When Almost Distance-Regularity Attains Distance-Regularity

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Abstract

Generally speaking, ‘almost distance-regular graphs’ are graphs which share some, but
not necessarily all, regularity properties that characterize distance-regular graphs. In
this paper we first propose four basic different (but closely related) concepts of almost
distance-regularity. In some cases, they coincide with concepts introduced before by
other authors, such as walk-regular graphs and partially distance-regular graphs. Here
it is always assumed that the diameter $D$ of the graph attains its maximum possible
value allowed by its number $d+1$ of different eigenvalues; that is, $D = d$, as happens in
every distance-regular graph. Our study focuses on finding out when almost distance-
regularity leads to distance-regularity. In other words, some ‘economic’ (in the sense
of minimizing the number of conditions) old and new characterizations of distance-
regularity are discussed. For instance, if $A_0, A_1, \ldots, A_D$ and $E_0, E_1, \ldots, E_d$ denote,
respectively, the distance matrices and the idempotents of the graph; and $D$ and $A$
stand for their respective linear spans, any of the two following ‘dual’ conditions suffice:
(\textit{a}) $A_0, A_1, A_D \in A$; (\textit{b}) $E_0, E_1, E_d \in D$. Moreover, other characterizations based on
the preintersection parameters, the average intersection numbers and the recurrence
coefficients are obtained. In some cases, our results can be also seen as a generalization
of the so-called spectral excess theorem for distance-regular graphs.

Keywords: Distance-regular graph, Walk-regular graph, Distance degree-regular graph,
Eigenvalues, Local spectrum, Predistance polynomial.

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