EXTREME RAINFALL RATES AND PROBABLE MAXIMUM PRECIPITATION

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ABSTRACT

One of the most important climatic features that characterize a territory is its pluviometric regime and, in particular, the extreme rainfall rates produced in the area. Their knowledge is essential for planning hydraulic works and for designing rainwater drainage systems and flood prevention. In addition, the behavior of this variable can be very useful for the evaluation of possible effects of the climatic change. In this chapter we summarize the methodology used in the calculation of the frequencies associated to maximum precipitations expected in an area according to rainfall duration. Results about the probable maximum precipitation in 24 hours for the 145 pluviometric stations of Catalonia are also shown. The obtained spatial distribution shows rainfall amounts in the range 200-550 mm, in a good concordance with the annual average distribution but with some exceptions caused probably by the different meteorological scales involved in each case.

Key words: Probable maximum precipitation, spatial rainfall distribution, IDF curves.

1. INTRODUCTION

The pluviometric regime is one of the most important climatic features that characterize a place or territory and among the many factors that define it stand out for their interest in the meteorological, hydrological and civil engineering, extreme rainfall intensities, their durations and the frequency with which they occur. Their knowledge is essential for planning hydraulic works, roads, sewage systems and for designing rainwater drainage systems in large installations and buildings in general, optimization of water resources in river basins and flood prevention. In addition, the behavior of this variable can be useful both for the detection of climate change and the assessment of their potential impact on a territory.
The behavior of the rain intensity averaged over time intervals of 24 hours or more is often studied from data provided by manual gauges (INM 1999; Lana et al., 1995). However, when it requires a finer knowledge of this variable we must use the data provided by rainfall rate gauges capable of recording its evolution during the course of the shower. In both cases the procedure is to determine the maximum expected precipitation amounts at selected time intervals for different return periods considered from the data sets provided by the meteorological observatories. If we need information in areas without data this information, usually available locally at each measuring point, has to be added territorially and interpolated or extrapolated.

From the point of view of water management, a specific application of these maximum expected precipitation maps is its usefulness in carrying out the prevention and management of the river flooding. Another example of its application can be found in waste management. In this case, to have a reliable and updated collection of maximum precipitation climate data let us to improve the design of the different treatment facilities, design properly waste disposal, and calculate and verify the sizing of pluvial networks.

Sometimes, for the design of certain hydraulic structures, one requires to know the amount of precipitation which can not be exceeded for a given duration. In this sense, the probable maximum precipitation (PMP) is defined as the amount of precipitation theoretically higher physically possible over a region (Hansen et al., 1982). Although initially the PMP is defined himself as maximum precipitation amount for a given duration, area and time of year which not can be exceeded (Wang, 1984), it was subsequently found that sometimes, the amounts of recorded precipitation had passed the PMP estimated previously. This clearly indicated that these amounts calculated as PMP could not be considered as zero risk (Koutsoyiannis, 1999).

This work presents the methodology currently used to calculate the frequencies of the maximum expected rainfall in an area depending on its duration and the results of its application to Catalonia.

2. INTENSITY-DURATION-FREQUENCY CURVES

From data provided by current gauges can calculate the maximum precipitation time intervals between 5 minutes and 24 hours. The relationship between the calculated rainfall intensities, its duration and frequency of occurrence is what is known as curves of Intensity-Duration-Frequency, or briefly IDF curves.

To calculate them we must first obtain the maximum precipitation series for each term and determine the theoretical statistical functions that best fit each of the experimental series. For example, in the case of Barcelona, from the database generated by digitizing the stripe charts recorded by the intensity pluviograph Jardí installed in the Fabra Observatory between 1927 and 1992 (Burgueño et al., 1994),
we have calculated the maximum rainfall amounts recorded at different durations from 5 minutes to 24 hours. For complete data sets, the best fit was obtained by the Gamma distribution, a function used extensively in engineering applications, limited to positive values and right asymmetry, whose density function is:

$$f(x) = \frac{\lambda (\lambda x)^{k-1}}{\Gamma(k)} e^{-\lambda x} \quad \text{for } x \geq 0$$  \hspace{1cm} (1)

where \(\lambda\) and \(k\) are the scale parameter and shape of the distribution.

From these functions the values of precipitation for different return periods can be obtained using the relationship between the frequency \(F\) and the return period given by equation:

$$F = 1 - \frac{N/T}{D}$$  \hspace{1cm} (2)

where \(N\) is the number of available years, 66 in our case, and \(D\) the number of values in the sample. The intensity values for each time \(t\) and the return period \(T\), ie, the points \(I(t, T)\), are represented in Figure 1.

Figure 1. Rainfall rate for each duration and return period considered (Casas et al., 2004)
The general equation obtained for the Intensity-Duration-Frequency curves for Barcelona:

\[ I(t,T) = \frac{19 \log T + 23}{(13 + t)^{0.87}} \]  

(3)

where \( I \) is the rainfall rate expressed in mm/min \( t \) is the rainfall duration in minutes and \( T \) is the return period \( T \) in years.

Figure 2 shows the IDF curves for the return periods of 1, 2, 5, 10, 15, 50 and 100 years.

![IDF curves for Barcelona](image)

Figure 2. IDF curves for Barcelona of return periods of 1, 2, 5, 10, 50 and 100 years (Casas et al., 2004)

3. **SPATIAL ANALYSIS OF EXTREME DAILY RAINFALL**

The knowledge of the maximum daily rainfall in an area for different return periods is necessary in many applications. This analysis is normally carried out from datasets of rain gauges installed in the area. One of the methods traditionally used for these
calculations is the use of distribution functions to fit analytic functions to the annual series of maximum daily rainfall, allowing then assign a frequency or recurrence period to each value the maximum daily precipitation in one place. When the objective is to determine the maximum daily rainfall that can fall anywhere in the region under study with an established frequency, one typically resorts to the scalar analysis of the amounts calculated from data sets available in the rainfall stations. This approach, although common, has some drawbacks that can cause large uncertainties in the results and even notable errors.

The main cause of the inherent difficulty in calculating maximum precipitation in a given region lies in the nature of weather events that produce them. In general, the cloud systems that cause high intensity rains pertain to microscale or mesoscale and the higher rainfall areas, within these organizations, are still smaller extension. This means that when in an observatory daily rainfall is recorded with large return period due to an extreme weather event, it is unlikely to be repeated in other observatories of mesoscale network and occurs much less if the density of stations corresponds to the synoptic scale or macroscale.

The high-resolution spatial analysis of extreme daily rainfall in Catalonia has been carried out from the series of annual maximum values of precipitation in 24 hours recorded in 145 rainfall stations that the Spanish Meteorological Agency (AEMET) has in this region (INM, 1999). The annual series of maximum daily rainfall selected have different lengths and correspond to the 1911-2001 period (Figure 3).

Since the approximate area of Catalonia is about 32 000 km$^2$ and the number of rain gauges of the network is N=145, the average distance between the rainfall stations is found to be about 15 km approximately. Some of the series of annual maximum precipitation available have a relatively short length (15-20 years), and in some cases there has been the presence of extraordinary extreme cases (outliers) Hershfield (1961a and b), WMO (1986), Nobilis et al. (1991)).

For these occasional showers, the traditional method of setting the Gumbel function can assign to some values of precipitation return periods much smaller than it really would apply if the sample cover a greater number of years. To minimize this effect, we have estimated the Gumbel distribution parameters using the method of moments L (Hosking, 1990; Hosking and Wallis, 1997), which are linear combinations of the weighted probability moments.

To calculate these maximum daily rainfall amounts, firstly we have determined the monthly precipitation corresponding to the rainiest month at every grid point using the multiple regression obtained by Ninyerola et al. (2000) and they have been normalized from the relationship between these quantities and those registered in 24 hours.
Figure 3. Rain gauges in Catalonia whose series have been analyzed. In white the stations used to test the analysis (Casas et al., 2008).

Figure 4 shows the analysis obtained from the maximum rainfall in 24 hours 25 years return period for Catalonia (Casas et al., 2007). Results show roughly the areas where you can expect a maximum of extreme daily rainfall are located in the eastern half of Catalonia, in the higher areas of the Pyrenees and the southern third, while the areas where we can expect the lowest extreme daily rainfall coincide largely with the central depression, extending from its western end to the highlands of Lluçanès and Plana de Vic.

In the eastern half of Catalonia the maximum values draw a line that follows the Sierra Prelitoral from Montserrat and Sant Llorenç de Munt to the Montseny and Guillerries and extending in a northerly direction along the transverse ridge until the Eastern Pyrenees, from where it continues west to the sector and eastward Moixeró by Alberas to the sea. The places where you can expect maximum values of extreme daily rainfall are Guillerries and Cape Creus (with values greater than 180 mm for a return period of 10 years), but also Though also there are prominent the values obtained in the zone between the headwaters of the rivers Ter and Muga.

In the Pyrenees highlight other areas where we can expect maximum daily rainfall important, coincident with higher altitudes. In the southern third of Catalonia is also obtained an area of maximum for extreme daily rainfall around Prelitoral hills, from the mountains of Prades to Montsià, where highlight a band oriented from west to east in which the maximum are more important.
4. **CALCULATION OF THE PROBABLE MAXIMUM PRECIPITATION**

The probable maximum precipitation (PMP) is defined as the greatest amount of precipitation meteorologically possible for a given length on a given storm area at a particular geographical location and the time of year, regardless of climate trends Long term (WMO, 1986). In hydrology PMP and their spatial and temporal distributions are used to calculate the probable maximum flood (PMF) which is one of the conceptual flood situations used in the design of hydrological structures for maximum reliability and safety.

Prior to the 50s, the concept of a potential upper limit of precipitation was known as

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Figure 4  Maximum precipitation in 24 hours for return period of 25 years (Casas et al., 2007).
maximum possible precipitation (MPP). The name was changed to PMP, reflecting the uncertainty involved in the estimation of maximum precipitation (Wang, 1984). Quoting Benson (1973): "The concept of "maximum probable" began as "maximum possible" because it was considered that maximum limits exist for all the elements that act together to produce rainfall, and that these limits could be defined by a study of the natural process. This was found to be impossible to accomplish - basically because nature is not constrained to limits". Procedures for determining the PMP are recognized as inaccurate, the results are estimates and a risk statement has to be assigned to them. The PMP concept "in no way implies actually zero risk" (Koutsoyiannis, 1999). The National Research Council (NRC, 1994) has estimated the return period of the PMP in U.S. between $10^5$ and $10^9$ years. Koutsoyiannis (1999) developed a method for assigning a return period PMP values obtained using the statistical of frequency factor method (Hershfield, 1961, 1965).

To estimate the PMP in a place, a variety of procedures depending on the situation of the basin have been proposed, the availability of data and other considerations (e.g. Wiesner, 1970; Schreiner and Reidel, 1978; WMO, 1986; Collier and Hardaker, 1996). Most of them are based on meteorological analysis while some are based on statistical analysis. PMP estimation techniques were listed by Wiesner (1970) as follows: [1] the storm model, [2] the maximization and transposition of real storms, [3] the use of precipitation data, their maximized duration and area from storms; [4] the use of empirical formulas determined from maximum precipitation data, duration and area, or theory, [5] the use of empirical relationships between variables in specific locations (only if detailed data are available), [6] statistical analysis of extreme rainfall. These methods are not entirely independent. Probably the easiest way to estimate a theoretical upper limit of precipitation in a basin for a given duration is the use of empirical formulas (methods [4] and [5]) to represent maximum values of local or global precipitation. Methods [2] and [3] involve the classification of storms by calculating their efficiency which is defined as the ratio between the maximum observed rainfall to the amount of precipitable water in the air column during the storm (NERC, 1975).

Among the statistical methods to estimate the PMP [6], the most commonly used is the method of Hershfield (1961b, 1965) which is based on the frequency analysis of the annual maximum rainfall data registered at the site of interest. The Hershfield technique for estimating the PMP is based on Chow’s general frequency equation Chow (1951):

$$\text{PMP} = \bar{x}_n + k_m \sigma_n$$  \hspace{1cm} (4)

$$k_m = \frac{x_M - \bar{x}_{n-1}}{\sigma_{n-1}}$$  \hspace{1cm} (5)

where $x_M$, $\bar{x}_n$ and $\sigma_n$ are the maximum value, the mean and the standard deviation respectively for a series of $n$ annual maximum of a given duration, and are the mean and standard deviation in said series but excluding the maximum of these each and $k_m$ is a frequency factor. To evaluate this factor, initially Hershfield (1961) analyzed
2645 stations (90% of them in USA) finding a value of 15 for $k_m$ which he recommended for PMP estimation using Equation (3). Later, Hershfield (1965) found that the value of 15 was too high for rainy areas and too low to the arid, whereas it is too high for rain durations shorter than 24 hours. Thus, he constructed an empirical nomograph (WMO, 1986) with $k_m$ varying between 5 km and 20 depending on the rainfall duration and the mean $\bar{x}_n$.

The methods discussed for PMP estimation can be used both for individual basins and larger regions covering several basins of different sizes. In the latter case, the estimates are known as widespread or regional estimates (WMO, 1986).

This section presents the results of the calculation of the PMP for Catalonia. As noted in the previous section, there are values of maximum precipitation in 24 hours and return periods from 2 to 500 years for 145 rainfall stations in Catalonia (Figure 3), calculated from their series of annual maximum daily precipitation. Following the technique of Hershfield, the statistical parameters $\bar{x}_n$, $\bar{x}_{n-1}$, $\sigma_n$ y $\sigma_{n-1}$ (means and standard deviations) involved in equations (4) and (5), and the coefficient of variation $CV = \sigma_n / \bar{x}_n$, have been calculated for all series.

Figure 5 shows the dependence between the average value of the series of annual maximum 24-hour precipitation and frequency factors $k_m$ observed for each of them. As recommended by WMO (WMO, 1986), to estimate appropriate values of the PMP is desirable to draw an enveloping curve that fits all cases, including the most extreme. The usual technique is to select the highest values of the sample and adjust to a curve. This process can be applied to the sample frequency factors $k_m$ calculated for a given duration (Dhar et al., 1981, Rakhecha et al., 1992). The same figure shows, jointly with the points $(k_m, \bar{x}_n)$, the enveloping curve representing the dependence between both variables.

In order to estimate the PMP for each station, the frequency factor $k_m$ has been used that bound associated with average precipitation in 24 hours $\bar{x}_n$ of each station. These values are in all cases higher than the original $k_m$ observed. With these theoretical values of $k_m$ that give the enveloping curve for each station, the mean $\bar{x}_n$ and the standard deviation $\sigma_n$, the PMP was calculated using equation (4).

Similarly to how we proceeded in section 3 the spatial analysis of PMP in 24 hours in Catalonia has been performed (Casas et al., 2008) using the method of Cressman (Cressman, 1959; Thiébaux and Pedder, 1987) in order to obtain a high spatial resolution of $1\text{km} \times 1\text{km}$. As a first approximation to the PMP at all points of a network of $1\text{km} \times 1\text{km}$ covering Catalonia, we will take the 24-hour precipitation with a return period of 100 000 years which is obtained at each point from the work of Ninyerola et al. (2000) and IDF curves obtained in paragraph 2 (Casas et al., 2004). These values are taken as the initial field of Cressman’s objective analysis, which will be modified with each iteration until to achieve convergence to the data.
Figure 5. Enveloping frequency factor curve for Catalonia, fitted to the four upper \( (\bar{x},\ k_m) \) points (thick dots). Dashed line fitted curve for the 4 extreme cases of the sample (Casas et al., 2008).

The result of the analysis of the PMP at 24 hours for Catalonia, after the filtering process to remove structures having a wavelength which can not be resolved by the density of the network is presented in Figure 6.
The PMP in 24 hours for Catalonia ranges from values less than 200 mm to values exceeding 550 mm, with a relative difference between the maximum and minimum over 150%. Higher values are expected in the eastern half of Catalonia, in the higher areas of the Pyrenees and the southern third, while the lowest were found on the Central Depression, from its western end to the Plana de Vic. The higher values of the PMP in the eastern half of Catalonia are the areas of Guilleries and Cape Creus. In the Pyrenees, the most notable area with high values of the PMP is the north of Cerdanya, among Perafita and Puigpedrós peaks. In the southern third of Catalonia, there is an area of high PMP around the Gulf of Sant Jordi. The main minimum of PMP are distributed in close agreement with the driest areas of Catalonia.
5. CONCLUSIONS

From the records of Jardí’s pluviograph of Fabra Observatory, between 1927 and 1992, we calculated the amount of maximum rainfall for durations from 5 minutes to 30 hours and also we have investigated the relationship between the maximum intensities of precipitation, duration and frequency, resulting in a revision of IDF curves for the city of Barcelona and its generalized equation

\[ I(t, T) = \frac{19 \log T + 23}{(13 + t)^{0.87}} \]

(rainfall rate in mm / min, duration t in minutes and return period T in years).

For the series analyzed the calculation of the parameters of the Gumbel’s extreme values distribution by the L moments method proposed by Hosking (1990), provides a more stable and realistic values of daily maximum precipitation for high return periods than when computed under the traditional procedure. With this method, the values of rainfall in 24 hours extraordinarily high registered in some observatories (La Pobla de Lillet, b0079, Cadaqués, g0433; Vimбедó "Riudabella" t0019; Bohí "Central", l9741) do not influence significantly in the fit to the Gumbel distribution function. This has contributed to obtain, for example, over 30% difference between the amounts of rain in 24 hours, for return periods exceeding 50 years, calculated in this work and those obtained by other authors in some areas of Catalonia (INM, 1999, Lana et al., 1995). The values obtained in our case are significantly lower than those that have been calculated using the traditional method of fit to Gumbel function from the mean and standard deviation of the data series. The calculated maximum rainfall amounts are more approximate the actual values because they have no so large dependence with the absolute maximum of the series, ie, if these data are dispensed the results obtained in this study suffer a lower variation than the estimated values applying the traditional technique.

The method applied to analyze the spatial distribution of extreme rainfall in Catalonia has provided a high spatial resolution (1 km x 1 km) through the joint use of an initial field of rain calculated from the multiple regression equation obtained by Ninyerola et al. (2000) and Cressman analysis algorithm (Thiébaux and Pedder, 1987). Using this initial field, which has a good correlation with the variable analyzed, the analysis improves resolution especially in the mountain areas of the Pyrenees and Sierra Transversal where the station density is not sufficient to adequately represent large variations associated with the irregularity of the terrain (Willmott and Robeson, 1995). Furthermore, the analysis allows to assign to each km² a numerical value calculated objectively by a mathematical algorithm, which significantly improves the approximate estimates that can be made from a map analyzed manually.

Using the statistical method of estimation, we calculated the Probable Maximum Precipitation (PMP) in 24 hours for 145 rainfall stations in Catalonia, from his series of annual maximum rainfall in 24 hours. It has obtained the equation that determines the maximum frequency factor \( k_m \) depending on rainfall in 24 hours \( \bar{x}_c \)
\[ k_m = -7.56 \ln \bar{x}_n + 40.5 \quad (\bar{x}_n \text{ in mm}), \]
as a calculated enveloping curve from the fit to the four points corresponding to the highest recorded extreme (Puigcerdà, La Pobla de Lillet, Capdella and Cadaqués). Over 90\% of the calculated values of the PMP in 24 hours have some return periods between \(10^4\) and \(10^8\) years, almost coinciding with the range established by the NRC (1994) for the PMP.

With the iterative application of a numerical analysis filter obtained by the Cressman method we have achieved to eliminate the structures of wavelength less than the average distance between the rainfall stations used, thereby adjusting to the density variability of the network observation. The technique maintains, however, the information provided by the use of an initial field of high resolution. Furthermore, the use of this filter ensures that the spatial resolution is homogeneous analysis thus avoiding that depends upon the density of the network of observation in each area and could result in false interpretations of the actual spatial variability of the analyzed variable, improving the result obtained when a method of automatic plotting isolines is applied directly.

From the calculated values of the PMP in 24 hours was performed by applying spatial analysis algorithm Cressman (Thiébaux and Pedder, 1987) to an initial field of precipitation in 24 hours with return period of 100,000 years in all points of a grid of 1 km of arm containing Catalonia. The spatial distribution obtained for the PMP in 24 hours has amounts ranging from values below 200 mm and others exceeding 550 mm and a relative difference between the maximum and minimum over 150\%. Also, just as with the maximum precipitation maps, the spatial distribution obtained shows good agreement with that of the average annual rainfall in Catalonia, with some exceptions attributable to the different meteorological scales involved in each case in our region. So while synoptic-scale organizations have a major influence on the distribution of annual precipitation, the local and mesoscale factors most affect the map of the PMP in 24 hours.

5. REFERENCES


