

SECOND ORDER ANALYSIS OF REINFORCED CONCRETE FRAMES

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SUMMARY

In this paper we propose a second order method to analyse concrete frames based on the imposed deformations method. The geometrical nonlinearity is introduced by - using imposed curvatures associated to the "second order isostatic forces" defined in this paper. In the same way material nonlinearity is introduced by means of imposed curvatures obtained from the axial force-moment-curvature diagram of each - cross section by compatibility of material properties requirements. The proposed - method can be considered to fall within the "General Method" proposed by the C.E.B. Several structures are analysed comparing the results with those obtained by other analytical methods and conclusions are drawn both relative to the method's proper - ties and the structural behaviour.

1. INTRODUCTION

The continuing development in the field of concrete technology has made possible to get high concrete strengths and also to build more slender structures in which the presence of high compression loads can lead to instability problems.

In the treatment of this problems the displacements of the structure can not be neglected, and hence the deflection line of the element must be included in the computation of the bending moment at each section. This is known as "second order analysis" and the increase in forces and displacements of the structure, relative to those obtained by linear theories, generated by the action of the axial force in the deflected line are called "second order effects".

This Kind of analysis presents, in the case of concrete structures, serious difficulties because of nonlinearity on the materials stress-strain relationships, cracking of concrete when tensile stresses reach the concrete tensile strength, creep of concrete under sustained loads, and other phenomena as tension stiffening, concrete confinement by reinforcement a.s.o.

The effect of material non linearity is twofold on reinforced concrete structures: On the one hand it causes redistribution in the structure and its sections forces.

On the other hand there is an important variation of the structural deformability due to the larger rotation capacity of the concrete sections than in case of elastic linear materials which affect directly to a second-order analysis.

Obviously the analytical treatment of both non-linearities presents serious difficulties and it is practically impossible even in the case of a simple member. On the other hand the extrapolation of the results obtained from the analysis of simple members to the case of members included in frames, that in the case of linear-elastic material, is performed by means of the effective length concept, is much more problematic in the case of reinforced concrete and their results can be discussed.

Finally, in many codes of practice, starting from certain values of column slenderness and certain structural requirements it is recommended to use a general method accounting for the phenomena above mentioned.

All this reasons lead to the need of having available general methods to be applied in certain specific conditions although these methods must gather all so simplicity, power and economy. This is the case of the second order method of analysis presented herein.

2. PROPOSED METHOD. GENERAL CHARACTERISTICS

The proposed method is applicable to reinforced concrete plane frames, acted upon by any kind of action (loads, end displacements, thermal effects ...). It is a nonlinear second order method of analysis which presents two well differenced stages:

- I Elastic linear analysis of the structure, acted upon by the considered actions, assuming elastic rigidities EI , EA for the cross sections.
- II Iterative procedure induced by the introduction in the structure of imposed deformations to account for the material and geometrical nonlinearities, whose values depend on the results of the previous iteration.
In this procedure the stiffness matrix of the structure remains unchanged being the same as that calculated in the first stage (linear analysis).

In addition to the material strength's own assumptions and those more usual in reinforced concrete, an specific assumption of this method is the following:

The variation of the axial force in a member from one iteration to other introduces negligible variations on the axial force-bending moment-curvature diagram $\bar{N}-\bar{M}$ of this member sections.

This assumption is verified in the examples shown in this paper and it allows to use the same N-M-C diagram along all the iterative procedure.

2.1. Treatment of the second order effects

Let us consider an slender structure acted upon by axial and lateral loads as shown in figure 1. The structure will be assumed to be hyperstatic, laterally unbraced and made out of a linear elastic material.

Figure 2 shows a member (A,B) of the structure with the end forces and displacements obtained from a linear first-order analysis.

If we assume that there are no lateral loads applied between joints in the member, the first order forces acting on an arbitrary section of abscissa x (in local coordinates) will be:

$$\begin{aligned} M_1(x) &= -M_{AB} + P_{AB,y} \cdot x && \text{(Moment)} \\ V_1(x) &= P_{AB,y} && \text{(Shear force)} \\ N_1(x) &= -P_{AB,x} && \text{(Axial force)} \end{aligned} \quad (1)$$

They will be designated together by $E_1(x)$.

Considering the effect of axial forces in the equilibrium of the deformed member, assumed isostatically supported, it must appear at the ends A and B force increments relative to those obtained from a linear analysis of value: (see figure 3)

$$\begin{aligned} \Delta M_A &= -N_1 \cdot \delta_1 \\ \Delta V_A &= N_1 \cdot \delta_1/L \\ \Delta N_A &= 0 \end{aligned} \quad (2)$$

In an arbitrary section of deflection: $Y_1(x)$ (with the axis shown in figure 3) the forces increments are:

$$\begin{aligned} \Delta M^{II}(x) &= -N_1 \cdot Y_1(x) + N_1 \cdot \delta_1 (1-x/L) \\ \Delta V^{II}(x) &= N_1 \cdot \delta_1/L \\ \Delta N^{II}(x) &= 0 \end{aligned} \quad (3)$$

This forces will be called "second order isostatic forces", and will be designated by ΔE_J^I , where I stays for the initial or "isostatic" and J the number of the iteration considered.

If we neglect the effects of shear deformations and being the flexural elastic rigidity EI , this moment increments can be associated to some deformations (curvatures) of value:

$$\Delta \varphi^I(x) = \frac{\Delta M^{\text{II}}(x)}{EI} = N_1/EI \left(-Y_1(x) + \delta_1 \left(1 - \frac{x}{L}\right) \right) \quad (4)$$

This deformations will be designated by $\Delta \varphi^I_J$, having I and J the same meaning as above. Knowing the second order isostatic forces and their associated curvatures, they can be introduced in the structure as actions (geometrical actions or imposed deformations) performing an analysis of the structure (with known characteristics) subjected to these actions. From a matrix point of view this is equivalent to a variation of the force vector, keeping constant the stiffness matrix of the structure.

The results of this calculation will be the forces and displacements called "second-order hyperstatic effects" (ΔE^H_J and $\Delta \varphi^H_J$) that appear to make compatible the imposed deformations, and which must be added to the "second order isostatic effects" to close the loop. That is to say, at the end of this first iteration - the so-called second order forces are:

$$\Delta E_1 = \Delta E_1^I + \Delta E_1^H$$

and the displacements:

$$\Delta \varphi_1 = \Delta \varphi_1^I + \Delta \varphi_1^H \quad (5)$$

which must be superimposed to the results of a first order analysis

$$E_2 = E_1 + \Delta E_1 \quad \text{and} \quad \varphi_2 = \varphi_1 + \Delta \varphi_1 \quad (6)$$

From here on it must be repeated the cycle because, although the obtained solution up to now is compatible, forces and displacements of the structure have been altered and so it must be reestablished the equilibrium in a second order treatment.

This should be made in spite of the structure being isostatic because of the nature of the second order forces.

With the final forces and displacements obtained from the first cycle we can initiate the 2nd iteration, obtaining the new deflection line on each member ($Y_2(x)$) and hence the new second order isostatic forces will be:

$$\begin{aligned} \Delta M_2^I(x) &= -N_2 \cdot Y_2(x) + N_2 \cdot \delta_2 (1-x/L) \\ \Delta V_2^I(x) &= N_2 \cdot \delta_2/L \\ \Delta N_2(x) &= 0 \end{aligned} \quad (7)$$

From here on we would obtain the associated curvatures to this forces and after a structural analysis under this imposed deformations, the superposition of isostatic and hyperstatic second order effects over the results of the 1st. order - analysis provide the final forces and displacements of the second cycle:

$$\begin{aligned} E_3 &= E_1 + \Delta E_2^I + \Delta E_2^H \\ \varphi_3 &= \varphi_1 + \Delta \varphi_2^I + \Delta \varphi_2^H \end{aligned} \quad (8)$$

The iterative procedure must continue until the results of two consecutive iterations be sufficiently similar (in case of convergence of the procedure). In this case one can ensure that it has been achieved an equilibrated in second order and

compatible solution of forces and displacements of the structure.

The critical system of loads (that produces the structural instability) can be obtained by increasing the compression load system step by step and keeping constant a perturbing load leading to the 1st. buckling mode. For each load step it must be performed the iterative procedure described above until its convergence - in which case it has been found a point on the load-displacement curve that represents the structural equilibrium.

Instability raises, in this method, by the divergence of the iterative procedure, giving rise to a monotonic series of values of the studied parameters. That is different from other kinds of divergence.

The proposed method is applicable both to laterally braced and unbraced frames. In practice the first case is a particular case of the second one, since in equation (3) making $\delta = 0$

$$\Delta M_j^{\text{II}}(x) = -N_j \cdot Y_j(x)$$

2.3. Unified treatment of material and geometrical nonlinearities

If the considered structure is made out of reinforced concrete we must include in the analysis the effects of material nonlinearity. For this purpose it will be used, as working tool the N-M-C diagram of each cross section (9) and (12). Let figure 4 represent the N-M-C-diagram of an arbitrary cross section. The treatment of material nonlinearity alone by the imposed deformations method has been performed in (1) and (2). In this case we include also the second order effects by the procedure presented above.

Let us suppose that the structure has been analysed using a linear flexural rigidity $E.I = K_0$ for this cross section (straight line passing through the origin - with slope K_0). The linear solution leads, in this section, to a Moment M_0 not equilibrated in second order which violates the material properties (it is not on the N-M-C curve, point 0 in figure 4). The introduction of the isostatic second order moments ΔM_1^{I} and its associated curvatures can be graphically seen in figure 4, which is equivalent to displace point 0 to 0'. To fulfil the material conditions it must be introduced and imposed curvature C_1 equal to the difference between the curvature corresponding to Moment M_0 according to elastic rigidity K_0 and that following the N-M-C diagram. The analysis of the structure under the combined action of the second order isostatic curvatures (C_1) and those due to material, leads to the hyperstatic response ΔM_1^{H} (straight line 0'-1) so that the final moment in this iteration will be the algebraic sum of M_0 , ΔM_1^{I} , ΔM_1^{H} . Continuing the iterative procedure we would find the final solution F which beside being equilibrated in second order and compatible fulfils material conditions.

3. APPLICATION EXAMPLES

Two examples will be presented. The first one is a reinforced concrete cantilever column. The object of this example is to compare the results obtained using the proposed method with those obtained by simplified methods of analysis. The second example is a simple reinforced concrete portic frame with fixed ends whose results can be compared with those obtained using other general methods.

This latter example is the same presented in C.E.B. Bulletin n° 103 (5)

3.1. Example nº 1

Figure nº 5 shows geometrical, mechanical and loading characteristics of the structure. Two loading systems have been considered. System 1 corresponds to an axial load F and lateral load H . System 2 substitutes the lateral load H by an applied moment M .

Structural instability has been achieved using a step by step procedure, increasing the perturbing load keeping constant the axial load.

Figure nº 6 shows the N-M-C diagram of the column cross section. The results of the analysis are shown in figure nº 7, where the top deflection (δ) is plotted versus the adimensional 1st order moment on the base of column, for the two loading systems. In both cases instability failure is reached before material failure, although the 2nd loading system is more unfavorable than the 1st one. (due to the influence of the 1st order moment distribution). In table 1 are compared these results with those coming from other methods (3), (5) and (15).

3.2. Example nº 2

Figure nº 8 shows schematically the analysed structure. The results of the analysis by the proposed method are compared with those obtained by General Frame Analysis (5), with those obtained by the method proposed by GRELAT(11) and with those obtained by the Model Column Method (6) and the simplified method of GRELAT (11).

The structure is a simple fixed feet portical frame (sway permitted) subjected to vertical and lateral loads. The columns cross sections is constant and the beam presents the same N-M-C diagram in its full lenght.

Cross sectional analysis provides the N-M-C diagrams of the columns and beam sections, shwon in figure nº 9.

The ultimate lateral load has been found keeping unchanged the vertical loads and increasing lateral loads. The results are shown in figure 10 where the lateral load are plotted versus the lateral top deflection of frame.

Instability failure is reached for a value of the lateral load of 6,26 T. In the same figure are shown the results coming from other methods of analysis. The value of ultimate lateral load is in good agreement in all cases, including those obtained by simplified methods, although these latter remain conservative.

4. FINAL CONSIDERATIONS. CONCLUSIONS

The proposed method of analysis presents the following remarkable characteristics:

- 1.- It is a general method of analysis of reinforced concrete plain frames. It can be applicable both manually and automatically and it includes, in a compact form, the material and geometrical nonlinearities.

It is valid both in service states and ultimate states. Between the ultimate states it can detect both the instability failure (of the whole structure or locals failures) and the material failure at any cross section of it.

- 2.- It can be considered an exact method because of the treatment of the material and geometrical nonlinearities. The plastic rotations are accounted for in all sections whenever they appear and are not concentrated in plastic hinges. With respect to second order effects, the actual deflection line of each member is calculated using the finite difference method, thus making no assumptions relative to the shape of the deflection line.

- 3.- Because of the invariability of the stiffness matrix of the structure along the iterative procedure the C.P.U. time is considerably reduced, which affects the economy of the analysis. Furthermore it is possible to analyze very complicated frames without introducing supplementary joints on members and hence without increasing the matrix stiffness size.
- 4.- To improve convergence of the iterative procedure the matrix stiffness used for linear-elastic analysis can be obtained including the effect of an assumed axial force in each member by means of stability functions c and s . This axial forces, in usual common structures are easy to compute by means of an isostatic distribution of vertical loads, with little error. In this case, the second order isostatic forces and curvatures to be introduced will be those generated by the variation of axial forces relative to the assumed ones N_0 , so that in the expressions 2, 3, 4 and 7 N_j must be substituted by $\Delta N = N_j - N_0$, being N_j the axial force computed on iteration j .
- 5.- The obtained results are in good agreement with those coming from other general methods, yet the proposed method giving a greater conceptual simplicity and a very simple adaptation to any standard computer program of matrix-analysis.

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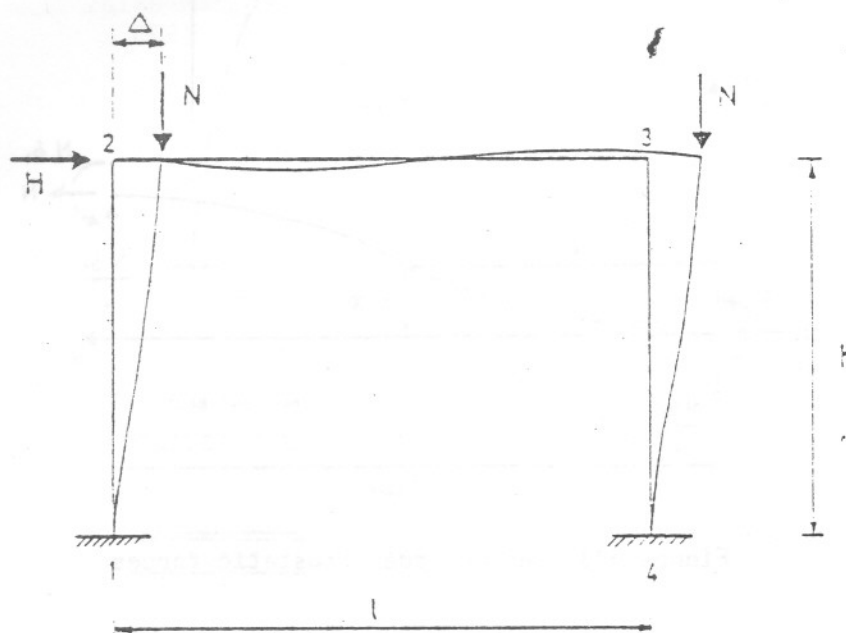


Figure n° 1. Deformed structure under combined axial and lateral loads.

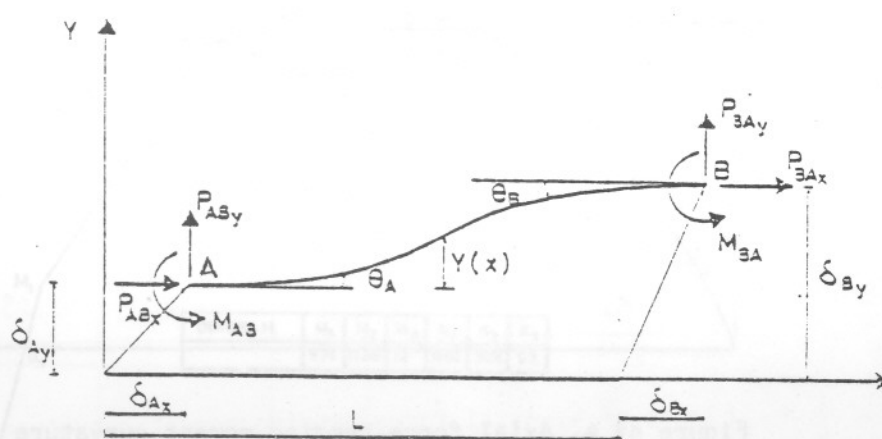


Figure n° 2. End forces and displacements obtained from a first order analysis.

Figure n° 3. Moment-curvature diagram utilized in example n° 1

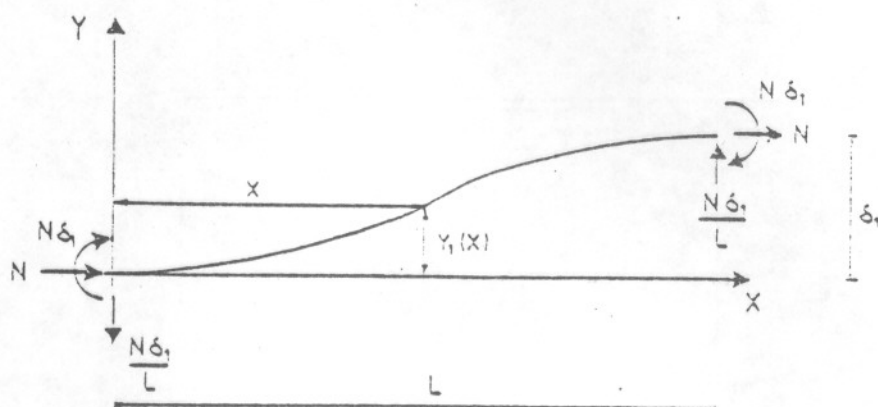


Figure nº3. Second order isostatic forces

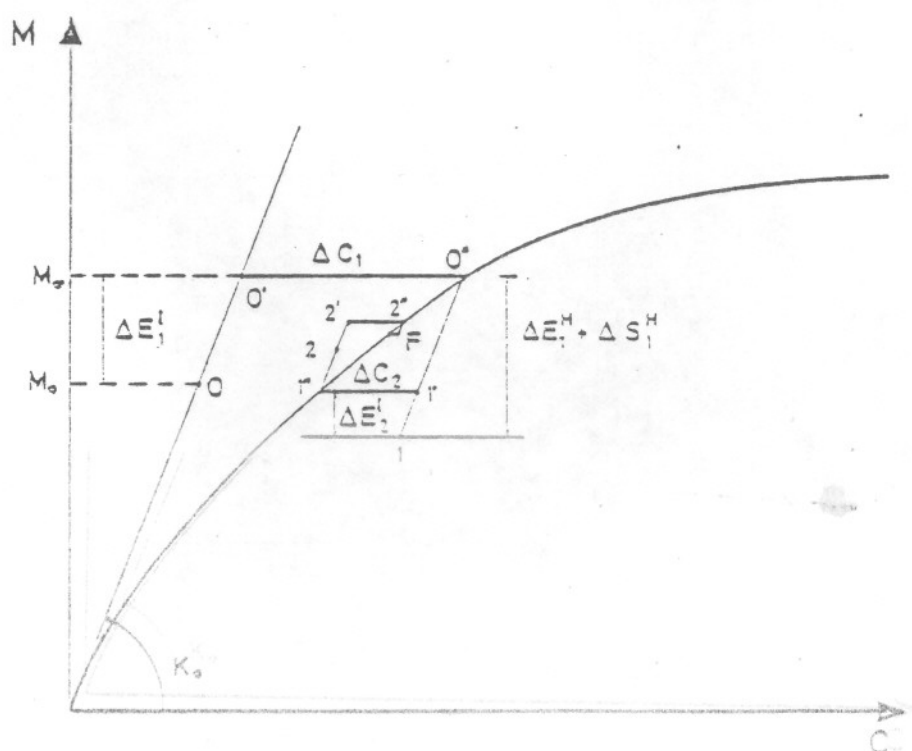


Figure nº 4. Axial force-bending moment-curvature diagram of an arbitrary cross section. Iterative procedure.

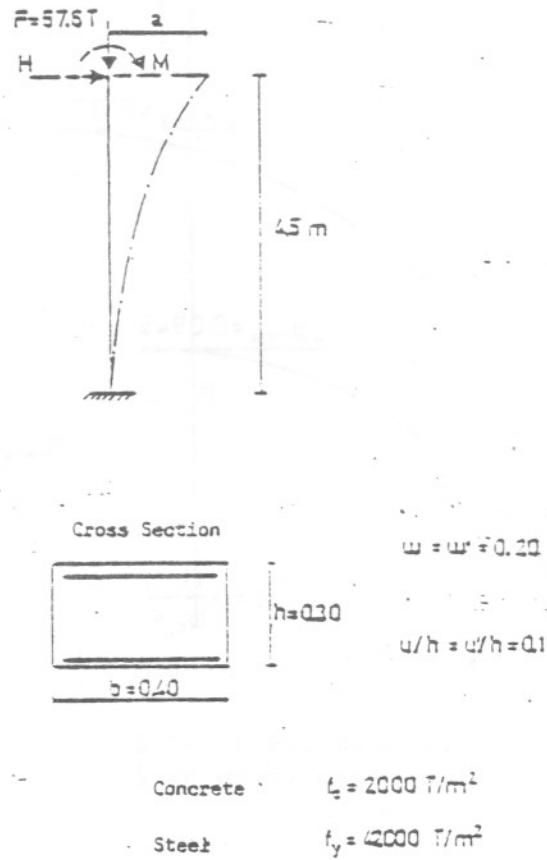


Figure nº 5. Example nº 1. Cantilever column.

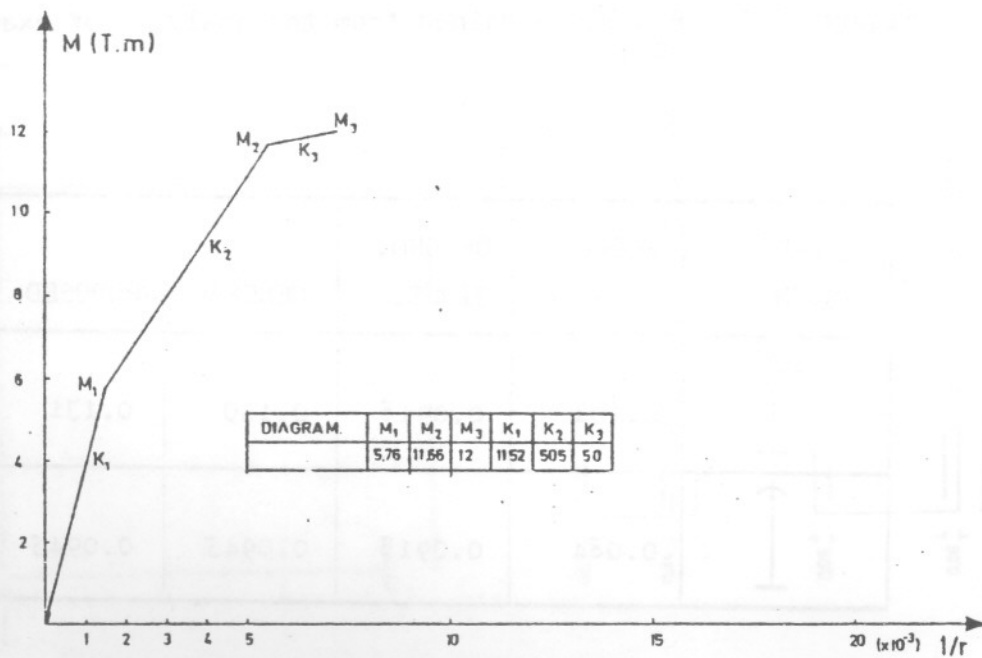


Figure nº 6. Moment-curvature diagram utilised in example nº 1

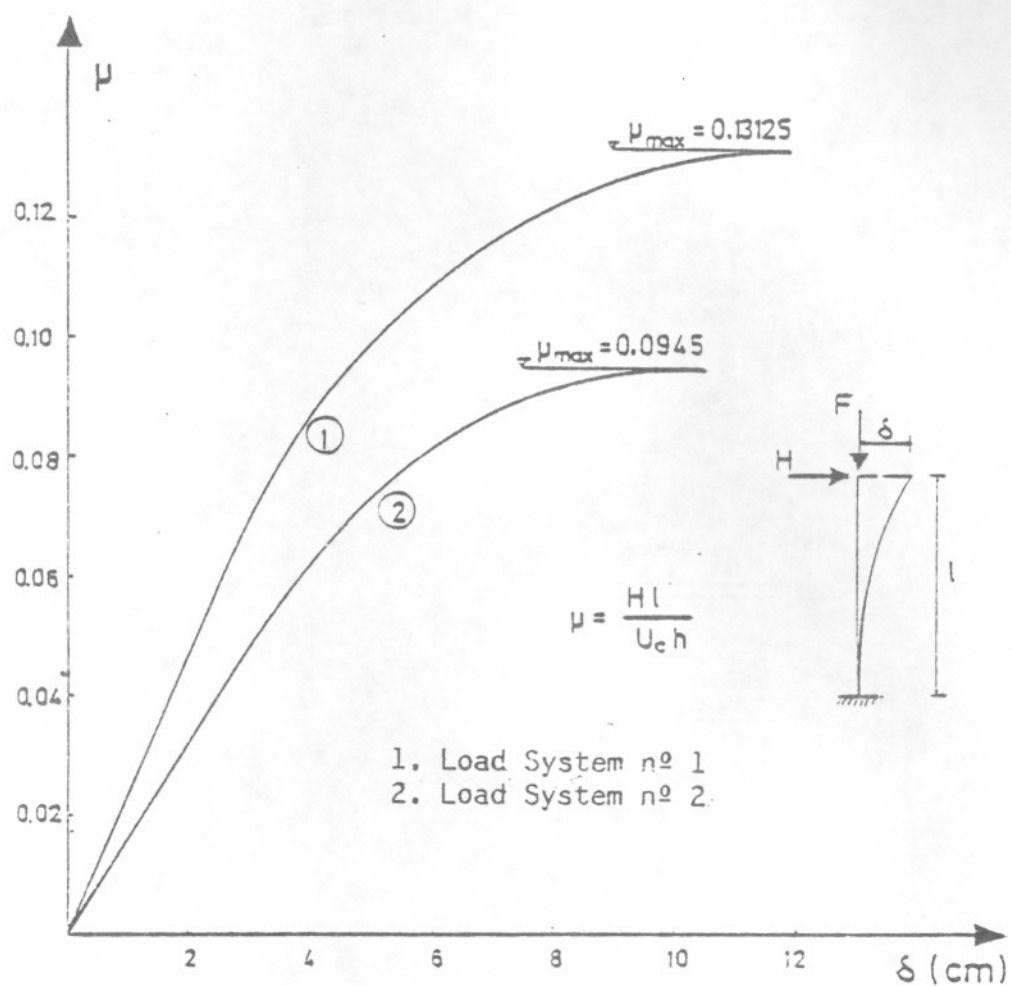
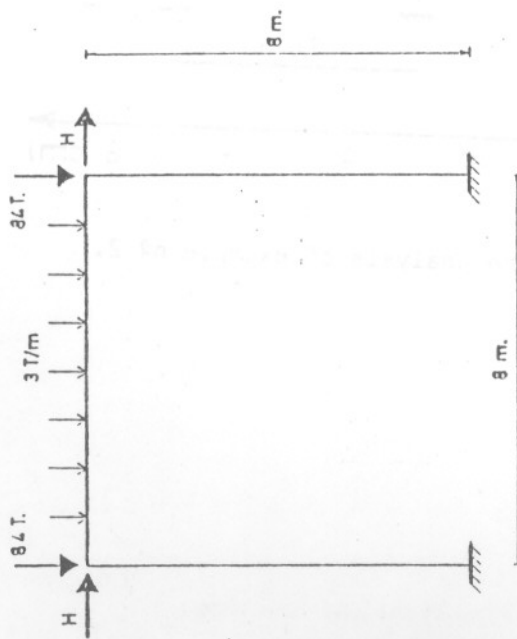


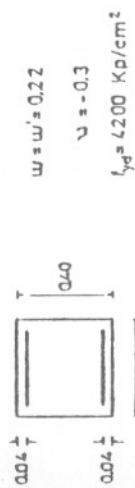
Figure nº 7. Results obtained from the analysis of example nº 1.

LOAD SYSTEM	MODEL COLUMN	DEFORM. SINUS.	GENERAL	PROPOSED
	0.104	0.0918	0.130	0.131
	0.084	0.0918	0.0945	0.0945

Table 1. Comparison of the results of example 1 with those obtained by other methods.



Columns cross section



Beam cross section

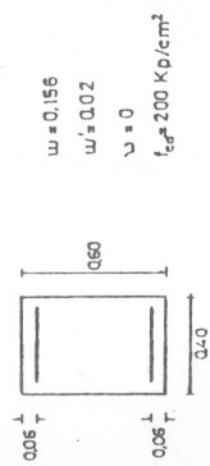
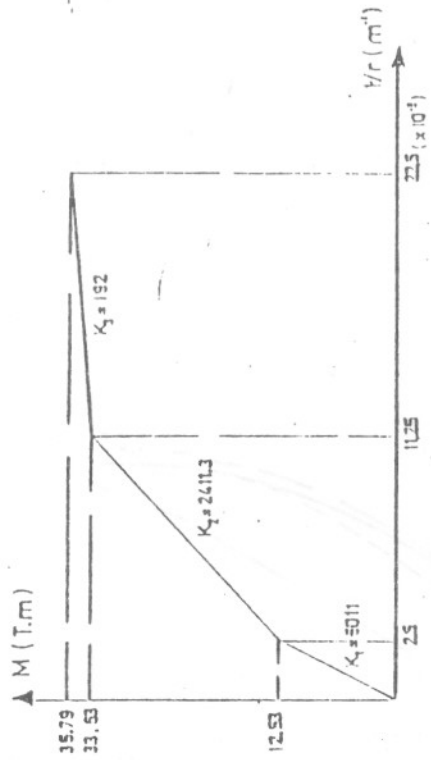
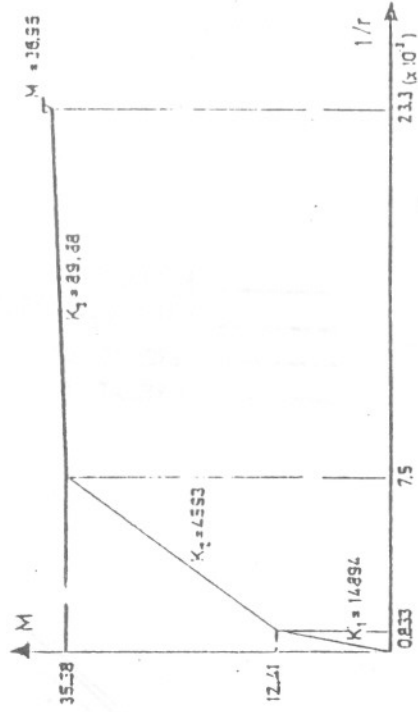


Figure nº 8. Structure analysed in example nº2.



a) Columns



b) Beam

Figure nº 9. Moment-curvature diagrams of the cross sections of the structure analysed in example 2.

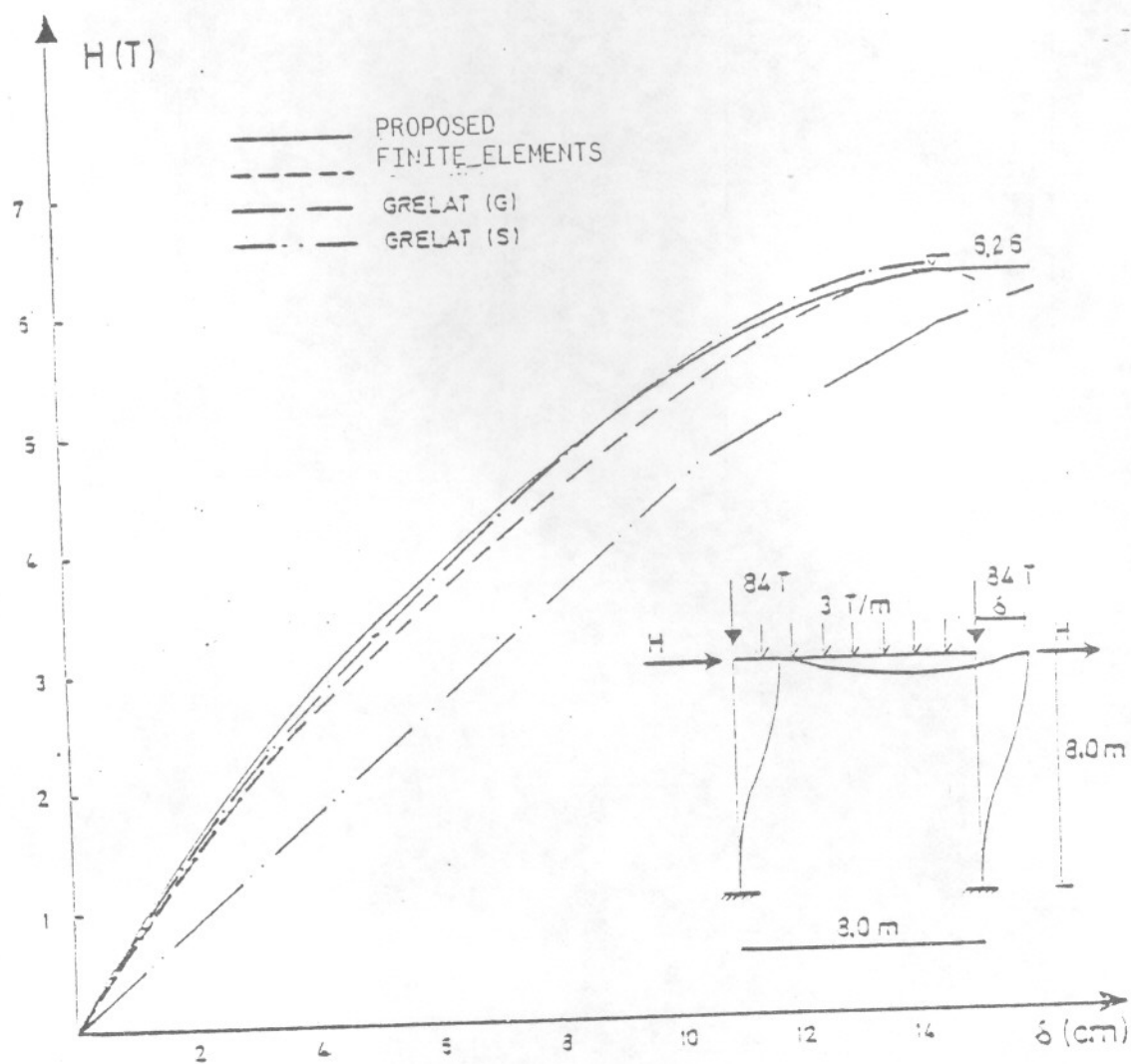


Figure nº 10. Results obtained from the analysis of example nº 2.