Network Model of Short-Term Optimal Hydrothermal Power Flow with Security Constraints

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Abstract—**Optimizing the thermal production of electricity in the short term in an integrated power system when a thermal unit commitment has been decided means coordinating hydro and thermal generation in order to obtain the minimum thermal generation costs over the time period under study. Fundamental constraints to be satisfied are the covering of each hourly load and satisfaction of spinning reserve requirements and transmission capacity limits. A nonlinear network flow model with side constraints with no decomposition into hydro and thermal subproblems was used to solve the hydrothermal scheduling. A novel thermal generation and transmission network is introduced and this transmission network is replicated with line outages in order to obtain a secure multi-interval short-term power flow. Computational results are reported.**

Keywords—Hydrothermal Scheduling, Optimal Power Flow, Line-Outage Constraints, Spinning Reserve, Electricity Generation, Nonlinear Network Optimization, Side Constraints

I. Introduction

The solution to short-term hydrothermal coordination indicates how to distribute the hydroelectric generation (cost-free) in each reservoir of the reservoir system and how to allocate generation to thermal units committed to operating over a short period of time so that the fuel expenditure during the period is minimized. The time period (e.g. 3 days) will be subdivided into short time intervals (e.g. of 1 hour) for which the generation and operating variables are optimized. In short-term hydrothermal coordination the predicted load at each hourly interval must be met, and a spinning reserve requirement to account for failures or load prediction errors must be satisfied. These load and spinning reserve constraints tie up hydro and thermal generation. On the other hand the initial and final water volumes at each interval in the reservoirs considered, which are used to calculate the hydrogenerations of the interval, tie up the variables of all intervals, and besides multi-interval emission constraints involve thermal generations over several intervals, thus all hydrothermal variables of the problem are in fact strongly coupled.

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Network flow techniques have come to be the most widely used tool for solving this problem. The literature on short-term hydrothermal optimization and coordination through network flows is rich [1,10]. The short-term hydrothermal scheduling problem has been researched intensively during recent years, either as the main problem [2,3,4] or as a subproblem of the short term hydrothermal coordination problem, which includes the commitment of thermal units [5,6]. The decoupled method followed by these papers consists in solving the hydro and thermal subproblems separately, coordinating these decoupled optimizations through *a)* the interchange of the marginal prices of the hourly load demand (from the thermal subproblem to the hydro subproblem) and *b)* the hydro generation in each time period (from the hydro subp. to the thermal subp.). In order to solve the hydro subproblems through efficient linear network flow codes, hydrogeneration , which appears in the objective function of the hydro subproblems, is usually approximated as a linear function of the discharges [2,3], or both the discharges and stored water [5] . Quadratic [6] and fully nonlinear formulation [7] of the hydrogeneration has also been reported. The thermal subproblem is usually posed as a set of independent thermal economic dispatch problems if the transmission network is not considered [5,6], or as a set of independent optimal power flow problems, either with a dc [3,4,7,8] or ac [9] approach. The thermal generation and power flows are optimized with a fixed value of the hydrogeneration that corresponds to the optimal solution of the last hydro subproblem. The models proposed in [2,3,4,6,7] take into account the load demands but neglect the spinning reserve, which is included in [5] .

Attempts to solve the hydro and thermal problems together are limited. In [9] a coupled model with an ac OPF solution and a very simplified model of the hydro system has been reported.

The decoupled procedure followed in previous works has to assume hydrogeneration values (to define constraints limits) for the thermal minimization and marginal prices of thermal production for hydro optimization. Since both hydrogenerations and marginal prices of thermal generation have unknown values at the optimizer, many solutions to the undecoupled problems will be needed until convergence, which is a clear disadvantage with respect to the undecoupled model.

In previous works by two of the authors [11,17] the network model usually employed for short-term hydrogeneration optimization was extended to include thermal units in an undecoupled way, imposing single load and spinning reserve constraints on both hydro and thermal generators and directly minimizing thermal production costs without decoupling the problem into hydro and thermal subproblems. When constraints are added to limit generation to pre-specified margins at each interval, or to satisfy a given spinning reserve requirement, pure network flow algorithms are no longer applicable. However if these constraints are linearized, efficient specialised algorithms for optimizing network flows with linear side constraints could be employed [12,13,17].

Transmission constraints and the economic effect of losses can also be taken into account through network flow techniques [10] . In [11] they were integrated into the thermal network developed.

The undecoupled solution to the short term hydrothermal coordination, taking into account a dc transmission network model, is nothing but a multi-interval dc optimum hydrothermal power flow, where the coupling effects of hydrogeneration over successive intervals are rigorously taken into account and optimized. The classical optimum power flow for a given interval finds values for thermal and for hydrogeneration but requires an "estimated value" of hydrogeneration determined beforehand, and it is not easy to choose values of hydrogeneration over successive intervals so that volumes and discharges in reservoir systems are all within limits and match natural inflows.

In this work the multi-interval dc optimum hydrothermal power flow concept is extended in two ways: line-outage security constraints are modeled and taken into account, and multi-interval emission constraints are also included in the problem. Computational results are reported using a nonlinear optimization package.

II. Short-Term Hydrogeneration Optimization Through Network Flows

Network flows can model any configuration of cascaded hydro stations along branched rivers and water transport delays between successive stations. Hydro variables such as initial and final stored water volumes in a reservoir at each interval and discharge and spillage over an interval are flows on replicated hydrostation networks. In a reservoir system, if the k^{th} of Nr reservoirs is of variable head we can compute its generation over the i^{th} interval as:

$$H_k^{(i} = \mu \rho_k{}^{(i} h_k{}^{(i} d_k{}^{(i} \quad (1)$$

where μ is the mechanical to electrical energy conversion constant and $\rho_k{}^{(i}$ is the efficiency of the k^{th} reservoir, $h_k{}^{(i}$ is its equivalent head —obtainable from the initial and final stored volumes— and $d_k{}^{(i}$ its discharge over the i^{th} interval. The efficiency $\rho_k{}^{(i}$ changes with water head and discharge. The total hydrogeneration over the i^{th} interval would be:

$$H^{(i} = \sum_{k=1}^{Nr} H_k^{(i} \quad (2)$$

The expression of the incremental spinning reserve of hydro units (the amount by which the current generation can be increased) in the i^{th} interval would be $\sum_{k=1}^{Nr} [\overline{H}_k^{(i} - H_k^{(i}]$ where $\overline{H}_k^{(i}$ would represent the maximum hydropower of the k^{th} reservoir over the i^{th} interval.

The total hydrogeneration in the i^{th} interval $\sum_{k=1}^{Nr} H_k^{(i}$ can be taken as the decremental hydro spinning reserve (amount by which the current generation can be decreased) in the interval.

Both the hydro incremental and decremental spinning reserve are assumed to be available within a short (relatively to that of a thermal unit) time lapse

III. Variables Associated to the Generation of a Thermal Unit

Let P_j be the power output of the j^{th} thermal unit and let $\overline{P}_j$ and $\underline{P}_j$ be its upper and lower operating limits.

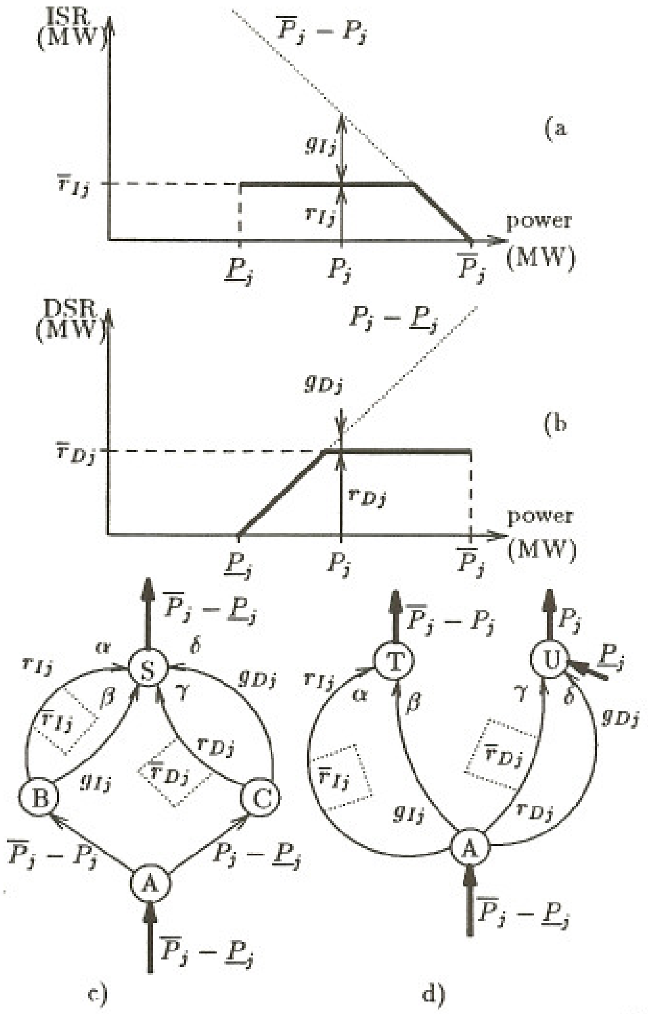

Fig. 1. a) Incremental Spinning Reserve (ISR) function of the j^{th} thermal unit
b) Decremental Spinning Reserve (DSR) function of the j^{th} thermal unit
c) Thermal network for the j^{th} thermal unit indicating limits on arcs α and γ
d) Thermal network for the j^{th} thermal unit showing node U with power output

$$\underline{P}_j \le P_j \le \overline{P}_j \quad (3)$$

The incremental spinning reserve (ISR) r_{Ij} of unit "j" is the amount of power by which the current generation P_j can be increased within a given time lapse. The maximum possible ISR $\overline{r}_{Ij}$ of the j^{th} unit is the product of the incremental power rate (MW/min) and the minutes of the specified time lapse. Similarly, the decremental spinning reserve (DSR) r_{Dj} of the j^{th} unit is the amount of power by which one can decrease the current power output P_j within a pre-determined time lapse. Its maximum value will be represented by $\overline{r}_{Dj}$. The ISR r_{Ij} and the DSR r_{Dj} of the j^{th} unit can be expressed as:

$$r_{Ij} = min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \quad (4)$$
$$r_{Dj} = min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \quad (5)$$

which are the thick lines of Fig. 1a) and 1b).

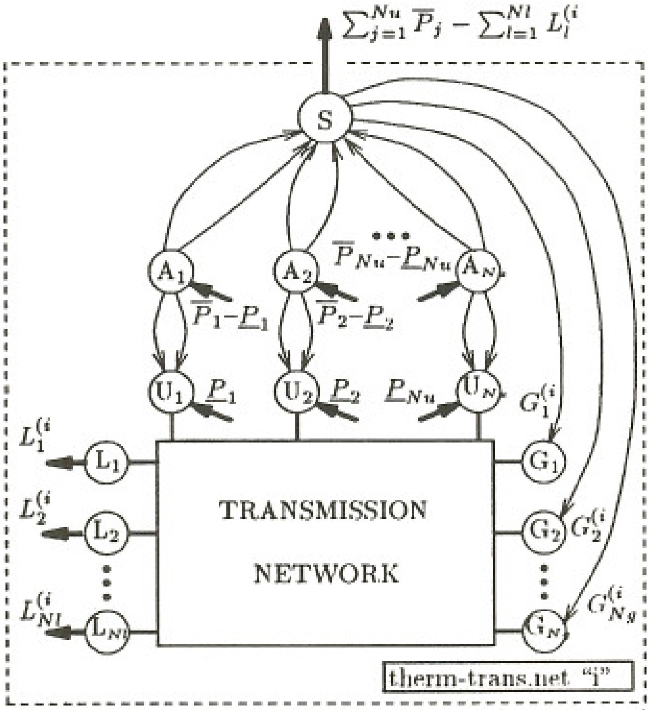

Fig. 2 Thermal network of ensemble of thermal units and connection to transmission network.

At power P_j we have an ISR r_{Ij} and a DSR r_{Dj}, and a power gap $g_{Ij} \geq 0$ from the ISR r_{Ij} to $\overline{P}_j - P_j$ and also a power gap $g_{Dj} \geq 0$ between the DSR r_{Dj} and $P_j - \underline{P}_j$:

$$r_{Ij} + g_{Ij} = \overline{P}_j - P_j \tag{6}$$
$$r_{Dj} + g_{Dj} = P_j - \underline{P}_j \tag{7}$$

The generation of a thermal unit, its ISR and DSR, the associated power gaps, and its operating limits lend themselves well to being modeled through network flows [11,17]. Fig. 1c) shows the directed graph having the variables described as flows on its arcs.

Node A has a power injection of $\overline{P}_j - \underline{P}_j$, which is collected at the sink node S. From the balance equations at nodes B and C, equations (6) and (7) are satisfied. Arcs α and β, both from node B to the sink node S, carry the power gap g_{Ij} and ISR r_{Ij} respectively, and an upper limit of $\overline{r}_{Ij}$ on arc α must be imposed to prevent the reserve from getting over its limit. From Fig. 1a) and 1b) it is clear that r_{Ij} and its gap g_{Ij} must be such that, for a given value of P_j, r_{Ij} takes the highest value compatible with $r_{Ij} < \overline{r}_{Ij}$ and with $r_{Ij} + g_{Ij} = \overline{P}_j - P_j$.

In fact the arc going from node A to node B in Fig. 1c) is useless and can be eliminated as in Fig. 1d) (since the flow on arc α plus that on arc β will amount to $\overline{P}_j - P_j$). The same happens to be so for the arc going from node A to node C, which can also be suppressed. However a (generally nonlinear) cost function of its flow $P_j - \underline{P}_j$ will have to be optimized, but it suffices to optimize the same function of the sum of flows on arcs γ and δ. Node S of Fig. 2 can be split into nodes T and U as in Fig. 1d). It can be noticed that if $\underline{P}_j$ is injected in node U, the outcome of this node will be just P_j, which is the generation of the j^{th} thermal unit. The simplified thermal network of Fig. 1d) can thus be employed. Only for explanatory purposes the notation $P_j - \underline{P}_j$, equivalent to $r_{Dj} + g_{Dj}$, will be maintained.

IV. Network Representation of the Ensemble of Thermal Units, Hydrogeneration and Transmission Network

Transmission lines with known characteristics and a maximum capacity connect generating units among themselves and to other (load or generating) nodes. The inclusion of the transmission network accounts for transmission limits, which may play an important part in shaping the thermal and hydrogeneration at some intervals.

It was shown in [17] that it is possible to combine the equivalent network of each thermal unit and a dc model of the transmission network, which takes power from hydro and thermal generating stations to the load nodes [10] , into a generation plus transmission network that ensures the satisfaction of load and transmission capacity limits, and where Kirchhoff's current law is satisfied. Kirchhoff's voltage law will also be imposed via linear side constraints on the flows of this network [10] .

The generations of thermal units coming out of nodes U (see Fig. 1d)) can be fed into a transmission network as in Fig. 2), where the generation of single reservoirs or of one or more reservoir systems must also be fed (it can be assumed that there are Ng nodes where hydrogeneration is fed in). There are Nl load nodes, with load $L_j^{(i}$, j=1,...,N_l, in the transmission network and thermal and hydrogeneration must balance the loads which draw from the network.

A sink node S is used to balance generation and load of a given interval "i". Node S collects thermal generation and supplies hydrogenerations ($G_j^{(i}$, j=1,...,N_g) through artificial arcs from the sink node S to nodes G_j (j=1,...,Ng). Nodes U_j,(j=1,...,Nu) receive the power output $P_j^{(i}$, which is fed into the transmission network. All nodes T of the equivalent network of each single thermal unit (see Fig. 1d)) are made to coincide with the sink S. Since the output of node T is $\overline{P}_j - P_j^{(i}$ (at interval "i") it is necessary to draw out of node S the flow $\sum_{j=1}^{Nu} \overline{P}_j - \sum_{l=1}^{Nl} L_l^{(i}$, which is constant, to balance the flows.

Hydrogenerations $H_k{}^{(i}$ (k=1,...,Nr) must correspond to the flows on arcs from the sink S to nodes G_j (j=1,...,Ng). In order for this to be so, Ng (linear) side constraints employing the linear approximation to hydrogeneration must be imposed:

$$G_j^{(i} = \sum_{k \in I_j} H_k{}^{(i} \quad j = 1, \ldots, Ng \quad i = 1, \ldots, Ni \tag{8}$$

where I_j is the set of reservoirs whose hydrogeneration enters into the transmission network through node G_j. In [17] there is a linearized version of this side constraint.

The transmission network is any set of connections between any of the nodes $U_1, \ldots, U_{Nu}, G_1, \ldots, G_{Ng}, L_1, \ldots, L_{Nl}$, either directly or via other nodes called transhipment nodes. Let $\mathcal{U}_j^+, \mathcal{G}_j^+, \mathcal{L}_j^+, \mathcal{T}_j^+$ be the sets of lines that have nodes U_j, G_j L_j and T_j as the origin nodes, and $\mathcal{U}_j^-, \mathcal{G}_j^-, \mathcal{L}_j^-, \mathcal{T}_j^-$ the sets of lines with the same nodes as destination nodes. Let $\overline{p}_{kl}$ be the capacity of transmission line (k, l).

Taking an arbitrary orientation on each arc of the transmission network, its balance flow equation will be:

$$\left.\begin{array}{ll}
\displaystyle\sum_{(k,l)\in\mathcal{U}_j^+} p_{kl}^{(i} - \sum_{(k,l)\in\mathcal{U}_j^-} p_{kl}^{(i} = P_j^{(i} & j=1,\dots,Nu \\
\displaystyle\sum_{(k,l)\in\mathcal{G}_j^+} p_{kl}^{(i} - \sum_{(k,l)\in\mathcal{G}_j^-} p_{kl}^{(i} = G_j^{(i} & j=1,\dots,Ng \\
\displaystyle\sum_{(k,l)\in\mathcal{L}_j^-} p_{kl}^{(i} - \sum_{(k,l)\in\mathcal{L}_j^+} p_{kl}^{(i} = L_j^{(i} & j=1,\dots,Nl \\
\displaystyle\sum_{(k,l)\in\mathcal{T}_j^+} p_{kl}^{(i} - \sum_{(k,l)\in\mathcal{T}_j^-} p_{kl}^{(i} = 0 & j=1,\dots,Nt \\
\quad -\overline{p}_{kl} \le p_{kl}^{(i} \le \overline{p}_{kl} \quad \forall(k,l) &
\end{array}\right\} i=1,\dots,Ni \tag{9}$$

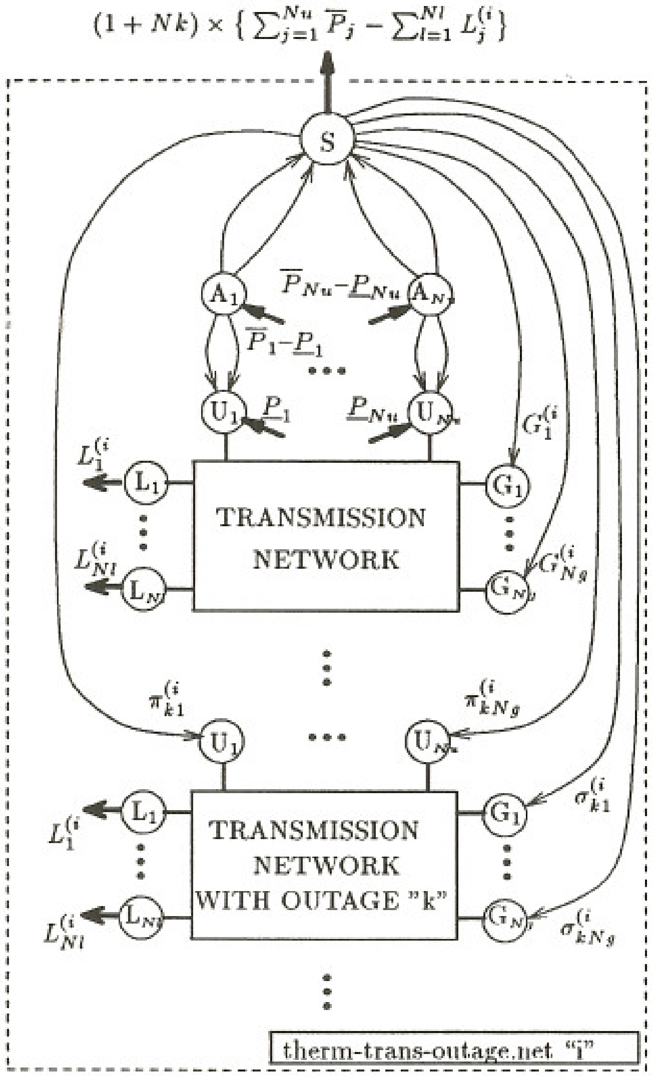

Fig. 3. Thermal network of ensemble of thermal units and connection to transmission network and to outage transmission networks

Since in a dc network model power and current flows measured in per unit (p.u.) coincide, the balance of flow at the nodes of the transmission network ensures the satisfaction of Kirchhoff's current law. Kirchhoff's voltage law must be imposed on all basic loops in the transmission network [10] . Imposing these constraints makes the flows in the transmission network more realistic.

Let x_{kl} be the p.u. reactance of the transmission line corresponding to the arc of the equivalent network going from node k to node l, and let p_{kl} be the power flow from k to l. The voltage drop along arc k–l can then be expressed as $x_{kl}p_{kl}$. Thus the expression of Kirchhoff's voltage law is:

$$\sum_{k,l\in \text{loop}j} x_{kl}p_{kl} = 0 \quad \text{for all basic loops} j \tag{10}$$

The network of Fig. 2 will be referred to as *therm-trans.net "i"*.

V. Line-Outage Security Constraints

An optimum power flow solution with security is such that, given a patern of loads at the load nodes, the generations fed in the network produce power flows that do not exceed the capacity limits neither in normal operation nor in case that any of a list of line-outage contingencies would take place.

It would be desirable to obtain a multi-interval optimal hydrothermal power flow solution with security and it will be shown here that an extension of the former model, as that in Fig. 4 is an adequate model of this problem. The transmission network should be replicated at each interval as many times as line-outage contingencies have to be considered. Each of these replicated transmission networks has all arcs (lines/transformers) but the outage one. Then artificial arcs from the sink to the hydro and thermal generation nodes of each contingency replicated network must be added and side constraints expressing that the flow in these arcs coincides with the generation flow in the analogous arcs of the non-contingency transmission network.

Assuming there are Nk contingencies to be considered the side constraints to be added are:

$$G_j^{(i} - \sigma_{kj}^{(i} = 0 \quad \left\{\begin{array}{l} k=1,\dots,Nk \\ j=1,\dots,Nu \\ i=1,\dots,Ni \end{array}\right. \tag{11}$$

$$P_j^{(i} - \pi_{kj}^{(i} = r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j - \pi_{kj}^{(i} = 0 \quad \left\{\begin{array}{l} k=1,\dots,Nk \\ j=1,\dots,Nu \\ i=1,\dots,Ni \end{array}\right. \tag{12}$$

The network of Fig. 3 will be referred to as *therm trans.-outage net "i"*.

VI. Hydro-Thermal-Transmission Extended Network (HTTEN) and Hydro-Thermal-Transmission-Outage Extended Network (HTTOEN)

All the variables taking part in the short-term hydrothermal scheduling are flows on arcs of a single network such as that in Fig. 4, called the *Hydro-Thermal-Transmission Extended Network (HTTEN)* and the *Hydro-Thermal-Transmission-Outage Extended Network (HTTOEN)*. A unique sink node S collects all the balance water plus the power supplied to the thermal networks. There is no problem in having a common sink node for the replicated hydro network and for the thermal network of each interval because each network is balanced in its own flow.

It should be stressed that the fact of using a common sink mode is just a means to reduce the number of balance equality constraints, but it does nor entail that the hydro optimization problem and the thermal optimization problem are coupled. What couples the two problems is the fact that their variables are optimized at the same time with respect to a unique objective function and, most

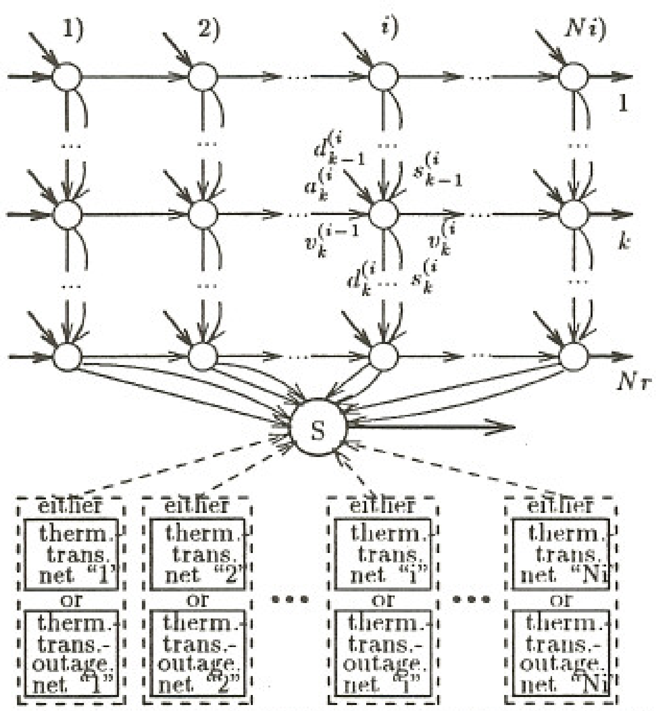

Fig. 4 Hydro-Thermal-Transmission Extended Network (HTTEN) or Hydro-Thermal-Transmission-Outage Extended Network (HTTOEN).

important, subject to common spinning reserve and load constraints where hydro and thermal variables take part.

VII. Generation cost of thermal units and losses in the transmission network

The production cost of the j^{th} thermal unit over the i^{th} interval, expressed as a second order polynomial with a linear and a quadratic cost coefficient c_{lj} and c_{qj} would be $c_{lj}P_j^{(i} + c_{qj}(P_j^{(i})^2$, which in terms of the network flow $P_j^{(i} - \underline{P}_j = r_{Dj} + g_{Dj}$ is $(c_{lj} + 2c_{qj}\underline{P}_j)(P_j^{(i} - \underline{P}_j) + c_{qj}(P_j^{(i} - \underline{P}_j)^2 + (c_{lj}\underline{P}_j + c_{qj}\underline{P}_j{}^2)$. The last parenthesis is of constant terms and can be excluded from the minimization. So the expression to be minimized is: $(c_{lj} + 2c_{qj}\underline{P}_j)(r_{Dj}^{(i} + g_{Dj}^{(i}) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i})^2$. Thus the thermal part of the cost function (corresponding to the i^{th} interval) to be minimized can be expressed as:

$$\min \sum_{j=1}^{Nu} [(c_{lj} + 2c_{qj}\underline{P}_j)(r_{Dj}^{(i} + g_{Dj}^{(i}) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i})^2] \quad (13)$$

When the equivalent network model of Section VI is considered, the transmission network model is a dc approach and losses are not included in the generation injected. Power losses cost can be estimated and added to the objective function to be minimized. $p_{kl}^{(i}$ being the p.u. value of power flow on the arc from node k to node l at the i^{th} interval, r_{kl} being the p.u. resistence of the transmission line corresponding to the arc, the losses on that line are approximately $r_{kl}[p_{kl}^{(i}]^2$, and $\kappa^{(i}$ being a price given to the losses over the i^{th} interval, the term to be added to the objective function would be:

$$\sum_{i=1}^{Ni} \kappa^{(i} \sum_{k,l \in \text{T.N.}} r_{kl}[p_{kl}^{(i}]^2 \quad (14)$$

where T.N. is the set of pairs of nodes that are the ends of all transmission lines.

VIII. Undecoupled Network Formulation of the Hydro-Thermal Scheduling

A. Objective Function.

The objective function to be minimized is

$$\min \sum_{i=1}^{Ni} \Big\{ \sum_{j=1}^{Nu} [(c_{lj} + 2c_{qj}\underline{P}_j)(P_j^{(i} - \underline{P}_j) + c_{qj}(P_j^{(i} - \underline{P}_j)^2] + \kappa^{(i} \sum_{k,l \in \text{T.N.}} r_{kl}[p_{kl}^{(i}]^2 \Big\}. \quad (15)$$

where the last term corresponds to the evaluation of the losses.

B. Network Constraints.

The network constraints are those related with the HTTEN or HTTOEN depicted in Fig. 4. Upper limits and lower limits, which are zero for most of the variables, exist for all the flows. They are taken into account by the specialised network codes.

C. Side Constraints.

Side constraints [12] (i.e.: constraints on the flows on the arcs different from the flow balance equations at each node) can be imposed and can be dealt with efficiently in specific network flow optimization methods [12,13]. Such side constraints are:

- Hydrogeneration side constraints (8).
- Line-outage Security side constraints (11,12)
- Kirchhoff's voltage law side constraints for the transmission network (10) and for each transmission network with outage "k".
- Incremental and decremental spinning side constraints. If $R_I^{(i}$ and $R_D^{(i}$ are the minimum ISR and DSR required respectively, these side constraints can be formulated as:

$$\left.\begin{aligned} -\sum_{j=1}^{Ng} G_j^{(i} + \sum_{j=1}^{Nu} r_{Ij}^{(i} &\geq R_I^{(i} - \sum_{k=1}^{Nr} \overline{H}_k^{(i} \\ \sum_{j=1}^{Ng} G_j^{(i} + \sum_{j=1}^{Nu} r_{Dj}^{(i} &\geq R_D^{(i} \end{aligned}\right\} i = 1, \ldots, Ni \quad (16)$$

D. Coupling between Hydro and Thermal System.

The ISR and DSR constraints (16) constitute the coupling side constraints between the hydro and the thermal variables of each interval. These variables are also coupled through the combined effect of the transmission network and the Hydrogeneration side constraints (8). The replicated hydro network involves a time coupling between the hydro and the thermal variables of all intervals.

E. Emission Side Constraints.

Single-interval and multi-interval emission constraints can be easily imposed, in a coupled formulation of the hydrothermal problem such as that described here, since all thermal generation variables are available. A linear multi-interval emission constraints referred to units using fule type "F.T." is;

$$\sum_{i=1}^{Ni} \nu_j \sum_{j \in \text{F.T.}} P_j^{(i} \leq \overline{E}_{\text{F.T.}} \quad (17)$$

Single interval, or nonlinear emission limits can also be easily formulated and incorporated.

IX. Computational Results and Case Examples

The network model put forward has been implemented to solve hydrothermal scheduling problems described in Tables I and II, where Nm and Nb stand for the number of lines/transformers and number of nodes of the transmission network. L.s.c. and n.s.c. indicate the number of linear and nonlinear side constraints respectively. There is one single multi-interval emission constraint in both problems A and B.

These problems could be solved efficiently through nonlinear network flow codes with side constraints. In a previous work by the authors [17] , the specialised nonlinear network flow program with linear side constraint NOXCB [14] was employed to solve the problem without outages, through successive linearization of the nonlinear hydrogeneration side constraints (8). Although the same methodology proposed in [17] can be applied to the problem with security constraints, the preliminary results presented here have been obtained using the general purpose nonlinear optimization code MINOS 5.3 [15,16]. The CPU times indicated in Table III have been obtained on a SUN Sparc 10/41 workstation.

Fig. 5 shows the effect of incorporating line-outage security constraints on the successive flows on one of the lines of the dc transmission network (case Bc: with line-outage constraints, against case B without). It can be apreciated that the successive flows of case Bc are more secure than those of case B in that they keep away of the capacity limit to make room for extra flow due to possible line-outages. The optimum cost obtained show that the extraprice paid for security is low (though, in the cases shown, the contingency list —of lenght Nk— is short). The number of saturated lines, "tn" in the transportation network, and "cn" in all Nk outage networks is indicated in Table III.

Table I : Power system size of case examples

Problem	Power system size					
	Nr	Nu	Nm	Nb	Ni	Nk
A	9	8	21	13	48	-
Ac	9	8	21	13	48	3
B	9	14	30	23	48	-
Bc	9	14	30	23	48	1

Table II : Problem dimensions of case examples

Problem	Optimization problem size			
	arcs	nodes	l.s.c.	n.s.c.
A	4704	1441	529	48
Ac	6816	2298	1453	48
B	7056	2209	480	48
Bc	9792	3314	1104	48

Table III : Computational results

Problem	CPU time (sec.)	iter.	Cost (10^6Pts.)	# sat. lin. tn	cn
A	6631.7	42361	196.951	1	-
Ac	11698.1	48211	197.015	0	4
B	20809.8	85699	47.775	5	-
Bc	27123.7	85763	47.789	0	21

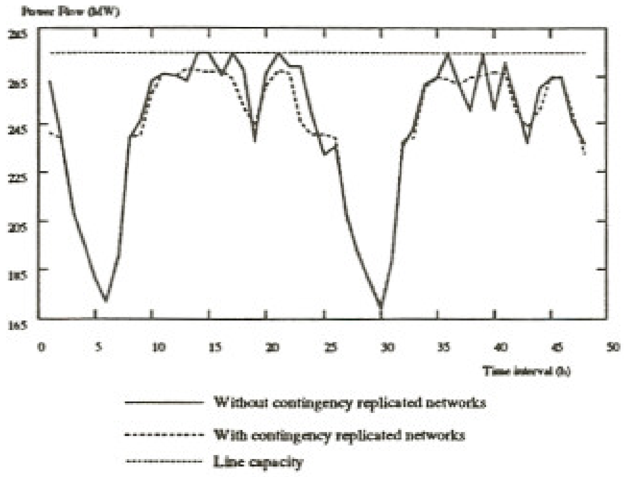

Fig. 5 Line flow values along intervals in a given line with or without contingencies considered.

X. Conclusions

An undecoupled formulation of the short-term optimal hydrothermal multi-interval power flows with security constraints has been presented and demonstrated computationally. Multi-interval emission constraints have also been imposed. The results obtained indicate that the solution to this problem is possible and that the computation resources required are moderate. The undecoupled formulation is more advantageous than the decoupled one because a single optimization leads to the optimum and there is no need to repeat optimizations with updated estimations of the Lagrange multipliers or of hydrogenerations, which could not converge, or converge slowly, to the optimum of the problem.

The dimension of the problem increases considerably when outage contingencies are taken into account.

The computation times required with a general purpose nonlinear optimization package are high but they could be reduced by an order of magnitude by employing an specialised nonlinear network flow code with linear and nonlinear side constraints.

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