HYBRID MODELING AND FRACTIONAL CONTROL OF A SCKAFO ORTHOSIS FOR GAIT ASSISTANCE

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ABSTRACT

SCKAFO, stance-control knee-ankle-foot orthosis, is a type of orthosis that permits free knee motion during swing while resisting knee flexion during stance, supporting thereby the limb during weight bearing. This orthosis specially assists patients who have incomplete spinal cord injury and allows them to walk with the aid of canes or crutches, maintaining a proper gait. In this paper, based on the human walking biomechanics, the SCKAFO hybrid modeling is proposed, which consists of eight different stages whose evolution is given by means of four planar sensors on each foot. In the model, it is considered that the patients can move their hip but not their knee that will be controlled using a DC motor. Two fractional order controllers are designed, following decision based control techniques, to control the knee angle. Simulation results are given in order to demonstrate the efficiency of the system performance.

INTRODUCTION

Spinal cord injuries cause paralysis of the lower limb in a major or minor degree based on the height of injury on the spinal cord. Incomplete spinal cord injured subjects can perform a high metabolic cost and low speed walking aided by crutches, bars or similar. In the literature, several devices have been proposed for assisting human gait to improve their walk. A survey on the current state of the art of such devices can be found in [1]. Indeed, the particular characteristics of the orthosis are based on the type of injuries, which will define a specific design process. For example, a biologic-based design is described in [2].

The stance control of the orthosis is a fundamental part of the design of the orthosis (refer to [3] for a review of this issue). Control tasks are necessary to identify gait cycle and establish locking and actuation stages. The design of the control will define the locomotor adaptation as is exposed in [4]. Fractional order controllers have received a considerable attention in the last years both from an academic and industrial point of view [7–11]. In fact, such controllers provide more flexibility during the controller design than the classic ones.

The aim of this paper is twofold. On the one hand, a hybrid model of the system operative is proposed based on the use of planar sensors located in the foot. In addition, the orthosis separates mechanical actuation and locking system on the knee, which allows reducing actuation requirements, and, consequently, the weight of the device. On the other hand, a fractional order controller is designed to control the knee angle, whose dynamics is based on the motor and load (orthosis) model. In addition, regarding to human walking biomechanics, a linear reference is proposed for each stage.

The rest of the paper is organized as follows. Biomechanics of walking is presented in the next section. Then, the reference for the knee angle is introduced. After illustration of mechanical design of the SCKAFO the control design is presented. Finally, the paper will be concluded with some remarks.

BIOMECHANICS OF WALKING

The gait cycle is defined as the time interval between two successive occurrences of one of the repetitive events of walk-
ing. Although any event could be chosen to define the gait cycle, it is generally convenient to use the instant at which one foot contacts the ground (‘initial contact’). If it is decided to start with initial contact of the right foot, then the cycle will continue until the right foot contacts the ground again. The left foot, of course, goes through exactly the same series of events as the right, but displaced in time by half a cycle. There seven major events (Initial contact, Opposite toe off, Heel rise, Opposite initial contact, Toe off, Feet adjacent, Tibia vertical) that subdivide the gait cycle into seven periods, four of which occur in the stance phase, when the foot is on the ground, and three in the swing phase, when the foot is moving forward through the air. The stance phase is also subdivided into:

1. Loading response,
2. Mid-stance,
3. Terminal stance,
4. Pre-swing.

The swing phase lasts from toe off to the next initial contact. It is subdivided into:

1. Initial swing,
2. Mid-swing,
3. Terminal swing.

The duration of a complete gait cycle is known as the cycle time, which is divided into stance time and swing time. In the stance phase the controller will lock the motor not to move and let the patient control his knees. And in the swing phase the controller will adopt himself to follow the reference. Fig. 1 shows when the controller should lock the motor (stance) and when the controller has to follow the reference (swing). In the double stance, both knees are locked.

Likewise, Fig. 2 shows a simplified diagram of human walking gait, with the terms that will be used.

Fig. 3 shows the sagittal plane angles at the hip and knee joint for the right leg showing joint angle for hip and knee extension motions during level-ground walking. These data are achieved for the right leg in a gate cycle [6].

**HYBRID DYNAMICS OF ASSISTED GAIT**

To control an orthosis to introduce an applicable reference is necessary. Therefore, four planar sensors will be used in the orthosis to find periods and then a linear reference will be introduced for each period. Planar sensors are ON/OFF sensors where 1 means that the leg is in touch with the ground and 0 refers to the contrary situation. Using the planar sensors and based on the human walking biomechanics (see Fig. 2) we will see eight different parts in each gate cycle of walking. The activation of the sensors in each part is shown in Table 1. In this table, P1 to P8 represent part 1 to 8 and PS1 to PS4 refer to each planar sensor.
Concerning those eight parts, seven periods for both legs can be achieved. Table 2 shows the relation of both legs in each part.

### TABLE 1. Planar sensor configuration in each part

<table>
<thead>
<tr>
<th></th>
<th>PS1</th>
<th>PS2</th>
<th>PS3</th>
<th>PS4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>P2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td></td>
<td>0</td>
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</tr>
<tr>
<td>P4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>P5</td>
<td>0</td>
<td>0</td>
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<td>1</td>
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<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>P8</td>
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<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As mentioned before a linear reference is introduced for each part which can be formulated as follows:

\[
\theta_{ri} = a_i(t_i - L_i) + \theta_{ri}, \quad i = 1, 2, ..., 8
\]  

where \( \theta_{ri} \in [\theta_{ri}, \bar{\theta}_{ri}) \), \( t_i \in [L_i, \bar{L}_i) \) and \( a_i = \frac{\bar{\theta}_{ri} - \theta_{ri}}{\bar{L}_i - L_i} \) are the reference value, a interval in the gate cycle and slope of the reference line, respectively. In addition, \( \theta_{ri}, \bar{\theta}_{ri} \) and \( L_i, \bar{L}_i \) are lower bounds and upper bounds of reference value and percentage of gate cycle which can be easily found using the human biomechanics of walking. Figs. 4 and 5 show the real values and proposed reference value for each part in a gate cycle. However, more exact reference can be achieved using curve fitting techniques but for simplicity we choose this simple reference. In addition, using simulation techniques it can be easily shown that this reference is applicable enough for knee movement. In this paper the obtained reference will be used in the simulation and the blocking phase will not be applied. Motor blocking will be considered in the future experimental platform.

### TABLE 2. Relation of right and left legs based on planar sensors

<table>
<thead>
<tr>
<th>Part</th>
<th>Perc. of gate cycle</th>
<th>Right leg’s period</th>
<th>Left leg’s period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-10</td>
<td>Loading response</td>
<td>Pre swing</td>
</tr>
<tr>
<td>2</td>
<td>10-23</td>
<td>Mid stance</td>
<td>Initial swing</td>
</tr>
<tr>
<td>3</td>
<td>23-35</td>
<td>Mid stance</td>
<td>Mid swing</td>
</tr>
<tr>
<td>4</td>
<td>35-50</td>
<td>Terminal stance</td>
<td>Terminal swing</td>
</tr>
<tr>
<td>5</td>
<td>50-60</td>
<td>Pre swing</td>
<td>Loading response</td>
</tr>
<tr>
<td>6</td>
<td>60-73</td>
<td>Initial swing</td>
<td>Mid stance</td>
</tr>
<tr>
<td>7</td>
<td>73-85</td>
<td>Mid swing</td>
<td>Mid stance</td>
</tr>
<tr>
<td>8</td>
<td>85-100</td>
<td>Terminal swing</td>
<td>Terminal stance</td>
</tr>
</tbody>
</table>

### FIGURE 4. Reference knee angle for the right leg. LR= Loading response; MS= Mid Stance; TS=Terminal Stance; PS= Pre Swing; IS= Initial Swing; MS= Mid Swing; TS= Terminal Swing.

### MECHANICAL DESIGN OF THE ORTHOSIS

In order to reduce the design parameters, this orthosis is considered specifically for patients with spinal cord injuries described by ASIA (American Spinal Injury Association) with order C and D. Those injuries decrease functionality, which means that patients with this kind of injury can maintain a pathological gait with low speed and high metabolic cost with the aid of crutches. For our group of interest, we must assist gait for patients who do not have muscular control in rectus femoris, so the priority movement to assist is extension. In that sense, the maximum torque in this phase will define the characteristics of the actuator and locking system. In the extension of the leg, knee must support an extension torque of 0.38 Nm/kg (approximately 30.4 Nm for a person of 80 kg) [5]. This load is supported by the locking system. Movement in swing phase must be assisted too,
but due to the fact that patients have control of the hip muscles, which also take part in flexion of the knee, the necessary torque is minor, about 0.18 Nm/kg (approximately 14.4 Nm for a person of 80 kg). Thereby, the actuator must provide this torque value.

According to the medical specifications for incomplete spinal cord injured patients, the proposed device must provide assistance in the swing phase and also must be locked in stance phase in safe conditions. This SCKAFO also has to avoid plantar flexion in swing phase due to the gait associated with this kind of injuries. Hence, the design of the orthosis is divided into two stages: on the one hand, the design of the ankle module and, on the other, the corresponding to the knee module. Next, both design will be described.

**Ankle module**

Ankle module (see Fig. 6) is based on a commercial passive orthosis AFO, which was modified to adapt an encoder on the articulation to measure the angular variation. This encoder is an Avago AEDA 3300 TAT model, with a resolution of 4096 CPR, whose rotation axis is joined to the axis of the articulation, and the mount is fixed to the stirrup, so the rotation recorded is foot over shank. This part of the orthosis is constituted by two aluminium bars, one on each side of the leg, making the function of support, which are adjusted to the leg by velcro tapes. A Klenzack device on each side of the ankle avoid dorsiflexion in swing phase. A metallic stirrup with an insole constitutes foot support which is inserted on the shoes. This orthosis can regulate the anti dorsiflexor torque by the adjustment of the spring which has a screw that controls its longitudinal dimension. This module also has a blocking mechanism to restrict flexion movement to a maximum of 20 deg. Planar sensors are installed over the insole in order to control the motor unit in the knee module. Those plantar sensors are MotionLab switches MA153.

**Knee Module**

The mentioned aluminum bars continue to the knee articulation, where they join with the knee articulation. Another pair of bars covers the thigh, giving a new supporting point. A velcro tape fixes the orthosis to the thigh. The knee module is composed by a commercial articulation on the external side and a commercial blocking system in the internal side, as can be observed in Fig. 7. The articulation is modified to adjust the actuation system which consists of a DC motor with a gearhead to obtain the proper torque in each moment of the gait cycle. The locking system provides the necessary lock on the knee during the stance phase. In this orthosis, we use the NEURO TRONIC W, which provides the adequate lock on stance phase in safe conditions. On the external side, a commercial articulation is used, which was also modified to include a DC motor: the EC 45 flat from Maxon Motor. This motor is associated with a plantery gearhead also from Maxon Motor, the GP 42 C, and the corresponding encoder to record the angular variation between the thigh and the shank. This group has a small weigh, about 580 g, and provides a torque of 10.88 Nm in continuous at 3.54 A with a velocity of 33.65 rpm at 18 V. In certain circumstances, the torque can be increased to reach the necessary 14.4 Nm mentioned in the specifications.

**CONTROLLER DESIGN**

Apart from the mechanical design, control of the orthosis is in interest of this paper. Two fractional order controller ($I^\alpha$ (PD) and PI$^\lambda$) are considered to control the knee angle. Both controller are compared for the same specification. The complete controlled system block is shown in Fig. 8. As can be observed, the output of the planar sensors together with left and right knee
angles ($\theta_{hl}$ and $\theta_{hr}$) are considered as a feedback to the controller. The aim is to introduce a proper input signal in order to track the reference in each period. The decision based controller will recognize which part of the movement is active and send the proper reference to the controller. Besides, introducing the reference value for the decision based controller will decide if the controller should be active or not. As it can be seen from Fig. 9, decision based controller will send a command to lock the motor when it is in stance phase and also activate the controller and motor to follow the calculated reference value when the movement is in swing phase. In normal conditions, in stance phase the patient injects more energy in the knees for resisting weigh-bearing so, in that way, the motor needs more energy to control the orthosis. Therefore, in this case in order to overcome this problem, the motor will be blocked so as to keep the knee angle constant. Hence, the orthosis dynamics in stance phase can be represented as:

$$\dot{\theta}_h = g(\theta_h, \theta_k),$$
$$\theta_k = \text{Const.} \quad (2)$$

And in the swing phase the dynamics of the system can be formulated as:

$$\dot{\theta}_h = g(\theta_h, \theta_k),$$
$$\dot{\theta}_k = f(\theta_h, \theta_k) + u_i (\theta_k - \theta_{kr}) \quad (3)$$

where $\theta_{kr}$ is the reference value in each period and $i$ represents each period. $g(.)$ and $f(.)$ are functions which depend on the orthosis. In order to design a controller, a DC motor model is used together with a load which is the mechanical part of orthosis (see Fig. 10). As mentioned before, in this model it is considered that the hip can be moved by the patient and the motor will control the movement of the knee.
FIGURE 10. Mechanical Orthosis model

can be represented as follows:

\[
\dot{x} = Ax + BV_{in}
\]

\[
A = \begin{bmatrix}
-\frac{R_m}{L_m} & \frac{K_b}{J_m} & 0 & 0 & 0 \\
\frac{L_m}{J_m} & \frac{b_m}{J_m} & -\frac{K}{J_m} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & \frac{b_o}{J_o} & \frac{K}{J_o} & \frac{b_o}{J_o} & \frac{b}{J_o} \\
0 & 0 & 0 & 1 & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
1/L_m & 0 & 0 & 0
\end{bmatrix}^T,
\]

where \(x = [i, \theta_m, \dot{\theta}_m, \dot{\theta}_o, \theta_o]\) are the states which \(\dot{\theta}_m\) and \(\theta_o\) are the motor and orthosis angle and \(i\) is the armature current. \(L_m\) and \(R_m\) represent the electric inductance and resistance and \(K_i\) and \(K_b\) are torque and back emf constants. \(J_m\) and \(J_o\) are the moments of inertia of the rotor and the orthosis. \(b_m\) and \(b_o\) are the damping ratios of the mechanical system for motor and orthosis link, respectively. \(K\) and \(b\) are the spring constant and damping for connection of rotor/orthosis link, respectively.

The parameters of the real motor and the orthosis are given in Table 3.

Using Table 3, the model can be easily reduced as:

\[
P(s) = \frac{\theta_o}{V_{in}} = \frac{3.58}{s(0.01s + 1)}.
\]  

### I\(^\alpha\)(PD) controller for ideal open loop

In order to control the orthosis, a fractional order controller i.e. I\(^\alpha\)(PD) will be designed. This controller will be designed based on a closed loop reference model expressing the dynamical and robust performances.

An ideal open loop transfer function in the form of a fractional integrator is considered here, with the form:

\[
P_{ref}(s) = \frac{\omega_{ref}}{s^{\alpha+1}} = \frac{1}{\tau_{ref} s^{\alpha}} , 0 < \alpha < 1.
\]  

Therefore, the relation between the reference model of the system and controlled open loop system can be,

\[
P_{ref}(s) = C(s) P(s)
\]

Using system dynamics \(P(s)\) and ideal reference model \(P_{ref}(s)\), then the controller \(C(s)\) can be easily achieved as,

\[
C(s) = \frac{(\tau s + 1)}{G \tau_{ref} s^\alpha},
\]  

which can be rewritten as:

\[
C(s) = \frac{1}{s^\alpha} (K_p + K_d s)
\]  

where \(K_d = \frac{\tau}{G \tau_{ref}}\) and \(K_p = \frac{1}{G \tau_{ref}}\) are classical PD coefficient and \(C(s)\) represent a PD controller composed with a fractional order integrator (I\(^\alpha\)(PD)). This controller increases the type of the system with the fractional integrator which eliminates the steady state error for the ramp reference (see Figs. 4 and 5).

### Table 3. Motor parameters

<table>
<thead>
<tr>
<th></th>
<th>Motor</th>
<th>Orthosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_m)</td>
<td>0.322 mH</td>
<td>(J_o)</td>
</tr>
<tr>
<td>(R_m)</td>
<td>0.413 (\Omega)</td>
<td>(b_o) 0.0154 Nms/rad</td>
</tr>
<tr>
<td>(J_m)</td>
<td>1.35e-5 Kgm(^2)</td>
<td>(K) 100 N/m</td>
</tr>
<tr>
<td>(b_m)</td>
<td>1.54e-3 Nms/rad</td>
<td>(b) 0.0001 Nms/rad</td>
</tr>
<tr>
<td>(K_b)</td>
<td>27.9e-5 Vs/rad</td>
<td>(K_t) 0.0251 Nm/A</td>
</tr>
</tbody>
</table>

\(I\)\(^\alpha\)(PD) controller for ideal open loop

In order to control the orthosis, a fractional order controller i.e. I\(^\alpha\)(PD) will be designed. This controller will be designed based on a closed loop reference model expressing the dynamical and robust performances.

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\]  

Therefore, the relation between the reference model of the system and controlled open loop system can be,

\[
P_{ref}(s) = C(s) P(s)
\]

Using system dynamics \(P(s)\) and ideal reference model \(P_{ref}(s)\), then the controller \(C(s)\) can be easily achieved as,

\[
C(s) = \frac{(\tau s + 1)}{G \tau_{ref} s^\alpha},
\]  

which can be rewritten as:

\[
C(s) = \frac{1}{s^\alpha} (K_p + K_d s)
\]  

where \(K_d = \frac{\tau}{G \tau_{ref}}\) and \(K_p = \frac{1}{G \tau_{ref}}\) are classical PD coefficient and \(C(s)\) represent a PD controller composed with a fractional order integrator (I\(^\alpha\)(PD)). This controller increases the type of the system with the fractional integrator which eliminates the steady state error for the ramp reference (see Figs. 4 and 5).
Fractional order PI Controller
A fractional order PI controller (FPI) will be designed in this section. Let us consider the FPI as follows,

\[ C(s) = K_p + K_i s^{-\lambda}, 0 < \lambda \leq 1, \quad (8) \]

where \( K_p \) and \( K_i \) proportional and integral coefficients. The phase and gain of the plant in frequency domain can be given by,

\[ \text{Arg}(P(j\omega)) = -\tan^{-1}(\omega T) - \frac{\pi}{2}, \]

\[ |P(j\omega)| = \frac{G}{\omega \sqrt{1 + (\omega T)^2}}. \]

PI\(^\lambda\) described by (8) can be written as,

\[ C(j\omega) = K_p + K_i \omega^{-\lambda} \cos \frac{\lambda \pi}{2} - j K_i \omega^{-\lambda} \sin \frac{\lambda \pi}{2}, \quad (9) \]

and the phase and gain are as follows:

\[ \text{Arg}(C(j\omega)) = -\tan^{-1} \left( \frac{K_i}{K_p} \omega^{-\lambda} \sin \frac{\lambda \pi}{2} \right), \quad (10) \]

\[ |C(j\omega)| = \sqrt{ \left( K_p + K_i \omega^{-\lambda} \cos \frac{\lambda \pi}{2} \right)^2 + \left( K_i \omega^{-\lambda} \sin \frac{\lambda \pi}{2} \right)^2 }, \quad (11) \]

and consequently the phase and gain of the controlled system are as follows:

\[ \text{Arg}(C(j\omega)P(j\omega)) = -\tan^{-1} \left( \frac{K_i}{K_p} \omega^{-\lambda} \sin \frac{\lambda \pi}{2} \right) - \tan^{-1}(\omega T) - \frac{\pi}{2}, \quad (12) \]

\[ |C(j\omega)P(j\omega)| = \sqrt{ \left( K_p + K_i \omega^{-\lambda} \cos \frac{\lambda \pi}{2} \right)^2 + \left( K_i \omega^{-\lambda} \sin \frac{\lambda \pi}{2} \right)^2 } \div \omega \sqrt{1 + (\omega T)^2}, \quad (13) \]

In order to tune the parameter the following specification will be considered [9]:

I) phase margin specification

\[ \text{Arg}(C(j\omega_c)P(j\omega_c)) = -\pi + \phi_m, \]

II) robustness to variation in the gain of the plant

\[ \frac{d}{d\omega} \text{Arg}(C(j\omega)P(j\omega)) \bigg|_{\omega=\omega_k} = 0 \]

III) gain crossover frequency specification

\[ |C(j\omega_c)P(j\omega_c)| = 1. \]

According to specification (I) and (12) following relation can be achieved:

\[ K_{ip} = \frac{-\tan(\phi_m + \tan^{-1}(\omega_k T) - \frac{\pi}{2})}{\omega_k^{-\lambda} \left( \sin \frac{\lambda \pi}{2} + \cos \frac{\lambda \pi}{2} \tan(\phi_m + \tan^{-1}(\omega_k T) - \frac{\pi}{2}) \right) }, \quad (14) \]

where \( K_{ip} = K_i/K_p \). From specification (II) following equation can be achieved:

\[ A\omega_k^{-2\lambda} (K_{ip})^2 + BK_{ip} + A = 0, \quad (15) \]

where, \( A = \frac{T}{1+(\omega_k T)^2} \) and \( B = 2A\omega_k^{-\lambda} \cos \frac{\lambda \pi}{2} - \lambda \omega_k^{\lambda-1} \sin \frac{\lambda \pi}{2} \). Solving (15) yields:

\[ K_{ip} = \frac{-B \pm \sqrt{B^2 - 4A^2 \omega_k^{-2\lambda}}}{2A\omega_k^{-2\lambda}}. \quad (16) \]

\( K_{ip} \) and \( \lambda \) can be easily achieved from (14) and (16). In addition, using specification (III) the controller parameters \( K_p \) and \( K_i \) can be obtained as follows:

\[ K_p = \frac{\omega \sqrt{1 + (\omega T)^2}}{G \sqrt{ \left( 1 + K_{ip} \omega^{-\lambda} \cos \frac{\lambda \pi}{2} \right)^2 + \left( K_{ip} \omega^{-\lambda} \sin \frac{\lambda \pi}{2} \right)^2 } }, \quad (17) \]

\[ K_i = K_{ip} K_p. \quad (18) \]

The gain crossover frequency is set as \( \omega_k = 100 \) (rad/s), and the desired phase margin is set as \( \phi_m = 40^\circ \). For the \( I^\lambda(PD) \) controller, regarding to the specification the reference system parameters obtained as, \( \tau_{ref} = 0.00077 \) and \( \alpha = 5/9 \). For the PI\(^\lambda\) controller, the parameters for the same specification obtained as,
$K_i = 86.87$, $K_p = 37.48$ and $\lambda = 0.68$. Bode diagrams of the ideal systems, shown in Figs. (11) and (12) clarify that both controller achieved the specifications. The simulation results are shown in Figs. 13 and 14. It must be mentioned the controllers are applied for both stance and swing phases in the simulation in order to show the performance of the controller. Whereas, in practice it should be blocked in the stance phase and controller will be activated in the swing phase. Mean square errors (MSE) of these two methods i.e. MSE(PI$\lambda$)=0.0025 and MSE(I$\alpha$(PD))=0.0031 imply that the response of PI$\lambda$ is quicker.

**FIGURE 11.**  Bode plot of controlled system with I$\alpha$(PD) controller

**FIGURE 12.**  Bode plot of controlled system with PI$\lambda$ controller

**FIGURE 13.**  Simulation result of controlled system with I$\alpha$(PD).

**CONCLUSIONS**

In this paper, a mechanical model and design of SCKAFO orthosis is studied. A hybrid model is introduced. In order to control the orthosis knee angle a linear reference is introduced using four planar sensor in the foot. A fractional order controller using the combination of classic PD and a fractional Integrator is proposed. This controller is not sensitive to the noise and eliminates the steady states error which a classical PD don’t. In addition, a PI$\lambda$ controller is proposed and the controller parameters are tuned using proper specifications. The simulation results show the efficiency of the controllers.

**ACKNOWLEDGMENT**

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FIGURE 14. Simulation result of controlled system with PI$^3$. 


