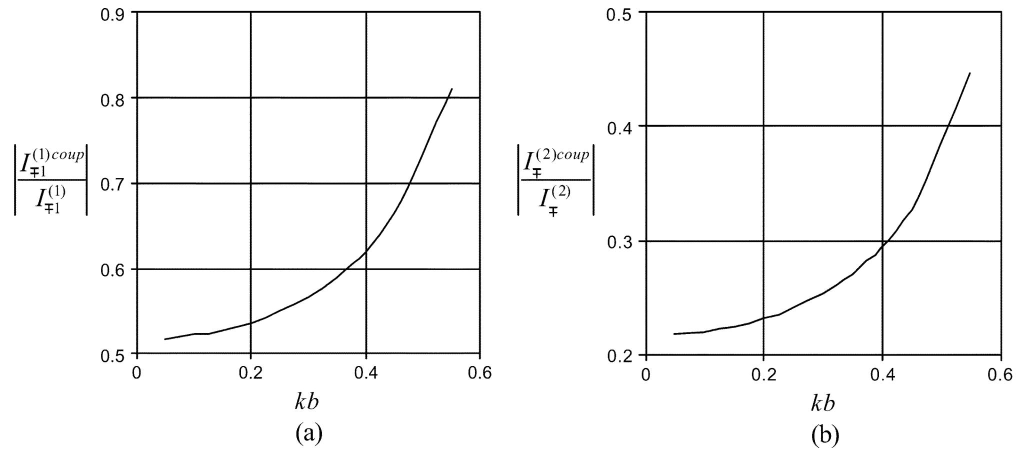

Fig. 1. Ratio of the first harmonic of coupling current to the first harmonic of current in the isolated loop.

$-(\alpha^c_{1,1})_{12}, (\alpha^c_{1,1})_{21} = (\alpha^c_{1,1})_{12}, (\alpha^c_{1,-1})_{21} = -(\alpha^c_{1,1})_{12}$ and the naming convention, $\alpha^c_{m,n} \equiv (\alpha^c_{m,n})_{12}$, (5) is rewritten as in (6) shown at the bottom of the previous page. From (6), it is apparent that $[X_{12}]$ is not equal to $[X_{21}]^T$ as stated in [1, eq. (50)]. Though there is symmetry between the submatrices, it is not with reference to the principal diagonal as required of transposed matrices. After a matrix inversion of (6), the correct equations for the coupling currents are obtained as

$$I^{(1)\text{coup}}_{\mp 1} = -\frac{j}{\pi\zeta}\left\{\left(\frac{2e^{\pm 2j\Phi_{e1}}(\alpha^c_{1,1})^2}{U} - \frac{2(\alpha^c_{1,1})^2}{U}\right)V^e_1 \pm \left(\frac{2e^{\pm j\Phi_{e1}}\cos\Phi_{e2}\alpha^s_1\alpha^c_{1,1}}{U}\right)V^e_2\right\} \quad (7)$$

$$I^{(2)\text{coup}}_{\mp 1} = -\frac{j}{\pi\zeta}\left\{\left(\frac{-2e^{\pm 2j\Phi_{e2}}(\alpha^c_{1,1})^2}{U} - \frac{(2\alpha^c_{1,1})^2}{U}\right)V^e_2 + \left(\frac{2je^{\pm j\Phi_{e2}}\sin\Phi_{e1}\alpha^s_1\alpha^c_{1,1}}{U}\right)V^e_1\right\} \quad (8)$$

where

$$U = (\alpha^s_1)^3 + 4\alpha^s_1(\alpha^c_{1,1})^2. \quad (9)$$

For the case where $\Phi_{e1} = \Phi_{e2} = 0$ and $V^e_1 = V^e_1 = V^e$, the ratio of the first harmonics of the coupling current to the first harmonics of the current in the isolated loop is given by

$$\frac{I^{(1)\text{coup}}_{\mp 1}}{I^{(1)}_{\mp 1}} = \pm\frac{2\alpha^c_{1,1}\alpha^s_1}{(\alpha^s_1)^2 + 4\left(\alpha^c_{1,1}\right)^2} \quad (10)$$

$$\frac{I^{(2)\text{coup}}_{\mp 1}}{I^{(2)}_{\mp 1}} = -\frac{4\left(\alpha^c_{1,1}\right)^2}{(\alpha^s_1)^2 + 4\left(\alpha^c_{1,1}\right)^2}. \quad (11)$$

The numerically calculated absolute values of the two ratios in (10) and (11) have been plotted in Fig. 1(a) and (b), respectively. These results have the same trend as those in [1], except that they are significantly larger in value. Using (6)– (11), the correct versions of [1, eqs. (51)–(73)] can easily be derived. We would like to also point out that a multiplication factor, $1/2\pi$, is missing in [1, eqs. (5), (6)] and that a minus sign is missing in [1, eq. (54)].

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Comments on "On the Relationship Between Fractal Dimension and the Performance of Multi-Resonant Dipole Antennas Using Koch Curves"

J. M. González-Arbesú, J. M. Rius, and J. Romeu

***Index Terms*—Fractals, multifrequency antennas, wire antennas.**

Reference [1] shows a rule for the design of generalized Koch curves ranging from fractal dimension 1 to fractal dimension 2. This design is very helpful for investigating the relationship (if any) between fractal dimension and several antenna parameters. Here, fractal dimension refers to the Haussdorf dimension of the limiting curve of infinite iterations.

The authors of [1] successfully analyze the variation of the first resonant frequencies and the radiation resistances for the initial iterations of several generalized Koch curves (with different fractal dimensions). This analysis is carried out through simulations and assessed with experimental validations (with the logical and expected frequency shifts due to dielectric substrates and the use of strips instead of wires). Nevertheless, and although we completely agree with the simulated and

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measured results, there are some aspects in the paper that we think deserve a further interpretation or comment to avoid extracting wrong conclusions about the relationship between fractal dimension and certain antenna parameters.

1) In the abstract of the paper the authors state, "These findings underscore the significance of fractal dimension as an important mathematical property of fractals that can be used as a design parameter for antennas." We cannot agree with this statement for two reasons. First, fractal dimension is a measure that can only be applied to fractal objects. The designs managed in the recent history of *fractal* antennas are actually *prefractals* (truncated versions of the infinite iterative procedure used to build them) obtained after a low number of iterations. Although we use to assign them the fractal dimension of the limiting fractal by convention, strictly speaking the family of generalized prefractal Koch curves studied in the paper have Haussdorf dimension equal to one.

   Second, the variation of the indentation angle (used to define the generalized Koch curves) determines how rapidly the wire length increases with the iteration. In this way, the length of the nth iteration of the dipole with indentation angle θ is given by

$$L_{\theta,n} = \left(\frac{2}{1+\cos\theta}\right)^n L_0 \tag{1}$$

   with L_0 being the length of the linear dipole with the same end-to-end length.

   In the end, it is this wire length $L_{\theta,n}$ that really produces the reduction on the resonant frequency of the antennas. Of course the geometry of the wire is also of importance. Works (not available to the authors of [1] at the time of writing the manuscript) about this topic have been published recently ([2], [3]).
2) When enumerating the performance of fractal shaped design, other parameters should be mentioned. Prefractal designs reduce the radiation resistance, radiation efficiency, and impedance bandwidth and increase the quality factor with the increasing iteration or with the increasing fractal dimension (both ruled by the growing length of the wire). Other nonfractal designs show better performance in terms of these parameters. Some examples can be found in [4] and [5].
3) Reference [1, Table II] shows the geometric interval between resonant frequencies for the generalized Koch dipole antenna. The authors of [1] note at the end of Section IV.B that the ratio of successive resonant frequencies remains nearly constant for different fractal iterations of the geometry of the same indentation angle (dimension). However, in [1, Table II], it is significant that for large fractal dimensions the resonant frequency reduction factor decreases as the fractal iteration is increased. We think that this difference should not be neglected because it is a proof of the stronger *shortcut* effect observed when the wire is highly convoluted [6], [7]. This effect helps to explain the stagnation in the reduction of the resonant frequency as prefractal iterations increases for any prefractal design (and also in other nonfractal convoluted designs). In a convoluted prefractal, and when the gaps among wire segments become small enough (in terms of wavelength), the electromagnetic wave *jumps* among segments. The wave does not follow the conductors (wires) of the antenna, arriving at its end sooner than following the electrical path through the wires. This effect has been assessed by numerical simulations.[1]

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Reply to Comments on "On the Relationship Between Fractal Dimension and the Performance of Multi-Resonant Dipole Antennas Using Koch Curves"

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***Index Terms*—Fractals, multifrequency antennas, wire antennas.**

The authors of the present comment [1] have tried to bring forth some issues that were possibly not clear in the original article [2]. We are glad at this second opportunity and thank them for the same. However, we disagree on most of their presumptions.

There are several ways to express fractal dimension, and it is widely accepted that many of these definitions do not always agree with one another. In the original article fractal similarity dimension was defined and used, and not Hausdorff dimension.

Regarding the correctness of using of fractal dimension, it may be contended that we do make approximations in all instances, whether they are obvious or not. In mathematics, these geometries (e.g., Koch curves) consist of lines—which have no width or thickness, and the strictest calculation of dimension is based on this. However, we cannot make antennas with conductors of zero width. Hence, truly speaking, the exact mathematical definition of dimension has little consequence in the present instance. On the other hand, the scientific community keeps defining and using this quantity, and considers it an important characteristic of fractals. It may be noticed that there are several in-

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