Stochastic and Robust design procedures applied to the optimization with uncertainties

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Summary

Engineers agree with the fact that uncertainty is an important issue to get a better model of real behavior. Uncertainty quantification techniques have been largely developed during the recent years; Stochastic and probabilistic collocation methods are clear examples of recent developments. This work is based on a more traditional uncertainty quantification method, as Monte-Carlo method and its extension to Latin Hypercube sampling techniques. Two definitions of the optimization problem have been analyzed. The first one is the called Stochastic procedure, while the second one is the Robust one. Both of them deal with uncertainty on the input parameters, but they manage the uncertainty effects from two different points of view. Applications to aerodynamics and aero-elastic problems have been described.

Keywords: Stochastic optimization, robust design optimization, uncertainty, Monte-Carlo, Latin Hypercube sampling, evolutionary algorithms.

1. Introduction

The necessity to deal with uncertainty is an accepted design issue. This topic is becoming more and more relevant in the research agendas around the world. A clear example is the NODESIM-CFD project\(^1\), which dealt with uncertainty quantification techniques and how to be applied to the design phase as a FP6 European Commission research project. It demonstrated the industrial interest the uncertainty management techniques have, as well as studied and developed several quantification techniques\(^2\)-\(^7\). One of those techniques was the Monte-Carlo method. The implementation of Monte-Carlo method, and its close friend the Latin Hypercube techniques, is one of the main tools used in the present research.

This communication presents two levels of the same problem. The first level is focused on the uncertainty quantification technique and its application to the aerodynamic and aero-elastic analysis. It may be considered a first step to the definition of uncertainty into the optimization problem due to the fact it helps to identify the most relevant parameters affecting the variability of the results. The second level is focused on the optimization problem.

Regarding the optimization problem definition, two procedures have been defined to deal with the uncertainty. The first one is the stochastic analysis procedure, which defines the uncertainty in the input variables, and only takes the mean value of the stochastic analysis of each design as the fitness function of the optimization analysis. The second procedure, the robust one, defines two objective functions: namely the mean and the standard deviation. Both of them are able to deal with parameters uncertainty in the input values, as required, and both of them takes into account the variability of the output values. The main difference is that the stochastic procedures provides an easy visualization of the effects of the uncertainty thanks to a low number of fitness functions, while the robust procedure doubles this amount of functions. On the other hand, the robust procedure ensures a better control of the variability induced on the outputs without a significant increment on the complexity of the definition.

This abstract communication is organized in four sections. The first one is the introduction it is about to end. The second one shows a brief description of the stochastic analysis and the results obtained from the two selected applications, the aerodynamic and the aero-elastic problem. The third one describes the optimization problem with uncertainties, as well as the two procedures. A brief comparison of both of them is discussed. Finally some conclusions are presented.
2. Stochastic analysis

2.1. Methodology

The methodology is better understood if it is compared to the standard deterministic analysis. The Figure 1 and Figure 2 show both the deterministic and the stochastic analysis flowchart. The first one is pretty simple. It defines the evaluation point, and gets the results from the solver. Only one point is evaluated so no variability can be analysed.

![Figure 1. Flowchart of a deterministic analysis](image)

On the other hand, Figure 2 shows the stochastic flowchart. Differences are all around. The initial definition of an evaluation point becomes now the random definition of the set of evaluation points through the use of the stochastic and statistical information about the uncertainties. The single evaluation of the deterministic analysis becomes now a set of evaluations, so the outcome is a set of result values. These values are used to statistically analyse the results.

![Figure 2. Flowchart of the Monte Carlo and Latin Hypercube analysis](image)

2.2. Results

If an aerodynamic analysis is performed to study the variability of aerodynamic coefficients (lift and drag, $C_l$ and $C_d$) when angle of attack (AoA) and Mach number (M) are defined under uncertainty conditions the results can be analysed comparing the mean and the standard deviation ranges for several cases, as shown in Figure 3. Or the physical meaning of the values can be plotted, as done in Figure 4. In this case it is easy to realize the effects of the variability. The standard deviation range reaches the point where the transonic shock appears, and it can be easily identified. The Figure 4 compares the deterministic and two stochastic cases, when AoA remains constant and when a when AoA is defined with a standard deviation equal to 0.5. Variable definition based on a Gaussian probability distribution is usually applied.

![Figure 3. $C_d$ ranges](image)

![Figure 4. $C_d$ physical representation](image)

Similarly, the aero-elastic case defines AoA and M under uncertainty. If the deterministic solution is compared with the stochastic one some effects, which the deterministic solution is not providing any clue about, can be identified.

![Figure 5. Angular oscillation in the deterministic case](image)
3. Stochastic and Robust optimization procedures

3.1. Methodology

The flowcharts of each procedure help to identify the main difference. The deterministic one, shown in Figure 7, is the usual evolutionary algorithm flowchart, which has been taken as the basic optimization method.

![Figure 7. Flowchart of a deterministic optimization](image)

The other two procedures, shown in Figure 8, modify the input values generation, the fitness function evaluation and the calculation of this fitness function. No significant differences can be identified between the stochastic and the robust procedures, only the fitness function definition.

![Figure 8. Flowchart of a stochastic/robust optimization](image)

3.2. Results

A brief description of the results is provided through the following figures. Figure 9 is a comparison between the three defined procedures. Once the optimum individuals have been obtained, one of them has been selected and analyzed along a range of values, which is centered to the mean values of the uncertain parameter. M equal to 0.7 is the mean value of the stochastic definition of the parameter. Figure 9 shows how the stochastic and the robust cases, which deal with uncertainty, are able to obtain a more stable behavior along the M range. The deterministic optimum point is not able to keep the best performance it obtains in the evaluation point outside this, while the stochastic and the robust do so. It is worst to mention that the optimum individual selected from the Pareto front can produce a completely different behavior, but all the Pareto members should obtain a more stable behavior when compared with the deterministic case.

![Figure 9. Comparison between the procedures](image)

For the aero-elastic problem, Figure 10 shows the physical representation of the angular oscillation for the...
Pareto members in the three optimization analysis, as well as the base-line design. The deterministic case shows a large variability among its Pareto members, while the stochastic and the robust cases restrict the variability to smaller values.

Figure 10. Comparison between the physical results

4. Conclusions

A brief and quick overview has been described regarding the analysis procedures and the results, but it is clear the great difference between considering or not the parameters uncertainty. Once the problem is taking into account the uncertainty a better control of the operating conditions is obtained.

It is well-known that Monte-Carlo method is an expensive one. But on the other hand, its computational cost remains constant whatever the number of uncertainties. The main drawback is the large amount of evaluation points required to deal with Monte-Carlo. In comparison, Probabilistic collocation method only requires few evaluation points in certain conditions. The debate is open and each method will fit to some application better than others.

The use of Monte-Carlo is based on its maturity and the ability to easily calculate the four statistical moments of the results, which can helps to characterize them.

The use of surrogate models to reduce the computational cost has been investigated. Also the use of self-Organized maps to better understand the solution space when its degree is higher than 3 has been investigated. Future work which leads to the parallelization and the ability to use high performance computing is also under investigation. Not only the parallelization of the solver but also of the optimizer has been taken into account.

5. References


