A MIXED STABILIZED FINITE ELEMENT FORMULATION FOR STRAIN LOCALIZATION ANALYSIS

M. Chiumenti, michele@cimne.upc.edu
M. Cervera, mcervera@cimne.upc.edu
Universidad Politecnica de Cataluña (UPC)
Departamento de Resistencia de Materiales y Estructuras en la Ingeniería,
c/ Jordi Girona 1-3, Edificio C1, 08034 Barcelona, Spain.

C. Agelet de Saracibar, agelet@cimne.upc.edu
Universidad Politecnica de Cataluña (UPC)
Departamento de Resistencia de Materiales y Estructuras en la Ingeniería,
c/ Jordi Girona 1-3, Edificio C1, 08034 Barcelona, Spain.

Abstract. *This work exploits the concept of stabilization techniques to improve the behaviour of either mixed* u/p *or mixed* ε/u *linear/linear triangular elements when strain localization analysis is required. Different stabilization methods suitable for either Mode-I or Mode-II structural failure are proposed and compared to check the global stability of the corresponding discrete finite element formulation. Both elasto-J2-plastic and isotropic damage models have been used for the constitutive behaviour of the material. In both cases, exponential softening has been introduced in the post-peak regime. The results obtained do not suffer from spurious mesh-size or mesh-bias dependence, comparing very favorably with those obtained with standard approach.*

Keywords*: Strain localization, Finite Element Technology, Mixed Stabilized Formulation.*

1. INTRODUCTION

Strain localization occurs in softening materials subjected to monotonic straining. This phenomenon leads to the formation of localization bands inside the solid because, once the peak stress is reached within a band, and under further straining, strains concentrate inside the band while the material outside the band unloads elastically. Upon continuing straining, the localization progresses, the width of the localization band diminishes and, unless there is a physical limitation, it tends to zero. The particular components of the strain tensor that localize during this process depend on the specific constitutive behaviour of the material. In Rankine-type materials, only normal elongations localize, eventually forming tensile cracks; in the so-called J2 materials, shear (or slip) strains concentrate, leading to slip surfaces (or lines).

It is generally accepted that the amount of energy released during the formation of a unit area of discontinuity surface is a material property, called the fracture energy (Mode I and Mode II fracture energies in Fracture Mechanics terminology). Dimensional analysis shows that if the elastic energy stored in the solid volume is released through the area of the fracture surface, the failure process leads to what is known as structural size effect (Cervera and Chiumenti, 2009).

Experimental evidence shows that, for a given structural geometry, ductile behaviour is observed in the small scale limit, when the energy dissipated by inelastic behaviour in the formation of the failure mechanism is much larger than the total stored elastic energy; contrariwise, brittle behaviour occurs in the very large scale limit, when the ratio between the dissipated inelastic and available elastic energies is close to one. The small scale limit is suitable for small laboratory specimens, and the large scale limit is appropriate for structures of very large dimensions or even for scales larger than man-made structures. Thus, it is of practical interest to develop analytical and numerical tools suitable to bridge the gap between perfectly ductile and perfectly brittle behaviour. This is called quasi-brittle failure (Bazant and Planas, 1998).

Quasi-brittle failure has been the object of intensive interest in computational solid mechanics during the last four decades. Even if the main motivation for this interest is the wide range of engineering applications connected to this field, academic concern has been sharpened by the unexpected numerical difficulties encountered. The fact is that most attempts to model strain localization in softening materials with standard, irreducible, local approaches fail is that the solutions obtained suffer from mesh-bias dependence in such a strong manner that it cannot be ignored. Consequently, many different, alternative, strategies have been devised to model strain localization and quasi-brittle fracture and the references in the bibliography are uncountable.

In the last 25 years, micropolar (de Borst, 1991 and Steinmann and Willam, 1992), gradient-enhanced (Aifantis 1984, Vardoulakis and Aifantis 1991, de Borst and Mulhaus 1992, Pamin 1994, Peerlings *et al.* 1998) and non-local , (Aifantis 1984, Pijaudier-Cabot and Bazant 1987, Pijaudier-Cabot and Huerta 1991, Jirásek 1998, de Borst 2001, Bazant and Jirásek 2002, among others) models have been proposed with the common basic idea of modifying the original continuous problem to introduce an internal length that acts as a localization limiter. On a different line, viscous-regularized, strain-rate dependent models (see Aifantis 1984, de Borst 2001, Needelman 1987) also attempt to solve the numerical difficulties by modifying the original continuous problem.

Common to all these approaches there is the understanding that the underlying standard boundary value problem associated with quasi-brittle failure is not well posed and it must be reformulated. However, this standpoint ignores the well-known fact that "well-aligned" finite element meshes produce good results when using the standard (irreducible and local) approach. This evidence strongly suggests that the "flaw" that produces spurious mesh-bias dependence of the discrete problem is in the spatial discretization procedure.

On one hand, the authors have applied *stabilized mixed displacement/pressure methods* (Chiumenti *et al.* 2002, Cervera *et al.* 2003-a, Cervera *et al.* 2003-b, Cervera *et al.* 2003-c, Chiumenti *et al.* 2004, Agelet de Saracibar *et al.* 2006, and Cervera and Chiumenti 2009) to the solution of J2 elasto-plastic and damage problems with simplicial elements. This formulation leads to a discrete problem which is fully stable, even for problems involving localization of shear strains and the formation of slip lines (Mode II failure mechanisms).

On the other hand, the authors have recently used a *stabilized mixed strain/displacement method* to extend these results to problems involving strain localization in Rankine-type materials and the formation of Mode I tensile cracks (see Cervera *et al.* part-I 2009 and Cervera *et al.* part-II 2009 for a detailed description of the formulation proposed).

In this work, we revised both mixed formulations, with focus on low order finite elements apply to *Mode-I and Mode-II strain localization problems*. The results obtained, both in terms of collapse mechanisms and global load-deflection responses, are practically mesh independent. Remarkably, the results obtained do not suffer from spurious mesh-bias dependence and therefore, the use of auxiliary *tracking techniques is not required*.

2. STRAIN LOCALIZATION IN J2 MODELS

The strong form of the continuum mechanical problem in mixed format to solve strain localization in J2 plasticity can be stated as: find the displacement field **u** and the pressure field *p*, for given prescribed body forces **f**, such that:

$$\begin{aligned} &\nabla\cdot\mathbf{s}+\nabla p+\mathbf{f}=\mathbf{0} \quad in\,\Omega \\ &\nabla\cdot\mathbf{u}-\frac{p}{K}=0 \qquad in\,\Omega \end{aligned} \tag{1}$$

where $p=\frac{1}{3}tr(\boldsymbol{\sigma})$ and $\mathbf{s}=dev(\boldsymbol{\sigma})$ are the volumetric and the deviatoric parts of the stress tensor, respectively, and Ω is the open and bounded domain occupied by the elasto-plastic solid. Eqs. (1) are subjected to appropriate Dirichlet and Neumann boundary conditions. In the following, we will assume these in the form of prescribed displacements $\mathbf{u}=\overline{\mathbf{u}}$ on $\partial\Omega_u$, and prescribed tractions $\overline{\mathbf{t}}$ on $\partial\Omega_t$, respectively. In the mixed formulation the value of the pressure is defined by the Neumann conditions or, alternatively, by prescribing its value at some point.

This mixed format is appropriate and even mandatory when incompressible elasticity or J2-plasticity (purely deviatoric inelastic deformations are allowed) models are used. In this work, a standard, locally defined, elasto-plastic constitutive model has been employed. Strain localization is induced via exponential strain softening, properly regularized according to the element size and the Mode-II fracture energy of the material.

The associated weak form of the problem (1) can be stated as:

$$\begin{aligned} &(\nabla^s\mathbf{v},\mathbf{s})+(\nabla\cdot\mathbf{v},p)-(\mathbf{v},\mathbf{f})-(\mathbf{v},\bar{\mathbf{t}})_{\partial\Omega_t}=\mathbf{0} \quad \forall\mathbf{v} \\ &(q,\nabla\cdot\mathbf{u})-\left(q,\frac{p}{K}\right)=0 \qquad \forall q \end{aligned} \tag{2}$$

where $\mathbf{v}\in V$ and $q\in Q$ are the variations of the displacements and pressure fields, respectively, and $(\cdot,\cdot)$ denotes the inner product in $L^2(\Omega)$, the space of square integrable functions in Ω.

As it is well known, to write the discrete finite element form of the mixed problem (2) it is necessary to be respect the severe restrictions posed by the inf-sup condition (Brezzi 1991) on the choice of the approximation spaces V_h and Q_h. For instance, standard mixed elements with continuous equal order linear/linear interpolation for both fields are not stable, and the lack of stability shows as uncontrollable oscillations in the pressure field that usually, and very particularly in non linear problems, pollute the solution entirely. Fortunately, the strictness of the inf-sup condition can be circumvented by modifying the discrete variational form appropriately, in order to attain the necessary global stability with the desired choice of interpolation spaces. Our proposal is the application of the orthogonal sub-grid scale stabilization method (OSGS) to the problem of incompressible elasto-plasticity as shown in previous works (Chiumenti *et al.* 2002, Chiumenti *et al.* 2004 and Cervera *et al* 2003). The resulting *stabilized* discrete form of problem (2) can be stated as:

$$
\begin{aligned}
&(\nabla^s \mathbf{v}_h, \mathbf{s}_h) + (\nabla \cdot \mathbf{v}_h, p_h) - (\mathbf{v}_h, \mathbf{f}) - (\mathbf{v}_h, \bar{\mathbf{t}})_{\partial\Omega_t} = \mathbf{0} && \forall \mathbf{v}_h \\
&(q_h, \nabla \cdot \mathbf{u}_h) - \left(q_h, \frac{p_h}{K} \right) - \sum_{e=1}^{nelem} \tau_e (\nabla q_h, [\nabla p_h - \Pi_h]) = 0 && \forall q_h \\
&(\nabla p_h, \pi_h) - (\Pi_h, \pi_h) = 0 && \forall \pi_h
\end{aligned}
\tag{3}
$$

where the stabilization parameter $\tau_e = c \dfrac{h_e^2}{2\overline{G}}$ is defined as a function of the characteristic length of the element h_e and the *secant* shear modulus $\overline{G}$ (Cervera *et al.* 2003-b). The third equation in (3) defines the Π_h as the projection on the finite element space of the discontinuous pressure gradient.

3. STRAIN LOCALIZATION IN RANKINE'S TYPE MODELS

In this case, the strong form of the continuum problem to solve Mode-I fracture failure can be stated as: for given prescribed body forces **f**, find the displacement field **u** and the strain field $\boldsymbol{\varepsilon}$ such that:

$$
\begin{aligned}
&-\mathbf{C} : \boldsymbol{\varepsilon} + \mathbf{C} : \nabla^s \mathbf{u} = 0 && in\, \Omega \\
&\nabla \cdot (\mathbf{C} : \boldsymbol{\varepsilon}) + \mathbf{f} = \mathbf{0} && in\, \Omega
\end{aligned}
\tag{4}
$$

where $\mathbf{C} = \mathbf{C}(\boldsymbol{\varepsilon})$ is the non-linear constitutive tensor which defines the (Rankine's type) constitutive behaviour. In this case, the first equation in (4) represents the constitutive equation while the second one is the balance of momentum.

In this work, a standard, locally defined, Rankine's type isotropic damage constitutive model has been employed. Strain localization is induced via exponential strain softening, properly regularized according to the element size and the Mode-I fracture energy of the material.

Following the standard procedure the associated weak form of the problem (4) can be stated as:

$$
\begin{aligned}
&-(\boldsymbol{\gamma}, \mathbf{C} : \boldsymbol{\varepsilon}) + (\boldsymbol{\gamma}, \mathbf{C} : \nabla^s \mathbf{u}) = \mathbf{0} && \forall \boldsymbol{\gamma} \\
&(\nabla^s \mathbf{v}, \mathbf{C} : \boldsymbol{\varepsilon}) - (\mathbf{v}, \mathbf{f}) - (\mathbf{v}, \bar{\mathbf{t}}) = \mathbf{0} && \forall \mathbf{v}
\end{aligned}
\tag{5}
$$

where $\mathbf{v} \in V$ and $\boldsymbol{\gamma} \in G$ are the variations of the displacement and strain fields, respectively. Note that the first equation in (5) represents the enforcement of the constitutive law in weak sense.

Also in this case, the inf-sup stability condition poses severe restrictions on the choice of the approximation spaces. If standard Galerkin procedure is used to discretize eq. (5) with continuous equal order linear/linear interpolation for strain and displacement fields, the resulting formulation is not stable, and the lack of stability shows as uncontrollable oscillations in the both displacement and stress fields that pollute the solution. Once more, it is possible to circumvent this condition using the residual-based sub-grid scale approach, which allows in particular the use of linear/linear interpolations for displacements and strains (Cervera *et al.* 2009-I and Cervera *et al.* 2009-II). The *stabilized* discrete form of problem (5) results in:

$$
\begin{aligned}
&-(1-\tau_\varepsilon)(\boldsymbol{\gamma}_h, \mathbf{C} : \boldsymbol{\varepsilon}_h) + (1-\tau_\varepsilon)(\boldsymbol{\gamma}_h, \mathbf{C} : \nabla^s \mathbf{u}_h) = \mathbf{0} && \forall \boldsymbol{\gamma}_h \\
&(1-\tau_\varepsilon)(\nabla^s \mathbf{v}_h, \mathbf{C} : \boldsymbol{\varepsilon}_h) + \tau_\varepsilon (\nabla^s \mathbf{v}_h, \mathbf{C} : \nabla^s \mathbf{u}_h) - (\mathbf{v}_h, \mathbf{f}) - (\mathbf{v}_h, \bar{\mathbf{t}}) = \mathbf{0} && \forall \mathbf{v}_h
\end{aligned}
$$

where $\tau_\varepsilon = c_\varepsilon \dfrac{h_e}{L_0}$ is the stabilization parameter and L_0 is a characteristic length of the problem.

This mixed formulation achieves better accuracy on the stresses (or strains) than the irreducible formulation. This may not be considered a discriminating argument, as this improvement is attained at the cost of using more degrees of freedom for the same number of nodes in the FE mesh. But rate of convergence is not the main issue in the case of strain localization problems. The real problem is *lack* of convergence of the irreducible formulation. Convergence estimates are usually global. Without additional regularity conditions, local estimates of convergence are expected to be one order smaller. This means that, using linear elements, convergence for the stresses (or strains) cannot be guaranteed in the irreducible formulation. Propitiously, the stabilized mixed formulation can guarantee first order convergence.

Given the intrinsic local nature of the strain localization problem, the discrete solution is largely affected by the local discretization error. In 2D and, obviously, 3D situations local discretization error affects both the pre and post strain localization regimes. This fact, inherent to the discretization process, is probably the major specific challenge in their solution, and it adds to the difficulties associated to the strongly nonlinear nature of the problem. The usual result of these combined difficulties is that, from all the possible localized solutions that the nonlinear discrete model has, the one obtained is mesh-biased and, therefore, apparently unrelated to the continuous case.

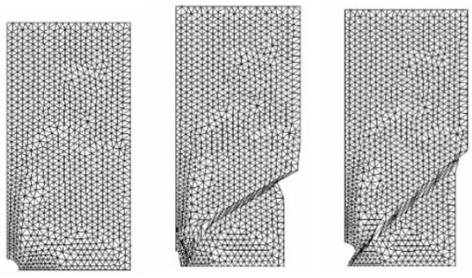
4. NUMERICAL EXAMPLES

The application of the stabilized formulations proposed to solve the problem of strain localization is illustrated below by solving two different benchmark problems. Relative performance of the irreducible displacement formulation and the stabilized mixed formulation is tested. The elements used will be: P1 (linear displacement), P1P1 (either linear displacement/ linear pressure (Mode-II) or linear strain/ linear displacement (Mode-I)). Only low order triangular elements are considered because they are more effective in problems involving sharp displacement and strain gradients. However, the proposed approach is very general.

4.1. 2D Perforated strip: shear band localization

The first example is a plane-strain perforated strip loaded through a rigid platen which sustains an axial central point load. Owing to the double symmetry, only one quarter of the domain (the top right quarter) needs to be discretized. Figure 1a depicts the geometry of the problem and the unstructured mesh used in the analysis: note that the mesh-generator used tends to introduce patches or equilateral triangles with predominant directions at -30º, +30 and 90º.

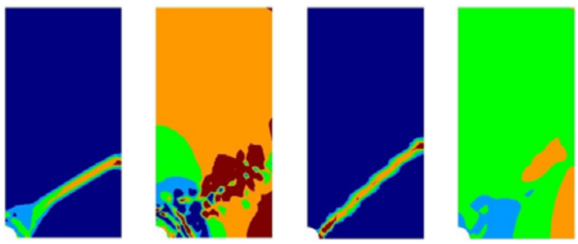
Figure 1b and Figure 1c depict the deformed shape and the collapse mechanism can be appreciated, both for the standard irreducible (3-noded with linear continuous displacement field) and the mixed u/p with OSGS stabilization (3-noded triangles with linear continuous displacement and pressure fields).



(a) (b) (c)

Figure 1. (a) FE mesh; (b) Deformed mesh using P1 elements;
(c) Deformed mesh using P1P1 mixed stabilized formulation.

Figure 2 shows the effective plastic strain and the pressure contour-fills once the plastic flow is fully developed. The differences in the responses are due to the different degrees of success of the formulations in removing the volumetric locking induced by the development of the plastic behaviour. Note how for the irreducible formulation the localization band forms an angle of 30º with the horizontal axis and it is completely influenced by the directional bias of the mesh. Correspondingly, using the stabilized mixed formulation the localization band forms a correct angle of approximately 45º with the horizontal axis and it is virtually free on any directional bias from the mesh. Also, note that the resolution of the shear band is optimal, as it is only one element across.



(a) (b) (c) (d)

Figure 2. (a) Equivalent plastic strain – P1; (b) Pressure – P1;
(c) Equivalent plastic strain – P1P1; (b) Pressure – P1P1

4.2. Rectangular strip under tension: tensile cracking test

The second example is a plane rectangular strip subjected to axial vertical stretching applied by imposing null vertical displacements at the bottom and increasing a uniform vertical displacement at the top. Two symmetrical notches are introduced close to the horizontal axis of symmetry of the strip to perturb the constant strain and stress fields and to ensure uniqueness of the strain localization problem.

This example is selected because it represents a sort of patch test for pure Mode-I fracture assessing the ability of the irreducible and mixed formulation to reproduce these ideal conditions and the dependence of the obtained results with respect the mesh-bias.

The mesh used consists of almost 3,600 rectangular triangles, but the mesh is *slanted* on purpose, so that the predominant directions are −13º, +32º and +90º with the horizontal axis (Fig. 3a). Therefore, the elements in this mesh do not have any of their sides parallel to the expected strain localization band.

When the irreducible formulation is used one damage band starts from each notch at an angle which is spuriously determined by the mesh bias. These two extension bands do not meet at the centre and they only change direction at a very advanced stage of the localization process (see Fig. 3b). Correspondingly, Fig. 3c depicts the deformed shape obtained using the stabilized mixed strain/displacement formulation. The localization band computed despite its strong unfavourable mesh-bias is remarkably correct. Here, the localization band zig-zags through the mesh to reproduce the expected horizontal crack.

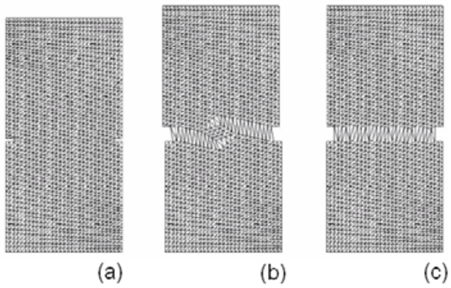


Figure 3. (a) FE mesh; (b) Deformed mesh P1 elements;
(c) Deformed mesh P1P1 mixed stabilized formulation

Figures 4 compare the results obtained with the irreducible and the mixed formulation when the localization band is well developed. Figure 4a and Figure 4c compare the contour-fills of the vertical displacement field, while Fig. 4b and Fig. 4d show the contours of the damage index. The performance and the differences between the irreductible and the mixed formulation are self-evident.

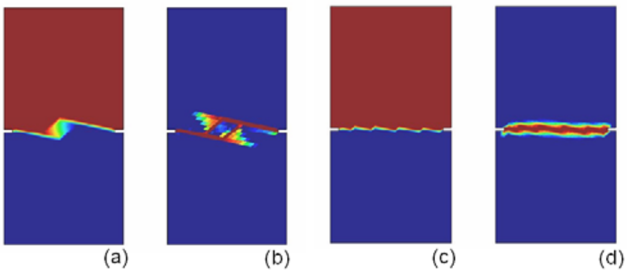


Figure 4. (a) Vertical displacements – P1; (b) Damage index – P1;
(c) Vertical displacements – P1P1; (b) Damage index – P1P1

5. CONCLUSIONS

This paper presents the application of stabilized mixed linear triangular elements to the solution of problems involving the capture of discontinuous solutions in either J2 plasticity or Rankine's type models. On one hand, by using a stabilized formulation which considers equal order interpolation of the displacement and pressure fields it was possible to design an element technology to allow the numerical simulation of shear band localization. On the other hand, tensile cracking failure mechanism has been simulated considering a stabilized mixed strain/displacement element technology. Both formulations exploit the orthogonal sub-grid scales (OSGS) approach. The stabilization scheme is shown to attain control on the pressure and displacement field, completely removing global and local oscillations.

The derived method yields a robust scheme, suitable for engineering applications in 2D and 3D. The numerical examples demonstrate the tremendous advantage of the mixed formulations over the irreducible one to predict correct failure mechanisms with localized patterns of the inelastic deformation, which are practically free from any dependence of the mesh directional bias yielding an improved global response in the softening regime.

Remarkably, the use of auxiliary *tracking techniques is not required*.

6. ACKNOWLEDGEMENTS

Financial support provided by the Spanish Ministry for Education and Science under the SEDUREC project: is gratefully acknowledged.

The invaluable help of Prof. R. Codina is also acknowledged.

7. REFERENCES

Brezzi, F. and Fortin, M., (1991). Mixed and Hybrid Finite Element Methods, Spinger, New York.

Cervera, M. and Chiumenti, M. (2009) Size effect and localization in J2 plasticity. Int. J. Solids Struct, doi:10.1016/j.ijsolstr.2009.04.025.

Bazant, Z. P. and Planas, J. (1998). Fracture and Size Effect in Concrete and Other Quasibrittle Materials. Boca Raton: CRC Press.

de Borst, R. (1991). Simulation of strain localization: a reppraisal of the Cosserat continuum. Engineering Computations, 8, 317-322.

Steinmann, P. and Willam, K. (1992). Localization witin the framework of micropolar elastoplasticy. In Advances in Continuum Mechanics. ed. V. Mannl et al. pp. 296-313. Springer Verlag, Berlin.

Aifantis, E.C. (1984). On the microstructural origin of certain inelastic models. Transactions ASME Journal of Engineering Materials Technology, 106, 326-330.

Vardoulakis, I. and Aifantis, E.C. (1991). A gradient flow theory of plasticity for granular materials. Acta Mechanica, 87, 197-217.

de Borst, R. and Mulhaus, H.B. (1992). Gradient-dependent plasticity: formulation and algorithm aspect. Int. J.. Num. Meths. in Engng., 35, 521-539.

Pamin, J. (1994). Gradient-Dependent Plasticity in Numerical Simulation of Localization Phenomena. Ph. D. Thesis, TU Delft, The Netherlands.

Peerlings, R.H.J., de Borst, R., Brekelmans,W.A.M. and Geers, M.G.D. (1998). Gradient-enhanced damage modelling of concrete failures. Mechanics of Cohesive-Frictional Materials, 4, 339-359.

Pijaudier-Cabot, G. and Bazant, Z.P. (1987). Nonlocal Damage Theory. Journal of Engineering Mechanics, ASCE, 113(10) 1512-1533.

Pijaudier-Cabot, G. and Huerta, A. (1991). Finite Element Analysis of Bifurcation in Nonlocal Strain Softening Solids, Comp. Meth. in Applied Mech. and Engrg., 90, 905-919.

Jirásek, M. (1998). Nonlocal models for damage and fracture: comparison of approaches. Int. J. Solids and Structures, 35, 4133-4145.

de Borst, R. (2001). Some recent issues in computational failure mechanics. Int. J. Num. Meths. in Eng., 52, 63-95.

Bazant, Z. and Jirásek, M. (2002). Nonlocal Integral Formulations of Plasticity and Damage: Survey of Progress. J. of EngineeringMechanics, ASCE, 128, 1119-1149.

Needelman, A., (1987), Material rate dependence and mesh sensitivity in localization problems. Comp. Meth. in Appl. Mech. and Eng., 67, 68-75.

Chiumenti, M., Valverde, Q., Agelet de Saracibar, C. and Cervera, M., (2002). A stabilized formulation for incompressible elasticity using linear displacement and pressure interpolations, Comp. Meth. in Appl. Mech. and Eng., 191, 5253-5264.

Cervera, M., Chiumenti, M., Valverde, Q. and Agelet de Saracibar, C. (2003-a). Mixed Linear/linear Simplicial Elements for Incompressible Elasticity and Plasticity. Comp. Meth. in Appl. Mech. and Eng., 192, 5249-5263.

Cervera, M., Chiumenti, M. and Agelet de Saracibar, C. (2003-b). Softening, localization and stabilization: capture of discontinuous solutions in J2 plasticity. Int. J. for Num. and Anal. Meth. in Geomechanics, 28, 373-393.

Cervera, M., Chiumenti, M. and Agelet de Saracibar, C. (2003-c). Shear band localization via local J2 continuum damage mechanics. Comp. Meth. in Appl. Mech. and Eng., 193, 849-880.

Chiumenti, M., Valverde, Q., Agelet de Saracibar, C. and Cervera, M., (2004). A stabilized formulation for incompressible plasticity using linear triangles and tetrahedra, Int. J. of Plasticity, 20, 1487-1504.

Agelet de Saracibar, C., Chiumenti, M., Valverde, Q. and Cervera, M., (2006). On the orthogonal subgrid scale pressure stabilization of finite deformation J2 plasticity. Comp. Meth. in Appl. Mech. and Eng., 195, 1224-1251.

Cervera, M. Chiumenti M. and R. Codina (2009-Part_I), Mixed Stabilized Finite Element Methods in Nonlinear Solid Mechanics. Part I: Formulation, Comp. Meth. in Appl. Mech. and Eng., Submitted.

Cervera, M. Chiumenti M. and R. Codina (2009-Part_II), Mixed Stabilized Finite Element Methods in Nonlinear Solid Mechanics. Part II: Strain Localization, Comp. Meth. in Appl. Mech. and Eng., Submitted. The author(s) is (are) the only responsible for the printed material included in this paper.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.