THE ROLE OF THE DEPTH-AVERAGED CONCENTRATION IN COASTAL MORPHODYNAMICS

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Abstract

In this contribution a discussion is presented on the development of self-organized coastal morphodynamic patterns which are due to the joint action of gradients in the depth-integrated concentration and the flow. This is done in the context of a depth-averaged shallow water model. Two physical mechanisms produce deposition-erosion patterns. Deposition either occurs where the current flows from high to low depth-averaged concentrations (1) or where the flow diverges (2). If flow conditions are quasi steady (i.e., the time scale on which bedforms evolve is much larger than the hydrodynamic time scales) only the former mechanism contributes to the formation of bottom patterns.

Key words: morphodynamics, hydrodynamics, sediment transport, sediment concentration, self-organization

1. Introduction

There are many beaches where the irregularities in their topography along the coast are small at the scales comparable to the surf zone width. Their cross-shore profile can be assumed to be approximately uniform along the coast and hence, alongshore gradients can be safely ignored. They are called 2D beaches. On the contrary, other beaches exhibit very clear morphological patterns in plan-view at those lengthscales. Their hydrodynamics and their morphodynamics is profoundly influenced by these patterns so that the assumption of alongshore uniformity is not applicable anymore. These beaches are called 3D beaches. These large scale patterns can range from quite irregular to fairly periodic in space along the coast. In the latter case they are referred to as rhythmic topography.

The most common rhythmic patterns are crescentic bars, shore-transverse bars, shore-oblique bars, finger bars or beach cusps are distinct morphological pattern pertaining to 3D beaches (Carter, 1988; Short, 1999). On the continental shelf, shoreface-connected ridges, tidal sands banks and sand waves (Huntley et al., 1993; Dyer and Huntley, 1999; Besio, 2006) are well known rhythmic morphological patterns. Although in some cases the rhythmic morphological patterns could be driven by hydrodynamic patterns (eg. edge waves), a very plausible explanation is that their origin lies in positive feedback mechanisms between the bathymetry and the hydrodynamics which render unstable the 2D morphology. These inherent instabilities are associated to self-organization processes in the coupling between flow and morphology that eventually lead to those patterns. In recent years there has been much research in this line, see for instance Christensen et al. (1994), Caballeria et al. (2002), Damgaard et al. (2002), Reniers et al. (2004), Garnier et al. (2005), Vis-Star et at. (2008).

In this contribution a general discussion is presented on the development of self-organized coastal morphodynamic patterns which are due to the joint action of gradients in the depth-integrated concentration (also called potential stirring) and the flow perturbations produced by the bedforms. This is done in the context of a depth-averaged shallow water model. In spite of its remarkable simplicity, this analysis has proven to be a powerful tool to get insight into the underlaying feedback mechanisms between the morphology and the hydrodynamics.

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2. The sediment stirring function

Sediment transport on coastal environment is a very complex process that depends on the sediment characteristics and the hydrodynamics induced by waves and currents. Currently this is a process that is not completely understood due to lack of observations in natural environments and the complexity of the hydrodynamics and the movement of sand grains. There have been many attempts to develop complex formulas to predict sediment transport, but these have limited applicability (SEDMOC, 2001). In most of the studies on coastal morphodynamics, as the referred above, sediment transport is described by simple formulas that work reasonably well, in the sense that they capture the overall characteristics of the process. These formulas can be summarized as

\[ q = \alpha v - \gamma \nabla z_b \]  

(1)

where \( q \) is the volumetric transport of sediment per unit width \( (m^2s^{-1}) \).

The first term on the right represents the advection of sediment (bedload, suspended load or total load transport) by the flow, in the sense that it represents the transport of sediment by the 3D flow of the 3D concentration of sediment. This is parameterized to be directed towards the depth-averaged horizontal current and \( \alpha \) corresponds to the total load, ie. depth-integrated concentration. Hereinafter we will be referred to \( \alpha \) as the depth-integrated sediment concentration. The \( \alpha \) depends nonlinearly on the hydrodynamics variables, as the magnitude of the current and near bed wave orbital velocity amplitude, and the sediment properties, but it can also depend on the water depth, grain diameter and other parameters. The \( \alpha \) function have been referred as ‘stirring function’ by some authors (eg, Falqués et al, 2000). The reason is that equation (1) may be interpreted as the sediment being picked up by the stirring due to the waves and the current up to a load \( \alpha \) and being transported by the current.

The second term on the right of equation (1) corresponds to the downslope transport term due to gravity. Here, \( z_b \) is the bed level, \( \gamma \) is another function which generally depends on the current, the wave orbital velocity and other parameters. The gradient is with respect to the horizontal coordinates. As we will show later on, this term is not relevant for our aim here and we will ignore it for the sake of simplicity.

The table 1 shows examples of the \( \alpha \) function for different transport formulas. Other sediment transport parameterizations used in coastal morphodynamics are the ones of Ackers and White (1973), Bijker (1968), Dijbajnia and Watanabe (1992) or Ribberink (1998). An illustration of different sediment parameterizations can be found in Camenen and Larroudé (2003).

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.04 ( C_d (s-1)^{1/4} \frac{U^4}{d_50} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engelund and Hansen (1974)</td>
<td></td>
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<tr>
<td>Bailard (1981), bedload</td>
<td>( \frac{c_s e_s}{g(s-1)\theta_b} U_b^2 )</td>
</tr>
<tr>
<td>Bailard (1981), suspended load</td>
<td>( \frac{c_f e_s}{g(s-1)w_s} U_b^2 )</td>
</tr>
<tr>
<td>Grass (1981)</td>
<td>( A_p \left( U^2 + 0.08 \frac{U^2_{max}}{C_d} \right)^{n+1/2} )</td>
</tr>
<tr>
<td>Soulsby – van Rijn (Soulsby,1997)</td>
<td>( A_p \left( U^2 + 0.018 \frac{U^2_{max}}{C_d} \right)^{1/2} - U_{crit} \right)^{2/4} )</td>
</tr>
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</table>
3. Bottom evolution equation

In this section we will derive an alternative form of the equation for the evolution of bottom for depth-averaged shallow water model. The coordinate system is shown in figure 1. We will assume a Cartesian coordinate system where \((x, y, z)\) are a cross-shore, longshore and vertical coordinate, respectively. Here, \(\nabla\) is the 2D horizontal nabla operator. The bed and the sea surface levels are represented by \(z_b(x, y, t)\) and \(z_s(x, y, t)\), respectively, and the total water depth is \(D = z_s - z_b\).

![Figure 1. Sketch of the coordinate system.](image)

Considering only the term \(\alpha v\) of equation (1), the bottom evolution equation becomes

\[
(1-p)\frac{\partial z_b}{\partial t} + \nabla \cdot (\alpha v) = 0
\]

(2)

where \(p (\approx 0.4)\) is the sediment porosity and \(t\) is the time. The mass conservation equation for water reads

\[
\frac{\partial D}{\partial t} + \nabla \cdot (Dv) = 0
\]

(3)

with \(D\) the total depth.

Combining equations (2) and (3) and by taking \(D = z_s - z_b\) into account, the bottom evolution equation is transformed into the following equation:

\[
\left((1-p)\cdot C\right)\frac{\partial z_b}{\partial t} + Dv \nabla C \cdot C \cdot \frac{\partial z}{\partial t} = 0
\]

(4)

where \(C = \alpha / D\) can be interpreted as the depth-averaged concentration. The different terms in the equation (4) are interpreted as follows:

- The first term represents the bottom changes (erosion or deposition), which is the interest of this study.
- The second term describes to bottom changes induced by spatial differences in depth-averaged concentration and the flow moves between regions of different depth-averaged concentration.
- The third term contributes to bottom changes where flow converges or diverges due to a change in the sea level. It describes an additional change in the bed level as a results of a change in the sea level to maintain the same concentration.

At this point it is important to take into account the scale of magnitude to assess the significance of each the terms. The depth-integrated sediment concentration, \(\alpha\), has characteristic values that ranges from \(\sim 10^{-6}\) m\(^3\)/m\(^2\), for bedload conditions, to \(\sim 10^{-3}\) m\(^3\)/m\(^2\), for total load transport (see Soulsby, 1997, for the value of
these orders of magnitudes). The depths at which bottom changes are of order ~1 m. This yields to characteristic values for the depth-averaged concentration, \( C \), that hardly can exceed \( \sim 10^{-3} \text{ m}^{3}/\text{m}^{3} \). Since \( C \ll (1-p) \), the depth-averaged concentration will be neglected in the first term of equation (4). Regarding the last term in equation (4), the scale of variations of the free surface, in the time scale of morphological changes, are much smaller that the morphological change. Therefore, this term is usually negligible. Only when the variation of the free surface is much larger than the variations of the bottom, convergence / divergence of the flow may have a role. This term is particularly important in the swash area, as shown by Dodd et al. (2008).

After all these considerations, the main balance in equation (4) is between the second term and the bottom change the one that and we can rewrite the bottom evolution equation (BEE) in terms of the gradient of the depth-averaged concentration and flow

\[
(1-p) \frac{\partial b}{\partial t} + Dv \cdot \nabla C = 0
\]

(5)

3.1. Remarks on the BEE

Several comments can now be made. If the second term in the r.h.s. of equation (1), \( \gamma \nabla z_b \), is also considered in equations (2) and (4), the BEE becomes:

\[
(1-p) \frac{\partial b}{\partial t} + Dv \cdot \nabla C - \nabla \left( \gamma \nabla z_b \right) = 0
\]

(6)

This term is a diffusive term and tends to damp the emerging bed features (Falqués et al., 1996, and references therein). For this reason, it will thereinafter be ignored.

Equation (6), also (5), assumes that the time scale on which bedforms evolve is much larger than the hydrodynamic time scales, the so called quasi-steady flow conditions. On these conditions, the erosion/deposition mechanisms due to flow divergences do not contributes to the formation of patterns.

As it has already been pointed out, the \( C \) function plays the role of a depth-averaged sediment concentration so that it can be referred to as ‘equivalent depth averaged concentration’. Some authors, following Falqués et al. (1996) and Coco et al. (2002), prefer call it ‘potential stirring’. They also found ‘potential stirring’ much simpler terminology that the other depth-averaged sediment concentration. Here we will stick using the later one and it has clear equivalence with field measurements.

Since there is no a unique parameterization for the function \( \alpha \) (depth-integrated sediment concentration or stirring function), the different sediment parameterizations that can be found in the literature will produce different depth-averaged sediment concentration, \( C \).

In the following sections some applications of this simple rule for erosion/deposition will be explored. As far as the authors know, a similar reasoning was first applied by Schielen et al. (1993) and latter by Trowbridge (1995) in simple and very particular situations (alternate bars in rivers and shoreface connected sand ridges in the inner shelf, respectively). A first general but linearized derivation of this equation for the nearshore was presented by Falqués et al. (1996) and revisited by Falqués et al. (2000) and Ribas et al. (2003). A nonlinear version was presented by Caballeria et al. (2002) and used by Coco et al. (2002) and Garnier et al. (2005).

3.2. Erosion/deposition processes

Equation (5) gives the time evolution of the bed level at any particular location as a function of the water depth, \( D \), the depth averaged current, \( v \), and the gradient of the depth-averaged concentration. In this sense it is not closed since it needs the knowledge of the flow and the sediment load patterns. It could be however used instead of equation (2) in a morphodynamic model together with the accompanying hydrodynamic equations. The main advantage of equation (5) is that it allows for an interpretation of the erosion/deposition processes, expressed in terms of depth-averaged concentration and current, which will be shown bellow.
If the current and the distribution of potential stirring are known (for instance, from field observations, from numerical simulations or just qualitatively from physical reasons), the morphodynamic effect of such a current can be easily inferred. Indeed, according to BEE, \( v \cdot \nabla C > 0 \), will imply \( \partial z_b / \partial t < 0 \) (erosion) and \( v \cdot \nabla C < 0 \), will imply \( \partial z_b / \partial t > 0 \) (deposition). In other words, any current having a component in the direction of the gradient of the potential stirring will produce erosion and any current with a component that opposes this gradient will cause accretion (see figure 2). Obviously, a current directed along the isolines of potential stirring will not produce any morphological change. Note that here only the effects of the current and that the total bed evolution would be obtained by adding the diffusive effects of gravity driven transport to the effects just mentioned.

Another physical explanation may be done by interpreting \( \alpha / D = C \) as a depth averaged concentration. It is necessary to realize that \( C(x, y, t) \) indicates the averaged concentration which is in equilibrium with the flow stirring at any position \((x, y)\) at time \(t\). Then, if \( C \) increases along the flow, water with little sediment load will be carried to places where the sediment load capacity due to the stirring allows for larger sediment load. Therefore, more sediment will be picked up from the bed underneath the control volume which will hence be eroding. Just the contrary will happen if \( C \) decreases along the flow.

3.3. Initial growth of patterns

The initial growth of self-organized bedform patterns can be studied by analysing the dynamics of small perturbations of a steady state. We will assume an alongshore-uniform steady state (equilibrium state), represented by an alongshore current \( V_0 \), concentration \( C_0 \) and depth \( D_0 \) and its perturbations. Therefore, these variables read

\[
v = (0, V_0) + (u, v), \quad C = C_0 + c, \quad D = D_0 + d\tag{7}
\]

Substituting these expression into equation (5) and linearizing with respect the small quantities \((u, v, c, d)\) the linearized bottom evolution becomes

\[
\left(1-p\right)\frac{\partial h}{\partial t} = -D_0 \left( u \frac{dC_0}{dx} + V_0 \frac{\partial c}{\partial y} \right)\tag{8}
\]

From equation (8) it can be seen that the small changes of a known equilibrium state \((V_0, C_0, D_0)\) can be analyzed just from the perturbations that the bottom produces in the cross-shore component of the current and the gradients of the depth-averaged sediment concentration. Note that if \( V_0=0 \), the case of normally
incident waves, erosion/deposition from equation (8) only depends on the cross-shore perturbation of the current and the cross-shore gradients of the depth-averaged sediment concentration.

Flow perturbations just depend on the hydrodynamic module of the morphodynamic models. This part is quite robust and there is little difference between the different models even thought different parameterizations are used for the bottom stress, turbulent mixing, etc. However, regarding the computation of the concentration, different parameterizations lead to different results for C, which strongly affects the morphological changes. Since the bottom evolution depends on the gradients of the concentration, it is essential that the parameterizations of sediment transport not only adequately represent the distribution of concentration, but especially describe the concentration gradients in an adequate manner.

4. Example: Nearshore oblique sand bars.

As an illustration of how to use equation (8) to predict the orientation of nearshore oblique bars, consider an alongshore uniform beach where waves drive the alongshore current and where the combined action of the waves and the current produces a depth-averaged concentration with a cross-shore profile that has a maximum in the surf zone, as it is illustrated in figure 3. Now we will examine the erosion or deposition over an oblique bar in the inner surf zone for different bar orientations.

Consider first the case where the bar is up-current oriented, see figure 4. From both field observations and numerical modeling it is seen that a series of up-current oriented, ie. the shoreward end of the bar is shift upstream with respect its shore attachment, bars separated by troughs in the surf zone produce a meandering of the wave-driven longshore current which veers offshore over the bars and onshore at the troughs (Hunter et al., 197; Trowbridge, 1995; Ribas et al., 2003; Garnier et al., 2005). Since the gradient in the depth-averaged concentration is offshore directed on the bar region ($\partial C / \partial x > 0$), the first term on the right side of equation (8) predicts that for this sort of current deflection ($u > 0$) will cause erosion over the bar crest, and therefore a negative morphodynamic feedback. On the contrary, if the bar would be
oriented downstream would be deflected onshore \((u<0)\) and the bar would grow. Therefore, down-current oriented bars are expected to form in case of shoreward increasing depth-averaged concentration (Ribas et al., 2003; Garnier et al., 2005). For a depth-averaged concentration decreasing shoreward the orientation of the bars is expected to be up-current.

![Figure 5. Sketch of the migration processes for an oblique bar.](image)

Whilst the role of the first term on the right side of equation (8) is related with the growth or decay of the bedforms, the second term of equation (8) is related with the migration of the bars. Analysing the migration is more complicated because this depends on the gradients of the perturbations of the depth-averaged concentration. In general we can say that the direction of migration depends on the phase shift between the bathymetry and the perturbation of the depth-averaged concentration. If its maximum is located at the upstream side of the bar then the migration would be against the current. On the contrary if the maximum is located in the leeward side the bar would migrate in the same direction as the current.

The arguments presented above reveal why the results of morphodynamic models are so sensitive to the parameterization used for sediment transport. Regarding the grow of bedforms, different behaviors of the depth-averaged concentration are going to produce different patterns. Following the example of the oblique bars, if the depth-averaged concentration would decrease offshore inside the inner surf zone, the bar would grow with an upcurrent orientation. The migration of the bars is more sensitive because the phase lags, see Vis-Star et al. (2007), of the depth-averaged concentration with respect the bathymetry depend on the functional dependences of the depth-averaged concentration on the disturbances in the water depth, wave orbital velocities, current, friction parameterization, etc. Figure 6 summarizes the role of each term of equation (8) and the expected orientation of inner surf zone bars.

![Figure 6. Sketch of the migration processes for an oblique bar.](image)
We want to emphasize also that the information on the orientation of the bars can also be used to infer, in a qualitative manner, the distribution of the depth-averaged concentration of sediment. This may be particularly useful to discriminate the validity of different sediment transport parameterizations depending on weather conditions, bathymetry, sediment type and the morphological patterns observed.

5. Numerical model experiments

In this section the model presented by Garnier et al. (2006) is used to illustrate the relation between the shapes of the depth-averaged concentration and the orientation of oblique inner surf zone bars.

The model is forced by waves approaching to the coast with oblique incidence that drives a longshore current. The initial bathymetry is an alongshore uniform planar beach. For the same model configuration two sediment transport parameterizations are used. One is based on the Soulsby-van Rijn transport parameterization (Soulsby, 1997), see table 1, and the second assumes a constant sediment stirring, \( \alpha \), function. As can be seen in figure 7, the depth-averaged sediment concentration for the Soulsby-van Rijn parameterization first increases when moving offshore from the shore and then decreases, while for constant stirring the depth-averaged concentration always decreases moving offshore.

From the arguments of the previous section it is expected that a down-current bar would form close to the coast in the first case, and a up-current bar for the constant stirring case. These predictions are verified by the model. The bathymetry after a few days of morphological evolution from the initial bathymetry for the two sediment parameterizations is plotte in figure 8. It can be seen that the orientation of the bars agree with the prediction using the concepts elaborated in the previous sections. However there is not a full correspondence with those simple predictions. For instance, the depth-averaged concentration in the first case has a maximum, so it would be expected that the orientation of the bars change between both side of the maximum. This is not reproduced by the model. One of the reasons for this disagreement is that when analyzing equation (8) the contribution of each term has to be examined separately whereas actually both are coupled. For instance the second term on the right, which depends on the perturbations of the depth-averaged concentration, could contribute also in the growth of the bars and not only in the migration. In spite of these disagreements, it appears that using the depth-averaged concentration together with equation (8) can help to understand model results and predict sediment transport characteristics from the morphological patterns.
6. Concluding remarks

In this contribution a general discussion is presented on the development of self-organized coastal morphodynamic patterns which are due to the joint action of gradients in the depth-integrated concentration (also called potential stirring) and the flow perturbations produced by the bedforms. This is done in the context of a depth-averaged shallow water model. In spite of its remarkable simplicity, this analysis has proven to be a powerful tool to get insight into the underlying feedback mechanisms between the morphology and the hydrodynamics. The effect of the morphology on the hydrodynamics includes effects like wave refraction, diffraction or shoaling and deflection of currents. Summarising, two physical mechanisms produce deposition-erosion patterns: deposition either occurs where the current flows from high to low depth-averaged concentrations (1) or where the flow diverges (2). On quasi-steady flow conditions (i.e., the time scale on which bedforms evolve is much larger than the hydrodynamic time scales) only the former mechanism contributes to the formation of patterns.

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