Decentralized $H_\infty$ control of systems with information structure constraints

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Abstract—The paper deals with a class of continuous-time state-delayed uncertain systems considering $H_\infty$ control. The main objective is to design static output feedback controllers satisfying three requirements simultaneously: asymptotic stability for the closed-loop system, a minimum effect of the disturbance input on the controlled output, and the obtention of a gain matrix having an arbitrarily preassigned zero-nonzero structure. To solve this problem, a linear matrix inequality (LMI) delay-independent approach is derived. A numerical example illustrates the effectiveness of the proposed method.

Index Terms—Information structure constraints, uncertain time-delay systems, output feedback control, $H_\infty$ control, LMI’s.

1 INTRODUCTION

The analysis and control of complex systems requires frequently to cope with uncertainties, time delays, and constraints in the information that is available for the feedback loops. Each of these features, considered either separately or combined, has led to significant research efforts.

This paper deals with a class of linear continuous-time state-delayed uncertain systems. The parameter uncertainties are assumed to be norm-bounded. The system is to be controlled by a static output feedback control scheme defined by a gain matrix such that an arbitrary zero-nonzero structure is prescribed a priori. Moreover, this gain matrix can be perturbed by an additive uncertainty. The control problem is stated in terms of robust stability and robust $H_\infty$ performance and LMI tools are used to solve it. In this way, the context of this research is twofold: (i) that of control of systems with structural information constraints; and (ii) that of static output feedback robust control of delayed systems.

The structural constraints on the information available to the local controllers that jointly control a complex system can be naturally incorporated into the control design by prescribing a zero-nonzero structure on the feedback gain matrix. A typical case is when the feedback gain is a block-diagonal matrix, which means that only local measurements are available to the local controllers; in this case, the control is totally decentralized [17]. A natural extension is when some measurements are shared among the subsystems according to a specific information pattern as it is the case in the so-called overlapping control. The inclusion principle has given a formal and useful framework to design overlapping controllers [9], [17]. Following this principle, the original system is expanded into a virtual one where the overlapped subsystems appear as disjoint. For the expanded system, a decentralized controller is designed which is finally contracted to be implemented in the initial system. The inclusion principle has been applied to different classes of overlapped systems and problems as illustrated for instance in [1], [2], [3], [4], [5], [6], [15], [16]. Besides these specific scenarios, the more general problem of designing gain matrices with arbitrary zero-nonzero patterns has been studied only in a few papers, like the early works in [10], [13], [19] and the recent approaches presented in [18], [23], [24]. From a practical point of view, the problem is of interest to a variety of engineering areas, especially when considering the increasing availability of communication networks which is continuously extending the possibility of deploying large-scale interconnected systems with more flexible information exchange and feedback control structures. Examples can be considered in traffic networks, vehicle formations, electrical power systems, water distribution and irrigation channels, environmental systems and large flexible structures in aerospace and civil engineering.

There exists an extensive literature dealing with delayed systems with uncertainties. Specifically, the combination of $H_\infty$ control theory and the computational LMI’s tools has produced a powerful framework for control design [7], [11], [12], [14], [20], [21]. The objective of this paper is to design static output feedback robust controllers for the considered class of linear state-delayed uncertain system, satisfying three requirements simultaneously: (i) to provide closed-loop asymptotic stability; (ii) to guarantee a minimum disturbance attenuation in the sense of $H_\infty$ control; and (iii) to derive a gain matrix with a priori prescribed zero-nonzero structure. To the best of our knowledge, this overall problem has not been solved in the literature. The solution is provided in two steps. First, we develop an LMI procedure guaranteeing

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the two first requirements; in the second step, new variables are introduced in the LMI which allow us to obtain a gain matrix with the prescribed zero-nonzero pattern. As a basis of our study, we have considered a class of continuous-time delayed uncertain systems under stability and $H_{\infty}$ robustness objectives. Of course, other classes of systems and control specifications could be considered following an analogous strategy.

The paper is organized as follows. Section 2 presents the problem formulation. It is divided in three parts, corresponding to the system model, the control objectives and the control design. Section 3 presents the problem formulation. Section 4 is devoted to the problem formulation. It is divided in three parts, corresponding to the system model, the control objectives and the control design. Section 5. Finally, Section 6 concludes the paper.

Notation. Throughout this paper, the 2-norm for $w: [0, \infty) \rightarrow \mathbb{R}^n$ belonging to the space $L_2^\infty [0, \infty)$ is noted as $\|w(t)\|_2 = \sqrt{\int_0^\infty w^T(t)w(t)dt}$. Moreover, $\|W\| = \sigma_{\text{max}}(W)$ denotes the matrix norm of $W$, the largest singular value of $W$. The notation $P > 0$ ($P \geq 0$, resp.) for $P \in \mathbb{R}^{n \times n}$ means that the matrix $P$ is real, symmetric and positive-definite (positive-semidefinite, resp.). Analogously, $P < 0$ ($P \leq 0$, resp.) means that $P$ is a symmetric negative-definite (negative-semidefinite) matrix.

2 PROBLEM FORMULATION

2.1 System Model

Consider a class of linear continuous-time delayed uncertain systems described by the state equation

$$
\dot{x}(t) = \tilde{A}(t)x(t) + \tilde{B}(t)u(t) + A_d(t)x(t-d) + B_dw(t),
$$

$$
y(t) = Cx(t) + Du(t),
$$

$$
z(t) = \phi(t), \quad -d \leq t \leq 0,
$$

where

$$
\tilde{A}(t) = A + \Delta A(t),
$$

$$
\tilde{B}(t) = B + \Delta B(t),
$$

$$
A_d(t) = A_d + \Delta A_d(t).
$$

The vector $x(t) \in \mathbb{R}^n$ corresponds to the state, $u(t) \in \mathbb{R}^m$ is the input control, $d > 0$ the time delay, $w(t) \in L_2^\infty [0, \infty)$ the disturbance input, $y(t) \in \mathbb{R}^q$ the output, $z(t) \in \mathbb{R}^l$ the controlled output and $\phi(t)$ is a given continuous initial function. $A, B, A_d, B_d, C, D$ are known, real and constant matrices of appropriate dimensions. $\Delta A(t), \Delta B(t)$ and $\Delta A_d(t)$ are real-valued matrices of uncertain parameters. Norm-bounded time-varying uncertainties are supposed in the form

$$
\Delta A(t) = H_{A} F_{A}(t) E_{A},
$$

$$
\Delta B(t) = H_{B} F_{B}(t) E_{B},
$$

$$
\Delta A_d(t) = H_{A_d} F_{A_d}(t) E_{A_d},
$$

where $H_{A}, E_{A}, H_{B}, E_{B}, H_{A_d}, E_{A_d}$ are known real constant matrices of appropriate dimensions and $F_{A}, F_{B}, F_{A_d}$ are unknown real time-varying matrices with Lebesgue measurable elements satisfying $F_{i}^T(t)f_{i}(t) \leq I$, for $i = A, B, d$.

Consider an output feedback controller in the form

$$
u(t) = [K + \Delta K(t)]y(t) = \tilde{K}(t)y(t)
$$

for the system (1), where $K \in \mathbb{R}^{n \times l}$. Suppose that the uncertain gain matrix $\Delta K(t)$ satisfies

$$
\Delta K(t) = H_{K} F_{K}(t) E_{K},
$$

where $H_{K}, E_{K}$ are known constant matrices and $F_{K}(t)$ is an unknown real time-varying matrix with Lebesgue measurable elements such that $F_{i}^T(t)f_{i}(t) \leq I$. The resulting closed-loop system taking into account (3) has the form

$$
S_{C}: \dot{x}(t) = [A + \Delta A(t) + [BK + \Delta B(t)K + B\Delta K(t) + \Delta B(t)\Delta K(t)]C_y]x(t) + [A_d + \Delta A_d(t)]x(t-d) + B_1w(t),
$$

$$
z(t) = [C + D[K + \Delta K(t)]C_y]x(t).
$$

2.2 Control Objectives

The main objective of the paper is to design an output feedback controller as given in (3) satisfying three requirements:

(i) the closed-loop system (5) is asymptotically stable when $w(t) = 0$,

(ii) it guarantees the disturbance attenuation of the closed-loop system from $w(t)$ to $z(t)$, i.e.

$$
\|z(t)\|_2 \leq \gamma \|w(t)\|_2, \quad \gamma > 0,
$$

for all nonzero $w(t) \in L_2^\infty [0, \infty)$ and zero initial conditions, paying special attention to the reduction of $\gamma$, and

(iii) the gain matrix $K$ has a predetermined zero-nonzero structure.

2.3 Control Design

The objectives (i) and (ii) can be reached by means of a linear matrix inequality approach. Related to the objective (iii), two problems arise: firstly, the gain matrix $K$ cannot be isolated from the equation $WX^{-1} = KC_y$, where $W$ and $X$ will be unknown matrices involved in the LMI. Secondly, a desired zero-nonzero structure cannot be imposed on $K$. These two problems could be simultaneously overcome by using appropriate transformations in the LMI.

3 STABILITY $H_{\infty}$ PERFORMANCE ANALYSIS

3.1 Asymptotic Stability

Having the previous ideas in mind, and in order to achieve the control objectives (i), (ii), and (iii), we present a constructive process starting with a Lyapunov function in the form

$$
V(x, t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(\sigma)[I + E_d^T E_d]x(\sigma) d\sigma,
$$
where \( P > 0 \) and \([I + E_d^T E_d] > 0\) are \( n \times n \) dimensional matrices. For simplicity, denoting \( x = x(t), \ x_d = x(t - d)\), and \( w = w(t)\), the time derivative of \( V(x, t) \) along the trajectory of the closed-loop system (5), is given by

\[
\dot{V}(x, t) = x^T P \dot{x} + x^T P x + x^T R x - x_d^T R x_d = \]
\[
= x^T P [A + B K C_y + H_n F_n(t) E_n K C_y] + \]
\[
+ [A + B K C_y + H_n F_n(t) E_n K C_y] x + \]
\[
+ 2 \epsilon_x^2 T P \left[ H_n F_n(t) E_n K_c + B H_r K_f(t) E_n K_c \right] + \]
\[
+ H_n F_n(t) E_n K_c + 2 \epsilon_x^2 T P \left[ I + E_d^T P \right] x_d + 2 \epsilon_x^2 T P B \dot{w} - \]
\[
x_d^T \left[ I + E_d^T P \right] x_d. \quad (6)
\]

Consider the following useful inequalities:

\( P = \sum \epsilon_i I \geq 0 \) for all \( \epsilon_i > 0 \).

\( \psi(x, t) = \sum \epsilon_i \| y(t) \|^2 \geq 0 \) for all \( \epsilon_i > 0 \).

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If the inequality (11) holds, then the asymptotic stability of the closed-loop system \( S_c \) given in (5) is guaranteed. In this case, the control objective (i) is achieved.

This result is stated in the following proposition.

**Proposition 1:** Consider the closed-loop system \( S_c \) given in (5). Suppose that \( w(t) = 0 \) for all \( t \). If there exist a symmetric positive-definite matrix \( P \), scalars \( \epsilon_i > 0 \), and a gain matrix \( K \) satisfying \( \parallel S^* + P A_d A_d^T P \parallel < 0 \), then the closed-loop system (5) is asymptotically stable.

**3.2** \( H_{\infty} \) Performance: An LMI Approach

In this paper, we consider the problem of designing static output feedback controllers to reduce (minimize) the effect of the disturbance input on the controlled output. Let us introduce the following index:

\[
J = \int_0^\infty [\dot{z}(t) - \gamma^2 \dot{w}(t) w(t)] dt = \]
\[
= \parallel z(t) \parallel^2_2 - \gamma^2 \parallel w(t) \parallel^2_2, \] \( \quad \) under zero initial conditions. Then,

\[
\parallel z(t) \parallel_2 \leq \gamma \parallel w(t) \parallel, \Leftrightarrow \parallel z(t) \parallel^2_2 - \gamma^2 \parallel w(t) \parallel^2_2 \leq 0 \Leftrightarrow J \leq 0.
\]

For any \( w(t) \neq 0 \) such that \( w(t) \in L_2^2(0, \infty) \), we get

\[
J \leq \int_0^\infty [\dot{z}(t) - \gamma^2 \dot{w}(t) w(t) + V(x, t)] dt. \quad (12)
\]

Substituting \( z(t) \) and \( \dot{V}(x, t) \) given in (5) and (8), respectively, in (12) and taking into account the inequalities

\[
[DH_K F_K(t) E_n K_c]_y \parallel C + D K C_y\parallel \]
\[
\leq \epsilon_2 [DH_K F_K(t) E_n K_c]_y \parallel C + D K C_y\parallel \]
\[
\leq \epsilon_2 [C + D K C_y]^T \parallel C + D K C_y\parallel, \forall \epsilon_2 > 0,
\]

\[
[DH_K F_K(t) E_n K_c]_y \parallel C + D K C_y\parallel \leq \epsilon_2 [C + D K C_y]^T \parallel C + D K C_y\parallel, \forall \epsilon_2 > 0,
\]

\[
whence \quad J \leq \int_0^\infty [\dot{z}(t) - \gamma^2 \dot{w}(t) w(t) + V(x, t)] dt.
\]

It is well known that the closed-loop system (5) is asymptotically stable if \( \dot{V}(x, t) < 0 \). Then, assuming that \( w(t) = 0 \) for all \( t \), we impose

\[
\dot{V}(x, t) \leq \left[ x_{\dot{x}} \right]^T \left[ S^* \right. \left. (A_d^T P - I) \right] \left[ x_{\dot{x}} \right] = \leq 0, \quad (10)
\]
with
\[ G = S + \alpha_2(1 + \epsilon_5)[E_K C_y]T[E_K C_y] + (1 + \epsilon_1)^2[C + DK C_y]T[C + DK C_y]. \]

By the Schur complement, \( \Psi \leq 0 \) is equivalent to \([G + P A d A_d^T P] < 0 \), which implies \( J < 0 \), and consequently \( \| z(t) \| < \gamma \| w(t) \| \). Moreover, \([G + P A d A_d^T P] < 0 \) and, therefore, \([G + P A d A_d^T P] < 0 \) ensures the asymptotic stability of the closed-loop system (5) with \( H_\infty \) bounded norm \( \gamma \).

In order to transform the condition \([G + P A d A_d^T P] < 0 \) in terms of LMI's, we can note that it is equivalent to \([P^{-1} G P^{-1} + A d A_d^T] < 0 \), for any matrix \( P > 0 \). Thus,
\[ \begin{align*}
[\epsilon_1 - &\epsilon_3 - \epsilon_1] H_B H_B^T + \epsilon_1 H_A H_A^T + \\
&+ \epsilon_2^{-1} [B H K][B H K]^T + H_d H_d^T + \gamma^{-2} B_1 B_1^T + P^{-1} P^{-1} + \\
&+ \epsilon_1 [E_B K C_y]^T[E_B K C_y] P^{-1} + [B K C_y P^{-1}] + \\
&+ [B K C_y P^{-1}] + P^{-1} E_d E_d P^{-1} + P^{-1} E_d E_d P^{-1} + \\
&+ [+ (1 + \epsilon_1)] P^{-1}[C + DK C_y]^T[C + DK C_y] P^{-1} + A d A_d^T < 0.
\end{align*} \]

Using the following changes of variables
\[ X = P^{-1}, \quad W = K C_y X, \quad W_2 = C X + D W, \]
we obtain
\[ \begin{align*}
[P^{-1} G P^{-1} + A d A_d^T] &< 0 \iff \\
\Rightarrow &AX + X A^T + (\epsilon_1 - \epsilon_3) H_B H_B^T + \epsilon_1 H_A H_A^T + \\
&+ \epsilon_2^{-1} [B H K][B H K]^T + H_d H_d^T + \gamma^{-2} B_1 B_1^T + X X + \\
&+ \epsilon_1 [E_B W]^T[E_B W] + [B W] + [B W]^T + X E_d E_d X + \\
&+ \epsilon X E_d E_d X + (\epsilon_2 + \epsilon_3) [E_K C_y]^T X [E_K C_y]^T X + \\
&+ (1 + \epsilon_1) W_d^T W_d + A d A_d^T < 0 \iff \\
\Rightarrow \quad &W_1 + X \begin{bmatrix}
\epsilon_a & E_d \\
E_d & E_K C_y
\end{bmatrix} \begin{bmatrix}
\epsilon_a & E_d \\
E_d & E_K C_y
\end{bmatrix}^T X + \\
&+ \epsilon_1 [E_B W]^T[E_B W] + (1 + \epsilon_1) W_d^T W_d < 0,
\end{align*} \]

where
\[ \begin{align*}
W_1 = &AX + X A^T + (\epsilon_1 - \epsilon_3) H_B H_B^T + \epsilon_1 H_A H_A^T + \\
&+ \epsilon_2^{-1} [B H K][B H K]^T + H_d H_d^T + \gamma^{-2} B_1 B_1^T + \\
&+ [B W] + [B W]^T + A d A_d^T.
\end{align*} \]

Then, \( u(t) = [K + \Delta K(t)] y(t) \) is an output feedback controller satisfying (4) such that the resulting closed-loop system (5) is asymptotically stable with \( H_\infty \) bounded norm \( \gamma \). Moreover, \( W X^{-1} = K C_y \).

Theorem 1, with the change of variable \( \eta = \gamma^{-2} \), can be transformed into the following convex optimization problem:

\[ \begin{align*}
\text{Problem 1:} \\
\max & \eta \\
\text{s.t.} \ & \eta > 0, \beta > 0, \beta_1 > 0, X > 0, \text{ and the LMI in Fig. 1}.
\end{align*} \]

Remark 1: In Theorem 1, \( \gamma > 0 \) is supposed to be given a priori. If for such value \( \gamma \) the LMI in Fig. 1 is feasible, then the above optimization Problem 1 is also solvable, hence a maximum \( \eta > 0 \) exists. With that, the control objective (ii) is achieved.

4 Design of Structured Gain Matrices

From Theorem 1, when the linear matrix inequality given in Fig. 1 is satisfied, it is possible to obtain matrices \( X \) and \( W \) so that \( W X^{-1} = K C_y \). However, in order to apply a state output feedback controller \( u(t) = [K + \Delta K(t)] y(t) \) in the system \( S \), it is necessary to isolate the gain matrix \( K \). Moreover, we are interested in a particular zero-nonzero pattern on the gain matrix \( K \), [22], [24].

With these ideas in mind, consider the following transformations:
\[ \begin{align*}
X &= \alpha X_0 + Q X \epsilon Q^T, \\
W &= W_c y X_0,
\end{align*} \]

where \( \alpha \) is an unknown scalar, \( X_0 \) is a constant symmetric matrix selected a priori, \( X_0 \) is an unknown symmetric \((n-l)\times(n-l)\) matrix and \( W_c \) is an unknown \((m\times l)\) matrix. By assuming that rank \((C y) = l \), it is always possible to choose a constant \(n \times (n-l)\) dimensional matrix \( Q \) with full rank \((n-l)\) such that \( Q^T C_y = 0 \). Further, we will impose \( X = [\alpha X_0 + Q X \epsilon Q^T] > 0 \) in the LMI. Now, it can be observed that
\[ X C_y = \alpha X_0 C_y + Q X \epsilon Q^T C_y = \alpha X_0 C_y, \]

which implies
\[ \alpha^{-1} C_y = C_y X_0^{-1}. \]
From (14)-(16), we have

$$WX^{-1} = W_cC_yX_0X^{-1} = \alpha^{-1}W_cC_y.$$  

Consider the Theorem 1 with $X_0$ and the new variables $\alpha$, $X_c$ and $W_c$ given in (14). If the LMI given in Fig. 1 is feasible for these variables, then $WX^{-1} = KC_y$ implies $\alpha^{-1}W_cC_y \equiv KC_y$. Thus, a matrix $K$ satisfying this equality can be chosen in the form

$$K = \alpha^{-1}W_c.$$  

**Remark 2:** Theorem 1 gives not explicit conditions on the properties of matrix $X_0$, and only its symmetry is required. However, it is worth noting that the selection of the matrix $X_0$ is crucial for the feasibility of the LMI given in Fig. 1. Consequently, the number of variables in the LMI is twofold: first, the gain matrix $X$ required. However, it is worth noting that the selection of the matrix $X_0$ is crucial for the feasibility of the LMI given in Fig. 1. Consequently, the number of variables in the LMI given by Fig. 1 can decrease by using these transformations. This decrease may be specially significant if rank($C_y$) is high. In this case, the corresponding LMI can be infeasible. However, when the LMI has a feasible solution, the benefit of the proposed transformations (14) is twofold: first, the gain matrix $K$ can be isolated; and second, the zero-nonzero structure on the matrix $K$ can be specified a priori by imposing a desired structure on $W_c$ in the corresponding LMI. This solves the objective (iii).

### 5 Numerical Example

Consider the problem (1) satisfying (2) with the following initial data:

$$A = \begin{bmatrix} -5 & -1 & -2 & 0 & -1.5 \\ -3 & 0 & 2.5 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 4 & 0 \\ 1 & 0 & -5 & 0 \\ -3 & 0 & 2.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1.5 & 0 & 0.3 & 0.3 \\ 0.3 & 0 & 0.3 & 0.3 \\ 0 & 1.2 & 0.3 & 0.3 \\ 0 & 0 & 0.6 & 0.6 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0.1 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.1 & 0.1 \\ 0 & 0.1 & 0.2 & 0 \\ 0 & 0.1 & 0.1 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 & 0 \\ 0.4 & 0.1 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$  

The corresponding minimum disturbance attenuation value is $\gamma_0 = 0.9479$. The following numerical example is introduced to illustrate the procedure presented in Remark 2.

**Case a)** Impose a structure on the matrix $K$ given by $K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. According to the procedure offered in Section 4, the structure of the matrix $W_c$ has the form

$$W_c = \begin{bmatrix} w_{c;11} & 0 & 0 \\ 0 & w_{c;22} & w_{c;23} \\ 0 & 0 & w_{c;33} \end{bmatrix},$$

and the resulting numerical gain matrix is

$$K = \begin{bmatrix} 0.5342 & -0.2922 & -0.4918 \\ 0 & 0 & 0.6207 \\ 0 & 0 & -0.1502 \end{bmatrix}.$$  

In this case, the minimum disturbance attenuation value is $\gamma_0 = 0.9479$.

**Case b)** Consider the objective of obtaining a diagonal gain matrix $K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then, we impose on $W_c$ the same diagonal structure:

$$W_c = \begin{bmatrix} w_{c;11} & 0 & 0 \\ 0 & w_{c;22} & 0 \\ 0 & 0 & w_{c;33} \end{bmatrix}.$$  

Computing the gain matrix $K$, we get

$$K = \begin{bmatrix} 0.5323 & 0.0069 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.1503 \end{bmatrix}.$$  

The corresponding minimum disturbance attenuation value is $\gamma_0 = 0.9479$, the same value than $\gamma_0$.
$I_5$, the following results for both case $a$) and case $b$) are obtained:

Case $a$)

$$K = \begin{bmatrix} 0.3582 & 0 & 0 \\ -0.2888 & -0.0843 & -0.1641 \\ 0 & 0 & -0.1493 \end{bmatrix}.$$  

In this case, the minimum disturbance attenuation value is $\gamma_a=0.3287$.

Case $b$)

$$K = \begin{bmatrix} 0.3729 & 0 & 0 \\ 0 & -0.1907 & 0 \\ 0 & 0 & -0.1493 \end{bmatrix}.$$  

The corresponding minimum disturbance attenuation value is $\gamma_b=0.3288$. We can observe that, in this example, if $X_5=I_5$ is selected, then the minimum disturbance attenuation values $\gamma_a$ and $\gamma_b$ are better.

All computations have been performed using the Matlab LMI Control Toolbox [8].

6 CONCLUSIONS

The paper has dealt with a class of continuous-time state-delayed uncertain systems. The main goal has been the design of static output feedback controllers satisfying three conditions: asymptotic stability, minimum effect of the disturbance input on the controlled output, and a gain matrix having a preassigned zero-nonzero structure. An LMI approach has been developed to solve this problem. An illustrative example has been supplied to show the advantages of this approach.

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