Solving a Mixed-Model Sequencing Problem with Production Mix Restriction by Bounded Dynamic Programming

Joaquín Bautista · Rocío Alfaro-Pozo · Cristina Batalla-García · Alberto Cano
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Abstract: In this article, we propose a hybrid procedure based on bounded dynamic programming (BDP) assisted by linear programming to solve the mixed-model sequencing problem with workload minimization (MMSP-W) with serial workstations, free interruption of the operations and with production mix restrictions. We performed a computational experiment with 23 instances related to a case study of the Nissan powertrain plant located in Barcelona. The results of our proposal are compared with those obtained by the Gurobi solver and previous procedures.

MSC:90C39, 90B35

Keywords: Mixed model sequencing; Dynamic programming; Mixed integer linear programming; Hybrid metaheuristics; Industrial application.

1. Introduction

Product-oriented manufacturing systems are very common in production environments related to the automotive sector. In such systems, the manufacturing process of a product (engines, stamp forging, body welding, body painting and trim and chassis lines, for example) is conceived as a set of consecutive stages or manufacturing processes (due to the product orientation) that add value from raw materials to the final product (automobile).

This production type, which is product-oriented, culminates in flexible manufacturing systems composed by cells and modules or workstations arranged in series in assembly lines. In this last type of system, in addition to the line balancing problems, we can encounter the batch or product-unit sequencing problems, where the units are not completely identical, and their manufacture may require different consumption of components and different resource use at each manufacturing stage.

Sometimes, the processing times of these mixed products are very different at each stage. In these situations, we encounter sequencing problems for which the scientific basis is found in the literature under the name "flow shop" (usually, we encounter the case known as permutation) with and without buffers between production stages. In one of the most popular version of the problem, known as the permutation flow shop problem (PFSP), the storage capacity between two consecutive phases of the process, where the jobs can wait until they can be processed by the
following machine, is assumed to be unlimited (an up-to-date review can be found at Ruiz and Vazquez-Rodriguez (2010)). Some recent works regarding this problem are Fernandez-Viagas and Framinan (2015) and Vanchipura, Sridharan, and Babu (2014), among others. In contrast, in the variant known as blocking flow shop problem (BFSP), the buffer capacities between stages are limited and the jobs must wait in the previous stage until sufficient space is released. Recent works regarding this variant include Ribas and Company's (2015) and Lin and Ying (2013).

In other problems, processing times depend on the number of units that constitutes a batch of pieces, which is determined by a balance between the setup and holding costs, as is the case in line sequencing of parts to stamp car bodies. This group of problems is known as the economic lot scheduling problem (ELSP), and one of the pioneering works regarding this problem is Elmaghraby Salah (1978). A recent review of the heuristics used to solve the ELSP is Raza and Akgunduz (2008).

Finally, when the processing times of mixed products differ slightly at each stage, we are faced with problems similar to those that are known in the literature under the name of mixed product sequencing (homogeneous units). In these problems, the objective is to establish a production order of the products. Frequently, this order must be maintained from process to process whenever possible at all stages of the manufacturing and supply chain of the production systems governed by the Just in Time (JIT, Toyota) and Douki Seisan (DS, Nissan) philosophies.

Focusing on assembly lines, the order is conditioned by the line characteristics, the manufactured products and the most important elements of the production systems to establish optimization criteria. Among these elements we state the following: (1) component and product stocks, (2) human resources, and (3) special options within the products (e.g., sunroof, long body, or reinforced frame) that can generate bottlenecks in the assembly line.

Considering the stock as a relevant element of the system, a reasonable objective is to establish a product sequence that minimizes the stocks levels of products and components. To do this, we can either limit or minimize the variation of the production rates, as is the case in the product rate variation problem (PRVP), which was introduced by Miltenburg (1989), or limit or minimize the variation of the product components rates, as is done in the problem proposed by Monden (1983), which is called the output rate variation problem (ORVP). In both cases, the objective is to keep these rates constants over time.

In contrast, if we consider human resources (HR) as the relevant element of the manufacturing system, then a reasonable objective is to minimize the work overloads that can appear when the mixed-product units treated by the line require different processing times at each stage or, more concretely, at each workstation. To achieve this, we can minimize the total work overload or maximize the total work completed, as in the mixed model sequencing problem with work-overload minimization (MMSP-W), which was proposed by Yano and Rachamadugu (1991). A recent work regarding this problem is Bautista, Alfaro and Batalla (2015).

Finally, if the bottlenecks generated by special options of some products are the relevant element of the manufacturing system, then a reasonable objective is to minimize the number of subsequences of products with special options (units segments), which can be detrimental to the production line because more work or space (compared with the standard) is required consecutively at each workstation. One of these types of problems is the car sequencing problem (CSP), which was originally proposed by Parello, Kabat and Wos (1986), in which the constraint...
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consists of sequencing a set of units with special options while respecting the number of allowed options within subsequences. Some works regarding the CSP include Golle, Rothlauf and Boysen (2014) and Morin, Gagné and Gravel (2009). Among the variants of the CSP, we can find the following: (1) a version that considers the problem as an optimization problem rather than a constraint satisfaction problem (Bautista, Pereira, Adenso-Díaz, 2008a) and (2) an extended version that incorporates restrictions to allow a minimum number of products with special options in a subsequence of products (xCSP: extended CSP (Bautista, Pereira, Adenso-Díaz, 2008b)).

Sometimes, as in real environments, the problems are treated as multi-objective problems. Several authors have used this perspective. For example, Drexl, Kimms and Matthießen (2006) incorporated into the CSP conditions from the level scheduling (which is related to ORV and PRV). Additionally, Fattahi and Salehi (2009) incorporated conditions such as the minimization of the total utility work and idle costs into the mixed model assembly line (MMAL). Focusing on mixed model sequencing, Tsai (1995) incorporated the minimization of the utility work into the mixed model sequencing problem (MMSP). There also exist more recent works: for example Bautista, Cano and Alfaro (2012a) and Manavizadeh, Tavakoli, Rabbani et al. (2013) proposed incorporating conditions from the PRV into the MMSP-W. This objective can be achieved through regularizing the work or the work overload using pmr (product mix restrictions), as in the case of the MMSP-W-pmr, for example.

A survey of some of these sequencing problems can be found in Boysen, Fliedner and Scholl (2009).

This paper examines a variant of the MMSP-W, the MMSP-W-pmr. The original problem, MMSP-W, is an NP-hard problem (Yano and Rachamadugu, 1991) for which several alternative solutions have been proposed. These solutions include exact procedures based on branch-and-bound (Bolat, 2003), dynamic programming (Yano and Rachamadugu, 1991; Bautista and Cano, 2001; Bautista, Cano and Alfaro, 2012b), heuristic procedures based on local search (Yano and Bolat, 1989; Bautista and Cano, 2008), greedy algorithms with priority rules (Bautista and Cano, 2008; Bolat and Yano, 1992), meta-heuristics (Scholl, Klein and Domschke, 1998) and beam search (Erel, Gocgun and Sabuncuoglu, 2007). Several studies have also considered the multi-criteria option (Aigbedo and Monden, 1997; Kotani, Ito and Ohno, 2004; Ding, Zhu and Sun, 2006; Rahimi-Vahed and Mirzaei, 2007).

Given the complexity of the problem and the size of the case study related to Nissan Barcelona powertrain plant presented in (Bautista and Cano, 2011), our objective is to find a computationally competitive procedure to solve the problem. For this paper, we use a hybrid procedure based on bounded dynamic programming (BDP) assisted by linear programming. This procedure combines features of dynamic programming with features of branch-and-bound algorithms. The principles of the BDP have been described by Bautista, Companys and Corominas (1996). A complete review of hybrid metaheuristics in combinatorial optimization can be found in Blum, Puchinger, Raidl et al. (2011).

Our proposal contains the following: (1) a model for the problem; (2) to solve this problem, procedures based on dynamic programming, which are referred to in this article as BDP-2/1 and BDP-2/2 (two versions), that use linear programming to obtain bounds for the problem; (3) a mathematical model to obtain the work overload of a given subsequence for use as part of the lower bound of the problem; (4) reduction of the search space of the procedure through pseudo-
dominances; and (5) a computational experiment with real instances from a case study of Nissan such that we can compare the results yielded by BDP-2 procedures with those offered by integer linear programming.

This paper is organized as follows: Section 2 presents a model for the MMSP-W with serial workstations, unrestricted interruption of the operations and production mix restrictions. Section 3 presents an illustrative example. Section 4 describes the basic elements and the application of the proposed BDP procedure. Section 5 describes the computational experiment with a case study related to the Nissan powertrain plant. Finally, Section 6 presents the conclusions of the study.

2. Model for the problem

The MMSP-W consists of sequencing \( T \) products, of which \( d_i \) are of type \( i \) \((i = 1, \ldots, |I|)\). A unit of product type \( i \) requires from each processor (e.g., operator or robot) of workstation \( k \) \((k = 1, \ldots, |K|)\) a processing time, \( p_{i,k} \), assuming the processor works at its normal work pace or activity level. The standard time assigned to each processor to work on any product unit is the cycle time \( c \). When a cycle ends at workstation \( k \in K \), the processor can work on the product in progress for an additional positive time \( l_k - c \), where \( l_k \) is the time window.

When it is not possible to complete all of the work required by the demand plan, work overload is generated. The objective of the problem is to maximize the total work performed, which is equivalent to minimizing the total work overload generated (see Theorem 1 in Bautista and Cano, 2011).

For the MMSP-W with serial workstations, unrestricted interruption of the operations, production mix restrictions (pmr) and work overload mininimization, we take as reference the \( M_{4U3} pmr \) model proposed by Bautista et al. (2012a), whose parameters and variables of are presented below.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>Set of workstations ((k = 1, \ldots,</td>
</tr>
<tr>
<td>( b_k )</td>
<td>Number of homogeneous processors at station ( k )</td>
</tr>
<tr>
<td>( I )</td>
<td>Set of product types ((i = 1, \ldots,</td>
</tr>
<tr>
<td>( d_i )</td>
<td>Programmed demand for product type ( i )</td>
</tr>
<tr>
<td>( p_{i,k} )</td>
<td>Processing time required for a unit of type ( i ) at station ( k ) for each homogeneous processor (at its normal activity level)</td>
</tr>
<tr>
<td>( T )</td>
<td>Total demand; obviously, ( \sum_{i=1}^{</td>
</tr>
<tr>
<td>( t )</td>
<td>Position index in the sequence ((t = 1, \ldots, T))</td>
</tr>
<tr>
<td>( c )</td>
<td>Cycle time, the standard time assigned to workstations to process any product unit</td>
</tr>
<tr>
<td>( l_k )</td>
<td>Time window, the maximum time that each processor at workstation ( k ) is allowed to</td>
</tr>
</tbody>
</table>
work on any product unit, where $l_k - c > 0$ is the maximum time that the work in process is held at workstation $k$.

Ideal rate of production for product type $i$, $\hat{d}_i = d_i / T$ ($i = 1, ..., |I|$).

### Variables

- $x_{i,t}$: Binary variable equal to 1 if a product unit $i$ ($i = 1, ..., |I|$) is assigned to the position $t$ ($t = 1, ..., T$) of the sequence and to 0 otherwise.
- $s_{k,t}$: Starting instant for the $t^{th}$ unit of the sequence of products at workstation $k$ ($k = 1, ..., |K|$).
- $\hat{s}_{k,t}$: Positive difference between the start instant and the minimum start instant of the $t^{th}$ operation at workstation $k \in K$. $\hat{s}_{k,t} = [s_{k,t} - (t + k - 2)c]^+$ (with $[x]^+ = \max\{0, x\}$).
- $v_{k,t}$: Processing time applied to the $t^{th}$ unit of the product sequence at station $k$ for each homogeneous processor (at its normal activity level).
- $w_{k,t}$: Work overload generated for the $t^{th}$ unit of the product sequence at station $k$ for each homogeneous processor (at its normal activity level); measured in units of time.
- $\rho_{k,t}$: Processing time required for the $t^{th}$ unit of the sequence of products at workstation $k$ for each homogeneous processor (at its normal activity level).

Under these conditions, we can define the following mathematical model, $M_{AU3_pmr}$:

$$\text{Min} \quad W = \sum_{k=1}^{\|I\|} \left( b_k \sum_{t=1}^{T} w_{k,t} \right) \quad \Leftrightarrow \quad \text{Max} \quad V = \sum_{k=1}^{\|I\|} \left( b_k \sum_{t=1}^{T} v_{k,t} \right)$$

subject to:

1. $\sum_{t=1}^{T} x_{i,t} = d_i \quad i = 1, ..., |I|$ (2)
2. $\sum_{j=1}^{\|I\|} x_{i,t} = 1 \quad t = 1, ..., T$ (3)
3. $v_{k,t} + w_{k,t} = \sum_{j=1}^{\|I\|} \rho_{j,k} x_{j,t} \quad k = 1, ..., |K|; t = 1, ..., T$ (4)
4. $\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + v_{k,t-1} - c \quad k = 1, ..., |K|; t = 2, ..., T$ (5)
5. $\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + v_{k-1,t} - c \quad k = 2, ..., |K|; t = 1, ..., T$ (6)
6. $\hat{s}_{k,t} + v_{k,t} \leq l_k \quad k = 1, ..., |K|; t = 1, ..., T$ (7)
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\[ \hat{s}_{k,t} \geq 0 \quad k = 1, \ldots, |K|; \ t = 1, \ldots, T \] (8)
\[ v_{k,t} \geq 0 \quad k = 1, \ldots, |K|; \ t = 1, \ldots, T \] (9)
\[ w_{k,t} \geq 0 \quad k = 1, \ldots, |K|; \ t = 1, \ldots, T \] (10)
\[ x_{i,t} \in \{0,1\} \quad k = 1, \ldots, |K|; \ t = 1, \ldots, T \] (11)
\[ \hat{s}_{ij} = 0 \] (12)
\[ \sum_{t=1}^{T} x_{i,t} \geq \left[ t \cdot d_i \right] \quad i = 1, \ldots, |I|; \ t = 1, \ldots, T \] (13)
\[ \sum_{t=1}^{T} x_{i,t} \leq \left[ t \cdot d_i \right] \quad i = 1, \ldots, |I|; \ t = 1, \ldots, T \] (14)

In the model, the equivalent objective functions (1) are represented by the total overload \( W \) and total work performed \( V \). Constraint (2) requires that the programmed demand to be satisfied. Constraint (3) indicates that only one product unit can be assigned to each position of the sequence. Constraint (4) establishes the relation between the processing times applied to each unit at each workstation and the work overload generated by each unit at each workstation. Constraints (5)-(8) constitute the set of relative starting instants of the operations at each station and the processing times applied to the products for each processor. Constraints (9) and (10) indicate that the processing times applied to the products and the generated work overloads, respectively, are nonnegative. Constraint (11) requires the assigned variables to be binary. Constraint (12) fixes the start of operations. Constraints (13) and (14) are those that incorporate the preservation property of the production mix desired in the JIT (Toyota) and Douki Seisan (Nissan) philosophies.

Additionally in this work, we measure the non-regularity of a sequence using the following quadratic function:

\[ \Delta_Q(X) = \sum_{t=1}^{T} \sum_{i=1}^{T} \left( X_{ij} - t \cdot d_i \right)^2 \] (15)

where \( X_{ij} = \sum_{t=1}^{T} x_{i,t} \) (\( \forall i = 1, \ldots, |I|; \ \forall t = 1, \ldots, |T| \)) is the cumulative production.

3. An Illustrative Example

To illustrate the model formulated above, we present the following example: There are six units of product \( T = 6 \), of which three are type \( A \), one is type \( B \) and two are type \( C \), with a total work required \( V_0 = 104 \). The units are processed at three workstations \( |K| = 3 \) with different numbers of processors \( b_k \); the processing times for each processor (at its normal activity level) for each type of unit \( i (A, B, \text{ and } C) \) at each workstation \( k (m_1, m_2, \text{ and } m_3) \) are listed in Table 1.
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<table>
<thead>
<tr>
<th></th>
<th>A ($d_A=3$)</th>
<th>B ($d_B=1$)</th>
<th>C ($d_C=2$)</th>
<th>$b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$m_2$</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$m_3$</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>19</strong> ($V_0(A)=57$)</td>
<td><strong>15</strong> ($V_0(B)=15$)</td>
<td><strong>16</strong> ($V_0(C)=32$)</td>
<td><strong>$V_0=104$</strong></td>
</tr>
</tbody>
</table>

Table 1: Number of homogeneous processors ($b_k$) at each station and processing times ($p_{ik}$) for each processor (at its normal activity level) required for each type of unit at each station or module.

Furthermore, $c = 4$ (cycle time) and $l_k = 6 \; \forall k$ (length of workstation or time window).

Fig. 1 shows a Gantt diagram of the optimal solutions offered by models $M_{4U3}$ (top) and $M_{4U3_pmr}$ (bottom). The sequence of products that yield the minimum total work overload for $M_{4U3}$ is $C-B-A-A-A$. The total work performed is $V = 101$, and the work overload, which is concentrated between workstations $m_t$ and $m_2$, is $W = 3$ (the grey area in Fig.1). The non-regularity for $M_{4U3}$ is 9.05. The sequence of products that yields the minimum total work overload for $M_{4U3_pmr}$ is $C-A-B-A-C-A$ (the sequence is affected by the production mix restrictions). The total work performed is $V = 101$, and the work overload, which is concentrated between workstations $m_t$ and $m_2$, is also $W = 3$, whereas the non-regularity for $M_{4U3_pmr}$ is 2.05.

Fig 1. Gantt chart for the optimum solutions for the example provided by $M_{4U3}$ (top) and $M_{4U3_pmr}$ (bottom).
4. BDP for the MMSP-W with PMR

This section presents the basic elements of the BDP procedure applied to the MMSP-W with serial workstations, unrestricted interruption of the operations and production mix restrictions (here, we use BDP-2).

4.1 Graph associated with the problem

Similar to Bautista, Cano and Alfaro (2014) we can build a linked graph without loops or direct cycles of $T + 1$ stages. The set of vertices at level $t$ ($t = 0, ..., T$) is denoted as $J(t)$. $J(t, j)$ ($j = 1, ..., |J(t)|$) is a vertex of level $t$, which is defined by the tuple $J(t, j) = \{(t, j), \bar{q}(t, j), \pi(t, j), W(\pi(t, j)), LB_R(t, j), \Delta_q(X(\pi(t, j)))\}$, where:

- $\bar{q}(t, j) = (q_1(t, j), q_2(t, j), ..., q_{|J(t)|}(t, j))$ is the vector of satisfied demand.
- $\pi(t, j) = (\pi_1(t, j), \pi_2(t, j), ..., \pi_s(t, j))$ is the partial sequence of $t$ units of product associated with the vertex $J(t, j)$. 
- $W(\pi(t, j))$ is the partial work overload generated by the sequence $\pi(t, j)$.
- $LB_R(t, j)$ is a lower bound on the work overload generated by the unsequenced products, $d_i - q_i(t, j)$ ($i = 1, ..., |J|$).
- $\Delta_q(X(\pi(t, j)))$ is the non-regularity of production generated by the sequence $\pi(t, j)$.

Obviously, to obtain a global bound on the work overload associated with vertex $J(t, j)$, we can set: $LB_W(t, j) = W(\pi(t, j)) + LB_R(t, j)$.

The vertex $J(t, j)$ has the following properties:

$$\sum_{i=1}^{|J(t)|} q_i(t, j) = t \quad (16)$$

$$\left[t \cdot \bar{d}_i\right] \leq q_i(t, j) \leq \left[t \cdot \bar{d}_i^*\right] \quad \forall i \in I \quad (17)$$

At level 0 of the graph, there is only one $J(0)$ vertex. Initially, we may consider that at level $t$, $J(t)$ contains the vertices associated with all of the sub-sequences that can be built with $t$ products that satisfy properties (16) and (17). However, it is easy to a priori reduce the cardinality of $J(t)$ by establishing the following definitions of pseudo-dominance:

- **Definition 1. PSD_1**: Given the sequences $\pi(t, j_1)$ and $\pi(t, j_2)$ associated with the vertices $J(t, j_1)$ and $J(t, j_2)$, then $\pi(t, j_1)$ pseudo-dominates $\pi(t, j_2)$ if:


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- Definition 2. PSD$_2$: Given the sequences $\pi(t, j_i)$ and $\pi(t, j_2)$ associated with the vertices $J(t, j_i)$ and $J(t, j_2)$, then $\pi(t, j_i)$ pseudo-dominates $\pi(t, j_2)$ if:

$$\pi(t, j_i) < \pi(t, j_2) \iff \begin{cases} \left[ \tilde{q}(t, j_i) = \tilde{q}(t, j_2) \right] \land \left[ LB_\text{W}(t, j_i) \leq LB_\text{W}(t, j_2) \right] \land \left[ \Delta_\varrho \left( X(\pi(t, j_i)) \right) \leq \Delta_\varrho \left( X(\pi(t, j_2)) \right) \right] \end{cases}$$

The reduction of $J(t)$ through the pseudo-dominances defined in (18) or (19) cannot guarantee the optimality of the solutions.

4.2 Bounds for the problema

Given a vertex of stage $t$ reached through a partial sequence $\pi(t) = \{\pi_1(t, j), \pi_2(t, j), \ldots, \pi_t(t, j)\}$, the overall bound on $W$ and a partial bound on the complement $R(t, j)$ associated with the sequence or segment $\pi(t, j)$ can be determined according to the schema presented in Fig. 2.

![Fig 2. Bound scheme for a partial sequence $\pi(t, j)$ at vertex $J(t, j)$](image)

To obtain the work overloads associated with $\pi(t, j)$, in each stage of the procedure, we use a mathematical model. Given a subsequence $\pi(t, j) = \{\pi_1(t, j), \pi_2(t, j), \ldots, \pi_t(t, j)\}$ of products, the processing times for each workstation $k \in K$ of the $t^{th}$ ($\tau = 1, \ldots, t$) units of the subsequence $\pi(t, j)$ are $p_{k, \tau} = p_{\pi_\tau(t, j), k}$ and are foreknown. We can define a mathematical model ($LP_\text{W}(\pi(t, j))$) in which the assignment variables have been removed:

$$\text{Min} \quad W(\pi(t, j)) = \sum_{k=1}^{|K|} \left( b_k \cdot \sum_{\tau=1}^{t} w_{k, \tau} \right)$$ (20)
subject to:
\[ \rho_{k,\tau} = p_{\pi(t,j),k} \quad k = 1, ..., |K|; \quad \tau = 1, ..., t \]  
\[ \rho_{k,\tau} - w_{k,\tau} \geq 0 \quad k = 1, ..., |K|; \quad \tau = 1, ..., t \]  
\[ \hat{s}_{k,\tau} \geq \hat{s}_{k,\tau-1} + \rho_{k,\tau-1} - w_{k,\tau-1} - c \quad k = 1, ..., |K|; \quad \tau = 2, ..., t \]  
\[ \hat{s}_{k,\tau} \geq \hat{s}_{k-1,\tau} + \rho_{k-1,\tau} - w_{k-1,\tau} - c \quad k = 2, ..., |K|; \quad \tau = 1, ..., t \]  
\[ \hat{s}_{k,\tau} \geq 0 \quad k = 1, ..., |K|; \quad \tau = 1, ..., t \]  
\[ w_{k,\tau} \geq 0 \quad k = 1, ..., |K|; \quad \tau = 1, ..., t \]  
\[ \hat{s}_{1,1} = 0 \]  
\[ (31) \]

The result of the proposed mathematical model corresponds to \( W(\pi(t,j)) \).

To obtain a bound on the work overload associated with the complement \( R(t,j) \), we use the combination of three lower bounds.

Given a workstation \( k \) and vertex \( J(t,j) \), the available time to complete the pending operations, for each homogeneous processor at its normal activity level, is:
\[ TD_k(t,j) = (T - t - 1) \cdot c + l_k \quad k = 1, ..., |K| \]  
\[ (29) \]

whereas the required time to complete these operations is:
\[ TP_k(t,j) = \sum_{j=1}^{p_k} p_{j,k} \cdot (d_j - q_{i}(t,j)) \quad k = 1, ..., |K| \]  
\[ (30) \]

Using (29) and (30), we can define a lower bound on the work overload of \( R(t,j) \) as
\[ LB1(t,j) = \sum_{k=1}^{K} b_k \cdot [TP_k(t,j) - TD_k(t,j)]^+ \]  
\[ (31) \]

However, if we consider the minimum work overload that a product of type \( i \) can generate, we have:
\[ LB2(i) = \left[ \sum_{k=1}^{K} b_k \left( p_{i,k} - c \right) - b_{[i]} (d_{[i]} - c) \right]^+ \quad i = 1, ..., |I| \]  
\[ (32) \]

Thus, a bound on the work overload of \( R(t,j) \) is the following:
\[ LB2(t,j) = \sum_{i=1}^{I} (d_i - q_{i}(t,j)) \cdot LB2(i) \]  
\[ (33) \]

A more refined bound on the minimum work overload that a unit of product type \( i \) can generate can be obtained using the following mathematical model:
LP\_LB3(i): \quad MIN \quad LB3(i) = \sum_{k=1}^{\#} b_k \cdot w_{k,i} \quad (34)

subject to:
\begin{align*}
\hat{s}_{k,j} &\geq \hat{s}_{k-1,j} + p_{k-1,i} - w_{k-1,i} - c \quad k = 2, \ldots, |K| \\
\hat{s}_{k,j} + p_{k,j} - w_{k,j} &\leq l_k \quad k = 1, \ldots, |K| \\
p_{k,j} - w_{k,j} &\geq 0 \quad k = 1, \ldots, |K| \\
\hat{s}_{k,j} &\geq 0 \quad k = 1, \ldots, |K| \\
w_{k,j} &\geq 0 \quad k = 1, \ldots, |K| \\
\hat{s}_{1,j} &= 0
\end{align*} \quad (35)\quad (36)\quad (37)\quad (38)\quad (39)\quad (40)

Using the solutions of the previous mathematical model, we can determine the following bound on the work overload of \( R(t, j) \):

\[ LB3(t, j) = \sum_{i=1}^{\#} (d_i - q_i(t, j)) \cdot LB3(i) \quad (41) \]

To determine \( LB\_R(t, j) \), we use:

\[ LB\_R(t, j) = \max\{LB1(t, j), LB2(t, j), LB3(t, j)\} \quad (42) \]

Finally, we can obtain a lower bound on the total work overload associated with vertex \( J(t, j) \):

\[ LB\_W(t, j) = W(\pi(t, j)) + LB\_R(t, j) \quad (43) \]

4.3 Properties derived from the production mix restrictions (pmr)

In this section, we will study the properties of the product sequences that are derived from the incorporation of the restrictions to preserve the production mix (pmr) in the MMSP-W.

First, we must define how to measure the non-regularity of the production \( \Delta_q(X) \) at each vertex of the graph associated with the problem. In effect, given a vertex \( J(t, j) \), associated with a sequence \( \pi(t, j) = (\pi_i(t, j), \ldots, \pi_{|\pi|}(t, j)) \), let \( X_{i,\tau}(\pi(t, j)) = \{i\} \) \( i = 1, \ldots, |\pi|, \tau = 1, \ldots, t \) be the number of units of product type \( i \) sequenced at the first \( \tau \) positions of the sequence \( \pi(t, j) \), that is:

\[ X_{i,\tau}(\pi(t, j)) = \left| \{\pi_h(t, j) \in \pi(t, j) : (\pi_h(t, j) = \{i\}) \land (1 \leq h \leq \tau)\} \right| \quad (44) \]

Using the previous definition, let us define the non-regularity of the production associated with the sequence \( \pi(t, j) \) of vertex \( J(t, j) \) as:
\[
\Delta_{d_i}(X(t, j)) = \sum_{t=1}^{T} \sum_{i=1}^{I} (X_{t,i} - \tau \cdot \hat{d}_i)^2
\]  

(45)

The restrictions to preserve the production mix can be expressed as follows:

\[
[t \cdot \hat{d}_i] \leq X_{t,i} \leq \lfloor t \cdot \hat{d}_i \rfloor \quad 1, \ldots, |I|; t = 1, \ldots, T
\]  

(46)

where \( X_{t,i} \) is a variable that represents the total number of units of product type \( i \) sequenced during the first \( t \) production cycles.

The imposition of these restrictions on the sequences leads to a set of properties which are defined in Bautista, Cano, Alfaró-Pozo et al. (2013). These properties are the following:

**Theorem 2:** If \( \lfloor t \cdot \hat{d}_i \rfloor \leq X_{t,i} \leq \lfloor t \cdot \hat{d}_i \rfloor \quad \forall i \in I; t = 1, \ldots, T \), then \( X_{t,i} - X_{t,j} \leq \lfloor t \cdot \hat{d}_i \rfloor - \lfloor t \cdot \hat{d}_j \rfloor \quad \forall \{i, j\} \subseteq I; t = 1, \ldots, T \).

**Corollary 2:** If \( d_i \leq d_j \), then \( X_{t,i} - X_{t,j} \leq 1 \quad \forall \{i, j\} \subseteq I; t = 1, \ldots, T \).

**Theorem 3:** If \( \lfloor t \cdot \hat{d}_i \rfloor \leq X_{t,i} \leq \lfloor t \cdot \hat{d}_i \rfloor \quad \forall i \in I; t = 1, \ldots, T \), then the following is satisfied: \( X_{t,i} - X_{t,j} \geq \lfloor t \cdot \hat{d}_i \rfloor - \lfloor t \cdot \hat{d}_j \rfloor \quad \forall \{i, j\} \subseteq I; t = 1, \ldots, T \).

**Corollary 3:** If \( d_i \geq d_j \), then \( X_{t,i} - X_{t,j} \leq -1 \quad \forall \{i, j\} \subseteq I; t = 1, \ldots, T \).

**Corollary 4:** If \( d_i = d_j \), then \( X_{t,i} - X_{t,j} \leq 1 \quad \forall \{i, j\} \subseteq I; t = 1, \ldots, T \).

In addition, the fulfillment of the pmr restrictions combined with the demand variety, results in the following property:

**Theorem 4:** If \( \lfloor t \cdot \hat{d}_i \rfloor \leq X_{t,i} \leq \lfloor t \cdot \hat{d}_i \rfloor \quad \forall i \in I; t = 1, \ldots, T \), given the sequence \( \pi = \{\pi_1, \pi_2, \ldots, \pi_T\} \), where \( \pi_t = \{j\} \) with \( 2 \leq t \leq T \), the following is satisfied:

If \( \exists i \in I : (X_{t,i} > 0) \land (d_i \leq d_j) \Rightarrow X_{t,i} \leq X_{t,j} \quad \forall t = 2, \ldots, T \).

**Proof:** If we suppose that \( \exists i \in I : (X_{t,i} > 0) \land (d_i \leq d_j) \) such that \( X_{t,i} > X_{t,j} \), then \( X_{t,i} - X_{t,j} \geq 1 \).

In contrast, given \( \pi_t = \{j\} \), the following must be satisfied: \( X_{t,i} = X_{t-1} + 1 \) and \( X_{t,j} = X_{t-1} \). Thus, we can write \( X_{t,i} - X_{t,j} = X_{t-1} - X_{t-1} - 1 \geq 1 \Rightarrow X_{t,i} - X_{t,j} \geq 1 \). Furthermore, given that \( X_{t,i} - X_{t,j} \leq \lfloor (t-1) \cdot \hat{d}_i \rfloor - \lfloor (t-1) \cdot \hat{d}_j \rfloor \leq \lfloor (t-1) \cdot \hat{d}_i \rfloor - \lfloor (t-1) \cdot \hat{d}_j \rfloor \), we have \( \lfloor (t-1) \cdot \hat{d}_i \rfloor - \lfloor (t-1) \cdot \hat{d}_j \rfloor \geq X_{t-1} - X_{t-1} \geq 2 \), which is absurd. Thus, the hypothesis \( X_{t,i} > X_{t,j} \) is false and consequently, the following must be fulfilled: \( X_{t,i} \leq X_{t,j} \quad \forall t = 2, \ldots, T \) and \( \forall i \in I : X_{t,i} > 0 \), when \( \pi_t = \{j\} \).
Corollary 5: If \( \lfloor t \cdot d_j \rfloor \leq X_{i,t} \leq \lceil t \cdot d_j \rceil \) \( \forall i \in I; t = 1, ..., T \), given the sequence \( \pi = \{ \pi_1, \pi_2, ..., \pi_r \} \), where \( \pi_r = \{ j \} \) with \( 2 \leq t \leq T \), and if \( \exists i \in I : X_{i,t} > X_{j,t} \Rightarrow d_i > d_j \).

Evidently, from Theorem 3, \( d_i \leq d_j \Rightarrow X_{i,t} \leq X_{j,t} \), which negates the hypothesis \( (X_{i,t} > X_{j,t}) \); therefore, it must be that \( d_i > d_j \).

4.4 Rules to discard vertices

At stage \( t \), let \( X_i(\pi(t-1,h)) \) be the satisfied demand for product type \( i \in I \) associated with the sequence \( \pi(t-1,h) \) of the vertex \( J(t-1,h) \).

Assuming that an extension of the vertex \( J(t-1,h) \) is built by adding at stage \( t \) a product type \( j \) to the sequence, let \( X_i(\pi(t,h')) \) be the resulting vertex for the partial sequence \( \pi(t,h') = \pi(t-1,h) \cup \{ j \} \). The satisfied demands must fulfill the following:

\[
X_i(\pi(t,h')) = X_i(\pi(t-1,h)) \quad \forall i \neq j
\]

\[
X_j(\pi(t,h')) = X_j(\pi(t-1,h)) + 1 \quad \text{with} \ \pi_j(t,h') = \{ j \}
\]

Under these conditions, the vertex \( J(t,h') \) can be discarded from the exploring process if any one the following rules is satisfied:

Block 1 (pmr constraints):

\[
\forall i \in I, \text{If } \exists j : [X_j(\pi(t,h')) < \lfloor t \cdot d_j \rfloor] \lor [X_j(\pi(t,h')) > \lceil t \cdot d_j \rceil] \rightarrow \text{Discard } J(t,h')
\]

Block 2 (Theorems 2 and 3):

\[
\forall i \neq j \in I, \text{If } \exists i : [X_i(\pi(t,h')) - X_j(\pi(t,h')) > \lfloor t \cdot d_j \rfloor - \lceil t \cdot d_j \rceil] \lor
[X_j(\pi(t,h')) - X_i(\pi(t,h')) < \lfloor t \cdot d_j \rfloor - \lceil t \cdot d_j \rceil] \rightarrow \text{Discard } J(t,h')
\]

Block 3 (Corollaries 2, 3 and 4):

\[
\forall i \neq j \in I, \text{If } \left[ \left[ d_i < d_j \right] \land \left[ X_i(\pi(t,h')) - X_j(\pi(t,h')) > 1 \right] \right] \lor
\left[ \left[ d_i = d_j \right] \land \left[ X_i(\pi(t,h')) - X_j(\pi(t,h')) > 1 \right] \right] \lor
\left[ \left[ d_i > d_j \right] \land \left[ X_j(\pi(t,h')) - X_i(\pi(t,h')) > 1 \right] \right] \rightarrow \text{Discard } J(t,h')
\]

Block 4 (Theorem 4):

Given the partial sequence \( \pi(t,h') \) associated with vertex \( J(t,h') \), with \( \pi_j(t,h') = \{ j \} \)

\[
\forall i \in I : X_i(\pi(t,h')) \geq 1; \text{If } \left[ \left[ d_j < d_j \right] \land \left( X_j(\pi(t,h')) < X_j(\pi(t,h')) \right) \right] \rightarrow \text{Discard } J(t,h')
\]
4.5 The use of BDP

The BDP procedure combines features of dynamic programming (determination of extreme paths in graphs) with features of branch-and-bound algorithms. The principles of BDP have been described by Bautista et al. (2014) and the procedure is described below:

\[ \text{BDP-2-MMSPW} \]

Input: \( T, |I|, |K|, d_i (\forall i), l_k (\forall k), b_k (\forall k), p_{i,k} (\forall i, \forall k), c, Z_0, H \)

Output: list of sequences obtained by BDP

Initialization: \( t = 0; \ LBZ_{\text{min}} = \infty \)

1. Generate_model();
2. While \((t < T)\) do
3. \( t = t + 1 \)
4. Add_constraints(t)
5. While (list of consolidated vertices in stage \( t-1 \) not empty) do
6. Select_vertex(t)
7. Develop_vertex(t)
8. Filter_vertices \((Z_0, H, LBZ_{\text{min}})\)
9. End while
10. End_stage()
11. end while

end \text{BDP-2-MMSPW}

In the procedure, the following functions appear:

- **Generate_model():** this function generates the initial model \( LP\_W(\pi(t,j)) \) to obtain the optimal solution \( W(\pi(t,j)) \) for \( t = 0 \).

- **Add_constraints(t):** this function adds the new constraints associated with the new stage \( t \) to the existing model.

- **Select_vertex(t):** this function selects one of the vertices consolidated in stage \( t-1 \) following a nondecreasing ordering of the \( LB\_W(t,j) \) values.

- **Develop_vertex(t):** this function develops the vertex selected in the previous function by adding a new product unit with pending demand. Vertices that do not satisfy properties (16) and (17) are not generated. This is performed by incorporating the rules contained in the blocks to discard vertices Block 1 and Block 3 into this phase.

- **Filter_vertices \((Z_0, H, LBZ_{\text{min}})\):** this function chooses, from all the vertices developed in the previous function, a maximum number \( H \) of the most promising vertices (according to the lowest values of the lower bound \( LB\_W(t,j) \)) and removes those vertices for which the lower bound is greater than \( Z_0 \) (known initial solution) and those that are pseudo-dominated, as defined in (18) or (19).

- **End_stage():** this function consolidates the most promising vertices in stage \( t \) (\( H \) vertices...
are the maximum number of vertices selected).

4.6 An example of the graph reduction

Figure 3 represents the vertex exploration of the graph associated with the problem to solve the illustrative example through the BDP procedure described in this paper; here, we do not perform the elimination of vertices allowed by the incorporation of the pmr restrictions into the MMSP-W. In the example, an initial solution $Z_0 = 4$ and a window width $H = 6$ have been used.

Figure 4 represents the same exploration when the rules, to discard vertices associated with Block-1 and Block-3 to assure the fulfillment of the pmr restrictions, are incorporated to the BDP procedure. For this graph, $Z_0 = 4$ and $H = 6$ have also been used (although $H = 3$ is sufficient).

In the figures, we can see the vertices’ elimination states:

1) Dominated vertex ($d$). For the example in figure 3, the representative vertex of the partial sequence $(B,C)$, with $\Delta_q(X) = 2.6$, is dominated by $(C,B)$, with $\Delta_q(X) = 2.3$. Additionally, in the same figure, we can see that vertex $(B,A)$ is dominated by $(A,B)$.

2) Removed vertex ($r$). The limitation of the window width to $H = 6$ contributes to selecting the most promising vertices (best value for $LB_{-W}$) to be developed at each stage $t$. For example, at stage $t = 3$ of figure 3, the vertices that correspond to the partial sequences $(A,A,A)$, $(A,C,C)$, $(C,B,C)$ and $(C,C,B)$ are removed and only six vertices are developed to reach stage $t = 4$.

3) Discarded vertex ($Z_0$). The discarded vertices are those for which their development cannot finish at solution that is better than the best known solution $Z_0$. For example, the sequence $(A,C,B,A,A,C)$ in figure 4 does not improve the best known solution $Z_0 = 4$.

4) Breaker vertex ($pmr$). This is a vertex for which the sequence does not satisfy the restrictions, in our case, the pmr restrictions. For example, in figure 4, the partial sequences $(A,A)$, $(B,C)$, $(C,B)$ and $(C,C)$ do not satisfy the pmr restrictions.
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Fig. 3. Original graph for the example using $Z_0 = 4$, pseudo-dominance 1 and $H = 6$.

Fig. 4. Graph using the pmr restrictions, $Z_0 = 4$, pseudo-dominance 1 and $H = 6$. 
5. Case study related to the Nissan powertrain plant

To analyze the validity of the \( BDP\)-2/1 and \( BDP\)-2/2 procedures for industrial applications, an assembly line from the powertrain plant of Nissan Spanish Industrial Operations (NSIO) in Barcelona, Spain, was investigated. These results were compared with those obtained by the Gurobi solver (Bautista et al., 2012a).

We used a line with 21 serially distributed modules or workstations in which nine types of engines \((p_1, \ldots, p_9)\), with different characteristics, are assembled: the first three are placed in 4x4 vehicles, models \(p_4\) and \(p_5\) are destined for vans, and the last four are placed in commercial vehicles (trucks) of medium tonnage.

The number of elementary tasks for the assembly of one of the engines is approximately 380. These tasks were grouped into 140 operations for which the balancing of the line was performed based on average processing times for the mix with equal numbers of the nine types of engines. The balancing, considering time and space restrictions, resulted in 21 stations, \(k = 1, \ldots, 21\). More details can be found at http://www.nissanchair.com/TSALBP.

Once the operations were assigned to the stations, the processing times for each type of engine at each of the stations, \(p_{i,k}\) \((i = 1, \ldots, 9; k = 1, \ldots, 21)\), were calculated. These data are shown in Table 4.

For the experiment, an effective cycle time of \(c = 175s\) was used. The chosen time window, \(l_k = 195s\ \forall k\), was identical for all workstations. This ensured a safety margin for the cycle time of great than 10%. These data indicate that the instances with \(T = 270\) were associated with a single workday with an effective time of 13.125 hours distributed over two shifts.

We considered an identical number of processors at each station \(b_k = 1\); the processor at each station were teams of two workers with identical skills and tools and the required auxiliary equipment.

To study the behavior of the \( BDP\)-2 procedures, we assumed different demand plans (see Table 5) to analyze the repercussions that variations in the production mix had on the work overload of the engine assembly line (see more details in Bautista and Cano, 2011).
A computational experiment using the procedures BDP-2/1 and BDP-2/2 was performed. The primary results are collected in Tables 6 and 7.

Regarding CPU times (see Table 6), BDP-2/1 and BDP-2/2 using a window width of $H = 126$ (the largest used in this experiment) improved the CPU times required compared with that required by the Gurobi solver by 4 and 7 times, on average.

Table 7 presents the best values for $W$ and $\Delta_0(X)$ of the 23 instances for the problem reached by Gurobi and the BDP-2/1 and BDP-2/2 procedures for the four window widths ($H = 1,36,81$ and 126). Moreover a relative percentage deviation is used to calculate the gain of one procedure over another, that is the $RPD_1$:
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\[ RPD_1(f, \varepsilon) = \frac{f(\hat{S}_G(\varepsilon)) - f(\hat{S}_2(\varepsilon))}{f(\hat{S}_o(\varepsilon))}; \varepsilon \in E \] (47)

Where \( \hat{S}_G(\varepsilon) \) (optimal for \( W \)), is the best solution found for the instance \( \varepsilon \in E \) using the Gurobi solver and \( \hat{S}_2(\varepsilon) \) is the best solution found for the instance \( \varepsilon \in E \) using the BDP-2 procedure, for each of its two variants.

The highlighted results are the following:

1) Both versions of the BDP-2 improved, on average, the best solutions for \( W \) and \( \Delta_\phi(X) \) that were obtained compared with obtained using Gurobi. The values for the improvements obtained using BDP-2/1 for \( W \) and \( \Delta_\phi(X) \) were 6.78\% and 15.07\%, respectively. Using BDP-2/2, the improvements were 2.20\% and 0.37\%, for \( W \) and \( \Delta_\phi(X) \) respectively.

2) When compared with Gurobi, BDP-2/1 improved the value of \( \Delta_\phi(X) \) for the 23 instances and the work overload value (\( W \)) for 16 instances.

3) BDP-2/1 dominated to BDP-2/2 with respect to the value \( \Delta_\phi(X) \). BDP-2/2 obtained better values for \( W \) than did BDP-2/1 for only 4 of the 23 instances, three of which are worse than the solution given by Gurobi.

Observing the results of this computational experiment, we can conclude that BDP-2/1 is more competitive, on average, than the remainder of the procedures.
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<tr>
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<td>16.6</td>
<td>20.8</td>
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<td>12.3</td>
<td>13.2</td>
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<td>15.1</td>
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<td>19.9</td>
<td>18.3</td>
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<tr>
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<td>10.8</td>
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<td>316</td>
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<td>950</td>
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Table 7: Results of the case study of the Nissan powertrain plant obtained for W and Δq(X) using the **Gurobi** solver (limited to 7200 s) and **BDP-2** procedures (with H=1, 36, 81, and 126). Values for **RPD** for W and Δq(X). The symbol “*” denotes an optimal solution.
6. Conclusions

We have proposed a hybrid procedure based on the BDP, the BDP-2 (two versions), for the MMSP-W problem that minimizes the total work overload or maximizes the total completed work and considers serial workstations, parallel processors, free interruption of the operations and restrictions to preserve the production mix in the manufacturing sequence.

The proposed procedure use global bounds based on linear programming. A mathematical program that minimizes the work overload given a subsequence of operations at any instant $t$ has been formulated. In addition, the proposed procedure incorporates pseudo-dominances between partial solutions to limit the search space. These pseudo-dominances consider the preservation of the production mix in the partial solutions. From both versions of these pseudodominances, PSD_1 and PSD_2, we proposed two versions of the BDP-2 procedure (BDP-2/1 and BDP-2/2).

We performed a computational experiment corresponded to a case study of a Nissan powertrain plant in Barcelona. 23 instances corresponding to different demand plans, one production day and two shifts were considered. Both BDP-2 procedures were competitive in terms of CPU times and in terms of the results for $W$ and $\Delta_q(X)$ compared with Gurobi (Bautista et al., 2012a) because we always found an improvement, on average, for these indicators. Between BDP-2/1 and BDP-2/2, the first variant performed better that the second and found the best solutions in most cases.

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