Mejora de la transformada inversa de Abel: aplicación a occultaciones ionosféricas de FORMOSAT-3/COSMIC

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Outline

1. Introduction
2. Classical Abel transform applied to:
   - GPS phase combination $L_1$ & $L_2$
   - $L_1$ bending angle
3. Improved Abel transform
4. Results for Ascension, Arenosillo & Ebre
5. Results for general performance
6. Conclusions
Electron density from occultation scenarios

Basic observable:
- Linear combination of dual frequencies (\(L_1 = L_1 - L_2\))
- Single frequency (\(L_1\))

Assumption of \(L_1\) and \(L_2\) on the same path
Clock calibration

The GPS receiver on the LEO observes the change in the delay of the signal path between the GPS and the LEO satellite.

This change in the delay includes the effect of the atmosphere which delays and bends the signal.
The basic measurement is the phase path:

\[ L = \int_{GPS}^{LEO} n ds \]

From it, the excess phase is defined:

\[ \Delta L = L - |\vec{r}_{LEO} - \vec{r}_{GPS}| \]

The change rate of the excess phase, called excess Doppler, is what is going to become our input observable:

\[ \Delta D = \frac{d(\Delta L)}{dt} \]

The projection of satellite orbital motion along signal ray-path produces a Doppler shift at both the transmitter and the receiver. The fundamental observable is the signal Doppler shift, which is different than expected from only velocities due to the satellite and receiver clock drifts and the atmospheric bending of the signal (ionosphere and troposphere).
The signal path is curved due to the changes in the refractive indexes along the ray trajectory according to Snell’s law.

Locally, in a spherical symmetric medium, Snell’s law is replaced by Bouger’s law imposing an extra constraint.

- Satellite positions and velocities known (both GPS and LEO) from navigation messages.
- Clock drifts required (for instance double differencing).
- Excess Doppler known derived from observations.
- With local spherical symmetry assumption, the bending angle will be determined associated to each impact parameter. An extra assumption needed: refractive index equal to 1 at LEO and GPS starting occultation positions i.e. void, with a negligible error [Hajj and Romans, 1998].
- Using inversion techniques, the refractive index will be derived from the knowledge of bending angles.
Classical Abel transform applied to bending angles

Each GPS occultation event is independently solved

The classical spherical symmetry hypothesis can be expressed as:

$N_e(LT, LAT, H) = \Phi(H)$

- Recursive solution starting from the outer ray.
- $\alpha_i$ corresponds to the bending angle of the ray with impact parameter $p_i$. 

Unknown to be solved is $N_e$
Improved Abel transform

A more general approximation than the spherical symmetry is assumed:

\[ N_e(LT, LAT, H) = VTEC(LT, LAT) \cdot F(H) \]

New unknown instead of \( N_e \)

VTEC information externally provided

Shape function
FORMOSAT-3/COSMIC mission

✓ Constellation Observing System for Meteorology Ionosphere and Climate (ROCSAT-3)
✓ 6 Satellites launched in April 2006
✓ Orbits: alt=800km, Inc=72deg, ecc=0
✓ Weather + Space Weather data
✓ Global observations of:
  ➢ Pressure, Temperature, Humidity
  ➢ Refractivity
  ➢ TEC, Ionospheric Electron Density
  ➢ Ionospheric Scintillation
✓ Demonstrate quasioperational GPS limb sounding with global coverage in near-real time
✓ Climate Monitoring
✓ Geodetic Research

Information available at www.cosmic.ucar.edu
Profiles retrievals
Profile retrievals: Intercomparison

Spherical Sym.
Separability

Rest of colors: co-located calibrated ionosonde data
1st Analyzed Scenario: January 6th to 15th, 2007

For the studied period, the Solar activity was at very low to low levels and the geomagnetic field ranged from quiet to minor storm conditions. The graphics on the right show the status of the following indexes: solar 10.7 flux, the Ap and daily Kp indexes during the selected days.
Assessment of Ionosonde reference values: Ascension

Period: January 6th to 15th, 2007

Relative Error

Relative error between foF2 from SPIDR website vs calibrated.

Maximum co-location distance set to 2000 km and 1 hour LT time span.
Results: Ascension

Period: January 6th to 15th, 2007

![Chart showing Relative error in Ascension with Day as X-axis and Number of samples as Y-axis.](chart.jpg)
Results: Ascension

Period: January 6th to 15th, 2007
Results: Ascension

Period: January 6th to 15th, 2007
Results: Arenosillo

Period: January 6th to 15th, 2007
Results: Arenosillo

Period: January 6th to 15th, 2007

![Graph showing relative error and separability over days]

**Relative error [%]**
- Sph. Sym.

**Separability**

**Number of samples**
Results: Arenosillo

Period: January 6th to 15th, 2007

![Graph showing relative error and separability over days from January 6th to 15th, 2007.](image-url)
Results: Ebre

Period: January 6th to 15th, 2007
Results: Ebre

Period: January 6th to 15th, 2007

<table>
<thead>
<tr>
<th>Day</th>
<th>Relative error [%]</th>
<th>Sph. Sym.</th>
<th>Separability</th>
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</table>

Number of samples
Results: Ebre

Period: January 6th to 15th, 2007

![Graph showing Relative error [%] and Separability over days 6 to 15.](image)
2nd Analyzed Scenario: 12th to 18th of December 2006

COSMIC/FORMOSAT-3 RO data used. In spite of 2006 being a quiet year in terms of ionospheric variability, a solar flare on Dec. 13th produced strong radio blackouts and an associated solar storm, a geomagnetic storm on Dec. 14th and strong to severe geomagnetic storming through Dec. 15th.
Ascension: Assessment of Ionosonde reference values
Period: Doy 346 to 352 of 2006

Maximum co-location distance set to 2000km and 1 hour LT time span

Relative error between foF2 from SPIDR website vs calibrated.

Local Time (h)
Results: Ascension

Period: from 346 to 352 of 2006
Results: Ascension

Period: from 346 to 352 of 2006

![Graph showing results of ascension measurements during the period from 346 to 352 of 2006. The graph includes data on relative error, spherical symmetry, and separability.](image-url)
## Results: Ascension

Period: from 346 to 352 of 2006

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- **Relative error [%]**
  - Sph. Sym.

- **Separability**

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*Influencia de la anomalía ecuatorial de la ionosfera, Madrid, 24-26 Septiembre, 2008*  
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Results: Arenosillo

Period: from 346 to 352 of 2006
Results: Arenosillo

Period: January 6th to 15th, 2007
Results: Arenosillo

Period: January 6th to 15th, 2007
### Results: Ebre

**Period:** January 6th to 15th, 2007

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**Relative error [%]**

- Sph. Sym.

**Separability**

- Number of samples

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Period: January 6th to 15th, 2007
Results: Ebre

Period: January 6th to 15th, 2007

![Graph showing relative error and separability over days.]

- Relative error [%]
- Sph. Sym.
- Separability

Number of samples
General results: Bending angle

Performed comparisons:

- **Frequency estimations**: $f_0F_2$ of profiles given by FORMOSAT-3/COSMIC data directly compared to ionosonde measurements

- Height estimations: No direct measurements of $h_mF_2$ with ionosonde ➔ Use of Dudeney formula for $h_mF_2$ [Dudeney, 1983]

- **Comparison criteria**:
  1. Max. co-location distance set to **2000km**
  2. Max. time difference set to **1 hour (LT)**
Results: foF2 from bending angle

F2 layer critical frequency comparisons using ionosonde foF2 parameter

Typically: Separability 45% better

Co-location ≤ 2000km

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relative RMS difference (%)
Results: foF2 from bending angle

Low latitudes vs. mid & high latitudes

Separability vs. Spherical Sym.

RMS is reduced

Typically: Separability 45% better
Results: foF2 from bending angle

Low latitudes vs. mid & high latitudes

Separability vs. Spherical Sym.

RMS is reduced
Typically: Separability 20% better

Mid & high latitudes 2000 km separability vs. Spherical Sym.

Night

Day & dusk & dawn

0 10 20 30 40 50 60 70 80 90 100
relative RMS difference (%)
Conclusions

The results from this study using FORMOSAT-3/COSMIC occultation data show:

- Usefulness of implementation of **Separability technique** for the retrieval of electron densities from **bending angles**.

- For ionospheric sounding, the equivalence of using LI and L₁ bending angle as main input has been shown on the analyzed data set. This confirms previous results in which the different path between L₁ & L₂ cannot be considered a drawback when inverting profiles [Tsai et al., 2001].

- The **Improved Abel transform** provides more accurate determination of foF₂ (≈45% **better**) regardless of the chosen observable, either LI or L₁ bending angle.

- When working with bending angles, clocks can be calibrated by means of the ionospheric-free combination of carrier phases avoiding double differencing strategies (valid for ionospheric heights).
References

- Hernandez-Pajares, M., J.M. Juan, J. Sanz, Real Time MSTID Modeling on Wide Area and Regional Networks and Improvement of Precise GNSS Navigation, ION GNSS, Fort Worth, USA, September 2006b.
Gràcies!