IAC-12-C1.6.5

METHODOLOGY AND ESTIMATION COSTS FOR PROXIMITY MANEUVERING SPACECRAFT FORMATIONS IN THE VICINITY OF LIBRATION POINTS

Laura Garcia-Taberner
Departament d'Informàtica i Matemàtica Aplicada, Universitat de Girona, Escola Politècnica Superior, 17071 Girona, Spain.
laura.garcia@ima.udg.edu

Josep J. Masdemont
IEEC & Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, Diagonal 647, 08028 Barcelona, Spain.
josep@barquins.upc.edu

ABSTRACT

The aim of this paper is to study and provide estimations of the cost of reconfiguration maneuvers in the neighborhood of L_1 and L_2 libration points of the Sun-Earth system. The problem is considered as a function of the parameters of the formation (baseline length, security distances, orientation, reconfiguration time) and simple formulae to evaluate the costs are given.

I INTRODUCTION

In this paper we consider formation flying as a technology that consider several spacecraft having to maintain constrained relative positions between them. Some of the most known projects of formation flying, like the Terrestial Planet Finder [1] of the NASA, the Darwin project [2] of the ESA or the TechSat-21, were projects on which the spacecraft would form a virtual telescope. One of the objectives is to obtain a larger instrument than the one that one could have obtained with a single spacecraft (both for the cost of launching a big spacecraft and for the possibility to obtain a large baseline of hundreds of meters). But this is not the only application of formation flying. Other missions are thought in order to use one of the spacecraft of the formation to eclipse a big target light (like the Sun) and focus on the surroundings or to examine relative big regions of the space. The main drawback of formation flying is that there are still many technological issues that have to be solved, such as how to maintain the formation in the same relative position with a very small error range in position. Unfortunately, the Terrestial Planet Finder and Darwin project were canceled due to the problems and cost associated to this new technology. Nonetheless some formations of two spacecraft like CALIPSO and CloudSat or Terra and Aqua are currently working.

Ideally, the formations would spend most of the time in tight formation, performing observations. Some papers in the literature have studied the maintenance cost of the spacecraft in formation [3, 4]. However in this paper we focus in the reconfiguration process between the observational periods. Natural reconfigurations would be for instance rigid-body like rotations to point the instruments to another goal, contingency maneuvers due to failure of one of the components, to change the pattern of the formation, to include new spacecrafts in it, etc. For all these complex maneuvers it is important to have a systematic methodology that provides optimal solutions.

There are some methodologies to perform the reconfigurations of formations of spacecraft. The methodology we use, the FEFF methodology, has been developed in [5, 6]. It uses finite elements in time to obtain the trajectories for each spacecraft. Vasile [7] also uses finite elements to find the trajectories, but other methodologies have been proposed.

We apply the FEFF methodology to a number of general maneuvers depending on the parameters that define their geometry. More precisely, among these parameters we consider the position of the spacecraft in the libration point orbit, the orientation of the formation, the security distance between spacecraft and the time span associated to the maneuver. The final objective of this paper is to give good estimators for basic maneuvers in order to easily evaluate the cost of the "pieces" of more complex missions. In this way, one can be able to have a first estimation of the needs and length of a particular lifetime mission.

II RECONFIGURATION OF SPACECRAFT

The objective of this paper is to make an study of the influence of the parameters of a formation in the cost of reconfiguration. Given a formation, we study the changes of the cost of reconfiguration depending on the position of the formation with respect to the halo orbit, the baseline length of the formation, the security distance between spacecraft or the reconfiguration time.

The cost of each reconfiguration is computed using a variational numerical methodology based on finite elements, that we summarize here, but that is fully presented in [8, 5]. This methodology computes the total cost of reconfiguration for a given formation of N spacecraft, with given initial and final states and a fixed reconfiguration time T. Although the methodology applies to orbits considered with full perturbations, it is simpler to be described using the linearized equations for the relative motion about a nominal halo orbit of the restricted three body problem (RTBP). They have the form

$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t), \tag{1}$$

where $\mathbf{A}(t)$ is a 6×6 matrix and $\mathbf{X}$ is the state of the satellite. The origin of the reference frame for the $\mathbf{X}$ coordinates is the nominal point in the halo orbit at time t and the orientation of the axis is parallel to the ones of the RTBP.

Since Halo orbits are periodic orbits, $\mathbf{A}(t)$ is also a periodic matrix. The matrix $\mathbf{A}(t)$ has as well some properties related to the characteristics of this kind of orbits: for a fixed value of t, it has six eigenvalues, two of them are real with opposite sign (the ones which give the hyperbolic part to the Halo orbit) and the other 4 ones are pure imaginary numbers and conjugated in pairs (the ones which are related with the rotations about the orbit), as can be seen in [9]. In the case of other libration orbits, this is not exactly in this way, but the hyperbolic and rotation characteristics are maintained.

Since the procedure must perform reconfigurations of a set of spacecraft, each spacecraft must be subjected to a control. Then the equations of motion for each of the spacecraft are,

$$\dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \bar{\mathbf{U}}_i(t),$$

where the control $\bar{\mathbf{U}}_i(t)$ only affects to the acceleration, i.e. it is of the form $\bar{\mathbf{U}}_i(t) = (0, 0, 0, \bar{u}_i^x(t), \bar{u}_i^y(t), \bar{u}_i^z(t))^t$.

Adding the initial and final states of the spacecraft in the reconfiguration problem, we obtain the equations,

$$\left\{ \begin{array}{l} \dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \bar{\mathbf{U}}_i(t) \\ \mathbf{X}_i(0) = \mathbf{X}_i^0 \\ \mathbf{X}_i(T) = \mathbf{X}_i^T \end{array} \right. \tag{2}$$

where $\mathbf{X}_i^0$ and $\mathbf{X}_i^T$ stand for the initial and final state of the i-th spacecraft of the formation. The goal is to find optimal controls, $\bar{\mathbf{U}}_1, \ldots, \bar{\mathbf{U}}_N$, subjected to given constraints, being a fundamental one the collision avoidance.

Using the properties of the halo orbit, the equations can be split in a set of six uncoupled equations of the form

$$\left\{ \begin{array}{ll} \ddot{x}(t) + \lambda(t)\,\dot{x}(t) + \tau(t)\,x(t) = u(t), & \\ x(0) = x_0, \qquad x(T) = x_T, & \\ \dot{x}(0) = v_0, \qquad \dot{x}(T) = v_T. & \end{array} \right. ,$$

where x refers now to a variable that is a function of the state of the spacecraft. We note that this process simplifies the problem and gives smaller computation time than without uncoupling, but it is not strictly necessary.

The reconfiguration problem then is reduced to find the controls $u(t)$ for the six equations. They are obtained with a methodology based on the finite element methodology: the total reconfiguration time $[0, T]$ is divided in M elements, which are subintervals of time of the domain. The border of each element, which is shared with the border of the neighboring one, is a node. The methodology finds a control in form of delta-v at the nodes that perform the reconfiguration.

Using the finite element theory, we can reduce the reconfiguration problem to an optimization problem, where the variables of the problem are the states of the spacecraft at the nodes, the functional is related to the total cost of the reconfiguration and collision avoidance enters in the problem as constraints.

In order to avoid the collision between spacecraft, we consider each spacecraft as a point surrounded by an sphere of radius R. The spheres corresponding to two different spacecraft cannot collide in all the reconfiguration time. The formulation of the problem, via the finite element methodology, makes easy to check if the spheres are colliding: the distance between each pair of spacecraft on each element must be greater than $2R$.

We note that the final cost for each reconfiguration depends on the parameters inherent to the reconfiguration problem, but it also depends on the mesh used to compute the cost. We have applied a remeshing strategy, that is fully explained in [10]. The essential idea of this strategy is that we compare the modulus of the estimated error of the mesh, $||e||$, with the total gradient of the solution. This methodology tends to obtain a bang-bang solution when there are no collision risks, and obtains a low-thrust based solution with a maximum error when there are collision risks.

As it has been previously stated, the methodology considers a linearized model about a nominal halo orbit, but we note that, as we work with small formations, this model gives us a good approximation for the nonlinear model. In [6] the authors have made an study of the influences of the nonlinear terms in the model, and the errors provided by the nonlinearities are smaller than the differences in cost obtained with the different parameters of the model.

III RECONFIGURATION PARAMETERS

For a particular formation and an associated reconfiguration procedure, we have considered different parameters to study the cost of the maneuver. We have taken into account four parameters: the length or size of the formation, the security distance, the reconfiguration time and the orientation of the formation. We have studied the cost of reconfiguration depending on each of the parameters independently or combining two of these parameters.

The size of the formation, d

For a particular pattern we can study the reconfiguration cost of a formation varying the distances between spacecraft in a proportional way. This gives us the idea of how the cost of the reconfiguration is growing when the formation grows, and also gives us an idea of the expected cost of a reconfiguration depending on its size.

We note that the length of the formation is independent from the security distance between spacecraft, and the increment on this size does not necessary give the same reconfiguration scaled.

The security distance between spacecraft, R

One of the most important parameters in the optimization of the trajectories is the security distance between the spacecraft. This parameter, which depends on the size of the spacecraft and other parameters inherent to the formation, is critical when doing reconfigurations with possible collisions. We study here how the cost of the reconfiguration is growing as this distance increases.

We note that R has a maximum. R is defined as the radius of the exclusion spheres about the spacecraft. Since the spheres cannot intersect, R can be as much as half of the initial or final minimum distance between spacecraft. We also note that, when approaching the maximum value of R, we might not converge to a solution, or we might obtain a solution with an associated high cost.

The reconfiguration time, T

The cost of the reconfiguration also depends on the reconfiguration time: in the usual ranges of T, the bigger the T, the smaller the cost is. Our objective is to know which is the model that gives the cost depending on T, or how many fuel can be saved when increasing T.

The orientation of the formation with respect to the halo orbit

The last parameter is the orientation of the formation with respect to the halo orbit. We consider a fixed formation (d, R and T also fixed) and we compute the cost of the reconfiguration as the formation rotates its initial and final positions.

IV RESULTS

As it is stated in [11], we can have two different kinds of reconfigurations: when a bang-bang control trajectory for each spacecraft is free of collisions, this is already the final optimal solution to the problem; on the other hand, when there are possible collisions between the spacecraft, we end up with a low thrust arcs solutions. All the reconfigurations we can have are either one of these or a combination of both kinds of maneuvers.

In order to obtain some different reconfigurations that make a vademecum of all of them, we have considered four sets of reconfigurations:

- The shift of one spacecraft. A single spacecraft is not a formation, but when the optimal trajectory does not collide with other spacecraft, the obtention of the optimal trajectory for this spacecraft is independent from the others. Then, the reconfiguration problem can uncouple in so many single problems like this one as spacecraft the formation have. The shift consists on taking an spacecraft from a position d meters far from the halo orbit and put it on the symmetrical point with respect to the halo orbit in a fixed time T.

- The swap of two spacecraft. The formation consists on two spacecraft with symmetrical positions with respect to the halo orbit. The two spacecraft are swaped. Notice that if the spacecraft would follow the bang-bang optimal trajectories, they would collide in the halo orbit in half the reconfiguration time.

- The rotation of three spacecraft. We consider three spacecraft in the vertex of an equilateral triangle, and the reconfiguration maneuver considered consists on a rotation where each spacecraft ends at the initial position of another spacecraft. When we consider a small security distance, the spacecraft do not have collision risk for the optimal bang-bang trajectory, but when the security grows, the collision is present. This reconfiguration is an example of a mixed problem: sometimes the optimal solution is a bang-bang control and sometimes is low thrust arc. It is easy to see that in this case, with linear trajectories at a constant velocity, there is no collision unless the security distance for each spacecraft is greater than $d\sqrt{3}/4$, where d is the initial distance from each spacecraft to the halo orbit.

- A double-swap of two spacecraft. This example is introduced in order to put a little more complexity to the swap of two spacecraft. The spacecraft are initially in the vertices of a square centered on the halo orbit. The objective is to swap each spacecraft with the one that is symmetrical to it with respect to the halo orbit. This generates a pair of spacecraft swappings. Each spacecraft must avoid collision inside its pair, but also with the components of the other pair.

In all the simulations we consider that the origin of the reference frame for the **X** coordinates is the nominal point in the halo orbit at time t and the orientation of the axis is parallel to the ones of the RTBP. The RTBP equations of motion are usually considered in a non inertial adimensional reference frame, known as the synodic system. The origin of this coordinate frame is located in the center of mass of the system. The X axis is defined by the instantaneous line joining the two primaries directed from the smallest primary to the larger one. The Z axis is normal to the orbital plane of the primaries, in the direction of their angular momentum and the Y axis is chosen orthogonal to the previous ones in order to have a positively oriented coordinate system. Also a-dimensional coordinates are selected in the way that the unit of mass is the sum of the masses of the primaries, the distance between the primaries is the unit of distance and the time unit is such that the sidereal period of the primaries is equal to 2π.

Cost depending on the orientation of the formation

We have considered the simulation of a shift of one spacecraft from an initial position to the symmetrical one respect to the halo orbit, with an initial and final distance to the halo orbit of d. In order to check if the cost of the reconfiguration depends on the orientation on the halo orbit, we consider an sphere of radius d and we make a map of the cost of the reconfiguration on spherical coordinates, ϕ and θ. Let us take the initial position of the spacecraft $x = d\cos(\phi)\cos(\theta), y = d\sin(\phi)\cos(\theta), z = d\sin(\theta)$, and the velocities the same as the halo orbit. The final position is the symmetrical point with respect to the center of the sphere. We note that we can obtain all the possible configurations with $\phi \in [0, \pi]$ and $\theta \in [-\pi, \pi]$. We have considered different halo times and different lengths of the formation, to compute the different costs for the range of ϕ and θ.

In figure 2, we present the results in terms of the cost that we obtain with $d = 100m$ and a reconfiguration time of 8 hours. Even we have computed the same cost using different halo times, the results are similar and we include the results with halo times: 0 and 1, which are far in the

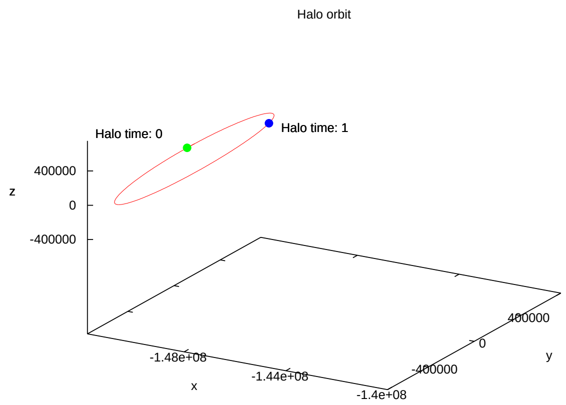

Figure 1: Halo orbit of z-amplitude 120000 km about L_2. The marked points are the ones used on figure 2 to show the cost of the shift of a spacecraft.

halo orbit and with different orientations (see figure 1). We observe that there are no preferred regions, and the cost is roughly the same in all the directions.

Given that the results are similar to the ones on figure 2 for different halo times, we can conclude that the orientation of the spacecraft in the halo orbit does not have influence in the total reconfiguration cost. From now on, we are going to study the cost with a fixed orientation of the formation and a fixed halo time.

Cost depending on the size of the formation

Now we fix the reconfiguration time, the orientation of the formation and the security distance (when needed) and study how the cost of reconfiguration grows with the length of the formation.

We have considered the 4 base examples, and we have increased the length of the formation from 100 meters to 500 meters. The results are plotted on figure 3.

We note that, in the case of the rotation of three spacecraft we also change the security distance between spacecraft. The reason is that if we maintain it fixed, as we are increasing the length of the formation, when d reach $4R/\sqrt{3}$, there is no collision, and for all the formations with a length greater than this the model would be the same as the parallel shift. In the other hand, we cannot take a big constant R (for instance, $R = 217m$, which would give collision between spacecraft when $d = 500$), because then we would have unfeasible problems for smaller formations (when $d = 100m$, the initial distance between spacecraft is 173 m). We have taken the security distance growing with the length of the formation, $R = 1.5d\sqrt{3}/4$, just to see the behavior of the model.

The cost of the reconfiguration is a linear function

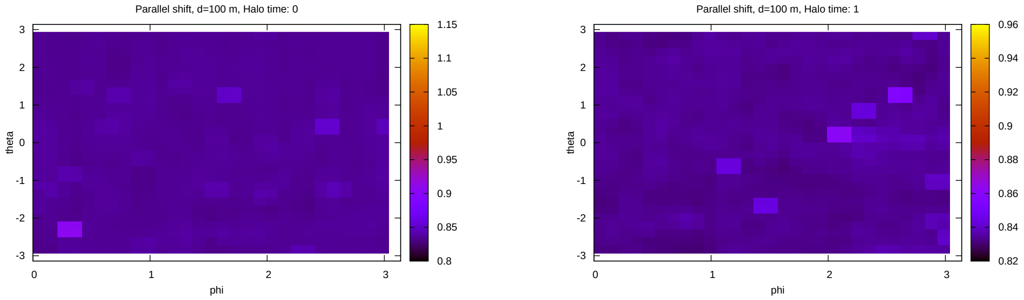

Figure 2: The costs of reconfiguration depending on the orientation of the formation on the halo orbit, with different positions of the halo orbit. We can see that the costs are the same in all the directions.

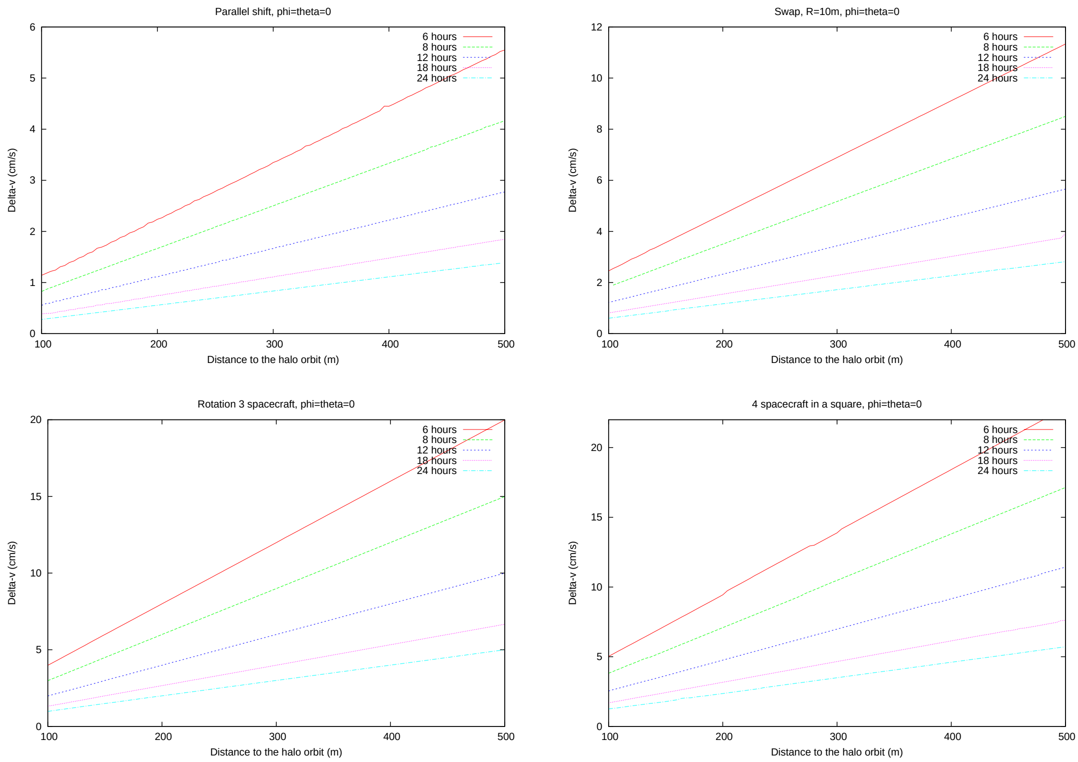

Figure 3: Cost of the reconfigurations when we increase the length of the formation from 100 m to 500 m. The security distance is fixed to 10 m and there is a set of different configuration times for each simulation. From left to right, and from top to bottom, the simulations are: the shift of one spacecraft, the swap of two spacecraft, the rotation of three spacecraft and the double swap of spacecraft.

Model	$a(T)$	$b(T)$
1 s/c	$\frac{0.0666334}{T}$	0
swap 2 s/c	$\frac{0.133341121}{T}$	$\frac{1.457528}{T} - 0.011$
rotation 3 s/c	$\frac{0.2399}{T}$	0
double swap 4 s/c	$\frac{0.279758}{T}$	$\frac{4.2822}{T}$

Table 1: Coefficients $a(T)$ and $b(T)$ in $cost(d,T) = a(T) \cdot d + b(T)$ that give the cost of the reconfiguration for each of the models with a security distance of 10 meters depending on the length of the formation (d) and the reconfiguration time (T).

Figure 6: Cost of the swap of two spacecraft, when we change both the length of the formation and the security distance. The length of the Halo orbit is from 100 m to 300 m (the initial distance between spacecraft is from 200 to 600 meters). The security distance if from 0 to half the length of the formation. Reconfiguration time is 8 hours.

which depends on the distance. The cost of the reconfiguration is then, for each model, and for each reconfiguration time,

$$cost(d) = a \cdot d + b.$$

But we obtain a different expression for a and b depending on the reconfiguration time,

$$cost(d,T) = a(T) \cdot d + b(T),$$

that we can find on table 1.

The results are obtained via linear regression, and the coefficient of determination is greater than 0.99.

We can conclude that the cost of reconfiguration increases linearly with respect to the length of the formation, with a coefficient that changes with the reconfiguration time on the form k/T. We note that the coefficient $a(T)$ grows almost linearly with the number of spacecraft and $b(T)$ depends highly on the complexity of the collisions to be avoided (the greater term is in the example with two swaps, and when there is no collision is 0). Finally, we remark that we do not have taken into account the security distance to compute these models (in all the examples we have used $R = 10m$).

Cost depending on the reconfiguration time

As we have seen in the study of the cost of reconfiguration depending on the length of the formation, the cost decreases when the reconfiguration time grows, following a law of the form k/T. On figure 4 we have the cost of reconfiguration changing the reconfiguration time.

Cost depending on the security distance

Finally, we study how the cost grows as we increase the security distance. We note that the security distance can not be greater than half the initial or final distance between each pair of spacecraft, or the problem would be unfeasible.

In the case of the swap of two spacecraft, when the security distance is still small, the cost grows linearly (see figure 5 on the left), but when the distances are big with respect to the initial distance between the spacecraft, it grows in an exponential way (figure 5 right).

We can also compute the cost of the reconfiguration when we change both the length of the formation and the security distance. As it was expected, the cost grows both when increasing the length of the formation and the security distance, as can be seen on figure 6.

Fixing a reconfiguration time of 8 hours, and changing the security distance from 5 to 25 meters, we obtain that the cost is also linear in d. We can compute the function

$$cost(d,R) = a(R) \cdot d + b(R).$$

The coefficients $a(R)$ and $b(R)$ obtained are in table 2. The model which gives the cost depending on d and R is

$$\begin{aligned} cost(d,R) = \quad & (0.0167360 - 0.0000072R) \cdot d + \\ & 0.01980R - 0.03205. \end{aligned}$$

In the case of the rotation of three spacecraft, we know that when the security distance is less than $d\sqrt{3}/4$ there is no collision risk. We can see in figure 7 that with small security distance, the cost is constant with R. In particular, the security distance on which they start colliding is $25\sqrt{3}$ (43.3m). When increasing the security distance from this value, the cost grows exponentially with the length:

$$cost(R) = 0.7190 \cdot 1.0230^R.$$

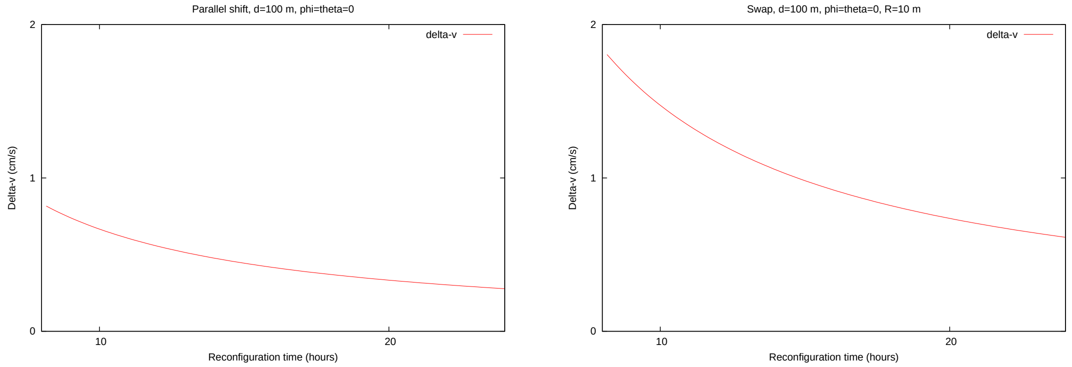

Figure 4: Cost of the reconfigurations when we increase the reconfiguration time, in the cases of parallel shift and the swap of two spacecraft. The length of the formation is $100m$ and the security distance is $10m$.

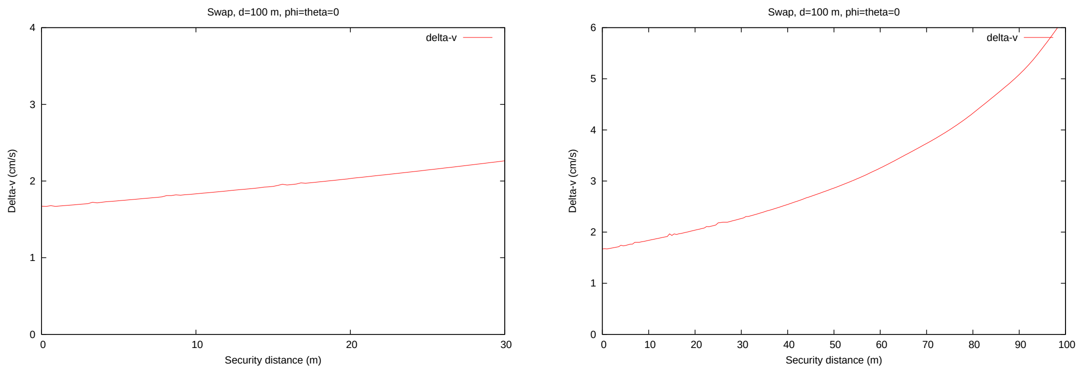

Figure 5: Cost of the reconfigurations when we increase the security distance. The initial distance between the spacecraft is 200 meters (each one 100 meters far from the orbit). On the left hand plot there is the cost with small security distances, and on the right hand plot the results are with big security distances (the maximum feasible is 10 meters).

Sec. dist.	$a(R)$	$b(R)$
5	0.01666	0.07819
10	0.01666	0.17103
15	0.01664	0.25665
20	0.01658	0.36527
25	0.01656	0.46482

Table 2: Coefficients $a(R)$ and $b(R)$ in $cost(d, R) = a(R) \cdot d + b(R)$ that give the cost of the reconfiguration for each of the models with a reconfiguration time of 8 hours.

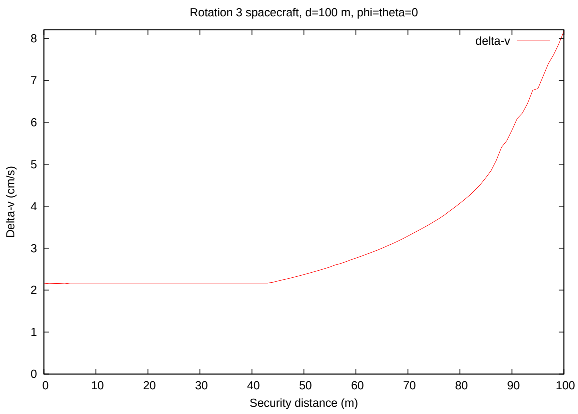

Figure 7: Cost of the rotation of 3 spacecraft, when increasing the security distance. The length of the formation is 100 meters, the reconfiguration time is 8 hours and the security distance in which the spacecraft start colliding is 43.3 meters.

The same behavior is obtained with bigger formations (figure 8). When the length of the formation is 250 m, the limit security distance is 108.3m, and for security distances bigger than this value, the cost is

$$cost(R) = 2.3325 \cdot 1.0074^R.$$

With a length of 500 m, the cost for security distances bigger than 216.5 m is

$$cost(R) = 4.7644 \cdot 1.0036^R.$$

As it is expected, the bigger the formation is, the bigger the cost is, and the bigger the formation, the less influence has the increase of the security distance.

We conclude that, when increasing the security distance, for small values of R the cost increases linearly, but with big R it grows exponentially.

V CONCLUSIONS

In this paper we use a methodology based on finite element methodology to compute the cost for different reconfigurations in the vicinity of libration points, taking into account the length of the formation, the reconfiguration time, the security distance between spacecraft and the orientation of the formation in the halo orbit. We have not find any evidence that the cost of the reconfiguration depends on the orientation of the formation or the position in the halo orbit. On the other hand, the cost grows linearly with the length of the formation, and decreases hyperbolically with the reconfiguration time, while the cost grows linearly with small values of R and exponentially with the bigger ones (in the case where there are possible collisions between the spacecraft of the formation). The results have been obtained in a halo orbit about L_2, but similar results can be obtained in other libration points due to symmetry and the isotropy of the space.

ACKNOWLEDGEMENTS

This research has been supported by the Spanish MCyT-FEDER grant, MTM2009-06973

REFERENCES

[1] The TPF Science Working Group. *The Terrestrial Planet Finder: A NASA Origins Program to Search for Habitable Planets*. JPL Publication 99-3, 1999.

[2] The Science and Technology Team of Darwin and Alcatel Study Team. *Darwin. The Infrared Space Interferometer. Concept and Feasibility Study Report*. European Space Agency, ESA-SCI 12, iii+218, July 2000.

[3] Beichman, C., Gómez, G., Lo, M.W., Masdemont, J. J. and Romans, L.. *Searching for Life with the Terrestrial Planet Finder: Lagrange Point Options for a Formation Flying Interferometer*. Advances in Space Reseach, 34, 637–644, 2004.

[4] Gómez, G., Lo, M., Masdemont, J., and Museth, K., *Simulation of Formation Flight near Lagrange Points for the TPF Mission,* American Astronautical Society, Paper 01–305, July 2001.

[5] Garcia-Taberner, L. *Proximity Maneuvering of Libration Point Orbit Formations Using*

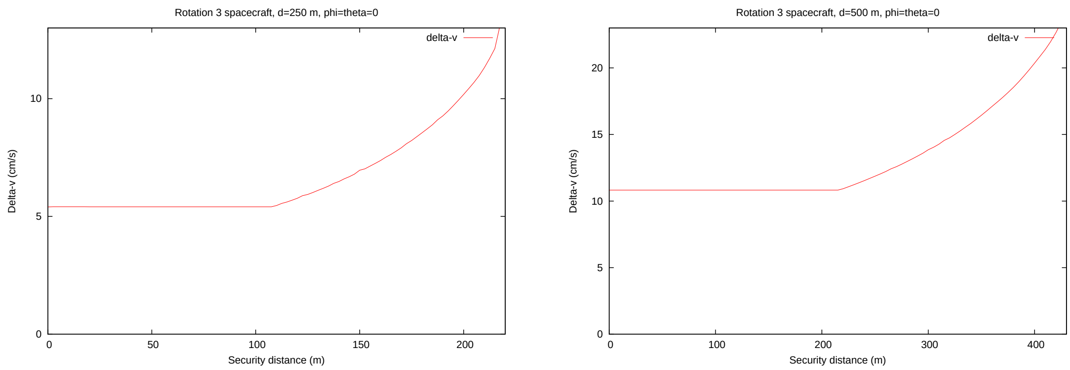

Figure 8: Cost of the rotation of 3 spacecraft, increasing the security distance. The length of the formation is 250 m (left) and 500 m (right). On both cases, the cost is constant until the limit of collision, and then it starts growing.

Adapted Finite Element Methods. PhD Dissertation. Universitat Politècnica de Catalunya. Barcelona, 2010.

[6] Garcia-Taberner, L, Masdemont, J. J. *FEFF Methodology for Spacecraft Formations Reconfiguration in the Vicinity of Libration Points*. Acta Astronautica, Vol. 67, Issues 7-8, 810–817, doi:10.1016, 2010.

[7] M. Vasile. *Finite Elements in Time: A Direct Transcription Method for Optimal Control Problems*. AIAA Paper 2010–8275, Aug. 2010.

[8] Garcia-Taberner, L. and Masdemont, J. J., *Maneuvering Spacecraft Formations using a Dynamically Adapted Finite Element Methodology*, Journal of Guidance, Control, and Dynamics, Vol. 32, No. 5, 1585-1597, 2009.

[9] Gómez, G., Llibre, J., Martínez, R. and Simó, C. *Dynamics and Mission Design near Libration Points. Vol 1, Fundamentals: The Case of Collinear Libration Points*, World Scientific Pub., xi+443, 2001.

[10] Garcia-Taberner, L., Masdemont, J. J., An Adaptive Remeshing Procedure for Proximity Maneuvering Spacecraft Formations, Mathematical Problems in Engineering, vol. 2012, Article ID 429479, 14 pages, 2012. doi:10.1155/2012/429479

[11] Garcia-Taberner, L. and Masdemont, J.J., *A Methodology Unifying Bang-Bang Controls and Low-Thrust Trajectories for the Reconfiguration of Spacecraft Formations*. American Astronautical Society, Paper 07–296, 2007.