Communications

An Approximate Expression to Estimate Signal-to-Noise Ratio Improvement in Cylindrical Near-Field Measurements

Jordi Romeu, Lluis Jofre, and Angel Cardama

Abstract—A very simple approximate expression for the process gain (PG) for the cylindrical case is derived. The different approximations and assumptions required to obtain this expression are shown. This expression might be useful for most practical cylindrical near-field measurements, providing a very simple mean to assess the near-field dynamic range requirements to obtain a desired far-field signal-to-noise ratio (SNR).

I. INTRODUCTION

Near-field measurements have been employed to obtain accurate antenna far-field patterns over the past years. The surface where the near-field data is acquired can be plane, cylindrical, or spherical. A common difficulty of the three near-field measurement techniques is to provide accurate estimates of the far-field errors due to near-field measurement errors. The planar case is probably the most evolved \[1\]--\[3\]; however, these developments are not valid for the cylindrical case due to the different nature of the transformation process.

A recent paper \[4\] has shown the contribution to the far-field errors of a white Gaussian near-field noise for the cylindrical case. This paper is an extension of \[4\], in which a simple approximate expression that relates near-field signal-to-noise ratio (SNR) to far-field SNR is presented. In this way, prior to the measurement, an assessment of the measurement conditions is possible. To provide this estimation, certain assumptions about the measurement probes and some a priori knowledge of the antenna under test (AUT) characteristics are required.

II. APPROXIMATE EXPRESSIONS FOR THE FAR-FIELD VARIANCE

Consider the case that the near-field measurement is contaminated by an additive complex white Gaussian noise, space stationary, with zero mean and variance \(\sigma^2\). Receiver floor noise responds to this model. In \[4\] it is shown that the far-field noise due to this near-field noise is Gaussian, with zero mean and a variance for each field component, given by

\[
\sigma^2_{\theta\phi}(k_z, \phi) = \sigma^2 \left[ \frac{k \gamma}{8 \pi^2 k_p} \right]^2 N_{\theta} N_{\phi} \left[ \frac{\Delta \phi \Delta z}{2 \pi} \right]^2 \sum_n \left| a^{(1)}_n (-k_z) b^{(2)}_n (-k_z) - a^{(2)}_n (-k_z) b^{(1)}_n (-k_z) \right|^2.
\]

(1)

where all the parameters are defined in \[4\].

The coefficients \(a^{(i)}_n (-k_z)\) and \(b^{(i)}_n (-k_z)\) are related to the cylindrical modal coefficients of the radiated fields by the measurement probes and can be computed as shown in \[5\] and \[6\]. Two near-field measurements with two different probes are required to perform the near- to far-field transformation. The superindex \((i)\) can be 1 or 2 to indicate to which of the two probes required in the measurement the coefficients are related.

From (1), two important conclusions can be drawn about the far-field noise variance:

1) The far-field noise variance is not constant for all space directions, but it has a dependence on the elevation angle \(\theta\).
2) The far-field variance depends on the measurement probe. Nevertheless, (1) can be simplified and generalized for most practical cases. In practice, typical probes usually meet the following:

1) The probes have linear polarization orthogonal to each other with a low crosspolar component. The crosspolar level is typically 30 dB below the copolar level.
2) The probes are not directive; usual choices are open-ended waveguides, small dipoles or small horns.

In \[7\], the effect of probe directivity and probe compensation on the constructed pattern was reported for cylindrical near-field case. If we assume that probe 1 is horizontally polarized and probe 2 is vertically polarized, both with a very low crosspolar component, it can be assumed that

\[
a^{(1)}_n \gg b^{(1)}_n,
\]

\[
b^{(2)}_n \gg a^{(2)}_n,
\]

\[
a^{(2)}_n \gg a^{(1)}_n,
\]

\[
b^{(1)}_n \gg b^{(2)}_n.
\]

(2)

and (1) can be simplified to

\[
\sigma^2_{\theta\phi}(k_z, \phi) = \sigma^2 \left[ \frac{k \gamma}{8 \pi^2 k_p} \right]^2 N_{\theta} N_{\phi} \left[ \frac{\Delta \phi \Delta z}{2 \pi} \right]^2 \sum_n \left| \frac{1}{b^{(2)}_n (-k_z)} \right|^2.
\]

(3)

\[
\sigma^2_{\theta\phi}(k_z, \phi) = \sigma^2 \left[ \frac{k \gamma}{8 \pi^2 k_p} \right]^2 N_{\theta} N_{\phi} \left[ \frac{\Delta \phi \Delta z}{2 \pi} \right]^2 \sum_n \left| a^{(2)}_n (-k_z) \right|^2.
\]

(4)

The coefficients \(b^{(2)}_n\) and \(a^{(1)}_n\) are related to the radiation pattern of the measurement probes; thus a certain pattern must be assumed in order to obtain a general expression for the far-field variance. A choice that leads to a simple closed expression is to consider the following copolar radiated field for both probes:

\[
E(\theta, \phi) = \cos \phi
\]

(5)
where the angles \( \theta \) and \( \phi \) are the conventional spherical angles. This radiation pattern is consistent with the assumption of a non-directive probe, and is a good approximation for most practical probes.

In [6] it is shown that the probe correction coefficients are related to the probe radiation pattern by the following expressions:

\[
\begin{align*}
\alpha_n^{(j)}(-k_\rho) &= -F(\psi_n) E_n^{(j)}(-k_\rho, \psi_n) \\
\beta_n^{(j)}(-k_\rho) &= jF(\psi_n) E_n^{(j)}(-k_\rho, \psi_n)
\end{align*}
\]  

where

\[
F(\psi_n) = \frac{1}{2k_\rho} \sqrt{\frac{2\pi}{k_\rho \rho_0 \cos \psi_n}} e^{-\gamma(k_\rho \rho_0 \cos \psi_n - \alpha(\psi_n + \frac{\pi}{2} - \frac{\gamma}{\lambda})}
\]

and

\[
\psi_n = -\arcsin \left( \frac{n}{k_\rho \rho_0} \right)
\]

and \( E_n^{(j)} \) and \( E_n^{(j)} \) are the radiated fields by probe \( (i) \) when the input current is normalized to unity. Considering a probe with a radiation pattern given by (5), it can be written that

\[
\sum_n \frac{1}{|a_n^{(j)}(-k_\rho)|^2} = \sum_n \frac{1}{|a_n^{(j)}(-k_\rho)|^2} \approx 2k_\rho^2 (k_\rho \rho_0)^2
\]

where \( \rho_0 \) is the measurement radius. Therefore, the far-field variance is the same for both field components, and can be written as

\[
\sigma_\phi^2(k_\rho, \phi) = \sigma_\psi^2 \left[ \frac{k_\rho}{2\pi \rho_0} \right]^2 N_\nu, N_\psi \left[ \frac{\Delta \phi \Delta \psi}{2\pi} \right]^2 2k_\rho^2 (k_\rho \rho_0)^2.
\]

The far-field variance is still dependent on the elevation angle \( \theta \) as \( k_\rho = k \sin \theta \), though in [4] it is shown that for a real probe the variation is less than 1 dB for an angle variation of \( \pm 30^\circ \) around the horizontal direction (\( \theta \approx 90^\circ \)). Cylindrical near-field measurement is an appropriate technique to measure directive antennas in the elevation plane, and therefore it is not an inconsistency to approximate the far-field variance for its value in the horizontal direction. In this case \( k_\rho = k \), and

\[
\sigma_\phi^2 = 2\sigma_\psi^2 N_\nu, N_\psi \left[ \frac{\Delta \psi}{2\pi} \right]^2 (f \cdot 10^{-7})^2
\]

where \( f \) is the frequency in hertz. Notice that this expression could be derived independently of the probe radiation pattern in the elevation plane, as is assumed in (5), as long as it is nondirective.

Not all parameters of (11) are independent, and by considering some of these interrelations the expression can be written in a simpler way. Some of the relations that can be established are:

1. The measurements are usually performed on a complete circular cylinder that encloses the AUT.
2. The vertical sampling spacing depends on the measurement frequency. The maximum allowed spacing to fulfill the sampling criteria will be considered.
3. The azimuthal sampling spacing is given by the radius \( a \) of the minimum cylinder that encloses the AUT.
4. In practice, the measurement radius \( \rho_0 \) and the minimum radius \( a \) do not differ much in relative terms.

All these relationships can be summarized as

\[
\Delta \phi = \frac{2\pi}{N_\phi}, \quad \Delta \psi = \frac{\lambda}{2}, \quad \Delta \phi = \frac{\lambda}{2\pi}, \quad k \cdot \rho_0 \approx k \cdot a = \frac{2\pi}{\Delta \phi} = \frac{\pi}{2\Delta \psi} = N_\psi.
\]

III. ESTIMATION OF THE AUT MAXIMUM RADIATED FIELD

In order to evaluate the far-field SNR it is necessary to estimate the AUT maximum far field. It is obvious that certain knowledge of the AUT characteristics is needed to estimate its maximum far field. Referred to the situation of Fig. 1, an equivalent plane aperture (EPA) can be defined by considering a plane orthogonal to the boresight direction and tangent to the measurement cylinder and with the field distribution produced by the AUT. Assuming that the AUT radiation pattern maximum in the boresight direction, the maximum far field \( E_{\text{max}}^{\text{max}} \) can be written as

\[
|E_{\text{max}}^{\text{max}}|^2 = \frac{1}{\lambda^2} S^2 |\psi_0| |E_{\text{max}}^{\text{max}}|^2
\]

where we have defined

\[
\chi = \int_0^{\int \frac{|E_{\text{max}}^{\text{max}}|^2 dS}{S}}
\]

This extremely simple expression is obtained considering that the vertical and azimuthal sampling spacing take the maximum allowed value by the sampling criteria. If the near field is oversampled by a factor \( \xi_\phi \), and \( \xi_\psi \) defined as

\[
\xi_\phi = \frac{\Delta \phi}{\lambda/2}, \quad \xi_\psi = \frac{\Delta \psi}{\lambda/2}\n\]

the far-field variance can be corrected by these factors and the following expression is obtained:

\[
\sigma_\phi^2 = \sigma^2 \cdot 112.5 \cdot N_\nu, N_\psi.
\]

Fig. 1. Equivalent plane aperture.
as a shape factor that relates the aperture radiated power to the maximum field on the aperture \( I_{m,\text{max}} \), and \( \eta_{\text{ap}} \) is the AUT aperture efficiency. Both parameters are assumed to be the same in the AUT and the EPA.

**IV. Process Gain in Cylindrical Near- to Far-Field Transformation**

The process gain (PG) is defined as the ratio

\[
\text{PG} = \frac{\text{(SNR)}_{\text{ff}}}{\text{(SNR)}_{\text{nf}}}. \tag{18}
\]

The far-field SNR can be approximately computed as

\[
\text{(SNR)}_{\text{ff}} = \frac{|E_{\text{ff, max}}|^2}{Y_{\text{nf}}} = \frac{1}{\frac{\lambda^2}{112.5} N_N N_o \xi_o \xi_\phi} \cdot \frac{|E_{\text{nf, max}}|^2}{\sigma^2}. \tag{19}
\]

The near-field SNR is defined as

\[
\text{(SNR)}_{\text{nf}} = \frac{|v_{\text{oc, max}}|^2}{\sigma^2}. \tag{20}
\]

where \( v_{\text{oc, max}} \) is the maximum open-circuit voltage measured during the near-field scan, and \( \sigma^2 \) is the near-field noise variance. In order to determine the PG, the relationship between \( v_{\text{oc, max}} \) and \( E_{\text{ff, max}} \) must be established. If the probe is nondirective, it is small compared to the wavelength. In the limit the relationship between \( v_{\text{oc, max}} \) and \( E_{\text{ff, max}} \) can be approximated by the case of an infinitesimal probe

\[
|v_{\text{oc, max}}|^2 = \frac{|E_{\text{ff, max}}|^2}{(2\pi)^2}, \tag{21}
\]

and the PG:

\[
\text{PG} = \frac{\sigma^2}{\lambda^2 \eta_{\text{ap}}} \cdot \frac{316}{N_N N_o \xi_o \xi_\phi}. \tag{22}
\]

This simple expression depends on some AUT characteristics that are usually known prior to the measurement, and some near-field scanning parameters.
TABLE III

<table>
<thead>
<tr>
<th>Distribution</th>
<th>S</th>
<th>nφ</th>
<th>x</th>
<th>Process gain (dB)</th>
<th>Process gain (dB)</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>64λ²</td>
<td>1</td>
<td>1</td>
<td>24.2</td>
<td>28</td>
<td>3.8</td>
</tr>
<tr>
<td>Taylor</td>
<td>64λ²</td>
<td>0.57</td>
<td>0.18</td>
<td>16.9</td>
<td>18.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Fig. 2. Variation of the electrical field magnitude along the x-axis for two different antenna field distributions.

V. SIMULATION

Numerical simulation has been used to validate the approximate expressions developed in the previous sections. First of all, we have defined seven different measurement situations; specifics are shown in Table I. Three different kinds of probes are tested: an ideal magnetic probe, a real probe, and probes of the type $\cos^n \phi$ over pedestal. The real probe is a choke horn that fulfills the different assumptions that have been made to obtain the approximate expressions. The simulations concerning the real probe are based on actual measured data of the probe characteristics. The probe has a linear polarization with a crosspolar level below $-35$ dB and a 10-dB beamwidth of $115^\circ$. The simulations with probes of the type $\cos^n \phi$ over pedestal have been used to assess the bounds of validity of (15) relative to the use of nondirective probes. In Table II the ratio between the near- and far-field variance computed with the exact expression (1) and the approximate expression (15) is shown for the seven cases described in Table I. The far-field variance estimation has an error smaller than 1 dB for all cases except for the probe $\cos^2 \phi$ over pedestal. As a matter of fact, the real probe pattern can be well approximated by a function of the type $\cos^2 \phi$. Therefore, for a probe with radiation pattern of the type $\cos^n \phi$ with $1 < n < 3$, (15) may provide a good estimate for the far-field noise variance. Although this is only a partial validation, the seven test cases are different enough to assume that at least the approximate expression has the correct dependency on the measurement parameters, and provides good estimates for quite different measurement situations. Also, a limit of validity is set regarding the directivity of the probe.

In order to validate the process gain expression given by (22), measurement case number 6 has been considered with two different AUT. In both cases a planar array of $16 \times 16$ small horizontal dipoles spaced $\lambda/2$ has been employed as AUT. In one case the AUT has a uniform distribution. In the other case, a Taylor distribution that results in a 40 dB and 30 dB sidelobe along the vertical and horizontal directions, respectively. Both cases were simulated. The results are shown in Table III. The error in the process gain estimation for the Taylor distribution is within allowable bounds for an approximate expression, while the error for the uniform case is much larger. The only difference between both cases is the AUT. In Fig. 2, the near-field magnitude produced by both AUT along the x-axis is represented. The field variation for the uniform distribution is much larger than for the Taylor distribution. As a matter of fact, referred to the field magnitude on the AUT, at the measurement distance of 5λ the field magnitude for the uniform distribution is 1.56 dB higher, while for Taylor distribution it is 0.26 dB smaller. This variation in the near-field magnitude leads to wrong estimates of the maximum far field, since it has been assumed throughout Section III that the field distribution in the EMA and the AUT is the same.

VI. CONCLUSION

The estimation of the process gain is based on obtaining simple expressions that relate the near-field noise variance to the far-field noise variance, and the near-field maximum measured open circuit voltage to the maximum far field. These expressions provide very good estimates considering their simplicity, though the estimation of the maximum far field can have an error of several decibels in certain cases. Nevertheless, the expression of the process gain offers the possibility to predict the performance of a cylindrical near-field range under dynamic range measurement limitations. The expression is valid for most practical measurement situations where non-directive probes are employed and relates the near-field measurement parameters to a commonly employed far-field measurement parameter, the far-field SNR.

REFERENCES