curve [9]. In the figure, we have considered the solution based on (16). The blue dot represents the achieved sum-rate for the particular values of Q_1 and Q_2 assigned. In the figure, the black line represents the target sum-rate, which in this particular case is greater than the initial case. In the plot,

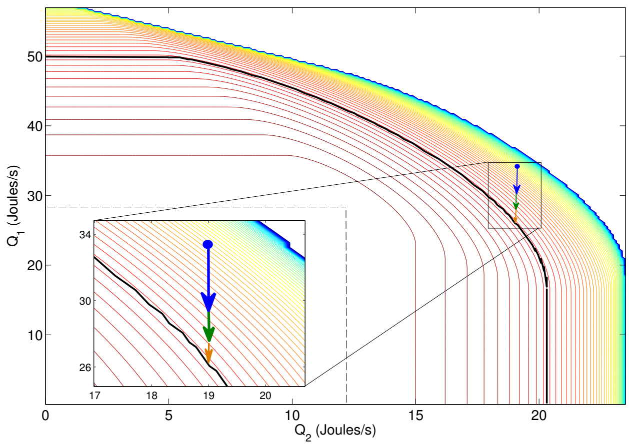

Fig. 4. Performance of the proposed algorithm based on (14).

we show the different iterations that the newton-like proposed algorithm in Table II performs. As it can be seen, in just 3 iterations we obtain a solution closer to 10^{-6} bits/s/Hz to the target sum-rate. Note that, when the algorithm converges, both users end up with the same amount of power to be harvested (approximately 20 J/s each one).

The performance of the solution based on (12) is presented in Figure 3. In this case, the figure shows the contour lines of the 3D *Rate-Energy* curve in order to better visualize the behavior of the algorithm. Also in this case, just 3 iterations are enough to yield a solution in the neighborhood of the target rate. Now, both harvesting users decrease their harvesting requirements by the same amount.

Finally, the behavior of the algorithm based on (14) is shown in Figure 4. As expected, just the user who has the largest harvesting requirement is modified while the other is left with its initial value.

VI. Conclusions

In this paper, we have proposed different strategies for managing the minimum energy to be harvested under the framework of simultaneous transmission of information and energy from the transmitter to multiple receivers. The procedures were derived from the sensitivity analysis of the duality theory where we also considered the effect on the system performance increase or decrease when adjusting the harvesting constraints.

Appendix A

The Lagrangian of the problem is $\mathcal{L}(\alpha,\mu,\sigma) = -\alpha + \mu\left(\alpha\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0 - \tilde{r}_t\right) - \sigma\alpha$. From the KKT conditions, If we take the partial derivative with respect to α we get $\frac{\partial\mathcal{L}(\alpha,\mu,\sigma)}{\partial\alpha} = -1+\mu\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0 - \sigma = 0$. By inspection we have that if $\alpha^\star = 0$, then $\alpha^\star\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0 = 0$ and $\alpha^\star\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0 < \tilde{r}_t$ which is not possible if $\tilde{r}_t > 0$. Thus, $\alpha^\star > 0$ which implies that $\sigma^\star = 0$ (from the complementary slackness $\alpha^\star\sigma^\star = 0$). As a consequence, $\mu^\star = \frac{1}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0} > 0$. Finally, substituting this value into the complementary slackness $\mu^\star\left(\alpha^\star\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0 - \tilde{r}_t\right) = 0$, yields $\alpha^\star = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q}^0}$.

Appendix B

Before attempting to obtain the optimum solution, let us characterize the sign of the perturbation $\mathbf{q}-\mathbf{q}^0$. We claim that, at the optimum, $\mathbf{q}^\star \preceq \mathbf{q}^0$, that is, $q_i^\star \leq Q_i^0$, $\forall i$. The proof is straightforward. Suppose $q_i > Q_i^0$. Then, constraint $C1$ can be rewritten as $\sum_{j\neq i}\lambda_j^\star q_j \leq \tilde{r}_t - \lambda_i^\star q_i$. Since $\lambda_i^\star \geq 0$, then we could reduce q_i (for example assign $q_i = Q_i^0$), the objective function would decrease its value and constraint $C1$ would become looser. We can proceed similarly by induction with the rest of variable to complete the proof. Thanks to this claim, we know the sing of the derivative of the term $\|\mathbf{q}-\mathbf{q}^0\|_1$ but we need to add explicitly $\mathbf{q}^\star \preceq \mathbf{q}^0$ to the original optimization problem. Thus, problem (14) is modified as

$$\begin{aligned} \underset{\mathbf{q}}{\text{minimize}} \quad & \|\mathbf{q}-\mathbf{q}^0\|_1 \\ \text{subject to} \quad & C1: \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q} \leq \tilde{r}_t \\ & C2: \mathbf{q} \preceq \mathbf{q}^0. \\ & C3: \mathbf{q} \succeq \mathbf{0}. \end{aligned} \tag{18}$$

The Lagrangian of problem (18) is $\mathcal{L}(\mathbf{q},\boldsymbol{\gamma},\mu) = \|\mathbf{q}-\mathbf{q}^0\|_1 + \mu\left(\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q} - \tilde{r}_t\right) + \boldsymbol{\gamma}^T(\mathbf{q}-\mathbf{q}^0)$. Let $\boldsymbol{\gamma} = (\gamma_1,\ldots,\gamma_M)$. Then, taking the derivative with respect to the primal variables yields $\frac{\partial\mathcal{L}(\mathbf{q},\boldsymbol{\gamma},\mu)}{\partial q_j} = -1 + \mu^\star\lambda_j^\star + \gamma_j^\star = 0 \implies 1 = \mu^\star\lambda_j^\star + \gamma_j^\star$. If $q_i < Q_i^0$, $\forall i \implies \gamma_i^\star = 0\ \forall i$. Then, it implies that $1 = \mu^\star\lambda_i^\star\ \forall i$ which is not possible since in general $\lambda_i^\star \neq \lambda_j^\star$. By inspection, we can claim that there is only one possible $\gamma_i^\star$ that could be equal to 0. Let $q_i < Q_i^0$ and $q_j = Q_j^0\ \forall j \neq i$. Then $\gamma_i^\star = 0$ and $\gamma_j^\star > 0$. We have that $\mu^\star = \frac{1}{\lambda_i^\star}$ and substituting back, $1 = \frac{\lambda_j^\star}{\lambda_i^\star} + \gamma_j^\star$. Since $\gamma_j^\star > 0 \implies \lambda_i^\star > \lambda_j^\star$ for the previous equation to be true. Finally, we have that $q_j^\star = Q_j^0$ and from constraint $C1$, $q_i^\star = \frac{1}{\lambda_i^\star}\left(\tilde{r}_t - \sum_{j\neq i}\lambda_j Q_j^0\right)$.

Appendix C

The Lagrangian of the problem is $\mathcal{L}(t,\mathbf{q},\boldsymbol{\nu},\boldsymbol{\gamma},\mu) = -t + \boldsymbol{\nu}^T(t\mathbf{1}-\mathbf{q}) + \mu\left(\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q} - \tilde{r}_t\right) - \boldsymbol{\gamma}^T\mathbf{q}$. Let us for the moment omit the positivity constraints ($\boldsymbol{\gamma}^T\mathbf{q}$) since (as it will be shown later) the solution will automatically satisfy them. Taking the derivatives with respect to the primal variables yields, $\frac{\partial\mathcal{L}(t,\mathbf{q},\boldsymbol{\nu},\mu)}{\partial t} = -1 + \boldsymbol{\nu}^{\star T}\mathbf{1} \implies \boldsymbol{\nu}^{\star T}\mathbf{1} = 1$ and $\frac{\partial\mathcal{L}(t,\mathbf{q},\boldsymbol{\nu},\mu)}{\partial q_j} = -\nu_j^\star + \mu^\star\lambda_j^\star(\mathbf{q}^0) = 0 \implies \nu_j^\star = \mu^\star\lambda_j^\star(\mathbf{q}^0)$. Summing at both sides, $\boldsymbol{\nu}^{\star T}\mathbf{1} = \mu^\star\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1}$ and so $\mu^\star = \frac{1}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1}}$. In this case, $\nu_j^\star = \frac{\lambda_j^\star(\mathbf{q}^0)}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1}} > 0$ and from the complementary slackness it is implied that $t^\star\mathbf{1} = \mathbf{q}^\star$. Multiplying both sides with $\boldsymbol{\lambda}^\star(\mathbf{q}^0)$ yields $t\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1} = \boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{q} \leq \tilde{r}_t$, and hence $t\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1} \leq \tilde{r}_t$. Given that, the maximum value of t is attained at $t^\star = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1}}$. Finally, we have $\mathbf{q}^\star = \frac{\tilde{r}_t}{\boldsymbol{\lambda}^\star(\mathbf{q}^0)^T\mathbf{1}}\mathbf{1}$ which fulfills the positivity constraint.

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