Target spinor calibration (TSC) for polarimetric radar cross-section measurements

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A new polarimetric calibration method is proposed based on scattering matrix decomposition, via Pauli matrices. This scheme operates with the Pauli target vectors instead of the usual calibrators such as a sphere, dihedral and 45º rotated dihedral. This fact allows direct and easy calculation of the error coefficients. An experimental validation of this method is presented.

Introduction: Owing to the increasing use of RCS polarimetric measurement systems, it has become indispensable to study and improve polarimetric calibration techniques with the objective of eliminating the measurement of systematic errors. The measured and correct scattering matrices are related by the expression

\[ [\mathbf{S}] = [\mathbf{M}] + [\mathbf{E}] \]

(1)

Where \([\mathbf{S}]\) is the measured target scattering matrix, \([\mathbf{M}]\) is the correct scattering matrix, \([\mathbf{E}]\) is the isolation error matrix, and \([\mathbf{R}]\) and \([\mathbf{T}]\) are the receiving and transmitting distortion matrices. Determination of the error coefficients and correct scattering matrix, may be achieved using different methods [2].

Calibration method: The unit matrix \(\mathbf{S}_0\) plus the three Pauli matrices \((\mathbf{\sigma}_x, \mathbf{\sigma}_y, \mathbf{\sigma}_z)\) form a complete set [3]. All complex \(2 \times 2\) matrices \([\mathbf{S}]\) may be written as

\[ [\mathbf{S}] = \sum_{i=0}^{3} k_i [\mathbf{\sigma}_i] \]

(2)

where

\[ k_i = \frac{1}{2} Tr([\mathbf{S}] [\mathbf{\sigma}_i]^T) \]

(3)

and \(Tr\) denotes the trace of a matrix.

The measured matrix \([\mathbf{M}] = [\mathbf{S}] - [\mathbf{E}]\) can be defined from eqn. 1, and after replacing the correct scattering matrix by its Pauli decomposition, \([\mathbf{M}]\) can be written as

\[ [\mathbf{M}] = \sum_{i=0}^{3} k_i^T [\mathbf{R}] [\mathbf{\sigma}_i] [\mathbf{T}] \]

(4)

From four complex numbers \(k_i, i = 0, \ldots, 3\), a quadridimensional complex vector \(\mathbf{k}\) (Pauli target vector) can be formed [3]. The measured Pauli target vector is

\[ k_i^m = \frac{1}{2} Tr([\mathbf{M}] [\mathbf{\sigma}_i^T]) \]

(5)

By substituting eqn. 4 into eqn. 5, the measured and correct Pauli target vectors are related through the expression

\[ C_{ij} = \frac{1}{2} Tr([\mathbf{R}] [\mathbf{\sigma}_i] [\mathbf{T}] [\mathbf{R}]^T [\mathbf{R}]^T) \]

(6)

Eqn. 6 represented in matrix form is

\[ \begin{bmatrix} k_0^m \\ k_1^m \\ k_2^m \\ k_3^m \end{bmatrix} = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} \\ C_{10} & C_{11} & C_{12} & C_{13} \\ C_{20} & C_{21} & C_{22} & C_{23} \\ C_{30} & C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} k_0^c \\ k_1^c \\ k_2^c \\ k_3^c \end{bmatrix} \]

(7)

Where \(I_n^m\) is the measured Pauli target, \(I_n^c\) is the correct Pauli target and \([\mathbf{C}]\) is a \(4 \times 4\) error coefficient matrix, where the coefficients \(C_{ij}\) are additions and substrations of \(R_{ij}\) terms such as

\[ C_{00} = \frac{1}{2} (R_{00} + R_{11} + R_{22} + R_{33}) \]

(8)

The equations that relate these error coefficients \(C_{ij}\) can be deduced from the nonlinear \(R_{ij}\) equations [4].

Monostatic case: For the monostatic case, \(S_0 = S_0\), and therefore \(k_0 = 0\). Eqn. 8 is transformed to

\[ \begin{bmatrix} k_0^m \\ k_1^m \\ k_2^m \\ k_3^m \end{bmatrix} = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} \\ C_{10} & C_{11} & C_{12} & C_{13} \\ C_{20} & C_{21} & C_{22} & C_{23} \\ C_{30} & C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} k_1^c \\ k_2^c \\ k_3^c \end{bmatrix} \]

(9)

These 12 \(C_{ij}\) error coefficients can easily be obtained from the expression

\[ C_{ij} = \frac{k_{ij}^{m(n+1)}}{k_{ij}^{c(n+1)}} \quad i = 0, 1, 2, 3 \quad j = 0, 1, 2 \]

(10)

where the calibration Pauli target vectors used correspond to the canonical base elements of the Pauli targets, in the \(hv\) base polarisation, and they coincide with the classical three calibrators such as a sphere \((I_s^s)\), a horizontal dihedral \((I_s^d)\) and 45º rotated dihedral \((I_s^d)\), where

\[ \mathbf{R}_s = k_s^s [1 \ 0 \ 0] \quad \mathbf{R}_d = k_d^s [0 \ 0 \ 1] \]

(11)

Once all the coefficients of \([\mathbf{C}]\) have been found, we can obtain the Pauli target of any target, from the measured Pauli target \(I_m^m\), via pseudo-inverse matrix

\[ \mathbf{R} = ([\mathbf{C}]^* [\mathbf{C}]^{-1} [\mathbf{C}]^*)^{-1} \]

(12)

TSC biaxial (TSCB): Returning to the general case posed in eqn. 8, there are different methods for solving the error equation, depending on the kind of calibrators that are used. Since the calibrators that we will use are three calibrators linearly independent and reciprocal \((k_0 = 0)\), only 12 coefficients can be obtained from the measurement of these calibrators. The error coefficients \(C_{ij}\), \(C_{ij}^c\), and \(C_{ij}^m\) must be deduced from coefficient relation equations.
Once \( [C] \) is known, the true Pauli target vector of any target can be obtained from the measured Pauli target vector by using:

\[
\hat{\mathbf{K}} = [C]^{-1} \hat{\mathbf{R}}^m
\]  

(14)

**TSC system with good polarisation isolation (TSCI):** If a system with good polarisation isolation is considered, where double coupling errors can be neglected, that is

\[
R_{00}T_{00} \simeq R_{00}T_{10} \simeq R_{01}T_{01} \simeq R_{01}T_{11} \simeq 0
\]

(15)

there exist only seven independent coefficients, \( C_{00}, C_{01}, C_{10}, C_{11}, C_{02}, C_{20} \), and for that reason we need only to use two calibrators, linearly independent and reciprocal, with Pauli target vectors

\[
\hat{\mathbf{K}}_{11} = k_1^0 [1 0 0] \quad \hat{\mathbf{K}}_{22} = k_1^2 [0 0 1]
\]

(16)

Once these seven error coefficients have been obtained, the other coefficients can be deduced from the coefficient relation equations. Once \([C]\) is known, the correct Pauli target vector is obtained in the same way as in the TSCB method.

**Experimental results:** Experimental results were carried out in a polarimetric compact range measurement facility installed in an anechoic chamber [5]. The kind of calibrators chosen were a disc, which has the same Pauli target vector as a sphere, a dihedral and a 45° rotated dihedral.

![Fig. 1 Polarimetric RCS of sphere, obtained from TSCM polarimetric calibration](image)

To illustrate the calibration method, the measurements corresponding to a metallic sphere of 74nm diameter are shown. Fig. 1 shows the polarimetric RCS of the metallic sphere in a bandwidth of 12 – 18GHz, calibrated with the TSCM technique. It is appreciated that the cross-polarisation purity is of the order of 30dB and the greatest deviation of theoretical behaviour is ~1.3dB. Similar results are obtained using the TSCB, TSCI, GCT [1] and Wiesbeck method [4]. This can be observed in Fig. 2, where \( \alpha_m \) is shown for all polarimetric calibration techniques.

![Fig. 2 Comparison of polarimetric calibration techniques TSC, GCT and Wiesbeck method from \( \alpha_m \) of sphere](image)

**Conclusions:** A new polarimetric calibration method (TSC) based on Pauli matrix decomposition has been described, where the base elements coincide directly with the usual calibrators. For this reason, the calculation of error coefficients is more direct and easier than in other calibration methods. The results obtained using TSC are similar to other general polarimetric calibration methods such as the GCT and Wiesbeck method, with the advantage of its simplicity in solving the polarimetric calibration.

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**In$_{10.25}$Ga$_{0.75}$As/AlAs resonant tunnelling diodes with switching time of 1.5ps**


Indexing terms: Gallium indium arsenide, Resonant tunnelling devices

Switching times of 1.5ps for In$_{10.25}$Ga$_{0.75}$As/AlAs resonant tunnelling diodes (RTDs) with 1.1nm wide barriers and a peak current density of \( 6.8 \times 10^{13} \text{A/cm}^2 \) are reported. To the authors’ knowledge, this is the fastest switching ever reported for any type of RTD. The authors confirm that the obtained switching time is close to the value calculated from the experimentally derived depletion-layer capacitance and average negative differential resistance.

**Introduction:** The study of the high speed characteristics of In$_{10.25}$Ga$_{0.75}$As/AlAs resonant tunnelling diodes (RTDs) is of great importance because of their possible applications in various ultra-high speed circuits. Recently, we developed an electro-optic sampling (EOS) technique that enables us to measure RTD switching times with a time resolution of <1ps [1]. Using this technique, we have systematically investigated the switching time of In$_{10.25}$Ga$_{0.75}$As/AlAs RTDs with barrier widths of 1.4 – 2.3nm [2]. However, switching speeds faster than those of InAs/AlSb RTDs, which showed a lower peak current density [4] than In$_{10.25}$Ga$_{0.75}$As/AlAs RTDs [2], have not been achieved.

We fabricated In$_{10.25}$Ga$_{0.75}$As/AlAs RTDs with very thin (1.1nm) barriers and measured the switching time. We then compared the obtained switching time with that calculated by using the measured depletion-layer capacitance and average negative differential resistance.

**Experiment:** The double-barrier structure used for fabricating RTDs was grown on semi-insulating InP substrates by molecular