QUANTUM MECHANICS

No more fields

A self-accelerating electronic wave packet can acquire a phase akin to the Aharonov–Bohm effect, but in the absence of a magnetic field.

Maciej Lewenstein

In 1959, Aharonov and Bohm\(^1\) predicted that the wave packet of a charged particle could acquire an electromagnetic-induced phase, despite never actually feeling any magnetic force. This explains why, for example, placing a solenoid between the two arms of an electronic interferometer would modify the interference phase profile. The mathematical explanation of this puzzling effect relies on the interaction of the charged particle with the vector potential, which may be present even in regions with vanishing magnetic field. Writing in *Nature Physics*, Ido Kaminer and colleagues\(^2\) propose a way of testing an analogous Aharonov–Bohm effect in free space — that is, without any potentials or fields.

The vector potential in question is a gauge-dependent quantity, namely a mathematical construct whose form is not uniquely defined. Physical quantities — such as the electromagnetic field — need to be gauge invariant, and for this reason the Aharonov–Bohm effect in the interferometer example can be more accurately described as arising from a non-vanishing enclosed magnetic flux. Recently, the possibility of creating artificial gauge potentials (Box 1) for ultracold atoms trapped in optical lattices offered an alternative way to test the Aharonov–Bohm effect experimentally.

Kaminer *et al.*\(^2\) took a very different approach by doing away with the need for gauge potentials and fields. This rather counterintuitive tactic relies on the construction of special solutions to the Dirac equation describing the relativistic propagation of wave packets. These solutions have two important properties: their transverse shape is invariant, and they are self-accelerating along a curved trajectory. Self-accelerating wave packets were first introduced in optics, where they are known as Airy beams. They propagate, shape unchanged, along a parabolic trajectory and, due to interference, the wave packet accelerates itself.

Kaminer *et al.*\(^2\) constructed a family of shape-invariant solutions describing self-accelerating electrons, or more generally, spin-1/2 fermions. The self-acceleration led to the accumulation of a phase, analogous to the Aharonov–Bohm effect, but in free space. The generation of such solutions requires the preparation of designed initial wave packets, which should be feasible in various experimental set-ups. One could imagine preparing a wave packet — for, say, an atom — by trapping it in the ground state of an appropriately designed optical trap and then releasing it from the trap while simultaneously applying a short laser pulse to imprint the desired initial phase pattern. With holographic masks, one can imprint practically any phase pattern on demand.

The effect predicted by Kaminer *et al.*\(^2\) turns out to be equivalent to a change of the particle’s proper time. In other words, as the particle accelerates its ‘internal clock’ ticks faster or slower during different stages of the propagation. This suggests a way

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**Box 1 | Abelian and non-Abelian artificial gauge fields.**

**Abelian gauge fields.** The simplest way to understand the concept of artificial gauge fields is to consider atoms in a rotating trap. In a rotating frame of reference, the atoms feel a Coriolis force that is mathematically equivalent to the Lorentz force acting on charged particles in a magnetic field. This is great in principle, but rather limited in practice. Recently, many experiments have focused on creating artificial gauge fields in optical lattices. This works by considering a particle (dark blue circle) hopping or tunnelling (dashed arrow) between the sites of a two-dimensional square lattice. If a ‘magnetic field’ (B) were to pierce the lattice, the wave function of the particle would acquire an Aharonov–Bohm phase (\(\phi_{\text{AB}}\)) equal to the flux of the field through the elementary square plaquette (thick blue square). In the standard experiment, the tunnelling probability amplitudes are real and positive, leading to the spread of the wave function. By using appropriately designed laser-assisted tunnelling, or lattice shaking and modulations, one can induce a complex tunnelling amplitude with a phase that realizes a ‘synthetic’ Aharonov–Bohm effect. In practice, hopping in the horizontal direction is induced by a laser running-wave in the vertical direction whose phase is ‘imprinted’ onto the hopping amplitude.

**Non-Abelian gauge fields.** The concept of non-Abelian gauge fields can be intuitively explained by considering the tunnelling of a particle (dark red circle) in a square lattice. Suppose that the particle now has an internal structure (see top right; for example, a spin or just a few internal states). In this case, tunnelling may lead to coherent transformation of internal states. The tunnelling probability amplitudes are now expressed by a unitary matrix describing the transitions of the particle from one site to another with the simultaneous change of internal states. When a particle goes around an elementary plaquette (red square) it acquires not only a phase, but in general undergoes a unitary transformation of its internal structure.
of using designed wave packets to mimic fundamental relativistic effects, such as time dilation or length contraction. And time dilation can, in turn, be exploited to prolong the lifetime or otherwise affect the decay of unstable particles. Kamener et al. proposed a concrete way of prolonging the lifetime of an unstable hydrogen isotope by preparing it in a certain initial motional wave packet.

In ultracold atom systems with artificial gauge potentials, it is possible to impose gauge invariance, enabling the measurement of gauge-dependent quantities. Interestingly, one can also create artificial non-Abelian gauge potentials (Box 1) enabling the realization of a non-Abelian Aharonov–Bohm effect. This immediately raises a natural question: could the approach of Kamener et al. also provide a solution for the non-Abelian case? This would involve multicomponent (multicolour or multilavour) relativistic theories, such as SU(N) symmetric
theories. Another interesting direction would be to look for generalizations of this effect in lattices with Dirac-like dispersion. Synthetic non-Abelian gauge fields lead to many novel phenomena and effects. Will this also be the case when considering shape-invariant self-accelerating solutions of wave equations with non-Abelian gauge symmetry?

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SPIN CHAINS

Long-distance relationship

Photons immediately spring to mind when we talk about long-distance entanglement. But the spins at the ends of one-dimensional magnetic chains can be entangled over large distances too — providing a solid-state alternative for quantum communication protocols.

Chiranjib Mitra

When the quantum state of two particles cannot be described independently, the particles are said to be entangled. Entanglement is at the heart of quantum communication and computation, with photons often the particles of choice. Spin chains, which are one-dimensional magnets, offer an intriguing solid-state alternative for short-range quantum communication, but entangling spins over practically useful distances has proven challenging. Sven Sahling and colleagues have now demonstrated that unpaired spins separated by hundreds of ångströms can be entangled through a series of spin dimers.

Any large-scale future quantum computer would likely be a hybrid system consisting of optical and solid-state components — optical components for long-range communication and solid-state components for connecting several quantum processors or gates on small scales. Networks or chains of spins could serve as solid-state-based channels for quantum information transfer, used to either directly transmit a quantum state or to share entanglement between two separated parties.

For bulk materials made up of spin chains, entanglement can be probed by measurements of macroscopic properties such as magnetic susceptibility and heat capacity — both measures of connected correlation functions. These correlation functions capture the quantum correlations below a characteristic temperature where the system becomes entangled. At extremely low temperatures, the system is in a pure state and is maximally entangled. At finite temperatures, but below the characteristic temperature, the system is in a mixed state. Both magnetic susceptibility and heat capacity can describe the mixed-state entanglement. By probing these macroscopic properties, Sahling et al. demonstrated long-range entanglement in a composite layered system consisting of alternating spin-ladder and spin-chain layers.

To understand their results, one can consider a toy model of antiferromagnetically coupled dimerized Heisenberg spin-1/2 chains. In this case, the interaction energy between any two neighbouring spins — say $S_a$ and $S_b$ — is given by the Hamiltonian $H = J(S_a \cdot S_b)$, where $J$ is the exchange interaction. For an antiferromagnetic interaction, the exchange interaction is positive and the energy is minimized if the two spins are oriented in opposite directions, say $|S_a S_b\rangle$ or $|S_b S_a\rangle$. However, neither of these states is an eigenstate of the Hamiltonian. The eigenstate that correctly captures the antiferromagnetic correlation is a superposition of these two, $1/\sqrt{2}(\ket{S_a S_b} - \ket{S_b S_a})$. This state is the singlet state and an entangled state, as it cannot be written in the form of a product of two states.

The energy level diagram of such a typical dimer as a function of the applied magnetic field is shown in Fig. 1a. For a critical magnetic field strength the excited triplet state, $|S_a S_b\rangle$, crosses the singlet state and becomes a new ground state. Thus, there is a change in the symmetry of the ground state at this critical field and the system, in the thermodynamic limit, is said to have undergone a quantum phase transition — a zero-temperature phenomenon that is driven by quantum fluctuations, as opposed to thermal fluctuation-driven transitions in classical systems.

The magnetic susceptibility, magnetization, and heat capacity data can all be understood in terms of this toy model and its energy level diagram. There are three types of interaction in the spin-1/2 chain. The strongest is the antiferromagnetic nearest-neighbour interaction ($J_1 = 115$ K) that causes the dimerization of certain copper spins. The next is interdimer coupling, which is a weaker ferromagnetic

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