Numerical LES models of Richtmyer-Meshkov and Rayleigh-Taylor instabilities

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Abstract

Experimental and numerical results on the advance of a mixing or non-mixing front occurring at a density interface due to gravitational acceleration are analyzed considering the fractal and spectral structure of the front. The experimental configuration consists on a unstable two layer system held by a removable plate in a box for the Rayleigh-Taylor fronts and a dropping box on rails and shock tube high Mach number impulse across a density interface air/SF_6.
The evolution of the turbulent mixing layer and its complex configuration is studied taking into account the dependence on the initial modes at the early stages and its spectral, self-similar information. Most models of the turbulent mixing evolution generated by hydrodynamics instabilities do not include any dependence on initial conditions, but in many relevant physical problems this dependence is very important, for instance, in Inertial Confinement Fussion target implosion. We discuss simple initial conditions with the aid of a numerical model developed at FIAN Lebedev which was compared with results of many simulations. The analysis of Kelvin-Helmholtz, Rayleigh-Taylor, Richtmyer-Meshkov and of accelerated instabilities is presented locally, and seen to dominate the turbulent cascade mixing zone differently under different initial conditions. Simulations and multifractal and neuron network analysis of Turbulent Mixing under RT and RM instabilities are presented for the different experiments and numerical simulations. In another paper in this proceedings volume further analysis on the numerical model is presented using wavelet preprocessing of the simulation results and neuron network presentation of the data with Kohonen map techniques (Stepanov et al.2004).

Abstract

Experimental and numerical results on the advance of a mixing or non-mixing front occurring at a density interface due to gravitational acceleration are analyzed considering the fractal structure of the front. The experimental configuration consists on a unstable two layer system held by a removable plate in a box. The initial density difference is characterized by the Atwood number. The evolution of the instability is non dimensionalized by $\tau = (Ag/H)^{½}$, as the plate is removed the gravitational acceleration, generates a combination of spikes and bubbles, which reach a maximum complexity and mixing efficiency before the front reaches the end walls. The instability produced is known as Rayleigh-Taylor (RT) instability, and in its simplest forms occurs when a layer of dense fluid is placed on top of a less dense layer in a gravitational field. In almost all practical circumstances, the instability forms a turbulent front between the two layers. A Large Eddy Simulation numerical model using FLUENT is used to predict some of the features of the experiments, different models on the interaction of the bubble generated buoyancy flux and on the boundary conditions are compared with the experiments. The aspect ratios of the bubble induced convective cells are seen to depend on the boundary and initial conditions applied to the front. The evolution of the Rayleigh-Taylor instability

develops into a turbulent mixing front that may be investigated further using the information that the fractal dimensions or Kolmogorov Capacities give as the flow evolves in time Two and three dimensional experiments are compared with Large Eddy Simulations of the same flow. The basic self-similar characteristics of the flow are compared and the evolution of the multi-fractal dimensions of density, velocity and vorticity contours provides indication that most mixing takes place at the sides of the dominant convective blobs. In the context of determining the influence of structure on mixing ability, multifractal and spectral analysis is used to estimate intermittency and determine the regions of the front which contribute most to molecular mixing.

Introduction

The stability of an interface between two superposed fluids of different density was studied by Lord Rayleigh and Taylor(1950) for the case when the dense fluid is accelerated towards the less dense fluid, the linear theory can be found in Chandrasekhar (1961). For inviscid fluids, the interface is always unstable, with the growthrate of the unstable modes increasing as their wavelengths decrease. The instability of the short waves can be reduced by dissipative mechanisms such as surface tension or viscosity, and then linear theory predicts the maximum growthrate to occur at a finite wavelength. For the viscous two-layer case, where the upper layer (density ρ_1) is denser than the lower layer (density ρ_2), the wavelength λ_μ of maximum growthrate is

$$\lambda_m \approx 4\pi\left(\frac{\nu^2(\rho_1+\rho_2)}{g(\rho_1-\rho_2)}\right)^{1/3}$$

where v is the mean kinematic viscosity of the two layers and g is the acceleration of gravity. The corresponding maximum growthrate is

$$n_m \approx \left(\frac{2g\pi(\rho_1-\rho_2)}{\lambda_m(\rho_1+\rho_2)}\right)^{1/2}$$

While the linear theory for two infinite layers is well established, the development of the instability to finite amplitude is not amenable to analytic treatment. There have been a number of semi-analytical and numerical studies in recent years, but they all involve simplifying assumptions which raise serious doubts about their validity particularly when applications to mixing are sought. An overview of the subject by Sharp(1984) characterized the development of the instability through three stages before breaking up into chaotic turbulent mixing. 1) a perturbation of wavelength λ_μ grows exponentially with growthrate n_μ. 2) when this perturbation reaches a height of approximately ½ λ_μ, the growthrate decreases and larger structures appear. 3) the scale of dominant structures continues to increase and memory of the initial conditions is supposedly lost; viscosity does not affect the latter growth of the large structures, This was observed not to be the case in the experiments of Linden et al.(1995) and in more recent experiments by Dalziel (1994) and Dalziel et al.(1999) with the two dimensional wake and the initial velocity structure affecting the growth of the RT mixing fronts for a long time.

The advance of this front is described in Linden & Redondo (1991), and may be shown to follow $\delta = 2cA\tau^2$ where δ is the width of the growing region of instability, g is the gravitational acceleration and A is the Atwood number defined as $A = \frac{\rho_1-\rho_2}{\rho_1+\rho_2}$

This result concerning the independence of the large amplitude structures on the initial conditions has led to consider that the width of the mixing region depends only on ρ_1, ρ_2, g and time,$\tau = t\text{-}t_0$. Then dimensional analysis may be used. The parameter c is considered to be a (universal) constant related to the accumulative mixing or entrainment produced by the front, although some dependence with the Atwood number and the initial conditions of the plate removal or random numerical fluctuations is expected.

The value of the parameter c, has been investigated experimentally and its value for experiments at different values of the Atwood number, A, do not show large variations, with a limit clearly seen for the larger A experiments performed. Values of c previously obtained experimentally have been in the range (0.03 - 0.035) (Read and Young 1984) in experiments with three dimensional effects and large density differences between the two fluids, $A \geq 1.5$. Redondo and Linden (1990) measured c for values of A in the range $1x10^{-4}$ to $5.0x10^{-2}$ and found values of $c = 0.035 \pm 0.005$. Numerical calculations in two dimensions (Young 1984) have given values of c in the range 0.02 - 0.025. The lesser values (Read 1984) have been explained in terms of two dimensional effects inhibiting the growth of the large scale.

Description of the RT experiments

The experiments consisted of a release of a dense fluid in a rectangular perspex tank of height H 0.50 m, length L 0.40 m and width W 0.20 m. The two fluids are initially separated by a removable stainless steal sheet, 1.5 mm thick,
in the centre of the tank. Fresh water is placed in the lower half of the tank and the sheet, sliding in tight fitting grooves, is pushed across and sealed with silicone grease, and finally the dense layer of brine is placed on top. The experiment is initiated by withdrawing the plate horizontally through a seal in the end wall of the tank. A 0.6 W argon laser light passing through an cylindrical lense was used to produce a 2 mm thick sheet of high intensity light. Fluorescein dye was added to one of the layers, and this produces a brilliant green image in the laser light. The leading edge of the advancing front was demarcated by this technique, and images of the small scale structures were obtained in the experiments described in Redondo and Linden (1993). Further analisis allowed for a large range of intensity values to be analysed and not just the Volume fraction isoline of 50%.

The fluorescein is only added in very small quantities so that it acts as a passive tracer. Using suitable orientations of the light sheet, side and end elevations and plan views of the flow were obtained. The second method of flow visualization which was used involved the use of a pH indicator to mark the flow where molecular mixing occurs. As discussed in Linden y Redondo (1991), it was necessary to have the Atwood number greater than 0.005 to ensure that the plate did not affect significantly the RT front. The perturbations induced by the plate were reduced using the method described in Dalziel(1994).

El avance del frente de mezcla C a lo largo de una escala de tiempo adimensional, definida como:

$$T = \sqrt{\left(\frac{Ag}{H}\right)}t$$

Introduction

In the context of determining the influence of structure on mixing ability, multifractal analysis is used to determine the regions of the front which contribute most to molecular mixing.

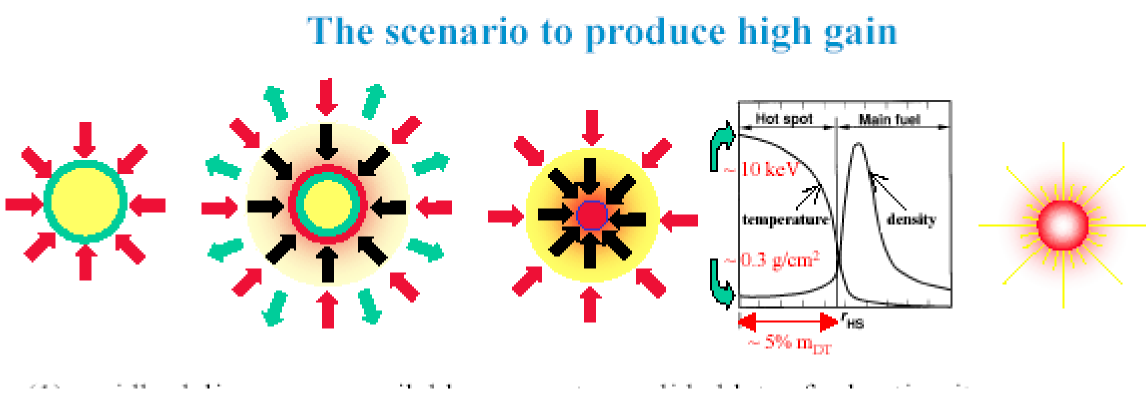
The objective of this studyis the comparison of models and experiments that model adequately describing process of excitation and subsequent evolution of Rayleigh-Taylor instability (RTI). In this project the hydrodynamical instability is driven by accelerating the mixing region between two gases of different densities, but much of what will be learnt may also be applied to incompressible flows and either externally imposed accelerations or the acceleration due to gravity.
The study will include:

Study of excitation and development of hydrodynamical Rayleigh-Taylor instability, resulting in formation of a turbulent mixing layer.

Stratification of this area caused by its subsequent deceleration.

A combination of experiments, theory and numerical simulations will be employed. The experiments will be performed with accelerations up to $1.5\times10^4 g_0$ (g_0 is the acceleration of gravity) and Atwood numbers ($At=(\rho a-\rho b)/(\rho a+\rho b)$, where ρa and ρb are the densities of used gases) of the initial mixing layer from -0.64 up to +0.74. The experimental results will be used to establish a model describing the instability, and to validate both this model and one-, two- and three-dimensional numerical simulations of the flow. The results will also be used to predict the behaviour of instabilities triggered in the compression of layered ICF targets.

We considered the modification of turbulent mixing model for the description of a separation at sign-variable acceleration. The model was obtained from the analysis of the equations for turbulent flows of concentration and density. Êå model [1] was added with items and the equation, containing the coefficient of heterogeneity Ã.

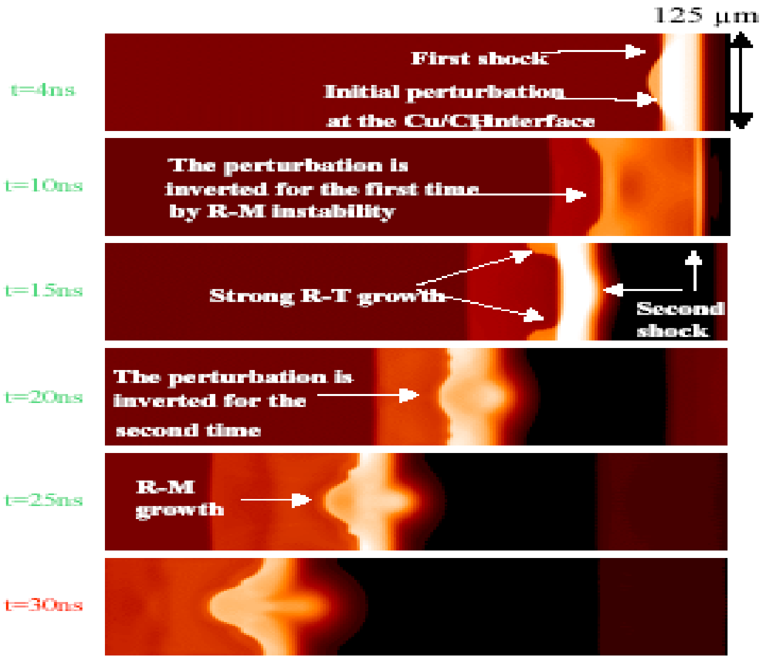


The model was investigated in case of two incompressible fluids in a field of gravity.
It is shown, that in an unstable case the dimensionless velocity of development of turbulent

zone depends on the coefficient of heterogeneity Ã. This fact is a possible explanation of a distinction of experimental data. The stable phase begins, when the sign of acceleration changes. The zone of turbulent mixing decreases in this case. This fact is in the consent with experimental data [2] and results of direct numerical simulation [3]. Development of a mixing zone depends from $\tilde{A}$ in this case. There is a full separation if Ã=1 and separation does not occur if Ã=0.

Introduction

The stability of an interface between two superposed fluids of different density was studied by Lord Rayleigh and Taylor(1950) for the case when the dense fluid is accelerated towards the less dense fluid, the linear theory can be found in Chandrasekhar (1961). For inviscid fluids, the interface is always unstable, with the growthrate of the unstable modes increasing as their wavelengths decrease. The instability of the short waves can be reduced by dissipative mechanisms such as surface tension or viscosity, and then linear theory predicts the maximum growthrate to occur at a finite wavelength. For the viscous two-layer case, where the upper layer (density ρ_1) is denser than the lower layer (density ρ_2), the wavelength λ_μ of maximum growthrate is



where ν is the mean kinematic viscosity of the two layers and g is the acceleration of gravity. The corresponding maximum growthrate has been described in Linden and Redondo(1991).
While the linear theory for two infinite layers is well established, the development of the instability to finite amplitude is not amenable to analytic treatment. There have been a number of semi-analytical and numerical studies in recent years, but they all involve simplifying assumptions which raise serious doubts about their validity particularly when applications to mixing are sought. An overview of the subject by Sharp(1984) characterized the development of the instability through three stages before breaking up into chaotic turbulent mixing. 1) a perturbation of wavelength λ_μ grows exponentially with growthrate n_μ. 2) when this perturbation reaches a height of approximately ½ λ_μ, the growthrate decreases and larger structures appear. 3) the scale of dominant structures continues to increase and memory of the initial conditions is supposedly lost; viscosity does not affect the latter growth of the large structures.

The advance of this front is described in Linden & Redondo (1991), and may be shown to follow $\delta = 2cA\tau^2$ where δ is the width of the growing region of instability, *g* is the gravitational acceleration and *A* is the Atwood number defined as $A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

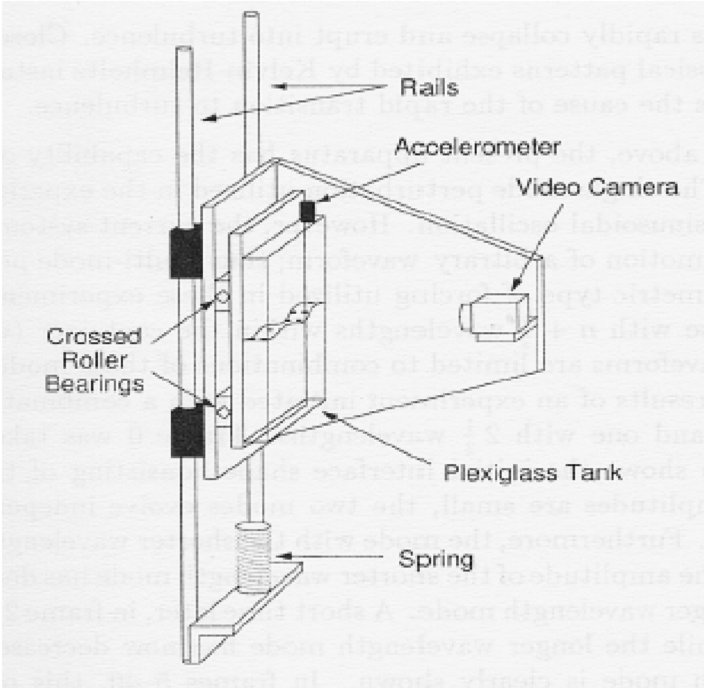
This result concerning the independence of the large amplitude structures on the initial conditions has led to consider that the width of the mixing region depends only on ρ_1, ρ_2, *g* and time, $\tau = t - t_0$. Then dimensional analysis may be used defining the relevant reduced acceleration driven time with respect to the height of the experimental box, H as:

$$\tau = \sqrt{H/2gA}$$

The proportionality factor *c* is considered to be a (universal) constant, although some dependence with the Atwood number and the initial conditions of the plate removal or random numerical fluctuations is expected, the key factor is the ratio of the potential and kinetic energy produced by the initial conditions to the Available potential energy of the whole mixing process, if this factor is small, then c tends to a constant value of 0.3, otherwise it will strongly depend on initial conditions and forcing scale.

The value of the constant *c*, has been investigated experimentally and its value for experiments at different values of the Atwood number, *A*, do not show large variations, with a limit clearly seen for the larger *A* experiments performed. Values of *c* previously obtained experimentally have been in the range (0.03 – 0.035) (Read and Young 1984) in experiments with three dimensional effects and large density differences between the two fluids, $A \geq 1.5$. Redondo and Linden (1990) measured *c* for values of *A* in the range $1x10^{-4}$ to $5.0x10^{-2}$ and found values of $c = 0.035 \pm 0.005$. Numerical calculations in two dimensions (Young 1984) have given values of *c* in the range 0.02 – 0.025. The lesser values (Read 1984) have been explained in terms of two dimensional effects inhibiting the growth of the large scale.

In a similar way as in the experiments a Rayleigh-Taylor mixing front has been simulated using FLUENT in the Large Eddy Simulation small scale parameterization mode, See figure 1 for sequences of the advance of the mixing front, using the non-dimensional time described above. The global aspects of the mixing front are seen to depend strongly on the mesh size used and on the random initial perturbation.


Rails
Accelerometer
Video Camera
Crossed
Roller
Bearings
Plexiglass Tank
Spring

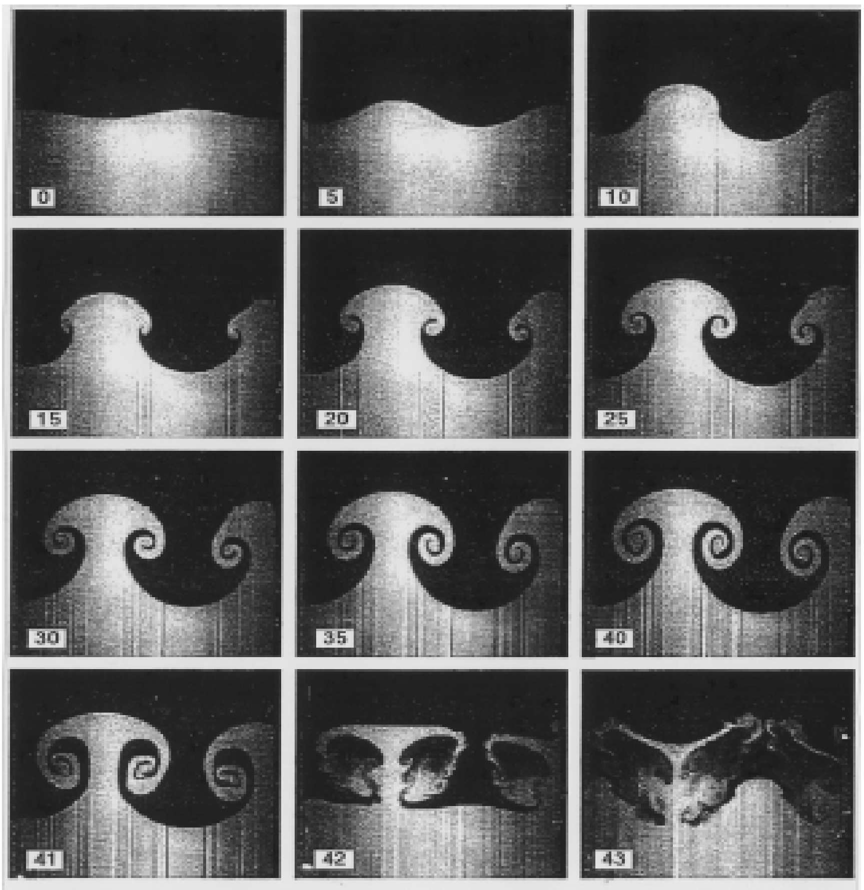
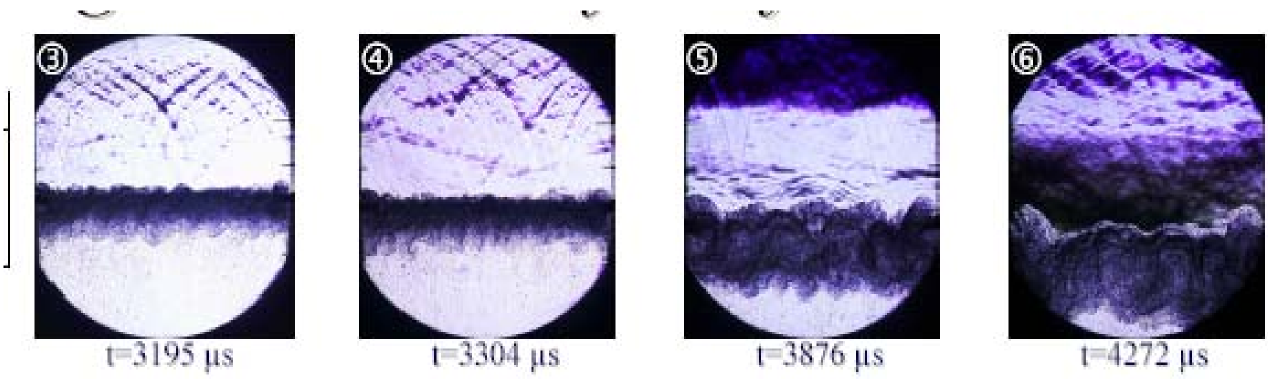
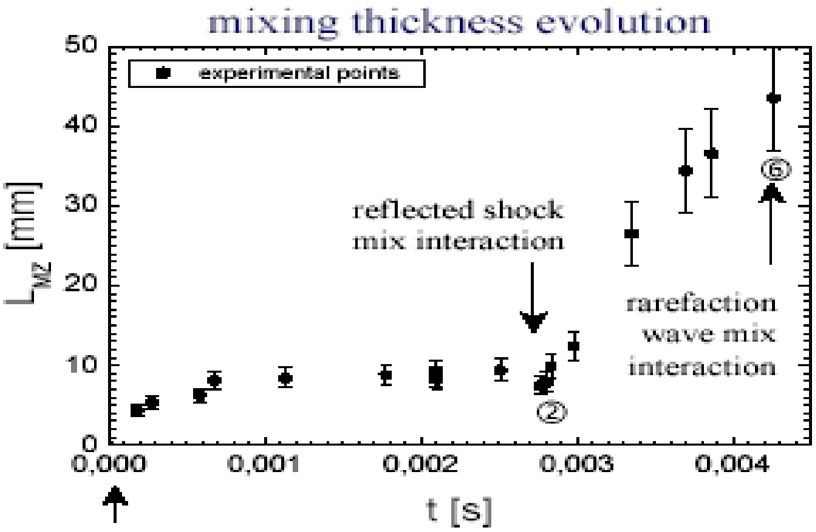
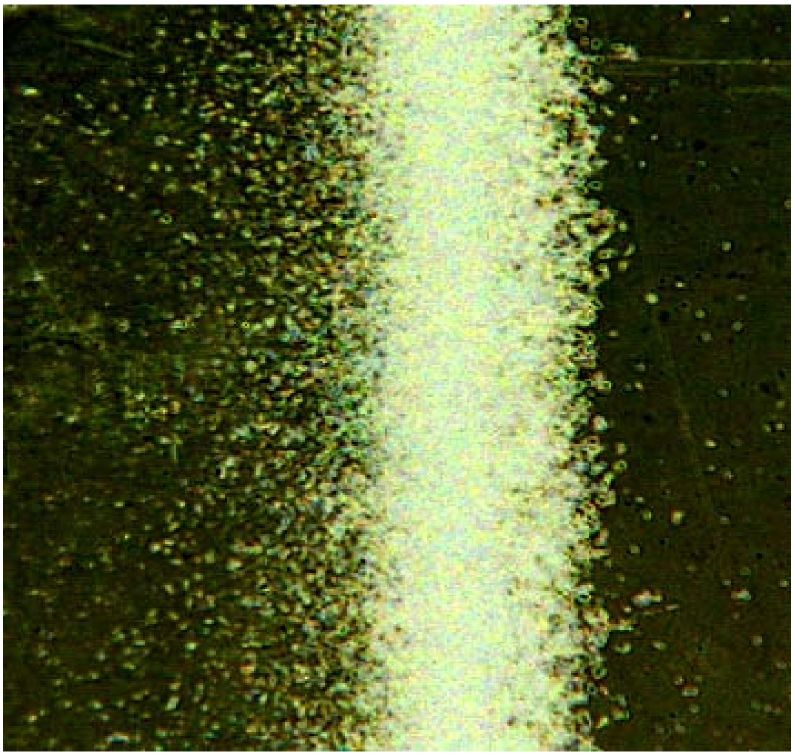


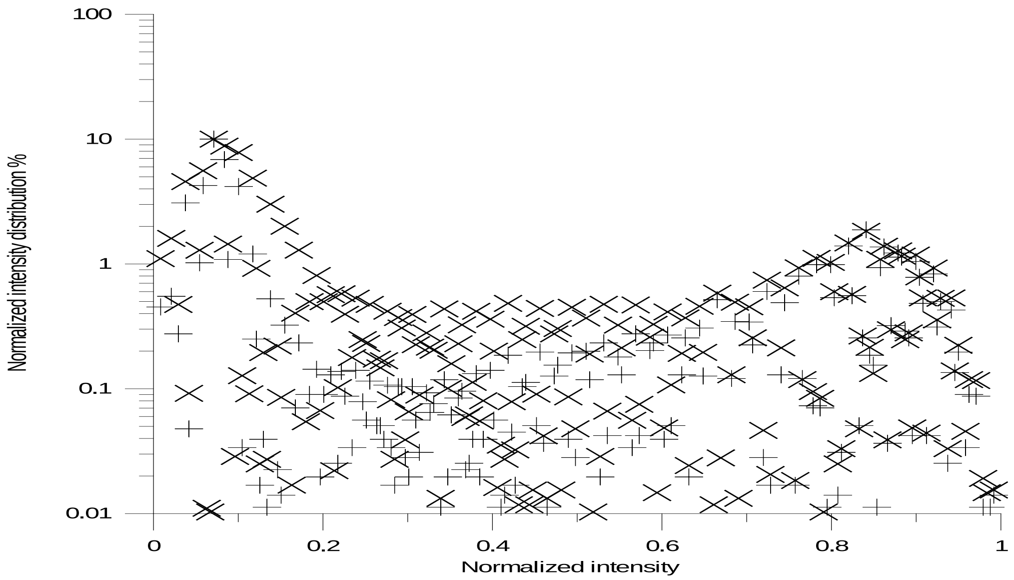
Figure . evolution of the structure of the RM front at times T=0-43 (From Jacobs IWPCTM6 1997).

The results describe the range of intensity values, where the isoconcentration lines exhibit a fully developed turbulent behavior indicated by a fractal dimension equal or greater that D=1.4 As expected, the turbulent self similar characteristics are shown only in the regions of strong contact between the two fluids.

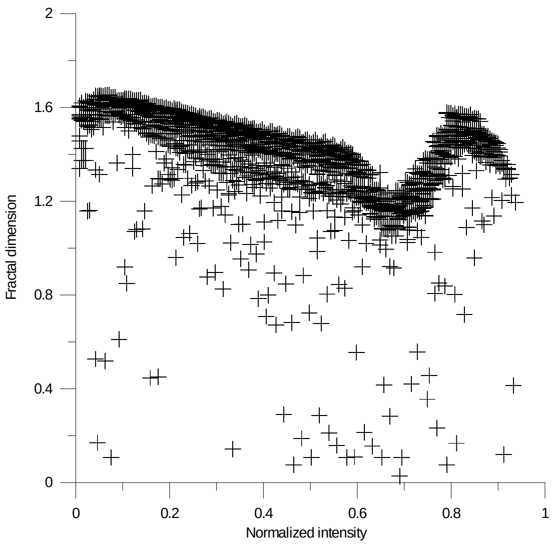



mixing thickness evolution
experimental points
L_{MZ} [mm]
t [s]
reflected shock
mix interaction
rarefaction
wave mix
interaction
2
6
0
10
20
30
40
50
0,000
0,001
0,002
0,003
0,004

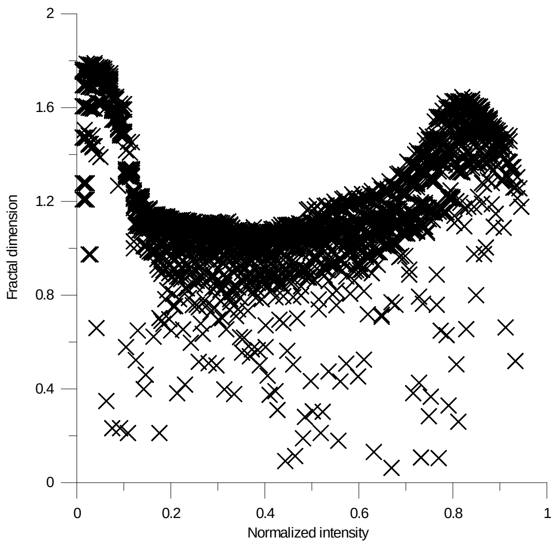




100
10
1
0.1
0.01
Normalized intensity distribution %
0
0.2
0.4
0.6
0.8
1
Normalized intensity



2
1.6
1.2
0.8
0.4
0
Fractal dimension
0
0.2
0.4
0.6
0.8
1
Normalized intensity



2
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Fractal dimension
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Normalized intensity

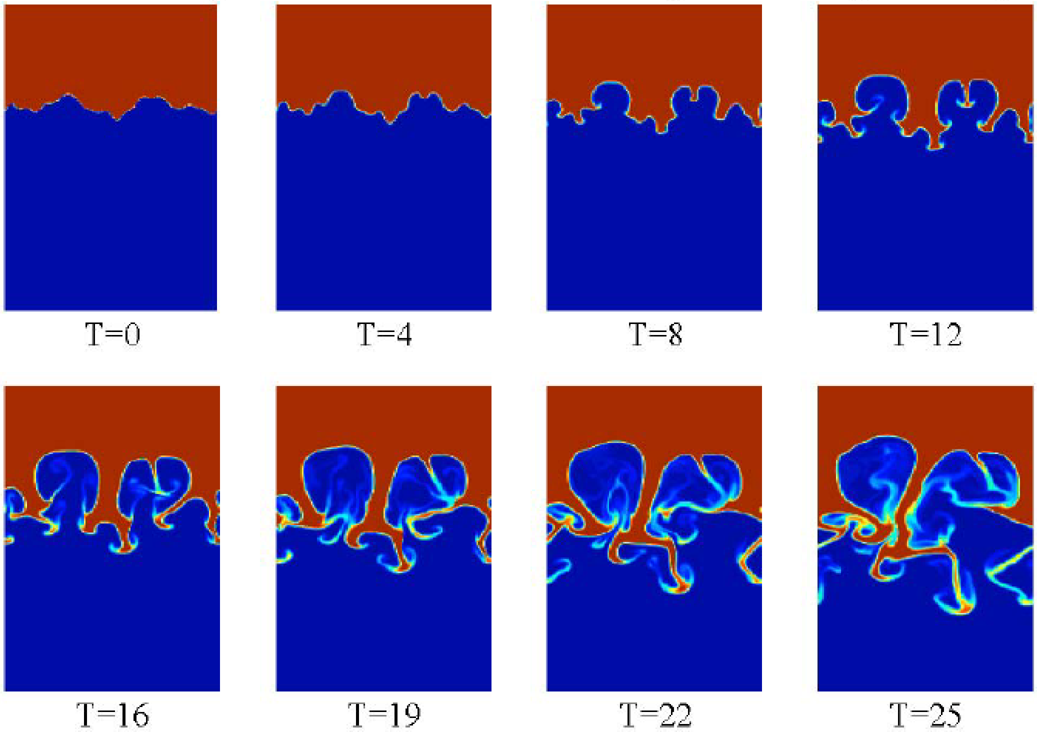

Figure 2. Evolution of the RM front in times/T=0, 4, 8, 12,16, 19,22 and 25

Fractal analysis

Using the variance of the signal ρ(t) defined in terms of the volume faction (or other relevant variable):

$$V(T) = < (\rho(t+T) - \rho(t))^2 >$$

where < > denotes the average over the sampling period T and taking into account the dependence for fractal time series $V(T) \approx T^{2H}$ (Voss 1985, Redondo et al. 2004).Using T = 1/f and the description of the spectral density function, S(f), we have the clasical spectral

$$S(T) \approx T^{\beta}$$

which will include intermittency effects and does not need to be in equilibrium. From the relationship between variance (or correlation) and spectra we may write

$$S \,\alpha\, T \int_0^T \rho^2(t)\, e^{-ift}\, dt \approx T V$$

so we can relate the spectra with the variance of a signal in terms of the maximum fractal dimension of the set of volume fraction values in a certain time period

$$S(f) \approx TV \approx T^{2H+1} \approx T^{2E+1-2D}$$

A similar analysis may be used with spatial correlations or coupled with ensemble averaging over many realizations

Figure 3. Fractal evolution of volume fraction, velocity and vorticity contours in the development of a RT front

Figure 2, shows the evolution of the multifractal dimension (calculated performing the box-counting algorithm) for each level of velocity modulus (a) and volume fraction (b). Much more relevant information can be extracted from these evolutions than from the maximum value presented by Linden et al.(1994), furthermore it is of great interest to study independently the fractal properties of velocity, volume fraction and vorticity fields as shown in figure 3 only for the intermediate values.

More work is still needed in order to fully interpret the results of the fractal analysis, but it is interesting to compare changes in the fractal dimension with other experimental set ups. Information about the mixing can be extracted from the thickening of the edges due to the phenolphthalein color change in Linden et al. (1995), or in the numerical simulations, and this thickness can be now analyzed with a digitizer system. For lower density runs with phenolphthalein, it was apparent that the vorticity originated by the plate increased mixing at the center of the vortices produced by it. This effect can be avoided using intermediate density differences.

Both in the experiments and in the numerical simulations the fractal evolution that indicates a transition to a turbulent flow is apparent as shown by Linden et al.(1995) by the increase in the maximum fractal dimension of the interface center (50/50 mixing ratio) between Dm = 1 and 1.4. The Spectra and fractal aspects of the numerical simulations are compared with the experiments, for example in figure 4 a scatter-plot of the multi-fractal dimension at two times of the different volume fractions of the front indicates its non-uniform curdling

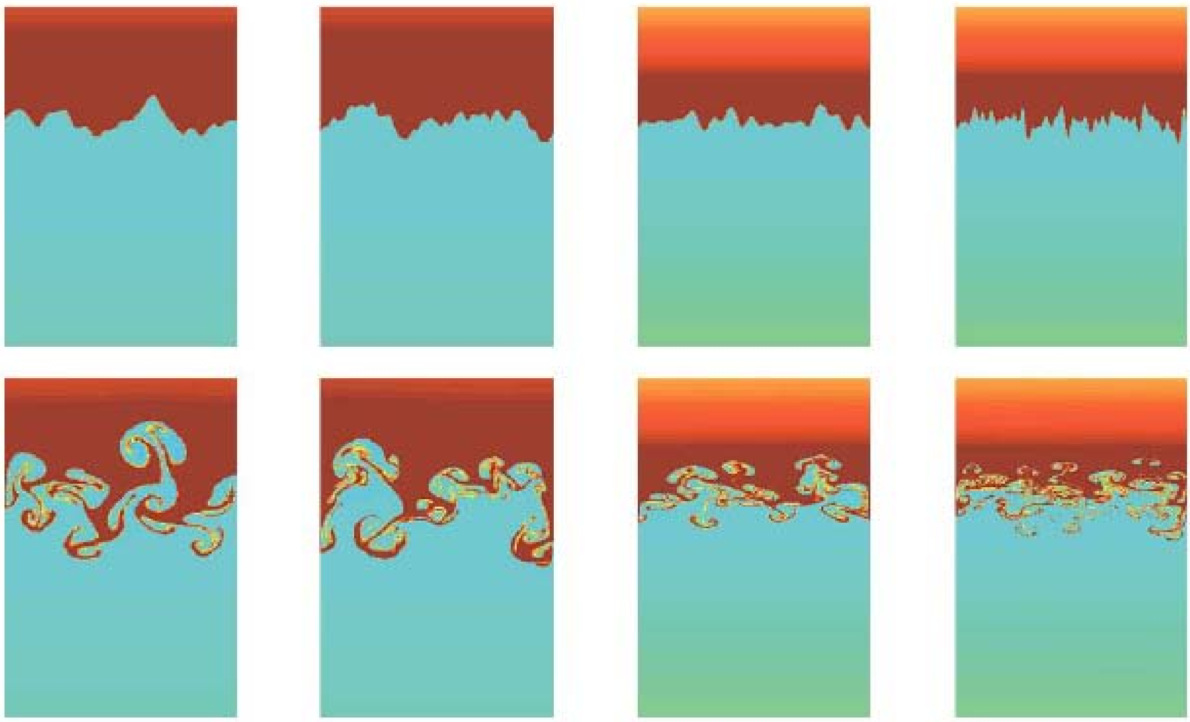

Figure 4. Comparison of the evolution of RT simulations with different initial waveform distributions A=0.941

The relation between fractal analysis and spectral Wavelet analysis can be very useful to determine the evolution of scales. Presently the emerging picture of the mixing process is as follows. Initially a pure RT instability with lengthscale λ_μ appears, together with the disturbances from the plate. The growth and merging of disturbances favors the appearance of several distinct blobs, bubbles or protuberances which produce shear instabilities on their sides. These sometimes develop further secondary shear instabilities. After 2/3 of the tank three dimensional effects have broadened the spectrum of lengthscales widely enough as to have a fractal structure in the visual range with dimensions ranging between 2.15 and 2.30. Some differences may be detected in the maximum fractal dimension evolution in time for experiments with different Schmidt or Prandtl numbers as described in Redondo (1996).

Acknowledgements

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