Active Control of Sound Transmission through an Aperture in a Thin Wall

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ABSTRACT

The reduction on sound transmission through an aperture by active means is theoretically investigated. The primary field is composed by a plane wave that arrives at a rectangular aperture placed in an infinite rigid thin wall. The sound transmission to the other side is calculated by the Rayleigh radiation equation after continuity conditions have been applied in the aperture plane. The secondary field is generated by a monopole source placed close to the aperture in the incidence side and it is tuned to get zero acoustic pressure at the error microphone, placed in the center of the aperture. The sound transmitted after active control is calculated by the superposition of the primary and secondary fields. The proposed model is compared with a BEM model in order to obtain the range of usability. Preliminary results show good agreement although some deviation is found in the low frequency range. Finally, an example of active control as function of frequency is developed, giving, as expected, better results for low frequency sounds and when the secondary source is placed far from the aperture.

1. INTRODUCTION

Active control strategies offer a possibility to reduce sound transmission through apertures, especially if the diameter of the aperture is small compared to the wavelength of the incident acoustic wave. Active control has been shown to be more effective if the control sources are located at a bottleneck in the flow path, rather than being located close to either primary source or receiver locations. An aperture in a wall or in an enclosure is an example of such bottleneck, thus the application of actively controlled apertures seems a suitable option. One example of such a noise reduction strategy has previously been demonstrated using be a hybrid passive/active system to control the exhaust noise radiated by a small generator. In this example, passive control was achieved using a steel rectangular enclosure lined with a layer of absorbing material, and active control was used to attenuate exhaust noise under 400 Hz. Other examples include the so-called hybrid window used for the natural ventilation of buildings near airports. Here, short ventilation ducts are installed below the window, inside which active control is applied to attenuate exterior noise. In each case, noise is forced to propagate through an area restriction, allowing for a more straightforward application of active control. Chen studied the active reduction of the sound transmission through a small width aperture in a wall by using a secondary source in the aperture. These duct devices, although quite effective, are somewhat
bulky for industrial enclosures or exterior building windows. Considering a more realistic thin wall (compared to wavelength), Emms et al.\(^5\) proposed the application of an active absorber at the aperture to control the sound transmission through it. Using BEM method as a calculation procedure, three types of active absorbers were compared regarding their ability to reduce sound transmission through a square aperture in an infinitesimally thin, infinitely large rigid wall. Romeu et al.\(^6\) demonstrated experimentally the effectiveness of the virtual earth configuration by locating an error microphone at the plane of the aperture and a secondary source, in the incident region, immediately upstream of the aperture. Nishimura and his coworkers\(^7\) have recently demonstrated that placing secondary sources in the aperture can attenuate the sound transmission into an enclosure, in the case of both a stationary and moving primary source.

The purpose of this investigation is to present an analytical model for the active control of sound transmission through an aperture in an infinite rigid wall, using a local control strategy. The aim is to cancel the sound pressure at the aperture boundary and, thus, to reduce sound transmission. The study focuses on the low frequency range where active noise control results in large quiet zones. At low frequencies, the thickness of the wall is assumed negligible compared to the wavelength of the incident waves at the aperture, as demonstrated by Serizawa\(^8\). The setup involves a secondary sound source located immediately upstream of the aperture and an error microphone placed at the aperture plane. The model is validated by comparing its results to those obtained by BEM model of the same problem. After that, some preliminary results of active control are given.

2. THEORY

The geometry of the test setup is illustrated in Fig. 1. The coordinate origin is placed at the center of the squared aperture of a size. In the region \(x < 0\), the primary field is an incident plane wave arriving at the aperture with normal or oblique incidence plus the plane wave reflected by the aperture. The model here is a simplification of previous one developed by some of the authors to approach the radiation from an aperture in a wall of an enclosure\(^9,10\). What is new here is that the incident field is composed by an incident plane wave instead of a modal sound field. Thus, assuming a time dependence of \(e^{i\omega t}\) for all sound fields, the incident pressure wave considered is:

\[
P_i = \hat{P}e^{ik(y \sin \theta_1 \cos \phi_1 + z \sin \theta_1 \sin \phi_1 + x \cos \theta_1)}
\]

where \(\theta_1\) and \(\phi_1\) denotes the angles of incidence of the primary field (Fig. 1) and \(k\) is the wave number. The pressure at the \(x < 0\) region is the sum of the incident and reflected pressure waves. Thus, considering that the aperture surface is not rigid, the pressure at the \(x < 0\) region could be expressed as

\[
P^- = \frac{2 \cos \theta_1}{\beta_1 + \cos \theta_1} \hat{P}e^{ik(y \sin \theta_1 \cos \phi_1 + z \sin \theta_1 \sin \phi_1 + x \cos \theta_1)}
\]

where \(\beta_1\) is the specific acoustic admittance of the opening surface. The pressure at the \(x > 0\) region of the aperture is expressed as

\[
P^+ = -\frac{ik\rho c}{2\pi} \int \int \mu_0(y',z') e^{ikR} dy' dz'
\]

where \(R = \sqrt{x^2 + (y-y')^2 + (z-z')^2}\) and

Noise-Con 2014, Ft. Lauderdale, Florida, September 8-10, 2014
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\[ y' = -a/2 < y < a/2 \]
\[ z' = -a/2 < z < a/2 \]  \hspace{1cm} (4)

\( u_o (y', z') \) is the fluid normal particle velocity in the opening and \( R \) is the distance between the (point) source and the field point.

Having the expressions for the pressure at both sides of the wall, continuity conditions of pressure and particle velocity have to be considered at the opening surface. And finally, normal particle velocity can be related to the pressure using the specific acoustic admittance \( \beta \) of the boundary surface.

The application of the boundary conditions results on the expression of the admittance of the aperture due to the primary field:

\[
\int_{S_{op}} e^{ikR} dy' dz' = -i k \beta \int_{S_{op}} \int_{S_{op}} e^{ikR} \frac{e^{iKR}}{2\pi R} dy' dz' dy dz
\]  \hspace{1cm} (5)

where \( R_i = y' \sin \theta_i \cos \phi_i + z' \sin \theta_i \sin \phi_i \). A Gauss numerical integration method has been used to obtain the admittance of the surface due to a plane incident wave at the \( x < 0 \) region. Once the admittance is known, pressure and particle velocity at the aperture can be calculated and finally, radiated pressure can be obtained.

By considering the acoustic field as a linear superposition of fields, it could be assumed that the pressure at one point, in the \( x < 0 \) region, is the sum of the primary and a secondary source (caused by a secondary source in the incidence region) pressure contributions plus the reflections caused by both incident waves. The purpose of this work is to investigate the potential effectiveness of an active noise control system, with one error microphone placed in the center of the aperture. Thus, the strength of the secondary source can be obtained making the acoustic pressure at the error microphone equal to zero:

\[
P_s (r_e) = \frac{2 \cos \theta_1}{\beta_1 + \cos \theta_i} \hat{P} e^{iKR_1} + \frac{2 \cos \theta_2}{\beta_2 + \cos \theta_i} Z_2 q \]  \hspace{1cm} (6)

where \( P_s (r_e) \) is the pressure at the error microphone position, \( Z_2 \) is

\[
Z_2 = -\frac{ik \rho c}{4\pi R_2} e^{iKR_2}
\]  \hspace{1cm} (7)

and \( R_2 \) is the distance between secondary source and error microphone.

3. VALIDATION

The problem of Fig. 1 considering only one primary source is also solved by using the BEM method. A point source is placed far enough from the wall to have an incident plane wave at the square-shaped aperture of dimension \( a=0.3 \) m placed in a rigid wall of dimensions large enough to avoid the diffraction effect at its edges. 45 different simulation cases have been solved, within a frequency range from 100 to 500 Hz, and an incident angle, \( \phi \) (Fig. 1), swept from \( 0^\circ \) to \( 80^\circ \). The incident angle \( \theta \) (Fig. 1) is maintained null in all simulations and the incident pressure, which is also kept invariable along the different simulation cases, is 0.0182N/m\(^2\).

The parameter to be compared is the transmission index of the aperture. The transmission index is defined as

\[
\]  \hspace{1cm} (8)

is the incident power calculated by

\[
\]  \hspace{1cm} (9)

where \( \hat{P} \) is the incident intensity estimated by

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and \( s \) is the aperture surface.

\( \mathcal{S} \) is the transmitted power, calculated from the pressure measured at twenty points distributed homogeneously over a hemispherical surface as in the ISO standard\(^{12} \):

\[
\mathcal{S} = \sum_{j=1}^{20} p_j^2
\]

\( \mathcal{S} \) is the transmitted power, calculated from the pressure measured at twenty points distributed homogeneously over a hemispherical surface as in the ISO standard\(^{12} \):

\[
\mathcal{S} = \sum_{j=1}^{20} p_j^2
\]

**Figure 1:** Co-ordinate system and location of the calculation points.

**A. Results**

The preliminary results obtained from the analytical and BEM model are shown in Tables 1 and 2.

**Table 1:** Transmission index of the aperture obtained with the analytical proposed model

<table>
<thead>
<tr>
<th>Incident angle, ( ^\circ )</th>
<th>( f=100\text{Hz} )</th>
<th>( f=200\text{Hz} )</th>
<th>( f=300\text{Hz} )</th>
<th>( f=400\text{Hz} )</th>
<th>( f=500\text{Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1655</td>
<td>0.4506</td>
<td>0.6603</td>
<td>0.7857</td>
<td>0.8565</td>
</tr>
<tr>
<td>10</td>
<td>0.1608</td>
<td>0.4394</td>
<td>0.6440</td>
<td>0.7650</td>
<td>0.8320</td>
</tr>
<tr>
<td>20</td>
<td>0.1474</td>
<td>0.4064</td>
<td>0.5968</td>
<td>0.7058</td>
<td>0.7624</td>
</tr>
<tr>
<td>30</td>
<td>0.1266</td>
<td>0.3544</td>
<td>0.5231</td>
<td>0.6153</td>
<td>0.6585</td>
</tr>
<tr>
<td>40</td>
<td>0.1005</td>
<td>0.2878</td>
<td>0.4292</td>
<td>0.5033</td>
<td>0.5336</td>
</tr>
<tr>
<td>50</td>
<td>0.0720</td>
<td>0.2125</td>
<td>0.3229</td>
<td>0.3799</td>
<td>0.4007</td>
</tr>
<tr>
<td>60</td>
<td>0.0444</td>
<td>0.1361</td>
<td>0.2130</td>
<td>0.2542</td>
<td>0.2692</td>
</tr>
<tr>
<td>70</td>
<td>0.0212</td>
<td>0.0678</td>
<td>0.1109</td>
<td>0.1364</td>
<td>0.1472</td>
</tr>
<tr>
<td>80</td>
<td>0.0056</td>
<td>0.0187</td>
<td>0.0323</td>
<td>0.0418</td>
<td>0.0469</td>
</tr>
</tbody>
</table>
Table 2: Transmission index of the aperture obtained with the BEM model

<table>
<thead>
<tr>
<th>Incident angle, (°)</th>
<th>f=100Hz</th>
<th>f=200Hz</th>
<th>f=300Hz</th>
<th>f=400Hz</th>
<th>f=500Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3926</td>
<td>0.3956</td>
<td>0.5448</td>
<td>0.6609</td>
<td>0.7715</td>
</tr>
<tr>
<td>10</td>
<td>0.3878</td>
<td>0.3893</td>
<td>0.5245</td>
<td>0.6397</td>
<td>0.7280</td>
</tr>
<tr>
<td>20</td>
<td>0.3727</td>
<td>0.3837</td>
<td>0.4881</td>
<td>0.5911</td>
<td>0.6476</td>
</tr>
<tr>
<td>30</td>
<td>0.3453</td>
<td>0.3455</td>
<td>0.4427</td>
<td>0.5149</td>
<td>0.5621</td>
</tr>
<tr>
<td>40</td>
<td>0.3042</td>
<td>0.2937</td>
<td>0.3754</td>
<td>0.4155</td>
<td>0.4642</td>
</tr>
<tr>
<td>50</td>
<td>0.2516</td>
<td>0.2316</td>
<td>0.2913</td>
<td>0.3125</td>
<td>0.3433</td>
</tr>
<tr>
<td>60</td>
<td>0.1406</td>
<td>0.1578</td>
<td>0.1987</td>
<td>0.2184</td>
<td>0.2412</td>
</tr>
<tr>
<td>70</td>
<td>0.0921</td>
<td>0.0971</td>
<td>0.1123</td>
<td>0.1204</td>
<td>0.1348</td>
</tr>
<tr>
<td>80</td>
<td>0.0514</td>
<td>0.0572</td>
<td>0.0496</td>
<td>0.0505</td>
<td>0.0502</td>
</tr>
</tbody>
</table>

The analytical model estimates the transmission index of the aperture quite accurately for frequencies above 200 Hz. In this fringe, the maximum differences in the transmission index are of 0.13 and are found at higher frequencies and with normal or almost normal incidence field, where the transmission itself is higher. The analytic transmission index is higher than the one obtained from the BEM model in all cases except when the angle of incidence is close to 80°, where the difference changes its sign. The transmission index results obtained from both models show the same tendency in front of frequency and angle of incidence: it increases with frequency and decreases with angle of incidence. For frequencies below 200 Hz the BEM model results show that the transmission index remains constant while the analytical model does not present that cutoff value and continues its decreasing tendency. Some other BEM simulation cases carried out with different aperture dimension (not included in this paper) reveal that the frequency from which the transmission index is constant depends on the dimension of the aperture. For the tested aperture dimension (a=0.3 m) the cutoff value takes place at 200 Hz leading to differences of the compared parameter in this fringe up to 0.23.

4. ACTIVE CONTROL

To estimate the reduction of the power transmitted through the aperture is calculated by the reduction of the squared pressure in a twenty calculation points (Fig. 1) distributed homogeneously over a hemispherical surface as in the ISO standard. The reduction R achieved when controlling is calculated by:

\[
R = 10 \log \frac{\sum_{\text{surface}} |P_{ps}|^2}{\sum_{\text{surface}} |P_{ac}|^2}
\]

where \(P_{ps}\) is the pressure at the point considered due to the primary source and \(P_{ac}\) is the pressure at the point considered after the application of the active control.

A. Results

Active control has been tested for different distances between the secondary source and the error microphone considering \(\theta_2=\phi_2=0\). Figure 2 shows the mean attenuation of the sound transmitted through the aperture, considering an incident primary wave arriving at the aperture with...
$\phi = \theta = 0$. It could be seen that the larger the distance from the secondary source to the error microphone, $r_{sc}$, the lower the transmission at the expense of an increase in the secondary source strength.

Guo\textsuperscript{13} established that the size of the zone of quiet increases as the distance from the cancelling point to the secondary source increases and as the distance between the primary and the secondary source decreases. The pressure map for a square aperture of $a=0.3$ m and an incident primary wave of 200 Hz, after applying active control just at the aperture, is represented at Fig. 3. This figure show that the larger $r_{sc}$, the bigger is the zone of attenuation at the aperture, and it suggests the following cancellation process: as the distance between the secondary source and the cancelling point increases, the radius of curvature of the spherically outgoing wave of the secondary source increases and the wavefront approaches that of a plane wave. Therefore, the outgoing wave of the secondary source is more similar to the incident wavefront producing a bigger destructive interference zone surrounding the error microphone.

Let us consider now a normal incidence for different dimensions of the aperture, different frequencies and different distances from the secondary source to the cancelling point. Fig. 2 shows the mean attenuation value versus normalized (against wavelength) $r_{sc}$ for different normalized dimensions of the aperture. It can be seen that the mean attenuation decreases as the dimension of the aperture increases. This is because, for a fixed distance between the secondary source and the error microphone, the zone of reduction at the aperture remains constant. Thus, when the dimension of the aperture increases, there is more aperture surface that is not affected by the active control, giving as a result more radiation surface. As it has been stated in the first analysis, the increase in the distance between secondary source and error microphone results in a greater attenuation of transmission, although it seems that for distances larger than $r_{sc}/\lambda > 0.2$, the increase of the distance between secondary source and error microphone causes small increase of attenuation. Finally, as the effect of frequency is not shown, because there are many parameters normalized by $\lambda$, the mean attenuation after active control versus the distance $r_{sc}$ for a squared aperture of $a=0.30$ m is plotted in Fig. 4. This figure shows that the mean attenuation decreases as frequency is increased. For high frequency sounds, the pressure caused by the secondary source at different points of the aperture can have important variations in phase between themselves. Thus, although cancellation at error microphone is always possible, zones of constructive interference with primary field could arise giving more radiation as a result.

![Figure 2: Mean attenuation versus normalized distance between secondary source and error microphone $r_{sc}/\lambda$ for three different normalized aperture dimensions $d=a/\lambda$.](image-url)
3. CONCLUSIONS

Preliminary results have shown that the proposed model can be satisfactorily applied to predict the sound transmission index through an aperture in a thin wall for frequencies above a certain value, which depends on the aperture dimension. Below this certain frequency value some deviation is found. The transmission index presents similar trends in the analytical and BEM model in front of frequency and angle of incidence.

Active control of transmission has been successfully demonstrated by locating an error microphone in the center of the aperture and a secondary source close to the aperture, in the incidence region. General results, limited to the low frequency range, have shown that local control strategy allows to increase the mean attenuation at the aperture without any increase of sound pressure at the receiving region.

In general terms, the distance between the secondary source and error microphone has strong effect on the attenuation, being the latter increased when distance increase as well, at the cost of a higher strength of the secondary source to achieve the control. The increase of the required secondary source strength may increase the sound pressure in the incident region, but this effect has not been studied in this work.
The increase of attenuation with the distance between the secondary source and the error microphone can be explained because as that distance increases, the radius of curvature of the spherically outgoing wave of the secondary source increases as well and its wave front approaches that of a plane wave. Therefore, the outgoing wave of the secondary source is more similar to the incident wave front producing a bigger destructive interference zone surrounding the error microphone. This mechanism would also explain the decrease of attenuation when frequency increases or when the dimension of the aperture increases as well: in these cases, the ratio between the area where the destructive interference occurs and the total aperture area decreases.

ACKNOWLEDGEMENTS
This work was supported by the Spanish National Research Plan, project BIA2011-24633.

REFERENCES
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