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Detection of nonlocality with two-body correlation functions

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Abstract. Nonlocality detection in multipartite quantum systems is of great interest. The most popular tool to detect nonlocality in quantum systems are Bell inequalities. Most of the provided constructions of multipartite Bell inequalities involve correlations between all parties which quickly becomes computationally intractable and hard to test experimentally in many-body quantum systems. Recently, *J. Tura* and collaborators have shown in Ref. [1] that detection of nonlocality in multipartite systems is possible with Bell inequalities involving only one- and two- body correlation functions. However, it is uncertain how efficient these new inequalities are. One of the objectives of the present work is to address this question by numerical means. The other objective is to show that these inequalities can also serve as device independent witnesses of different forms of entanglement such as genuine multipartite entanglement.

Keywords: Bell inequalities, nonlocality detection, GME, 2-body Bell correlators

1. Introduction

Entanglement can give rise to counter-intuitive phenomena like correlations between remote quantum systems that cannot be simulated with a *local hiden variable* (LHV) model, meaning that these correlations cannot be simulated by any local strategy assisted by shared randomness. This phenomenon is known as nonlocality. It comes as no surprise that efforts have been put into the study of nonlocality. Apart from its philosophical and fundamental interest, these nonlocal correlations have been turned into a powerful resource for groundbreaking tasks such as quantum key distribution [2].

In order to detect nonlocality in quantum systems the usual tools are Bell inequalities [3]. These are linear inequalities constructed from expectation values of tensor products of measurements performed by the local observers. Thus, if a Bell inequality is violated by some quantum state, then this state is nonlocal. Many constructions of multipartite Bell inequalities have been provided, but most of them involve correlations between all the parties. This makes them hard to test experimentally in multipartite quantum systems and also theoretical characterization quickly becomes computationally intractable for larger N. Recently, it has been shown in [1] that detection of nonlocality in multipartite systems is possible with Bell inequalities involving only 1- and 2-body correlation functions (we name such inequalities *2-body Bell inequalities*), opening a new possibility of experimental nonlocality detection in many-body quantum systems and to perform numerical tests to study its properties. It is nevertheless uncertain how efficient these new inequalities are, that is, how much of all multipartite nonlocal states they are capable to detect. The first goal of the present work is to address this question in multipartite quantum systems for which all two-body Bell inequalities can be determined using computer algorithms, that is, systems consisting of three, four and five parties. The second goal of the present work is, following an approach of [4, 5], to show that two-body Bell inequalities can be used as *Device Independent Entanglement Witnesses* (DIEW) that guarantee N-partite entanglement. In other words, these inequalities not only detect entanglement but are also capable of distinguishing different types of entanglement multipartite scenario features. In particular we will show that they are able to detect *genuine multipartite entanglement* (GME).

2. Preliminaries

In this section we summarize some known results and set up the notation we will use throughout the present work.

2.1. Multipartite entanglement

Here we introduce the notions of separability in the multipartite case. For this purpose, let us consider N parties $A_1 \ldots A_N$ sharing an N-partite quantum state $\rho_{\mathbf{A}}$ that acts on a Hilbert space $\mathcal{H}_{\mathbf{A}} = \mathcal{H}^{A_1} \otimes \ldots \otimes \mathcal{H}^{A_N}$. Let us divide the set $\mathbf{A} = \{A_1, \ldots, A_N\}$ into k pairwise disjoint groups $\mathcal{S}_i$ such that by adding them one recovers the set $\bigcup_{i=1}^{k} \mathcal{S}_i = \mathbf{A}$. Denoting by $\mathcal{S}_k$ the set of k-partitions and calling it a k-partiion of $\mathbf{A}$, we say that $\rho_{\mathbf{A}}$ is k-separable if it admits

$$\rho_{\mathbf{A}} = \sum_{\mathcal{S} \in \mathcal{S}_k} p_{\mathcal{S}} \sum_i q_{\mathcal{S},i} \bigotimes_{k=1}^{K} \left|\psi_{\mathcal{S}_{k,i}}\right\rangle\!\left\langle\psi_{\mathcal{S}_{k,i}}\right| \tag{1}$$

with $p_{\mathcal{S}}$ and $q_{\mathcal{S},i}$ are probability distributions and $\left|\psi_{\mathcal{S}_{k,i}}\right\rangle$ are pure states defined on the $\mathcal{S}_k$ subsystem. This k-separability in the multipartite case opens the door to distinguish different types of entanglement. On one extreme, a state can be N-separable (*i.e.*, *full separability*) meaning no entanglement. On the other extreme, there are states that do not admit any k-separability implying what is called *Genuinely Multipartite Entanglement* (GME). In between these extremes there will be all the possible k-separability. Take as illustrative examples the bipartite case where two parties AB can only be separated in one way (*e.g.* $A - B$) and the 3-partite case ABC where it can be: fully separable (*e.g.* $A - B - C$); biseparable (*e.g.* $A - BC$, $AB - C$, $C - AB$ and also their convex combinations); or non-separable providing GME.

2.2. Nonlocality in many-body systems

Consider the standard Bell-type experiment (N, m, d) in which N spatially separated parties $A_1, \ldots, A_N$ share some N-partite quantum state ρ and each party can perform m different measurements with d outcomes on their share of ρ. We are interested in the simplest scenario, $(N, 2, 2)$, where each party A_i freely chooses one out of two dichotomic measurements $\mathcal{M}_{x_i}^{(i)}$ $(x_i = 0, 1)$ each having two outcomes $a_i = \pm 1$. The correlations that arise in such an experiment are described by a collection of conditional probabilities $p(a_1, \ldots, a_N | x_1, \ldots, x_N)$ of obtaining results $a_1, \ldots, a_N$ upon measuring $\mathcal{M}_{x_1}^{(1)}, \ldots, \mathcal{M}_{x_N}^{(N)}$. Since we stick with the case where each party chooses between two dichotomic observables, it is more comfortable to work with a collection of correlation functions that we will refer to as *correlators*

$$\left\{ \langle \mathcal{M}_{x_{i_1}}^{(i_1)} \ldots \mathcal{M}_{x_{i_k}}^{(i_k)} \rangle \right\}_{i_1, \ldots, i_k; x_{i_1}, \ldots, x_{i_k}; k}, \tag{2}$$

where $x_{i_1}, \ldots, x_{i_k} = 0, 1$, $i_k = 1, \ldots, N$ and $k = 1, \ldots, N$. In particular we will be interested in the lowest order correlators, that is, one- and two-body correlators

$$\langle \mathcal{M}_{x_{i_1}} \rangle := P(a_i = 1 | x_{i_1}) - P(a_i = -1 | x_{i_1}), \tag{3.1}$$

$$\langle \mathcal{M}_{x_{i_1}}^{(i)} \mathcal{M}_{x_{j_1}}^{(j)} \rangle := P(a_i = a_j | x_{i_1} x_{j_1}) - P(a_i \neq a_j | x_{i_1} x_{j_1}). \tag{3.2}$$

It is known that the set of quantum correlations $\mathcal{Q}$ is convex [6] and that the classical (or *local*) correlations define a polytope $\mathbb{P}$ whose vertices correspond to the vectors constructed from (2) in which every correlator factorizes

$$\langle \mathcal{M}_{x_{i_1}}^{(i_1)} \ldots \mathcal{M}_{x_{i_k}}^{(i_k)} \rangle = \langle \mathcal{M}_{x_{i_1}}^{(i_1)} \rangle \cdot \ldots \cdot \langle \mathcal{M}_{x_{i_k}}^{(i_k)} \rangle, \tag{4}$$

where every local mean value is ± 1. This means that any vertex from this polytope $\mathbb{P}$ represents correlations in which each local measurement has a perfectly determined outcome. Bell was the first to recognize that the set of classical correlations can be constrained by certain inequalities, referred to as Bell inequalities [3]. In fact, classical correlations form a polytope $\mathbb{P}$ that can be fully determined by a finite number of *tight* Bell inequalities, *i.e.*, those corresponding to the facets of $\mathbb{P}$. Thus, correlations that violate these inequalities are called nonlocal. The problem of finding the facets of the polytope $\mathbb{P}$ can be fully solved for the simplest scenarios using computer algorithms such as the CDD algorithm [7]. However, since the dimension of $\mathbb{P}$ and the number of its vertices grow exponentially with N, it quickly becomes computationally intractable for larger N.

3. Bell inequalities from 2-body correlators

In order to simplify the computational complexity of the polytope $\mathbb{P}$, we are particularly interested in Bell inequalities involving only 1- and 2-body correlators (3.1) (3.2). We will refer to them as 2-body Bell inequalities and they take the general form

$$
\begin{aligned}
I := \sum_{i=0}^{n-1} \left(\alpha_i \langle \mathcal{M}_0^{(i)} \rangle + \beta_i \langle \mathcal{M}_1^{(i)} \rangle \right) + \sum_{0 \le i < j < n} \gamma_{ij} \langle \mathcal{M}_0^{(i)} \mathcal{M}_0^{(j)} \rangle + \\
+ \sum_{0 \le i \ne j < n} \delta_{ij} \langle \mathcal{M}_0^{(i)} \mathcal{M}_0^{(j)} \rangle + \sum_{0 \le i < j < n} \epsilon_{ij} \langle \mathcal{M}_0^{(i)} \mathcal{M}_0^{(j)} \rangle \quad \le \beta_c
\end{aligned} \tag{5}
$$

for some $\alpha_i, \beta_i, \gamma_{ij}, \delta_{ij}, \epsilon_{ij} \in \mathbb{R}$ and the constant term $\beta_c = \max_{c \in \mathbb{P}} I \in \mathbb{R}$ is the so-called classical bound, where we have denoted by c the vertices (4). Accordingly, $\beta_Q = \max_{\mathcal{Q}} I$ will be denoting the maximal quantum violation. Clearly, since any local correlation can be obtained from a separable state, $\beta_C \le \beta_Q$. If we get the case where $\beta_Q = \beta_C$, then the Bell inequality does not have quantum violation and we will call such an inequality to be *trivial*.

3.1. The symmetric polytope of 2-body correlations

Even having got rid of highest-order correlators and thus reducing the polytope $\mathbb{P}$, it is still computationally complex. Another way to simplify the problem and make it tractable is by taking the inequalities that obey a certain symmetries.

Here, following Refs. [1, 8] we will shortly introduce the *Permutationally Invariant* (PI) and *Translationally Invariant* (TI) 2-body Bell inequalities which are the subject of study in Section 5.

3.1.1. Permutationally invariant Bell inequalities Let us first define the symmetric correlators for the PI case built from 1- and 2-body expectation values

$$
\mathcal{S}_k := \sum_{i=0}^{n-1} \langle \mathcal{M}_k^{(i)} \rangle \,, \qquad \mathcal{S}_{kl} := \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \langle \mathcal{M}_k^{(i)} \mathcal{M}_l^{(j)} \rangle \tag{6}
$$

for $j \ne i$, $0 \le k \le 1$ and $0 \le k \le l \le 1$. Then, any 2-body Bell inequality obeying the permutational invariant symmetry, that is, one that is invariant under a permutation of any pair of parties, can be written as

$$
\alpha \mathcal{S}_0 + \beta \mathcal{S}_1 + \frac{\gamma}{2} \mathcal{S}_{00} + \delta \mathcal{S}_{01} + \frac{\epsilon}{2} \mathcal{S}_{11} \le \beta_c, \tag{7}
$$

with $\alpha, \beta, \gamma, \delta, \epsilon \in \mathbb{R}$ and $\beta_c \in \mathbb{R}$ being the corresponding classical bound. The PI 2-body Bell inequalities in $(N, 2, 2)$ scenario were derived and classified in equivalent classes for the 3-, 4- ,5- and 6-partite cases in Ref. [1] where they are listed. From now on, when referring to a specific class of inequality we will refer to those.

3.1.2. Translationally invariant Bell inequalities Now we will look at 2-body Bell inequalities that obey a less restrictive symmetry: translational invariance. This symmetry is the one generated by the full cycle: the permutation τ such that τ : $0 \mapsto 1 \mapsto 2 \mapsto \ldots \mapsto n-1 \mapsto 0$. *J. Tura* and collaborators checked in Ref. [8]

their quantum violation and fully classified into equivalent classes all 3- and 4- partite Bell inequalities of this kind for the $(N, 2, 2)$ scenario. In this case the translationally invariant correlators are

$$\mathcal{S}_k := \sum_{i=0}^{n-1} \langle \mathcal{M}_k^{(i)} \rangle \,, \qquad \mathcal{T}_{kl}^{(r)} := \sum_{i=0}^{n-1} \langle \mathcal{M}_k^{(i)} \mathcal{M}_l^{(i+r)} \rangle \,, \tag{8}$$

with $k \in \{0, 1\}$, $k \leq l \in \{0, 1\}$, $r = 1, \ldots, \lfloor n/2 \rfloor$ for $k = l$ and $r = 1, \ldots, n-1$ for $k < l$. The parameter r can be seen as an interaction range. The party indices are taken *modulo* n. Hence, any 2-body translationally invariant Bell inequality reads

$$\alpha \mathcal{S}_0 + \beta \mathcal{S}_1 + \sum_{r=1}^{\lfloor n/2 \rfloor} \left(\gamma_r \mathcal{T}_{00}^{(r)} + \epsilon_r \mathcal{T}_{11}^{(r)} \right) + \sum_{r=1}^{n-1} \delta_r \mathcal{T}_{01}^{(r)} \leq \beta_c. \tag{9}$$

4. Methodology

Before jumping into the main findings, here we present some of the tools and reasoning used to achieve the results. The numerical techniques follow the next procedures:

Efficiency: In order to know how efficient 2-body Bell inequalities are, first a random pure state is generated (see Section 4.1); then the 2-body Bell operator is built (see Section 4.2); and finally the expectation value of the Bell operator with the state is computed and optimized over measurements to see if the corresponding inequality is violated or not.

Quantum Bound: In order to find the quantum bounds β_Q of the inequalities, the 2-body Bell operator is built for the corresponding inequalities and this time the expectation value of the Bell operator is optimized over a generalized quantum state and over measurements.

4.1. Generating random pure states

For simplicity we consider only pure states. We sample them from $(\mathbb{C}^2)^{\otimes N}$ according to the unique unitary invariant measure induced by the Haar measure on the unitary group $U(2^N)$. To be more precise we generate a random unitary and our random state is its first row (or column) [9, 10, 11].

Since we are dealing with 2-body correlators, which limits the efficiency nonlocality detection, we will also consider some subclasses of states such as the W states, denoted $\mathcal{W}$, and 2 excitation Dicke states, denoted $\mathcal{D}$, whose general form is given by, respectively,

$$|\mathcal{W}\rangle = \alpha_{100\ldots0} |100\ldots0\rangle + \alpha_{010\ldots0} |010\ldots0\rangle + \ldots + \ldots \alpha_{00\ldots01} |00\ldots01\rangle \,, \tag{10.1}$$
$$|\mathcal{D}\rangle = \alpha_{110\ldots0} |110\ldots0\rangle + \alpha_{101\ldots0} |101\ldots0\rangle + \ldots + \ldots \alpha_{0\ldots011} |0\ldots011\rangle \,. \tag{10.2}$$

The reason behind choosing the subclass $\mathcal{W}$ is that it has been proven in Ref. [12] that W-states are uniquely determined among all states by their 2-body reduction. In particular, there are no other fully-separable states compatible with these reductions. In contrast, we expect no detection of GHZ-states-like $|\mathcal{GHZ}\rangle = \alpha\,|0\rangle^{\otimes N} + \beta\,|1\rangle^{\otimes N}$ with $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$ since its 2-body reduction coincides with the 2-body reduction of fully-separable states $|\alpha|^2\,|0\rangle\langle 0|^{\otimes N} + |\beta|^2\,|1\rangle\langle 1|^{\otimes N}$ with $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$. We take the subclass $\mathcal{D}$ out of curiosity.

4.2. Quantum violation

Let us denote by $\hat{\mathcal{B}}$ the so-called Bell operator corresponding to the operators that form the corresponding Bell inequalities (7) or (9) in which the measurements are now one-qubit operators given by $\hat{\mathcal{M}}_{x_i}^{(i)} = \vec{n}_{x_i}^{(i)} \cdot \sigma^{(i)}$, where $\vec{\sigma} := (\sigma_x, \sigma_y, \sigma_z)$ denotes the vector of Pauli matrices, and $\hat{n} := (x, y, z)$ is a unit vector; $\hat{n} \cdot \vec{\sigma} = x\sigma_x + y\sigma_y + z\sigma_z$, with $x^2 + y^2 + z^2 = 1$. Equivalently, x, y and z can be expressed in spherical coordinates in terms of sine and cosine functions. Also since each party can choose a measurement, we indicate that they act on the i-th subsystem. Then, in order to detect if a state violates an inequality the expression $\langle\psi|\hat{\mathcal{B}}|\psi\rangle$ will be optimized by finding the best angles that each party can choose in order to detect nonlocality in that state and then check if the resulting expectation value violates the corresponding inequality.

5. Results

Here we present the efficiency of 2-body Bell inequalities and then we show that 2-body Bell inequalities can be used as Device Independent Entanglement Witnesses and detect genuine multipartite entanglement in multipartite quantum states.

5.1. Efficiency of nonlocality detection of two-body Bell inequalities

We generate random entangled pure states consisting of 3, 4 and 5 qubits, check if they violate the 2-body Bell Inequalities and make statistics to see how efficient they are at detecting quantum nonlocality (see Section 4).

The main results are summarized in Table 1. The efficiency shown is taking into account all classes of each case, *e.g.*, if for a given sample the TI 4-partite case inequality (#3) does not detect nonlocality, we will look for another class in TI 4-partite case that does violate. Efficiencies for all the classes have also been looked at individually but the results are too large to show in the present work. The number of samples generated for each inequality are: more than 10000 for the 3-partite cases; between 500-1000 for the 4-partite cases; and 500 for the 5-partite cases.

5.1.1. 3-partite case Taking a look at the classical β_c and quantum bounds β_Q (Secion 4.2), tells us that for both PI and TI cases there is only one inequality that is non-trivial (that is, $\beta_c < \beta_Q$) and thus can detect quantum nonlocality. We have tested with trivial Bell inequalities (*i.e.*, those with $\beta_c = \beta_Q$) and, as expected, efficiency is 0%. Therefore,

Table 1. Efficiencies of nonlocality detection with 2-body Bell inequalities for the 3-, 4- and 5 partite cases in *PI* and *TI* symmetries. The number of class # is indicated when there is only one non trivial inequality for that particular case.

	Efficiency (%)				
	$N = 3$		$N = 4$		$N = 5$
State Form	PI[a](#2)	TI[b](#6)	PI	TI	PI
General	47.75	53.70	0.00	2.19	12.29
W state	90.30	98.20	85.07	100.00	72.50
Dicke State (2)	"	"	75.11	92.50	83.33

[a] Permutationally Invariant
[b] Translationally Invariant

we make use of the non trivial inequalities for the 3-partite case which we present in what follows as an illustrative example

$$-2\mathcal{S}_0 + 6\mathcal{S}_1 - \mathcal{S}_{00} - 3\mathcal{S}_{01} + 3\mathcal{S}_{11} \leq 18 \tag{11.1}$$

$$-\mathcal{S}_0 - 3\mathcal{S}_1 - \mathcal{T}_{00}^{(1)} + \mathcal{T}_{01}^{(1)} + 2\mathcal{T}_{01}^{(2)} + 3\mathcal{T}_{11}^{(1)} \leq 9 \tag{11.2}$$

corresponding to PI class #2 from Ref. [1] and TI class #6 from Ref. [8] respectively. Inequality (11.1) has a classical bound $\beta_C = 18.00$ and allows for maximal quantum violation $\beta_Q = 20.03$ while inequality (11.2) has $\beta_C = 9$ and $\beta_Q = 10.02$.

Looking at the results obtained for $N = 3$ shown in Table 1, we observe that in both cases almost half of the states are detected. As we have said in Section 4.1, we should expect high efficiency coming from $\mathcal{W}$ states and indeed there is a noticeable increase reaching almost 100% detection. In this case, the $\mathcal{W}$ states are not taken into account since they are the same as the $\mathcal{W}$ states up to a unitary transformation and it would be redundant. An overall observation is that the efficiency is higher in general for the TI inequality which coincides with the fact that it is built from a less restrictive symmetry than the PI.

Since the 3-partite case is the less expensive computationally we have used it to do some tests that might be of interest. First we have checked GHZ-states-like and, as expected in Section 4.1, the efficiency obtained is 0%. Up until now we have been using the general set of measurements mentioned in Section 4.2. We explored what happens when applying restrictions to the set of measurements. For instance, if we force parties to choose the same measurement —that is $\mathcal{M}_k^{(i)} = \mathcal{M}_k$, for $i = 1, 2, 3$ indicating the party and $k = 0, 1$ indicating one of the two measurements— in the PI tripartite with $\mathcal{W}$, we see a drop from 90.30% efficiency to 22.10% with the restriction. Restricting to use only real measurements the drop goes from 90.30% to 46.50% forcing parties to use the same real measurement results on a drop from 90.30% to 20.00%.

5.1.2. 4-partite case We first notice is that there are more non-trivial Bell inequalities that we can use. Precisely, looking at the quantum and classical bounds we see that for the PI case there are 2 classes of Bell inequalities that are non trivial (and thus provide detection of quantum nonlocality) while for the *TI* there are 78 classes of non trivial Bell inequalities.

The main results obtained for $N = 4$ are collected and summarized in Table 1. A general drop when increasing N is expected since higher-order correlators come into play while we stick with 2-body correlators. We see that the tendencies in $N = 3$ are repeated here, that is, in general TI efficiency is higher and selecting the particular subclasses $\mathcal{W}$ states and $\mathcal{D}$ states offer very high efficiency. In this case it makes more sense to look at particular subclasses of states since the looking at the whole vector space $|\psi\rangle \in (\mathbb{C}^2)^{\otimes N}$ provides an efficiency close to 0%. Again the TI with $\mathcal{W}$ states offers the highest efficiency even reaching 100%. Out of curiosity, if instead of looking at the all the non-trivial inequalities available we look at the behavior of specific classes, we notice a new tendency: when an inequality provides high efficiency of detecting $\mathcal{W}$ states then it will provide low efficiency of detecting $\mathcal{D}$ states and the contrary is also true. This tendency persists for all non-trivial classes of PI and TI 2-body Bell inequalities.

5.1.3. 5-partite case Here we just consider the PI case because the TI is already too computationally expensive. There are 22 non-trivial 2-body Bell inequalities. Remarkably, looking at the results from Table 1, we see that for $N = 5$ the whole vector space $|\psi\rangle \in (\mathbb{C}^2)^{\otimes N}$ offers more efficiency of detection than $N = 4$ which goes against the expected drop. This fact rises a lot of questions, some speculation could be that due to 2-body Bell correlators acting on pairs maybe we should compare between even and odd parties cases. Another speculation could be that for this particular case the inequalities obtained are better and counters the fact of increasing party. When looking at $\mathcal{W}$ states or $\mathcal{D}$ states, though, we experience the expected drop due to increasing the number of parties.

5.2. 2-body Bell inequalities as DIEW

Here we show that 2-body Bell inequalities can serve as DIEW. To simplify, we will study the 3-partite case where, as explained in Section 2.1, a state can be: fully separable, biseparable or GME. Our aim is to find a biseparable bound β_{BS} such that $\beta_C < \beta_{\mathrm{BS}} < \beta_Q$. To simplify more, in order to find the biseparable and quantum bounds we use qubit states since Ref. [13] proofs that in the $(N,2,2)$ scenario they are sufficient to find the maximum quantum value of a Bell inequality.

When detecting nonlocality of a given state, if the 2-body Bell inequality with bound β_{BS} is violated we will know that the state carries GME. So in this way, 2-body Bell inequalities are capable to serve as DIEW and detect GME. On the other hand, if the 2-body Bell inequality with the bound β_C is violated, but not with bound β_{BS}, we will know that the state carries entanglement but not which kind.

Taking the Bell operator from Section 4.2, this bounds can be found by numerically optimizing the Bell operator over states and measurements, *i.e.*,

$$\begin{aligned}\beta_C &= \max_{\mathbb{P}} I = \max_{|\psi_{\text{prod}}\rangle,\hat{\mathcal{B}}} \langle\psi_{\text{prod}}|\hat{\mathcal{B}}|\psi_{\text{prod}}\rangle \\ \beta_{BS} &= \max_{BS} I = \max_{|\psi_{BS}\rangle,\hat{\mathcal{B}}} \langle\psi_{BS}|\hat{\mathcal{B}}|\psi_{BS}\rangle \\ \beta_Q &= \max_{Q} I = \max_{|\psi_{Q}\rangle,\hat{\mathcal{B}}} \langle\psi_{Q}|\hat{\mathcal{B}}|\psi_{Q}\rangle\,,\end{aligned} \tag{12}$$

where $|\psi_{\text{prod}}\rangle$ denotes the product state, $|\psi_{BS}\rangle$ denotes a biseparable state and $|\psi_Q\rangle$ denotes a nonseparable state.

Notice that, in the 3-partite case, the product and the GME state have only one combination possible, but for the biseparable state there are several combinations depending on which partition we choose. Since we use inequalities that obey PI symmetries or TI symmetries, this partitions will be equivalent (*e.g.*, we could choose some partition like $A-BC$ or $B-AC$ and they would turn out to be equivalent due to *PI* and *TI* symmetries). As we increase the number of parties more combinations have to be taken into account.

Taking inequality (11.1) and inequality (11.2), the bounds found for the 3-partite case are:

$$\beta_C = 18.00 < \beta_{BS} = 19.10 < \beta_Q = 20.03 \tag{13.1}$$

$$\beta_C = 9.00 < \beta_{BS} = 9.19 < \beta_Q = 10.02 \tag{13.2}$$

for PI and TI respectively. We have generated random pure states in order to test them and for the PI case 8.60% were confirmed to be GME (*i.e.*, those states that its correlation surpassed the bound β_{BS} in the inequality) and for the TI case 33.27% were confirmed to be GME for the last.

This procedure can be generalized by N parties taking into account the k-separations mentioned in Section 2.1. We have done it for the 4-partite case and from the 80 non-trivial inequalities, at least 32 appear to be good candidates to serve as DIEW and detect GME.

6. Conclusions and outlook

In the present work we have addressed two questions supported by the research started in [1]. First we have determined how efficient the Bell inequalities involving only 1 and 2-body correlation functions are at detecting nonlocality coming from quantum pure states. And second we have explored the question that if the mentioned Bell inequalities can be used as DIEW to detect GME. By numerically generating random pure states we have found the efficiencies of nonlocality detection in the 3-, 4- and 5- partite cases obeying permutational and translational symmetries. We have studied some of their properties and tendencies detecting nonlocality in the different cases like a general drop in efficiency by increasing the number of parties. Noticeably, the efficiency is very high by some specific class of vectors which led us into find which subclasses of vectors were a smart choice. Finally, we have shown that 2-body Bell inequalities can be used as DIEW and certify if a quantum state with nonlocal correlations carries genuine multipartite entanglement.

For further development, since now we know that 2-body Bell inequalities are capable of detecting GME, it would be of interest to give an analytical proof and expand it to the general case. It could also be of interest to expand the efficiency study to more parties or to study the case with mixed states.

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