

## PRELIMINARY STUDY ON THE RESISTANCE OF FERRITIC STAINLESS STEEL CROSS-SECTIONS UNDER COMBINED LOADING

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### 1 INTRODUCTION

The classical approach for the determination of cross-sectional resistance in stocky cross-sections, based on section classification, underestimates real capacities, specially when non-linear metallic materials with high strain-hardening are considered. Alternatively, a more efficient design approach has been developed in the last few years: the Continuous Strength Method (CSM), based on the cross-section deformation capacity. The expressions for the determination of pure axial and bending capacities of cross-sections according to the CSM have been widely analysed.

The use of stainless steel for structural purposes in construction has been continuously increasing due to its corrosion resistance, appropriate mechanical properties and aesthetic appearance. Additionally, ferritic stainless steels contain only small quantities of nickel, condition that makes them cheaper and relatively price-stable compared to other stainless steels. Considering that these materials need a higher initial investment than carbon steel and are characterized by high non-linear stress-strain behaviour with important strain-hardening, the CSM can be considered a suitable design approach to exploit their features.

The assessment of the CSM has already been studied for austenitic and duplex stainless steel stocky cross-sections subjected to either axial compression or bending moment in isolation and the corresponding expressions have been developed. Some research has also been conducted in order to extend the CSM to more general loading conditions for austenitic stainless steel rectangular and square hollow sections (RHS and SHS) [7] and I-sections [8]. This paper presents a preliminary study on the determination of the ultimate resistance of ferritic stainless steel RHS and SHS subjected to combinations of axial compression and bending moment. Hence, the assessment of the interaction equations proposed in EN 1993-1-4 [1] and in literature for austenitic stainless steels are analysed in order to determine the most appropriate approach for ferritic stainless steels.

### 2 NUMERICAL SIMULATIONS

#### 2.1 Numerical model

A numerical study is presented in order to evaluate the ultimate resistance of ferritic stainless steel RHS and SHS subjected to combinations of axial compression and bending moment. This numerical analysis has been performed by the general purpose FEM software Abaqus and includes the different loading combinations needed for the definition of the biaxial bending and axial compression interaction surface.

All studied specimens show a total length of three times the largest element dimension in order to ensure that the element fails due to local buckling without any overall buckling interaction. The mid-surface of the cross-section has been modelled by using the four-node shell elements with reduced integration S4R, widely used when modelling cold-formed stainless steel cross-sections. After a mesh convergence study, and in order to guarantee computational efficiency, the analyses have been conducted with 5mm long shell elements. The non-linear analysis has been performed conducting a modified Riks analysis, where an initial imperfection, according to the first buckling mode shape obtained from a linear eigenvalue analysis, is introduced. *Fig. 1* shows some examples of the first buckling modes for some of the considered different loading conditions for the 120x80x3 cross-section, together with the failure modes.

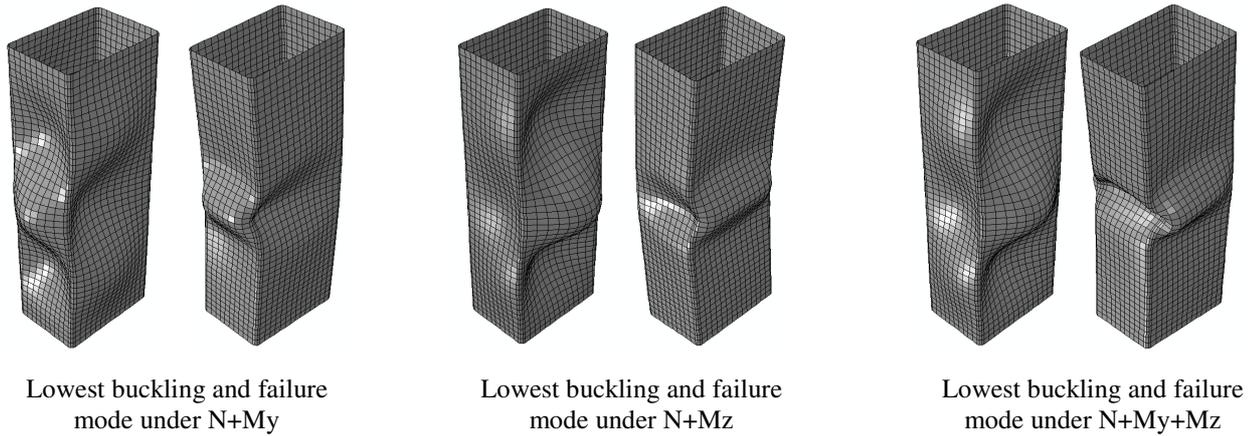


Fig. 1. Lowest buckling and failure modes for 120x80x3 under different combined loading conditions

The bottom edges of all specimens have been constrained to all displacements and rotations except for the studied axis where the bending moment is applied. The loads have been introduced as imposed displacements at the upper edges, resulting on pin-ended boundary conditions.

## 2.2 Analysed material, cross-sections and loading cases

In order to complement some similar previous works in austenitic RHS, the present study has been conducted under ferritic stainless steel grade 1.4003. The stress-strain curve obtained from typical material parameter values for this grade (see *Table 1*) and a two-stage material model described by the equations proposed by Mirambell-Real [2] has been considered in the models.  $E$  is the Young's modulus;  $\sigma_{0.2}$ , conventionally considered as the yield stress, is the proof stress corresponding to a 0.2% plastic strain;  $n$  and  $m$  are the strain-hardening exponents and  $\sigma_u$  and  $\epsilon_u$  are the ultimate strength and the corresponding ultimate strain, respectively.

Table 1. Adopted material properties

$E$ (MPa)	198000
$\sigma_{0.2}$ (MPa)	330
$\sigma_u$ (MPa)	480
$\epsilon_u$	17%
$n$	11.5
$m$	2.8

Table 2. Analysed cross-sections

S1-120x80x4	S7-60x60x3
S2-80x40x4	S8-70x50x2
S3-80x80x4	S9-100x100x3
S4-60x60x4	S10-100x100x3.5
S5-100x100x4	S11-120x120x5.5
S6-120x80x3	S12-80x80x2.5

A total of twelve different RHS and SHS cross-sections have been analysed (see *Table 2*) covering a wide range of cross-sectional slenderness, ranging from 0.26 to 0.76, in order to guarantee that the analysis considers the entire casuistry of stocky cross-sections. For each of the studied cross-sections 21 different compression and biaxial loading conditions have been considered, including the fundamental cases where a single load is applied (pure compression, pure bending and pure biaxial bending).

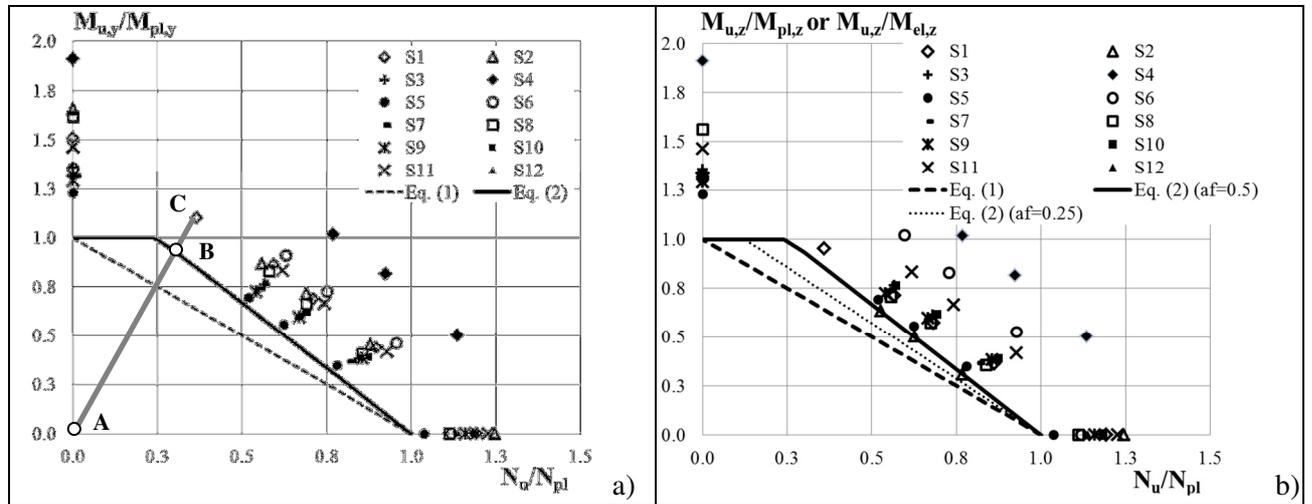
## 3 EN 1993-1-4 ASSESSMENT

EN 1993-1-4 [1] refers to the corresponding equations for carbon steel in EN 1993-1-1 [3] regarding cross-sectional resistance, which depends on section classification. For axial and bending moment interaction in slender cross-sections –Class 3 and 4– a linear equation is assumed, *Eq. (1)*, imposing that failure occurs when the maximum stress reaches the yield stress.  $N_{Ed}$ ,  $M_{y,Ed}$  and  $M_{z,Ed}$  are the applied loads; and  $M_{y,Rd}$  and  $M_{z,Rd}$  are the moment resistances about the principal axes. Concerning stocky cross-sections –Class 1 and 2–, some plastic response is allowed and the interaction between axial force and bending moments is governed by *Eq. (2)*, where  $M_{N,y}$  and  $M_{N,z}$  are the axial-reduced plastic moment resistances about the principal axes, given in EN 1993-1-1 [3]; and  $n=N_{Ed}/N_{pl}$ .

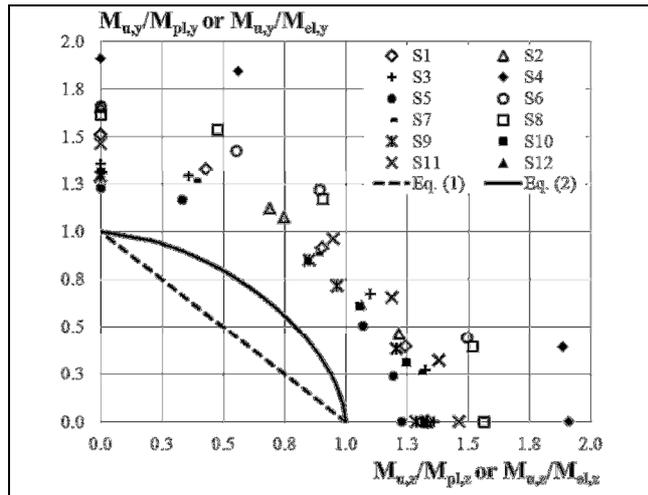
$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0 \quad \text{and} \quad \left( \frac{M_{y,Ed}}{M_{N,y,Rd}} \right)^{\frac{1.66}{1-1.13n^2}} + \left( \frac{M_{z,Ed}}{M_{N,z,Rd}} \right)^{\frac{1.66}{1-1.13n^2}} \leq 1.0 \quad (1) \text{ and } (2)$$

All the studied cross-sections are Class 1 or 2 when subjected to combinations of axial compression and bending moment as well as biaxial bending in absence of axial force, so the ultimate capacity of each cross-section is determined by *Eq. (2)*. On the other hand, regarding combined axial compression and minor axis bending, the same classification is considered, except for section S5, which is classified as Class 3, where the linear interaction expression, *Eq. (1)*, needs to be used.

*Figs. 2a, 2b* and *3* show the obtained numerical loads ( $M_u$  and  $N_u$ ) normalized by the resistances predicted in EN 1993-1-1 [3], together with the coded interaction expressions: linear interaction *Eq. (1)* and *Eq. (2)*. The reduced design plastic moment resistance  $M_{N,z,Rd}$  has been determined using an average  $a_f$  of 0.5 for SHS and 0.25 for RHS. Since the considered stocky sections can attain stress levels beyond the yield stress, Eurocode predictions are quite conservative given that EN 1993-1-1 does not make allowance for material strain-hardening, as shown in *Figs. 2a, 2b* and *3*.



*Fig. 2.* a) Interaction of axial compression and major axis bending ( $N+M_y, M_z=0$ ); b) Interaction of axial compression and minor axis bending ( $N+M_z, M_y=0$ )



*Fig. 3.* Interaction of biaxial bending ( $M_y+M_z, N=0$ )

*Table 3.* Comparison of different proposals with FEM data through  $U^*$  parameter

	Mean	COV.
EN 1993-1-1 [3], Eq. (1) and (2)	0.81	0.152
[7] proposal for RHS, Eq. (5)	0.91	0.163
[7] proposal for RHS, Eq. (6)	0.79	0.144
CSM in EN 1993-1-1, Eq. (7)	0.86	0.163

In order to assess the design provisions for combined loading,  $U^* = \overline{AB}/\overline{AC}$  ratios, by which design interaction curves exceed or fall short of the corresponding test data, have been calculated by assuming proportional loading (constant bending moment to axial compression ratio) (see *Fig. 2a*), where a value of  $U^*$  greater than unity indicates an unsafe result. The mean and coefficient of variation (COV) of the calculated  $U^*$  values for EN 1993-1-1 and other interaction expressions that will be studied in the following section of this paper are presented in *Table 3*.

#### 4 CSM ASSESSMENT

The Continuous Strength Method (CSM) is a new design approach based on the deformational capacity of a cross-section, allowing the consideration of the material's strain-hardening, therefore being ideal for stainless steel structural design. More in-depth basis of this method are described in [4]. The maximum strain that a cross-section can reach  $\epsilon_{CSM}$  is evaluated in terms of its relative slenderness  $\bar{\lambda}_p$  and the yield strain  $\epsilon_y$ , as shown in Eq. (3), which has been adjusted considering both stub column and beam test data. The relative slenderness for each fundamental loading case can be obtained from Eq. (4), where  $\sigma_{cr}$  is the critical buckling stress, obtained from the lowest buckling mode in an eigenvalue analysis. At the same time  $\bar{\lambda}_p$  can also be calculated according to Eurocode for the most slender plate element in the cross-section. It should be noted that the former procedure accounts for element interaction whereas the latter does not. The  $\bar{\lambda}_p = 0.68$  limit is adopted given that, beyond this limit, there is no significant benefit of considering material strain-hardening effects.

$$\frac{\epsilon_{CSM}}{\epsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}} \quad (3)$$

$$\bar{\lambda}_p = \sqrt{\frac{\sigma_{0.2}}{\sigma_{cr}}} \text{ but } \bar{\lambda}_p \leq 0.68 \quad (4)$$

Cross-sectional capacities are derived from the  $\sigma_{CSM}$  stress reached at the maximum strain  $\epsilon_{CSM}$ . The compression resistance  $N_{CSM}$  is calculated assuming an upper bound stress  $\sigma_{CSM}$  in the cross-section whereas the bending moment capacities about the strong and weak axes ( $M_{CSM,y}$  and  $M_{CSM,z}$ ) can be calculated by assuming that plane sections remain plane and normal to the neutral axis, therefore integrating the resulting linear strain distributions. The expressions for the determination of the resistances of the cross-sections subjected to pure compression or bending are gathered in [5].

In the very early version of the CSM, the limiting stress  $\sigma_{CSM}$  was determined from the non-linear Ramberg-Osgood [6] stress-strain model. Although this model provided accurate  $\sigma_{CSM}$  results, the process to calculate them was relatively complex. Hence, a simplified bilinear material model, which also considers material strain-hardening, was developed for austenitic and duplex stainless steels in [5]. This bilinear model has been found to be inaccurate for ferritic grades due to the lower ductility and ultimate strain of these materials. Therefore, a bilinear material model which accurately predicts the stress-strain behaviour of the studied material, with a strain-hardening coefficient  $E_{sh} = 2000 \text{ MPa}$  (approximately  $E/100$ ) has been considered, as shown in Fig. 4a. All the CSM predictions in this study have been determined according to this bilinear model.

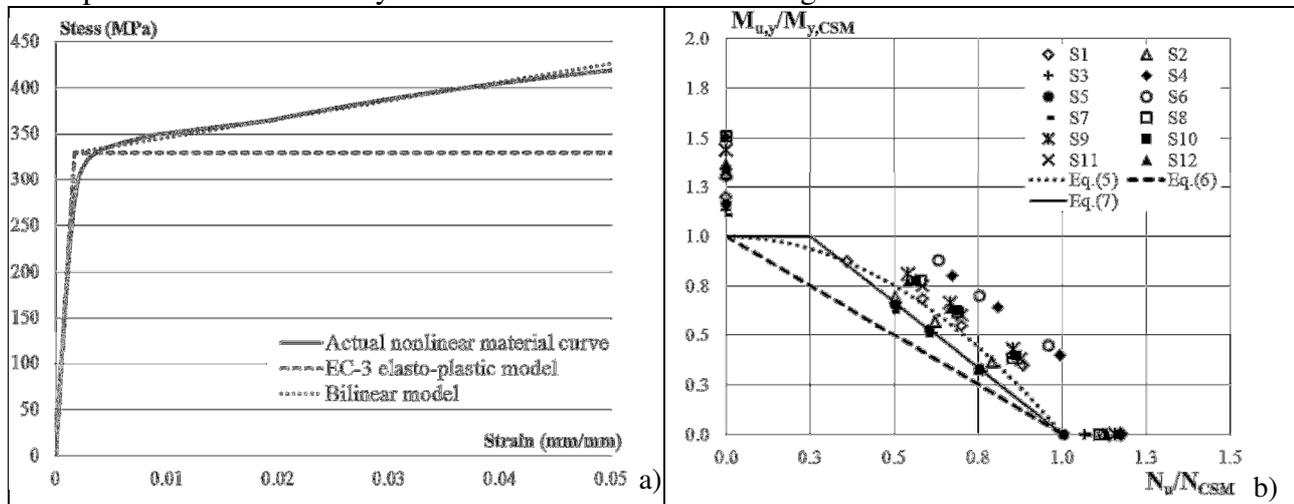


Fig. 4. a) Considered material models; b) Interaction of axial compression and major axis bending ( $N+M_y, M_z=0$ )

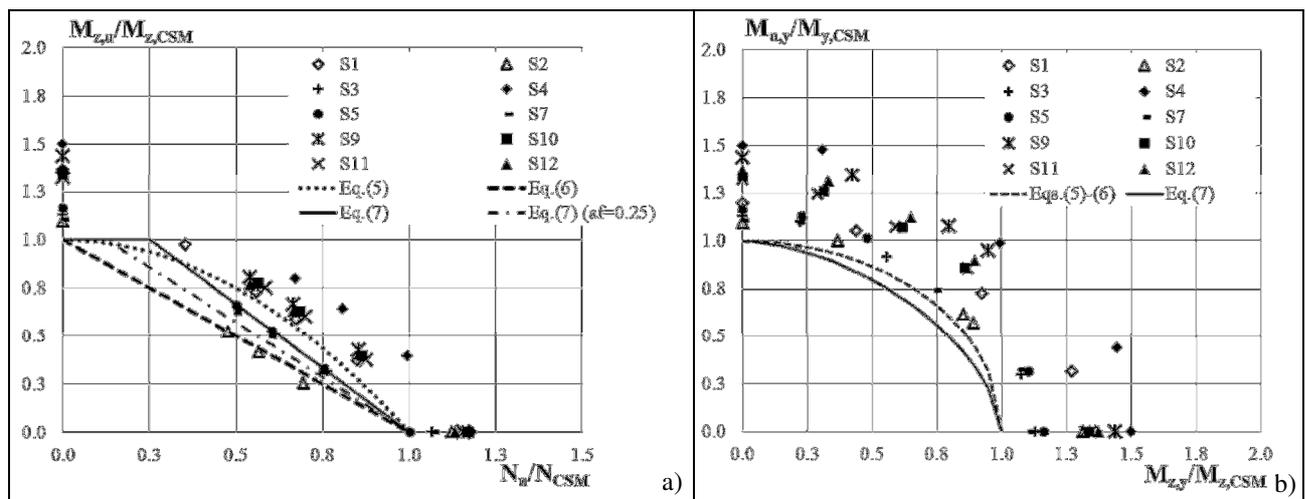
As it has been established before, [5] only proposes expressions for pure compression and bending for austenitic and duplex stainless steels cross-sections in isolation. General loading conditions, such as combinations of axial compression and bending moment, on austenitic stainless steel have already been analysed for RHS and SHS in [7] and for I-sections in [8] by conducting some numerical studies. Since all the cross-sections adopted in this paper are RHS and SHS, the interaction expressions proposed in [7], given in *Eq. (5)* and *(6)*, have been considered in the analysis. However, it is important to point out that the research carried out in [7] was based on austenitic stainless steel and not performed with the most recent and simplified CSM version presented in [5].

$$\left(\frac{N_{Ed}}{N_{CSM}}\right)^2 + \sqrt{\left(\frac{M_{y,Ed}}{M_{CSM,y}}\right)^2 + \left(\frac{M_{z,Ed}}{M_{CSM,z}}\right)^2} \leq 1.0 \quad \text{and} \quad \frac{N_{Ed}}{N_{CSM}} + \sqrt{\left(\frac{M_{y,E}}{M_{CSM,y}}\right)^2 + \left(\frac{M_{z,Ed}}{M_{CSM,z}}\right)^2} \leq 1.0 \quad (5) \text{ and } (6)$$

Nevertheless, the evaluation of the resistance of a cross-section subjected to combinations of axial compression and bending moment loading conditions by slightly modifying the interaction expression currently coded in EN 1993-1-1 [3], as described in *Eq. (7)*, was not included in previous analyses. The considered fundamental capacities have been determined according to CSM, where  $M_{N,CSM,y}$  and  $M_{N,CSM,z}$  correspond to the axial-reduced CSM moment resistances about both principal axes according to the expressions gathered in EN 1993-1-1 but considering  $N_{CSM}$ ,  $M_{CSM,y}$  and  $M_{CSM,z}$  instead of  $N_{pl}$ ,  $M_{pl,y}$  and  $M_{pl,z}$ .

$$\left(\frac{M_{Ed,y}}{M_{N,CSM,y}}\right)^{\frac{1.66}{1-1.13n^2}} + \left(\frac{M_{Ed,z}}{M_{N,CSM,z}}\right)^{\frac{1.66}{1-1.13n^2}} \leq 1.0 \quad (7)$$

*Figs. 4b*, *5a* and *5b* show the numerical ultimate loads normalized by the CSM predictions together with the aforementioned interaction expressions in order to assess their suitability and accuracy. As these figures demonstrate, the consideration of the CSM fundamental capacities in the expression coded in EN 1993-1-1, *Eq. (7)*, is the best approach for the prediction of the combined loading capacity of ferritic stainless steel RHS, providing safe and more accurate results than the ones predicted by the coded expression, *Eqs. (1)* and *(2)* (see *Table 3*). Due to the fact that ferritic grades are less ductile and show lower strain-hardening than other stainless steel families, the achieved improvement is lower for this ferritic stainless steel solution than for austenitic and duplex grades [5], yet the progress is considerable.



*Fig. 5.* a) Interaction of axial compression and minor axis bending ( $N+M_z$ ,  $M_y=0$ ); b) Interaction of major and minor axis bending ( $M_y+M_z$ ,  $N=0$ )

Although *Table 3* suggests that the interaction equation *Eq. (5)* proposed in [7] is the best option for the prediction of the cross-sectional resistance of the studied sections, this solution is not a suitable option because it overestimates the ultimate capacities for a number of cases, as shown in *Figs. 4b*

and 5a. Same figures also show that *Eq. (6)* provides safe results for all the studied cases, but with more conservative predictions than *Eq. (7)* does. Therefore, it can be concluded that the best approach for the determination of the capacity of ferritic stainless steel RHS under combined loading is *Eq. (7)*, which considers the CSM fundamental capacities in the expressions coded in EN 1993-1-1, notably improving the predicted resistance values without the need of including any additional equation into the code while slightly modifying the existing one.

## 5 CONCLUSIONS

A numerical study on ferritic stainless steel RHS subjected to combinations of axial compression and biaxial-bending moment loading conditions has been presented in this paper, showing that the interaction expression coded in Eurocode provides conservative predictions. The suitability of the Continuous Strength Method for combined loading conditions has also been assessed by analysing different interaction equations available in the literature. The present study demonstrates that the consideration of the CSM fundamental capacities in the expressions in EN 1993-1-1 is the best approach for the prediction of the combined loading capacity of ferritic stainless steel RHS and SHS, without the need of including an additional equation into the code. As the conclusions of this study have been reached by analysing a limited number of numerical data, some experimental tests are currently in progress to validate these results and will be followed by a more extensive FEM parametric study.

## 6 ANCKNOWLEDGEMENTS

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