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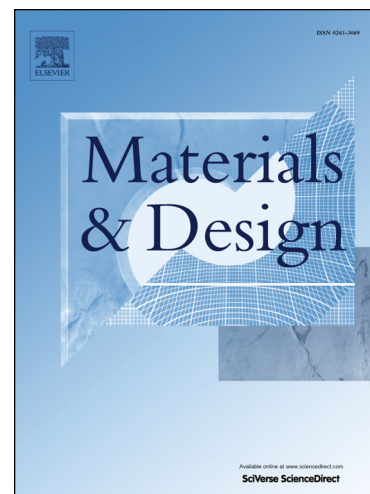
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A Simple Constitutive Model for Predicting Flow Stress of Medium Carbon Microalloyed Steel during Hot Deformation

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Abstract

The constitutive behavior of a medium carbon microalloyed steel during hot working over a wide range of temperatures and strain rates was studied using the Johnson-Cook (JC) model, the Hollomon equation, and their modifications. The original JC model was not able to predict the softening part of the flow curves and the subsequent modifications of the JC model to account for the softening stage and the strain dependency of constants were not satisfactory owing to the uncoupled nature of the JC approach regarding strain rate and temperature. The coupled effect of these variables was considered in the form of Zener-Hollomon parameter (Z) and the constants of the Hollomon equation were related to Z. This modification was found to be useful for the hardening stage but the overall consistency between the experimental flow curves and the calculated ones was not good. Therefore, a simple constitutive model was proposed in the current work, in which by utilization of the peak stress and strain into the Hollomon equation, good prediction abilities were attained. Conclusively, the proposed model can be considered as an efficient one for modeling and prediction of hot deformation flow curves.

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1. Introduction

Microalloyed or High Strength Low Alloy (HSLA) steels constitute an important category of steels estimated to be around 12% of total world steel production, which have been increasingly used in a variety of automotive components such as connecting rods, wheel hubs, suspension systems, crankshafts and driveline components [1-3].

Hot deformation is an important step in the production of microalloyed steels, which facilitates shaping, precipitation control, and grain refinement to achieve desired mechanical properties. Hot deformation in austenite recrystallization region refines coarse grains by repeated static recrystallization in the interpass times and also by dynamic recrystallization during deformation. Moreover, deformation in the non-recrystallization region increases ferrite nucleation sites through pancaking of austenite grains and creation of deformation bands. In this way, a fine microstructure will be produced after transformation [4,5].

In order to improve the properties, the parameters of the forming process must be controlled carefully. The understanding of the microstructural behavior of the steel under consideration is therefore required, together with the constitutive equations describing material flow [6-9].

Industrial hot deformation processing such as rolling for these steels is conducted in the temperature range of stability of austenite phase. Due to low stacking fault energy of austenite, the major restoration process during hot deformation is dynamic recrystallization (DRX) [10-14]. DRX is an important phenomenon for controlling microstructure and mechanical properties in hot working. The modeling of hot flow stress and the prediction of flow curves are important in metal-forming processes from the mechanical and metallurgical standpoints because this is an essential part of the

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numerical simulations in finite element codes. As a result, considerable researches have been focused on this subject in recent years [6,15-17].

The aim of this work is to introduce a simple but effective constitutive equation for modeling the flow curves during hot working of a medium carbon microalloyed steel.

2. Experimental Materials and Procedures

The chemical composition of the investigated steel is listed in Table 1. Cylindrical specimens with 11.4 mm in height and 7.6 mm in diameter were prepared from the microalloyed steel for the hot compression test, which carried out at deformation temperatures in the range of 900 to 1150 °C (1173 to 1423 K) and strain rates from 0.0001 to 3 s^{-1}. Previous to every compression tests, the samples were soaked at 1150 ℃ to put the microalloying elements into solution. The elastic region of flow curves was subtracted for subsequent flow stress analyses and modeling. More information about the hot deformation experiments on this material has been reported elsewhere [3,18] and are here revisited.

Table 1: The chemical composition of the investigated steel.

Element	C	Mn	Si	P	S	V	Al	Ti	N
Wt.%	0.34	1.52	0.72	0.025	0.025	0.083	0.0145	0.018	0.0114

3. Results and discussion

3.1. Flow curves

The obtained flow curves are shown in Fig. 1. These curves illustrate the conventional DRX behavior, showing a broad peak with subsequent flow softening. During initial stages of deformation, the dislocations multiplication and interaction result in an

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increase in the flow stress, during which dynamic recovery is underway. After the dislocation density reaches a critical value, the DRX becomes operative and because of softening that occurs by DRX process, the flow curve experiences a peak and afterward, a flow softening region appears and continues till reaching the steady-state stress.
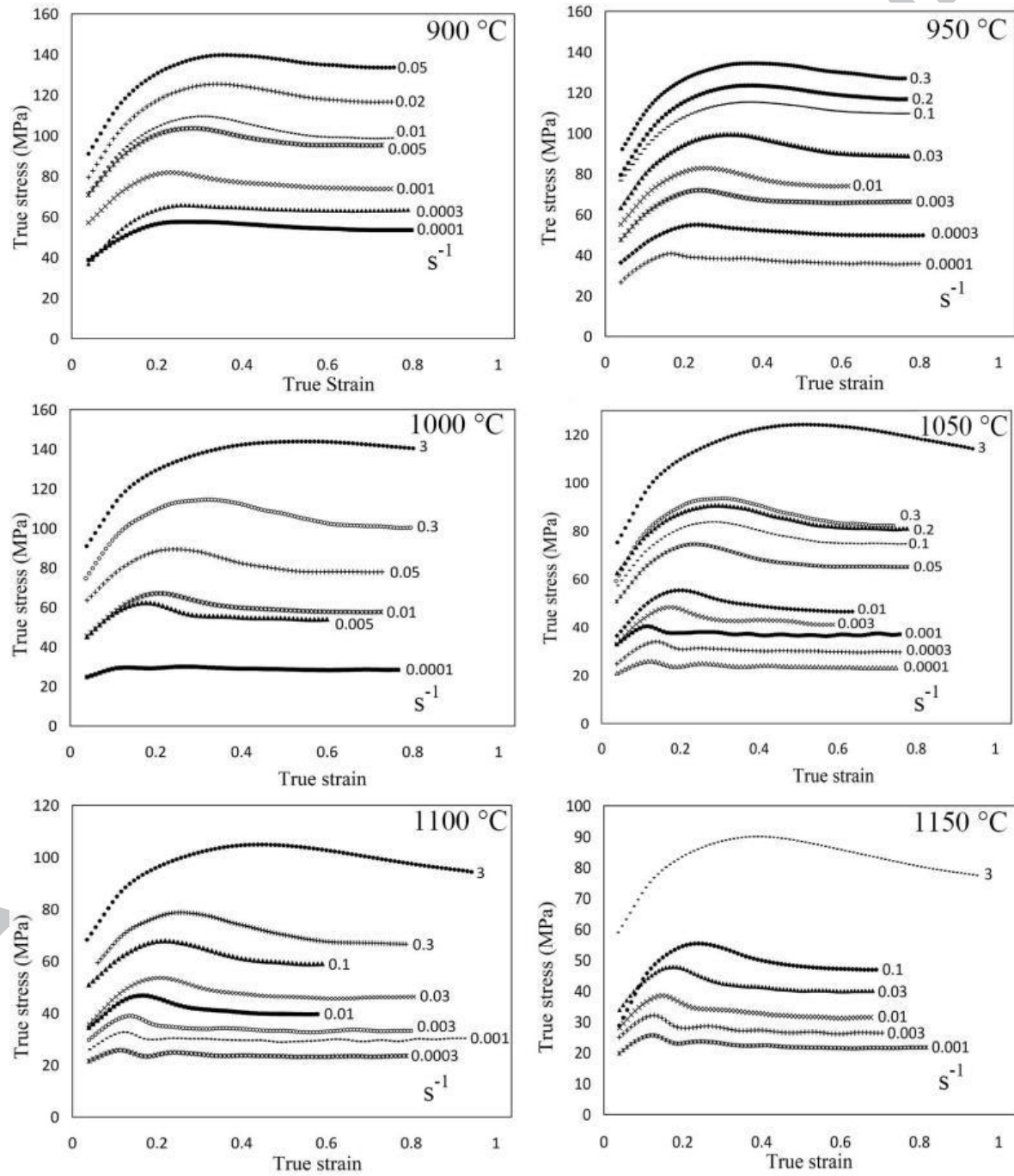

Fig. 1: The obtained flow curves.

The peak stress becomes less obvious when the strain rate is increased or the deformation temperature is decreased. However, the cyclic or multiple peaks DRX can be observed for high temperatures and low strain rates. The concurrent effect of temperature and strain rate can be represented by the Zener–Hollomon (Z) parameter in the form of $Z = \dot{\varepsilon} \exp(Q/RT)$, where Q is the deformation activation energy, R is the universal gas constant (8.314 J.mol^{-1}.K^{-1}), and T is the absolute temperature (in Kelvin). The value of Q was taken as 270 kJ/mol, which is the value reported for the lattice self-diffusion activation energy in austenite [6,18]. As shown in Fig. 1, by increasing Z (decreasing temperature and increasing strain rate), the stress values increases. This is also illustrated in Fig. 2 for peak stresses based on a power law relationship and it can be seen that a linear relation exists between lnσ_p and ln$Z$ and this shows that the Z parameter can appropriately predict the effect of deformation temperature and strain rate on the flow stress. This point will be revisited later. In the following sections, different models will be applied to model the flow curves.
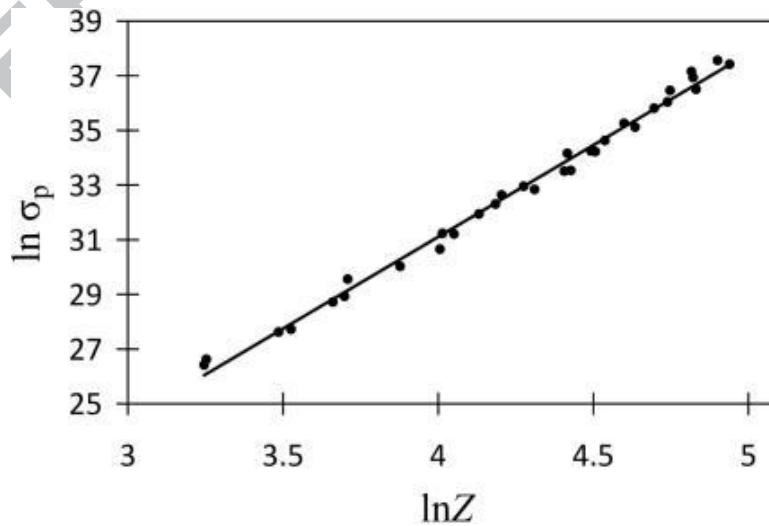

Fig. 2: The power law constitutive analysis.

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3.2. The Johnson-Cook (JC) model

The most common form of constitutive equation by consideration of the effects of strain, strain rate, and deformation temperature has been proposed by Johnson and Cook [19] as shown below:

$$\sigma = \left(\sigma_{0r} + B\varepsilon_{plastic}^{n}\right) \times \left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right) \times \left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^{q}\right) \tag{1}$$

where $\dot{\varepsilon}_r$ and T_r, T_m, and σ_{0r} are the reference strain rate, reference temperature, the melting point of the material (1538 °C in the present steel), and the yield stress at reference temperature and strain rate, respectively. Moreover, since the elastic strain can be neglected, it is usual to use the total strain (ε) instead the plastic strain ($\varepsilon_{plastic}$). However, it should be noted that the correct procedure is to remove the σ_{0r} term while applying the total strain (ε) into the latter equation [6,20].

In the Johnson-Cook equation, the three groups of terms in parentheses represent work-hardening (based on the constants n and B), strain rate (based on the constant C), and thermal (based on the constant q) effects, respectively [6]. In the current work, the lowest temperature and strain rate were considered as the reference values. Therefore, T_r = 900 °C and $\dot{\varepsilon}_r$ = 0.0001 s^{-1} and it was found that σ_{0r} = 21.7 MPa. At reference temperature and strain rate, Eq. (1) simplifies as $\sigma = \sigma_{0r} + B\varepsilon^{n}$ or $\sigma - \sigma_{0r} = B\varepsilon^{n}$. Taking natural logarithm from each sides of the latter equation gives $\ln(\sigma - \sigma_{0r}) = \ln B + n\ln\varepsilon$. Therefore, the slope and the intercept of the plot of $\ln(\sigma - \sigma_{0r})$ against $\ln\varepsilon$ (Fig. 3a) was used for obtaining the values of n = 0.128 and $\ln B$ = 3.602 (B = 36.67 MPa).

At the reference temperature, Eq. (1) can also be simplified as $\sigma / \left(21.7 + 36.67\varepsilon^{0.128}\right) = 1 + C\ln(\dot{\varepsilon}/0.0001)$. Therefore, the slope of the plot of

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$\sigma/\left(21.7+36.67\varepsilon^{0.128}\right)$ vs. $\ln(\dot{\varepsilon}/0.0001)$ at constant strains (0.1 to 0.7 at an interval of 0.1) and various strain rates by consideration of the intercept of 1 gives the value of C = 0.221 as shown in Fig. 3b.

At the reference strain rate, Eq. (1) is simplified as $1-\sigma/\left(21.7+36.67\varepsilon^{0.128}\right)=\left[(T-1173)/638\right]^{q}$. Taking natural logarithm from both sides of this equation gives $\ln\left[1-\sigma/\left(21.7+36.67\varepsilon^{0.128}\right)\right]=q\ln\left[(T-1173)/638\right]$. Therefore, the slope of the plot of $\ln\left[1-\sigma/\left(21.7+36.67\varepsilon^{0.128}\right)\right]$ vs. $\ln\left[(T-1173)/638\right]$ at constant strains (0.1 to 0.7 at an interval of 0.1) and various temperatures by consideration of the intercept of 0 gives the value of q = 0.444 as shown in Fig. 3c. Therefore, the JC equation can be summarized as follows:

$$\sigma=\left(21.7+36.67\varepsilon^{0.128}\right)\times\left(1+0.221\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{r}}\right)\times\left(1-\left(\frac{T-1173}{638}\right)^{0.444}\right) \qquad (2)$$

As it is clear from Fig. 3b and c, the consistency of the linear fitting method to obtain the values of C and q is very poor and C and q are dependent on strain. This dependency is in contrast to the hypothesis behind the JC model. This is a prevalent problem of the JC model and similar results have been also reported for other materials [6,21-23]. Based on these results, in the following section, the constants C and q were considered as functions of strain to address this issue.

The comparison between the experimental flow curves and predicted ones is shown in Fig. 4. As it is apparent, the original JC model, cannot adequately predict the flow curves at hot working conditions. When the difference between a given testing and the reference conditions increases, a significant deviation will be resulted and the prediction

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ability will be impaired. Moreover, due to the parabolic form of the model, the original JC equation cannot represent the softening stage resulted form DRX. Therefore, the Ludwik form in the first parenthesis should be applied separately for hardening and softening stages. As a result, two values for n and B should be considered.
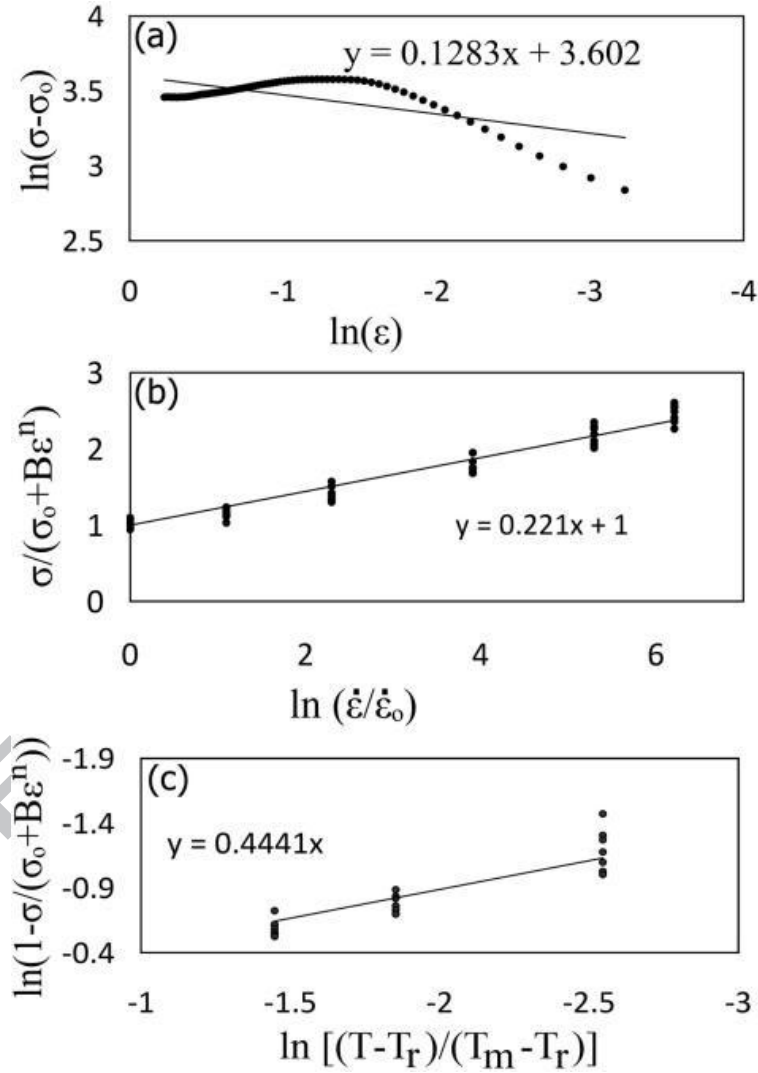

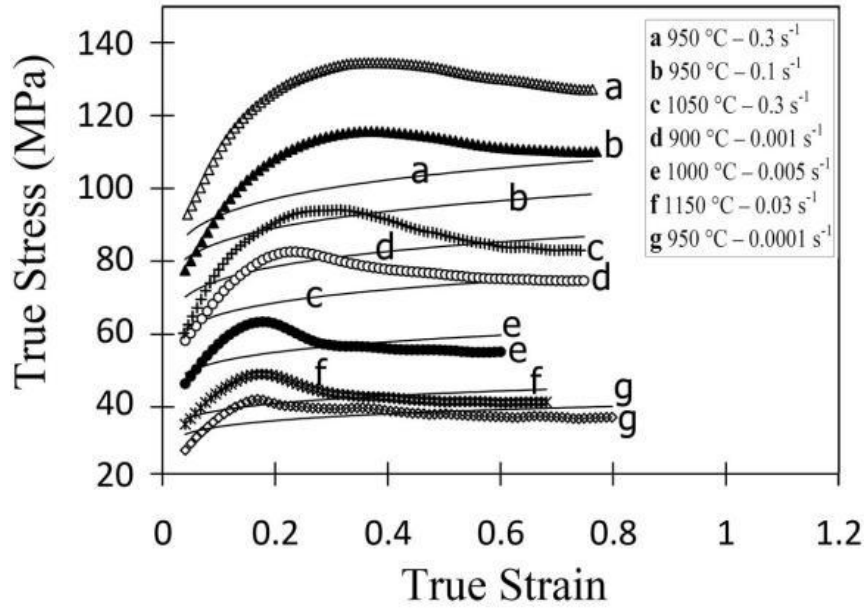Fig. 3: Plots used to determine the constants of the original JC model.

Fig. 4: Comparison between the experimental and the calculated flow curves by the
original JC model.

3.3. The modified JC model

To address the problems of the original JC model, two values for n and B were
considered before and after the peak stress (n_1, n_2, B_1, B_2) and the constants C and q
were considered to be strain dependent. The required plots for determination of n and B
values are shown in Fig. 5a and 5b, which resulted in the values of $n_1 = 0.418$, $n_2 = -0.143$, $B_1 = 66.93$, and $B_2 = 30.37$. The next step is attaining C and q as functions of
strain. For this purpose, the values of C and q were determined at constant strains (0.1 to
0.7 at an interval of 0.1) as shown in Fig. 5c and the equations $C = 0.0303 \times \varepsilon + 0.2028$
and $q = -0.1391 \times \varepsilon + 0.4894$ resulted.

The flow curves were calculated by consideration of n_1 and B_1 before and n_2 and B_2
after the peak point of each flow curve and the experimental values of the peak strain
were used for this distinction. This was also done in the models proposed in Sections

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3.4 and 3.5. The comparison between calculated flow curves with experimental ones is shown in Fig. 6 and, as it is obvious, this method cannot predict adequately either.
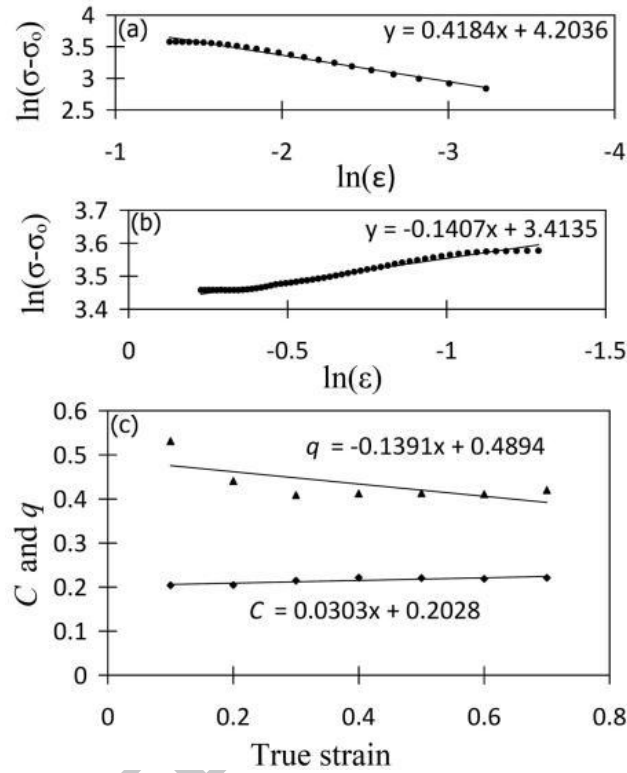

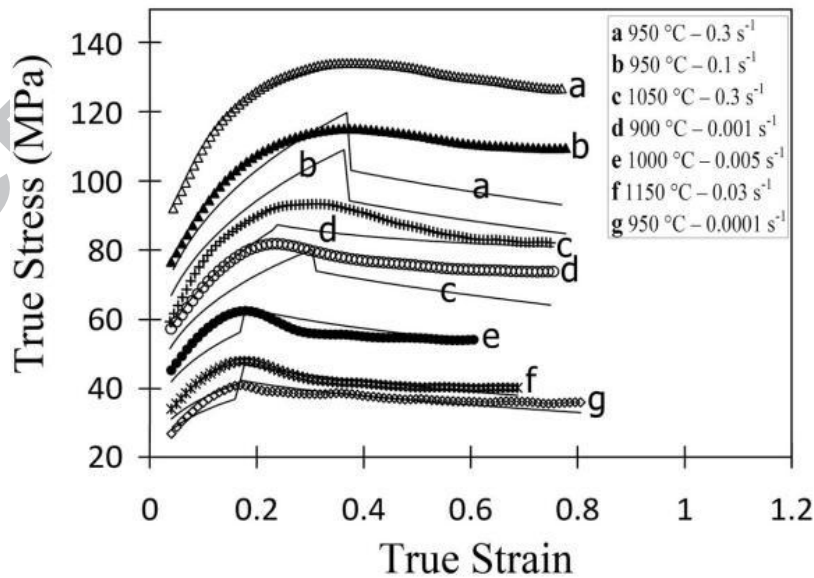Fig. 5: The plots used to determine the constants of the modified JC model.

Fig. 6: Comparison between the experimental and the calculated flow curves by the

modified JC model.

It should be noted that many other modifications were applied but the results were not satisfactory. Some of them are (1) higher order polynomial fitting for C and q, (2) changing the reference conditions, (3) removing σ_{0r}, (4) substituting the strain rate term of $\left(1 + C \ln \dot{\varepsilon} / \dot{\varepsilon}_r \right)$ with the Fields–Backofen form of $\left(\dot{\varepsilon}^m \right)$, (5) neglecting the thermal term and consideration of temperature dependency of B, C, and n.

From this section, it is easy to get that the effect of strain rate and temperature on the flow stress cannot be isolated from each other. Therefore, in previous research works, more sophisticated modifications were considered. For instance, Vural and Caro [23] acknowledged that the amount of strain hardening (B) decreases faster than predicted by thermal softening rate in the JC model and subsequently modified the strain-hardening coefficient. They also showed that the JC model exhibits unrealistically small strain-rate dependence at high temperatures essentially because of completely uncoupled nature of strain-rate sensitivity from thermal effects. Therefore, they also modified the strain rate sensitivity parameter (C) in order to include enhanced rate sensitivity at elevated temperatures, which is observed particularly in quasi-static strain rate regime, and also to introduce an enhanced rate-sensitivity in dynamic regime [23]. However, these modifications add extra complexities to the model. It can be concluded that a parameter that considers the coupled effect of temperature and strain rate should be employed to address this problem. As indicated before, the Z parameter is an appropriate one for this purpose and by removing the σ_{0r} and relating B and n with Z, the Hollomon equation can be evaluated for modeling the flow curves.

3.4. Modeling by the Hollomon equation

To apply the Hollomon equation ($\sigma = B\varepsilon^n$), n_1, n_2, B_1 and B_2 were determined for all flow curves and then their relations with Z was plotted in Fig. 7. It can be seen that the Z parameter fairly correlates well with the experimentally determined values. The obtained equations are also shown in Fig. 7 and it is obvious that, with increasing Z, the values of the strength coefficients B_1 and B_2 increase.

The comparison between the calculated flow curves with experimental ones is shown in Fig. 8 and although certain accuracy is noticed in the hardening stage, the flow stress prediction after the peak value (softening region) obviously fails. The worst issue of this method is the large separation between the two parts of the calculated flow curves, i.e., the lack of continuity at the peak stress. Conclusively, it is apparent that a successful model should compensate the separation at the peak point. In the following section, a new constitutive equation is proposed to address all of the mentioned problems.
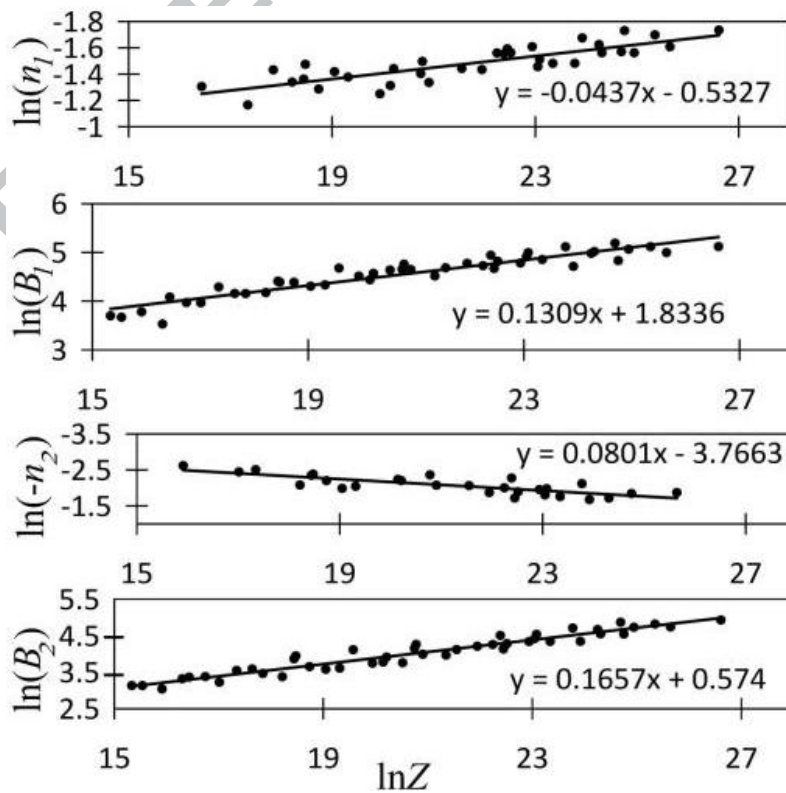
Fig. 7: Determination of the constants of the Hollomon equation as functions of Z.
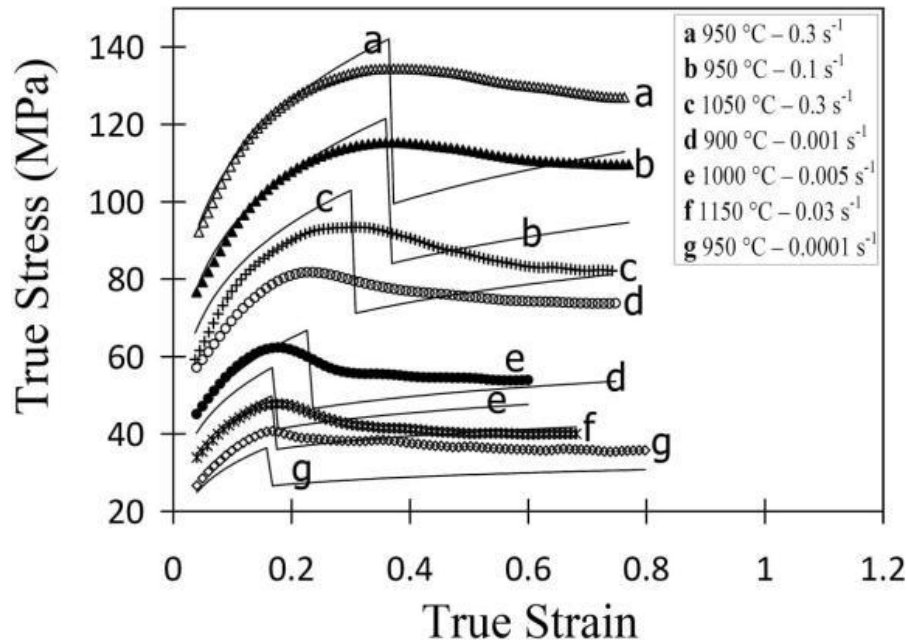

Fig. 8: Comparison between the experimental and the calculated flow curves by the

Hollomon equation.

Before proposing a new approach, it should be noted that Sheng and Shivpuri [24] have successfully modeled the hot flow curves by consideration of four different constitutive equations for the hardening stage (Hollomon equation by correlating its constants with Z using quadratic polynomial fits), stable stage, softening stage (by incorporation of peak stress), and steady-state region (by including the strain corresponding to the onset of steady-state regime) [24]. While this model can offer good results for different portions of flow curves but still add extra complexities to the model.

3.5. The proposed constitutive equation

To solve the problems of the original Hollomon equation, the following equation by incorporation of the peak stress (σ_P) and the corresponding strain (ε_P) was proposed:

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$$\sigma = \sigma_P - B\left|\varepsilon - \varepsilon_P\right|^n \tag{3}$$

For representation of hardening and softening stages, Eq. (3) was applied to flow data before and after the peak point separately, and therefore, the four Z functions of n_1, n_2, B_1 and B_2 were considered. Based on the formula, at $\varepsilon = \varepsilon_P$, the equation simplifies to $\sigma = \sigma_P$, which remedies the problem of separation at the peak point. Since $B\left|\varepsilon - \varepsilon_P\right|^n$ is always a positive value, the values of σ will be less than σ_P. In this way, it is anticipated that the proposed constitutive equation can appropriately predict the flow curves.
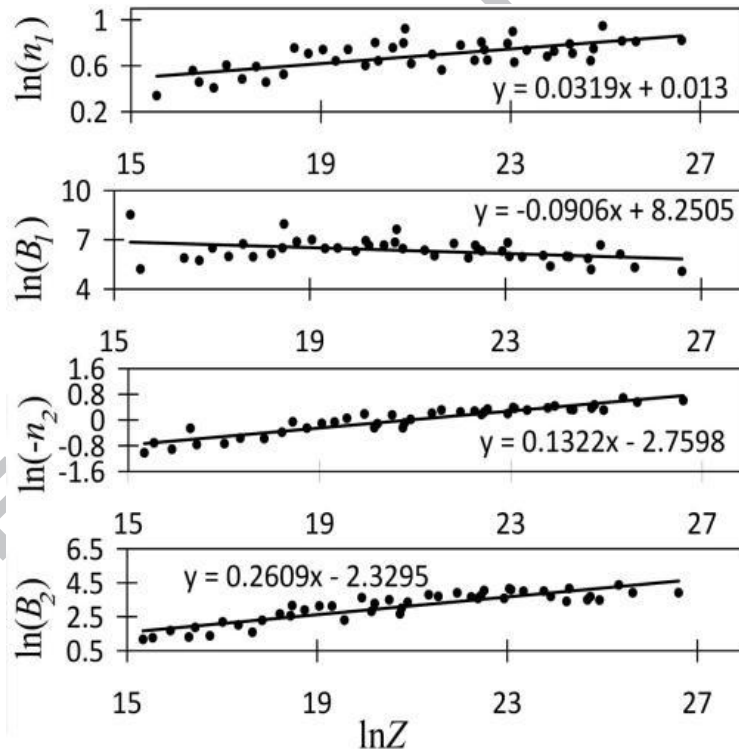

Fig. 9: The plots used to determine the constants of the proposed equation as functions of Z.

The obtained equations for n_1, n_2, B_1 and B_2 are shown in Fig. 9, and subsequently, the flow curves were calculated by consideration of n_1 and B_1 before and n_2 and B_2 after the peak point of each flow curve and the experimental values of the peak stress and strain

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were used for these calculations. The comparison between the calculated flow curves with experimental ones is shown in Fig. 10 and it can be seen that the consistency between the calculated flow stress and the experimental ones are satisfactory and the problem of separation at the peak point has vanished. Moreover, this model can adequately represent both the hardening and softening regions. The capability of the used methods has been summarized in Table 2 for 45 flow curves, by consideration of the root mean square error (RMSE) as defined below:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(t_i - y_i)^2} \tag{4}$$

where t_i and y_i are the experimental and calculated values, respectively. The average RMSE for the proposed approach is significantly lower than the other methods. This confirms the better applicability of the proposed equation (Eq. 3) for flow stress calculation, which is expressed in a simple form including typical parameters, i.e. peak stress and peak strain, and four Z functions of n_1, n_2, B_1 and B_2. It can be deduced that the proposed modification of the Hollomon equation with incorporation of the effect of the deformation temperature and strain rate into the Z parameter is an effective method for modeling of high-temperature flow curves. Nevertheless, it should be noted that due to the parabolic form of the Hollomon equation, it is not suitable for representation of the steady-state regime, basically.

Table 2: The RMSE for the various modeling techniques used in this study.

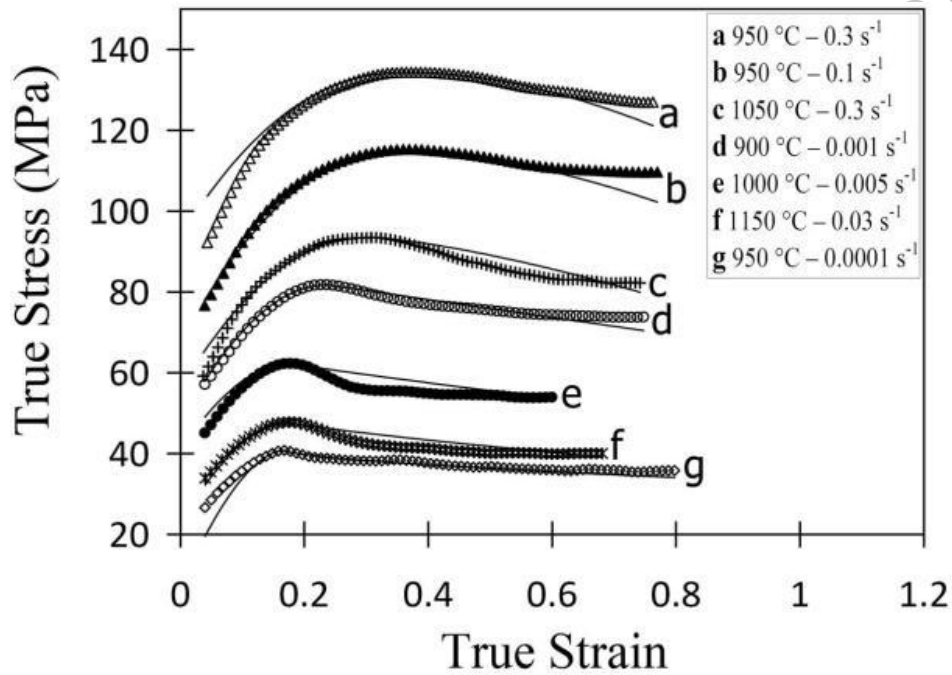| Method	Original JC	Modified JC	Hollomon Eq.	Proposed Eq.
RMSE (MPa)	10.1	9.6	12.2	2.8

Fig. 10: Comparison between the experimental and the calculated flow curves by the

proposed equation.

4. Conclusions

The constitutive behavior of medium carbon microalloyed steel during hot working over

a wide range of temperatures and strain rates was studied using the Johnson-Cook (JC)

model, the Hollomon equations and their modifications. The following conclusions can

be drawn from this study:

(1) The original JC model was not able to predict the softening part of the flow curves

and the subsequent modifications of the JC model to account for the softening stage and

strain dependency of the constants were not satisfactory owing to the uncoupled nature

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of the JC approach regarding strain rate and temperature. The average RMSE for the original and the modified JC methods were determined as 10.1 and 9.6, respectively. These high values show that the JC model is not an optimized one for modeling and prediction of hot deformation flow curves.

(2) The coupled effect of deformation temperature and strain rate was considered in the form of Zener-Hollomon parameter (*Z*) and the constants of the Hollomon equation were related to *Z*. This was found to be useful for the hardening stage but the overall consistency between the experimental flow curves and the calculated ones was not good. The worst issue of this method was the large separation between the hardening and softening parts of the calculated flow curves at the peak point.

(3) A simple constitutive model was proposed, in which by utilization of the peak stress and strain into the Hollomon equation in the form of $\sigma = \sigma_P - B|\varepsilon - \varepsilon_P|^n$, very good prediction abilities were attained. It was found that the proposed approach with incorporation of the effect of the deformation temperature and strain rate into the *Z* parameter and the peak stress and peak strain into the Hollomon equation is an effective method for modeling of high-temperature flow curves.

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Figure Captions

Fig. 1: The obtained flow curves.

Fig. 2: The power law constitutive analysis.

Fig. 3: Plots used to determine the constants of the original JC model.

Fig. 4: Comparison between the experimental and the calculated flow curves by the original JC model.

Fig. 5: The plots used to determine the constants of the modified JC model.

Fig. 6: Comparison between the experimental and the calculated flow curves by the modified JC model.

Fig. 7: Determination of the constants of the Hollomon equation as functions of Z.

Fig. 8: Comparison between the experimental and the calculated flow curves by the Hollomon equation.

Fig. 9: The plots used to determine the constants of the proposed equation as functions of Z.

Fig. 10: Comparison between the experimental and the calculated flow curves by the proposed equation.
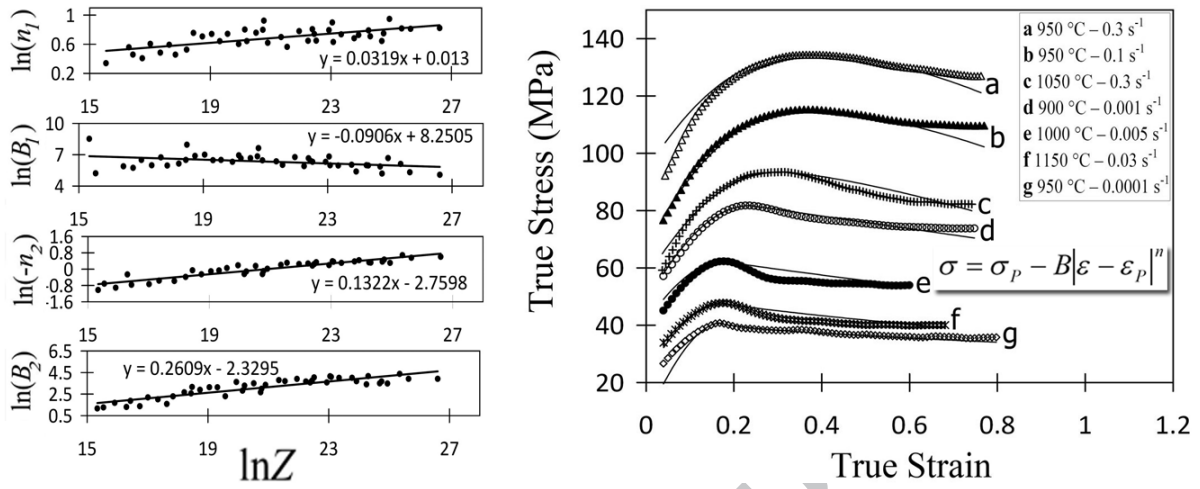
Table Captions

Table 1: The chemical composition of the investigated steel.

Table 2: The RMSE for the various modeling techniques used in this study.

Graphical abstract

Research Highlights

- Hot deformation behavior of medium carbon microalloyed steel

- The coupled effect of temperature and strain rate in the form of Z parameter

- A simple constitutive model for predicting flow stress during hot deformation