Robust Optimization based Energy Dispatch in Smart Grids Considering Simultaneously Multiple Uncertainties: Load Demands and Energy Prices

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Abstract: Solving the problem of energy dispatch in a heterogeneous complex system is not a trivial task. The problem becomes even more complex considering uncertainties in demands and energy prices. This paper discusses the development of several Economic Model Predictive Control (EMPC) based strategies for solving an energy dispatch problem in a smart micro-grid. The smart grid components are described using control-oriented model approach. Considering uncertainty of load demands and energy prices simultaneously, and using an economic objective function, leads to a non-linear non-convex problem. The technique of using an affine dependent controller is used to convexify the problem. The goal of this research is the development of a controller based on EMPC strategies that tackles both endogenous and exogenous uncertainties, in order to minimize economic costs and guarantee service reliability of the system. The developed strategies have been applied to a hybrid system comprising some photovoltaic (PV) panels, a wind generator, a hydroelectric generator, a diesel generator, and some storage devices interconnected via a DC Bus. Additionally, a comparison between the standard EMPC, and its combination with MPC tracking in single-layer and two-layer approaches was also carried out based on the daily cost of energy production.

Keywords: Smart grid, Model Predictive Control, Robust Optimization, Demand uncertainty, Energy prices uncertainty

1. INTRODUCTION

Complex systems such as smart grids whereby several heterogeneous components attempt to interact with each other, require definitely a well-defined proactive control strategy in order to optimize its efficiency, and avoid conflicting interactions. Furthermore, the complexity of smart grids increases in the presence of uncertainties.

Model uncertainty and noise are two important factors which need to be taken into consideration in the development of robust MPC based control strategies. In linear time-invariant (LTI) systems, the problem of model uncertainty and noise can be solved through enforcement of computational constraints as reported in Bemporad et al. (1999), Loefberg (2003), and Abate et al. (2004). However, this approach is usually raising the issue of tractability.

A better approach of tackling uncertainties is reported in Loefberg (2003) and Löfberg (2012) would be to consider optimization techniques such as Minimax MPC, even though Minimax is only based on worst case scenario.

Moreover, in Ben-Tal et al. (1998), Ben-Tal et al. (2004), and Löfberg (2012), Adjustable Robust Solutions have been proposed, which assume that adjustable control inputs can be made to depend affinely on the uncertainty parameters of the problem. This approach is more flexible, and is most of the time expected to result in a computationally tractable problem (Vandenberghe et al. 1996). In this work, we follow the technique of affine dependence to solve the problem of demand and energy price uncertainty in electrical micro-grid.

In Nassourou et al. (2016), uncertainty of load demand was taken into consideration, it was shown that the standard EMPC was not only superior to the standard MPC tracking, but also to the integration of both in single-layer and two-layer approaches.

In this paper, we repeat the study in Nassourou et al. (2016) by considering simultaneously the uncertainty of load demands and energy prices. Several studies (Ocampo-Martinez et al. 2009; Grosso et al., 2012a; Grosso et al., 2014; Limon et al, 2014) have dealt with the issue of tackling uncertainties separately using stochastic approaches. In this work, we use a deterministic approach namely robust optimization to model uncertainties. Considering both uncertainties at the same time, and using EMPC strategies, the optimization problem becomes non-linear and non-convex. The technique of using an affine dependent controller is used to convexify the problem.

Robust optimization based EMPC strategies for smart grids are discussed in this paper. We have explicitly included the uncertainty information into the Minmax optimization problem, by substituting the uncertain variables with their robust counterparts in the objective function, as well as in the constraints. Energy prices are split into actual prices and predicted ones.

We consider a hybrid system comprising some photovoltaic (PV) panels, a wind generator, a hydroelectric generator, a diesel generator, and some storage devices interconnected via a DC Bus, from which load demands can be satisfied.

EMPC based control strategies have been developed by using both single-layer and two-layer approaches. In the one-layer approach, standard EMPC strategy was applied to the hybrid system. After that, both the economic optimization and the tracking formulation were integrated in a single layer.

In the two-layer approach, the upper layer consists of an EMPC controller acting as the supervisory unit, which is in charge of scheduling the operation of the subsystems, and computing their power references. At the lower layer, we used standard MPC tracking controllers responsible for implementing the computed reference values for each subsystem.

2. PROBLEM FORMULATION

The main objectives of this work is the development of a controller based on EMPC strategies, that tackles load demand and energy prices uncertainties, in order to minimize economic costs and guarantee service reliability of the system. To achieve this aim, three operational goals have been considered:

Economic cost:

The total economic cost is given by:

$$f_E(k) = (\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2(k))^{T}\mathbf{u}(k)\Delta t \quad (1)$$

where: $\mathbf{u}(k)$ is a vector of control actions at time k;

Δt is the sampling time in seconds;

$\boldsymbol{\alpha}_1$ is a known vector related to economic costs of maintenance of generators and its accessories;

$\boldsymbol{\alpha}_2(k)$ is an unknown time-varying vector associated to the economic cost of power flows related to transmission and distribution.

In this study we consider $\boldsymbol{\alpha}_2(k)$ to be uncertain but with known bounds:

$$\boldsymbol{\alpha}_2^{min}(k) \le \boldsymbol{\alpha}_2(k) \le \boldsymbol{\alpha}_2^{max}(k) \quad (2)$$

where $\boldsymbol{\alpha}_2^{min}(k)$ and $\boldsymbol{\alpha}_2^{max}(k)$ are the lower and upper bounds of the energy prices respectively that are known functions.

Safety Storage Measures:

This function is used to penalize quadratically the amount of power that goes below the pre-specified security threshold $\boldsymbol{\delta}$ in (8). The safety measures are defined as:

$$f_S(k) = \boldsymbol{\varepsilon}(k)^{T}\boldsymbol{\varepsilon}(k) \quad (3)$$

where $\boldsymbol{\varepsilon}(k)$ is the amount of soft constraint violation. $\boldsymbol{\varepsilon} = 0$ means there is no violation.

Smoothness/Stability of the control action:

This function is used to avoid excessive power on the DC Bus.

$$f_{\Delta u}(k) = \Delta\mathbf{u}(k)^{T}\Delta\mathbf{u}(k) \quad (4)$$

where $\Delta\mathbf{u}(k)$ is the rate of change of control signal, defined as $\Delta\mathbf{u}(k)=\mathbf{u}(k) - \mathbf{u}(k\text{-}1)$.

3. CONTROL-ORIENTED MODELING

Smart grids could be viewed as instances of generalized flow-based networks. Basically every flow-based network is made up of some components (Ocampo-Martinez et al. 2009; Nassourou et al. 2016) e.g.: flow sources, links, nodes, storage, flow handling, and sink elements.

3.1. Control-oriented model

A smart grid consisting of n_x storage elements, n_u energy flow handling and source elements, n_d sinks and n_q intersection nodes is considered. The source elements are considered as inflows.

3.1.1. State space model

The hybrid power system is an example of a MIMO (multiple-input multiple-output) system, whose linear state space modelling is given by the following equations:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_d\mathbf{d}(k) \quad (5a)$$

$$\mathbf{E}_u\mathbf{u}(k) + \mathbf{E}_d\mathbf{d}(k) = 0 \quad (5b)$$

where:

$\mathbf{x} \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ stands for the vector of control inputs, $\mathbf{d} \in \mathbb{R}^{n_d}$ denotes the disturbances vector. $\mathbf{A} \in \mathbb{R}^{n_x \times n_x}$, $\mathbf{B} \in \mathbb{R}^{n_x \times n_u}$, $\mathbf{B}_d \in \mathbb{R}^{n_x \times n_d}$ are system matrices. $\mathbf{E}_u \in \mathbb{R}^{n_q \times n_u}$ and $\mathbf{E}_d \in \mathbb{R}^{n_q \times n_d}$ are matrices of suitable dimensions relating the supply and the load demand on the n_q DC Bus(ses).

3.1.2. Constraints

Control inputs are subject to some bounds (upper and lower limits):

$$\mathbf{u}^{min}(k) \le \mathbf{u}(k) \le \mathbf{u}^{max}(k) \quad (6)$$

($\mathbf{u}^{min}(k)$ is in this case always zero, because energy flow from the generators is positive).

The state of charge (SOC) of each storage element is subject to the following constraint:

$$\mathbf{x}^{min} \le \mathbf{x}(k) \le \mathbf{x}^{max} \quad (7)$$

where $\mathbf{x}^{min}$ and $\mathbf{x}^{max}$ are the lower and upper limit values of the state of charge respectively.

To guarantee availability of energy in the batteries we set:

$$\mathbf{x}^{min}(k) \ge \boldsymbol{\delta} \quad (8)$$

where $\boldsymbol{\delta}$ is the minimum quantity of energy that should always be available in the batteries.

The disturbance $\mathbf{d}(k)$ representing the load demand is uncertain but with known bounds:

$$\mathbf{d}^{min}(k) \le \mathbf{d}(k) \le \mathbf{d}^{max}(k) \quad (9)$$

where $\mathbf{d}^{min}(k)$ and $\mathbf{d}^{max}(k)$ are the lower and upper bounds of load demands respectively.

4. ROBUSTNESS IN EMPC STRATEGY

The performance and accuracy of MPC relies principally on the model used to predict the behaviour of the plant. Unfortunately for real systems, there are frequently uncertainties about the model parameters, as well as occurrences of external disturbances. These uncertainties degrade the performance of the controllers. They can be tackled using stochastic or deterministic approaches. In this

work, a deterministic approach namely robust optimization is selected, which offers some possibilities of bounding uncertain parameters and variables.
Load demand $\boldsymbol{d(k)}$ and the energy prices namely $\boldsymbol{\alpha}_2(k)$ (as explained in Section 2) are chosen to be uncertain.

4.1. Modelling uncertain energy prices

In this paper, we will consider that there is independence between the different uncertain variables. *i.e.* for the uncertain energy prices:

$$\alpha_{2,i}^{\min}(k) \leq \alpha_{2,i}(k) \leq \alpha_{2,i}^{\max}(k) \qquad \forall i = 1,\ldots,n_u \qquad (10)$$

Therefore, at every time instant k energy prices $\boldsymbol{\alpha}_2(k)$ can be bounded by a box $\boldsymbol{\Theta}_\alpha(k)$:

$$\boldsymbol{\alpha}_2(k) \in \boldsymbol{\Theta}_\alpha(k) = \left[\alpha_{2,1}^{\min}(k), \alpha_{2,1}^{\max}(k)\right] \times \cdots \times \left[\alpha_{2,nu}^{\min}(k), \alpha_{2,nu}^{\max}(k)\right]$$

Upper and lower limits of the price ($\alpha_{2,i}^{\min}(k)$ and $\alpha_{2,i}^{\max}(k)$) can be found by means of a forecast considering additive bounded error:

$$\alpha_{2,1}(k) = \alpha_{2,1}^0(k) + \delta_{\alpha_{2,1}}(k) \qquad (11)$$

where $\alpha_{2,1}^0(k)$ is the price forecast and $\delta_{\alpha_{2,1}}(k)$ is the additive error that is bounded by $\left|\delta_{\alpha_{2,1}}(k)\right| \leq \Delta_{\alpha_2}(k)$ with $\Delta_{\alpha_2}(k)$ a known function. Then, the prices upper and lower limits can be computed as:

$$\alpha_{2,1}^{\min}(k) = \alpha_{2,1}^0(k) - \Delta_{\alpha_2}(k); \quad \alpha_{2,1}^{\max}(k) = \alpha_{2,1}^0(k) + \Delta_{\alpha_2}(k)$$

4.2. Modelling uncertain load demands

The load demand $\mathbf{d}(k)$ is split into two parts: a nominal demand and an uncertain additive demand.
The uncertain additive load demand is bounded at every time instant k by a box $\boldsymbol{\Theta}_d(k)$:

$$\mathbf{d}(k) \in \boldsymbol{\Theta}_d(k) = \left[d_1^{\min}(k), d_1^{\max}(k)\right] \times \cdots \times \left[d_{nd}^{\min}(k), d_{nd}^{\max}(k)\right]$$

Upper and lower limits of the demand ($d_i^{\min}(k)$ and $d_i^{\max}(k)$) can be found by means of a forecast considering additive bounded error:

$$d_i(k) = d_i^0(k) + \delta_{d_i}(k) \qquad (12)$$

where $d_i^0(k)$ is the price forecast and $\delta_{d_i}(k)$ is the additive error that is bounded by $\left|\delta_{d_i}(k)\right| \leq \Delta_{d_i}(k)$ with $\Delta_{d_i}(k)$ a known function. Then, the demands upper and lower limits can be computed as:

$$d_i^{\min}(k) = d_i^0(k) - \Delta_{d_i}(k); \quad d_i^{\max}(k) = d_i^0(k) + \Delta_{d_i}(k)$$

Using the affine dependence method, it is possible to establish a relationship between the control and the load demands and the control inputs. A mathematical derivation of this relationship has been developed in Nassourou et al. (2016).

4.3. Parameterization of control inputs with respect to both uncertainties

Considering price and demand uncertainties simultaneously, and using the economic cost function defined in (1), we end up with a non-linear non-convex problem. In fact, equation (1) consists of a multiplication of the two uncertainties.
A non-convex optimization problem in the case of smart grids is not desirable, because of the fact that, there could be many local optimal solutions, which make the identification of a global optimal solution extremely difficult.
One optimal approach to deal with multiple uncertainties simultaneously would be to convert the non-convex problem into a convex one.
The relationship between control inputs and the disturbances is evident through the equation (5b). This justifies as well the affine dependence between the control inputs and the load demand (Abate et al. 2004). However, the relationship between the control inputs and the energy prices is not evident.
In this approach, the approach for dealing with the uncertainties based on an affine dependence between the control inputs and both uncertainties has been considered (Loefberg, 2003; Abate et al., 2004):

$$\underline{\mathbf{u}} = \underline{\mathbf{v}} + \mathbf{W}\underline{\mathbf{d}} + \mathbf{Z}\underline{\boldsymbol{\alpha}_2} \qquad (13)$$

where
$\underline{\mathbf{u}}=[\mathbf{u}(k|k),\ldots, \mathbf{u}(k+N-1|k)]^T$
$\underline{\mathbf{v}}=[\mathbf{v}(k|k),\ldots, \mathbf{v}(k+N-1|k)]^T$
$\underline{\mathbf{d}}=[\mathbf{d}(k|k),\ldots, \mathbf{d}(k+N-1|k)]^T$
$\underline{\boldsymbol{\alpha}_2}=[\boldsymbol{\alpha}_2(k|k),\ldots, \boldsymbol{\alpha}_2(k+N-1|k)]^T$

with $\mathbf{W}$ and $\mathbf{Z}$ matrices of proper dimensions and N the prediction horizon. With this parameterization (affine dependence between control inputs and uncertain variables) the non-linear non-convex problem is transformed into a convex problem.

It might be important to mention that two other strategies could be used:

- Transformation of the problem into a simpler and equivalent one;
- Control inputs do not dependent affinely on energy prices

These strategies will be discussed in our future works.

5. PROPOSED APPROACHES

All the problems are formulated using the worst-case robust optimization approach, namely the minmax format of the Wald's maximum model (Loefberg, 2003). The problems are solved using the robust convex optimization methods proposed in Loefberg (2012).
The goal is to minimize the costs of energy production in the presence of uncertain load demands and energy prices.

5.1. Economic MPC

The MPC objective function is given by using (1), (2) and (3)

$$\boldsymbol{J_{EMPC}} = \lambda_1 (\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2(k))^T \underline{\mathbf{u}}(k)\Delta t + \lambda_2 \boldsymbol{\varepsilon}(k)^T \boldsymbol{\varepsilon}(k) + \lambda_3 [\underline{\mathbf{u}}(k) - \underline{\mathbf{u}}(k-1)]^T [\underline{\mathbf{u}}(k) - \underline{\mathbf{u}}(k-1)] \quad (14)$$

where λ_1, λ_2, and λ_3 are weighting coefficients for prioritizing the objectives.
The EMPC optimization problem is formulated as follows:

$$\min_{\mathbf{u}(k)\mathbf{x}(k)} \max_{\boldsymbol{\alpha}_2(k)\mathbf{d}(k)} \sum_{k=0}^{N-1} J_{EMPC}(k) \quad (15)$$

s.t $\quad (5),(6),(7),(8),(9),(10)$

$\boldsymbol{\alpha}_2(k) \in \boldsymbol{\Theta}_{\alpha 2}(k), \mathbf{d}(k) \in \boldsymbol{\Theta}_d(k), \forall k$

5.2. Two-layer approach: EMPC and MPC tracking

The main idea behind the use of a two-layer approach is to overcome the problem of non-reachable reference trajectories (feasibility). The standard Economic MPC (EMPC) is used as the supervisory controller (upper layer), which computes the reference trajectory (set-points) for the lower layer comprising standard MPC tracking controllers responsible for driving the subsystems to desired set-points accordingly.

a) Upper layer: Economic MPC
This layer comprises the EMPC described in Section 5.1. The problem to be solved is expressed in (15).
b) Lower layer: MPC tracking
The lower layer consists of a standard MPC tracking. Instead of using manually selected reference trajectories, the computed states and control inputs by the upper layer are used.
The following objective function is used:

$$J_{MPC} = (\mathbf{x}-\mathbf{x}^r)^T \mathbf{Q} (\mathbf{x}-\mathbf{x}^r) + (\underline{\mathbf{u}}-\mathbf{u}^r)^T \mathbf{R} (\underline{\mathbf{u}}-\mathbf{u}^r) + \left(\mathbf{x}_{Np}-\mathbf{x}^r_{Np}\right)^T \mathbf{S}_\mathbf{x} \left(\mathbf{x}_{Np}-\mathbf{x}^r_{Np}\right) \quad (16)$$

$\mathbf{x}_{Np}$ is the vector of terminal state and $\mathbf{x}^r_{Np}$ its reference trajectory;
Q, R and $\mathbf{S}_\mathbf{x}$ are weights on the states, control inputs, and terminal state respectively.

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \mathbf{R} = \mathrm{diag}(c_b, c_b, c_h, c_h, c_g, c_g, c_d, c_{hy}, c_w, c_{pv}); \mathbf{S}_\mathbf{x} = \mathbf{W}_c.\mathbf{Q}$$

c_b, c_h, c_g, c_d c_d, c_{pv}, c_w, c_{hy} , are positive weight coefficients (≤ 1) for the lead-acid battery, hydrogen battery, grid connection, diesel, solar, wind, and hydroelectric generators respectively, and $\mathbf{W}_c$ is a positive scalar.

The optimization problem is formulated as follows:

$$\min_{\mathbf{u}(k)\mathbf{x}(k)} \max_{\mathbf{d}(k)} \sum_{k=0}^{N-1} J_{MPC}(k) \quad (17)$$

s.t $\quad (5),(6),(7),(8),(9)$

$\mathbf{d}(k) \in \boldsymbol{\Theta}_d(k), \forall k$

5.3. Single-layer approach: EMPC and MPC tracking

Contrary to the two-layer approach as defined above, the economic optimization (EMPC) and the tracking formulation (MPC tracking) are integrated in a single layer.
The problem to be solved is given as follows:

$$\min_{\mathbf{u}(k)\mathbf{x}(k)} \max_{\boldsymbol{\alpha}_2(k)\mathbf{d}(k)} \sum_{k=0}^{N-1} \left(J_{EMPC}(k) + J_{MPC}(k)\right) \quad (18)$$

s.t. $\quad (5),(6),(7),(8),(9),(10)$

$\boldsymbol{\alpha}_2(k) \in \boldsymbol{\Theta}_{\alpha 2}(k), \mathbf{d}(k) \in \boldsymbol{\Theta}_d(k), \forall k$

6. CASE STUDY

6.1. Description

In this subsection, we present a smart micro-grid that consists of: two storage elements (batteries), three sinks (loads) and one virtual sink (external grid connection), one node (DC Bus), four sources (PV, Wind, Hydroelectric, and Diesel generators), and one virtual source (external grid connection). Since all the components (excluding sinks) are connected to a single node (DC Bus) through flow handling elements, they are all considered as manipulated inputs. The states of the smart grid are defined to be the state of charge of the storage elements. The block diagram of the smart micro-grid is shown in Fig 1.

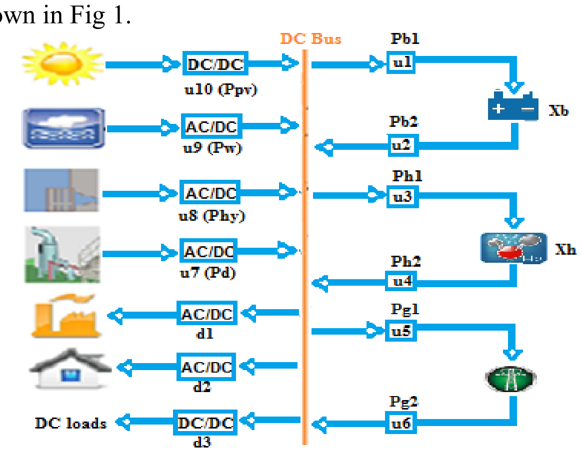

Fig 1. block diagram of the smart micro-grid

6.2. Control-oriented Model

State variables:
x_b and x_h are the state of charge of the batteries (lead-acid and hydrogen respectively). $\mathbf{x}(k) \triangleq [x_b(k), x_h(k)]^T$
Control input variables:
P_{b1} and P_{b2} are charged power and discharged power of the lead-acid battery;
P_{h1} and P_{h2} are the charged and discharged power of the hydrogen battery;
P_{g1} and P_{g2} are the exported and imported power into/from the external grid; P_d, P_{hy}, P_{pv}, and P_w stand for the power supplied to the DC Bus by the diesel, hydroelectric, wind, and photovoltaic generators respectively;

$$\mathbf{u}(k) \triangleq [P_{b1}(k), P_{b2}(k), P_{h1}(k), P_{h2}(k), P_{g1}(k), P_{g2}(k), P_d(k), P_{hy}(k), P_w(k), P_{pv}(k)]^T$$

Disturbance variables:
d_1 is the industrial load, d_2 is the residential load, and d_3 is the DC-load. The disturbance vector **d** consists of the three loads.
$\mathbf{d}(k) \triangleq [d_1(k), d_2(k), d_3(k)]^T$
The matrices and vectors that define the system and its constraints are given as follows:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ \boldsymbol{B} = \begin{bmatrix} \eta_{bc} & -\eta_{bd} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{hc} & -\eta_{hd} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\boldsymbol{B}_p = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix};\ \mathbf{x}^{min} = [0\ 0]^T;\ \mathbf{x}^{max} = [100\ 100]^T;$$

$$\mathbf{u}^{min} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T;\ \mathbf{u}^{max} = [35\ 10\ 35\ 18\ 18\ 18\ 18\ 18\ 18\ 18]^T;$$

$$\mathbf{E_u} = [-1\ 1\ -1\ 1\ -1\ 1\ 1\ 1\ 1\ 1];\ \mathbf{E_d} = [-1\ -1\ -1]$$

Table 1 presents system and control parameters as well as energy prices.

Initial values of the subsystems, as well as the state of charge of the batteries are set to zero. The simulations were made for 96 hours (4 days). The batteries were used during the first two hours of the day. They delivered 2 kWh in the first hour and 1 kWh in the second hour. 1 kWh was bought from the external grid during the second hour of the day. Each additive uncertain load demand (as explained in Section 4.2) is bounded as follows $|\delta_{\alpha_{2,1}}(k)| \leq 1$ kW. On the other hand, the prices forecast error (as explained in Section 4.1) is bounded $|\delta_{d_i}(k)| \leq 3$ e.u (economic unit).

System parameters		Control parameters		Energy prices (e.u)	
Parameters	Values (kW)	Parameters	Values	Lead-acid battery charging	2.2
P^{max}_{pv}	15	N_p	24	Lead-acid battery discharging	2.2
P^{max}_{w}	15	N_c	24	Hydrogen battery charging	2.2
P^{max}_{hy}	15	c_{pv}	0.2	Hydrogen battery discharging	2.2
P^{max}_{d}	15	c_w	0.3	External grid selling	3
P^{max}_{b1}	35	c_{hy}	0.4	External grid buying	3
P^{max}_{b2}	15	c_b	0.75	Diesel	3.3
P^{max}_{h1}	35	c_h	0.75	Hydroelectric	2.1
P^{max}_{h2}	15	c_d	1	Wind	2.1
P^{max}_{g1}	15	c_g	0.75	Solar	2
P^{max}_{g2}	15	Q	as defined previously		
η_{bc}	0.95	R	as defined previously		
η_{bd}	1	λ_1	2500		
η_{hc}	0.85	λ_2	12		
η_{hd}	1.0	λ_3	0.1		
Δ	$[35\ 35]^T$				

Table 1. System and control parameters; and energy prices

Figure 2 shows the profiles of the load demand. The additive uncertain demand is represented with the shadowed area.

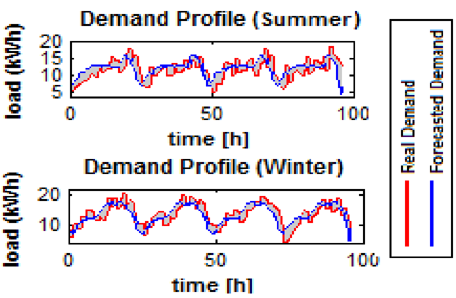

Figure 2. Load demands' profiles

Figure 3 shows the profile of the energy prices. The forecast error is the shadowed area.

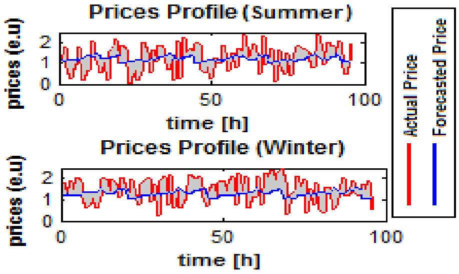

Figure 3. Profile of the energy prices

6.3. Simulation Results

The MPC controller implementations have been made using YALMIP (CPLEX and QuadProg solvers) (Löfberg, 2012) within the Matlab environment. In order to get a reasonable computational time, demands and prices in (13) have been considered unknown but constant during the prediction horizon *N*. One of the goals of this study is to minimize the energy production as much as possible in the presence of uncertainties (load demands and energy prices uncertainties). Figures 4a and 4b show a sample comparison of the energy production of the diesel and wind generators.

Figure 4 displays a sample plot of the batteries' state of charge trajectories.

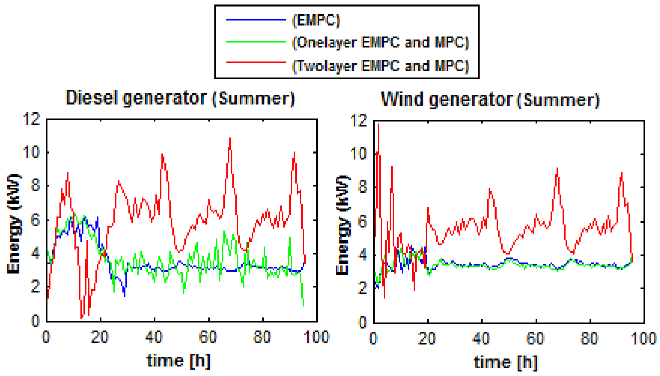

Figure 4a. Plots of the energy generation in summer

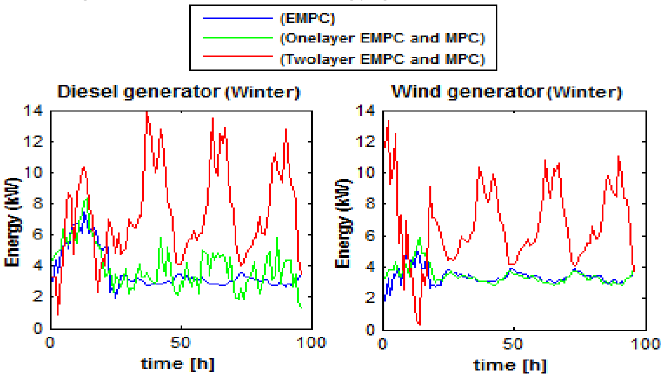

Figure 4b. Plots of the energy generation in winter

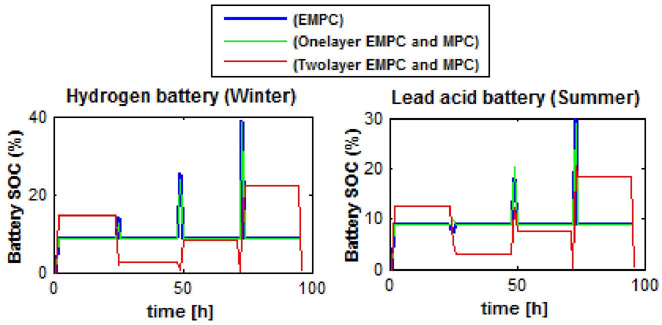

Figure 5. Plots of the batteries' state of charge trajectories

Figures 4 and 5 show that, the EMPC strategy offers a better result, since it yields the lowest energy production, and a higher energy saving in the batteries.
It can be stated that, one-layer approach is economically superior to a two-layer hierarchical scheme. This study confirms the results found in Nassourou et al. (2016). Similar result was obtained in Grosso et al. (2012a).
Finally, Table 2 shows that the EMPC produces the lowest overall economic costs, thereby proving its superiority to the other strategies.
Table 2 presents a comparison of the three EMPC strategies' economic costs (measured in economic unit (e.u)).

	EMPC	EMPC + MPC tracking (single-layer)	EMPC + MPC tracking (two layer approach)
Summer economic cost	7722.4	8421.9	8433.5
Winter economic cost	7891.7	8645.0	9312.4

Table 2. Quantitative comparison of the economic costs

7. CONCLUSION AND FUTURE WORK

In this study, we have presented the application of three variations of robust optimization based EMPC strategies for controlling energy dispatch in a smart micro-grid. Load demands and energy prices uncertainties have been simultaneously considered, and modelled using the affine dependence method. Several EMPC strategies have been discussed and compared. The optimization problems were solved using minmax worst case approach. It has been found that, a single layer approach is superior to a hierarchical scheme. Moreover the standard Economic MPC yields a better economic result. The study confirms that, uncertainties degrade the performance of the micro-grid, because the cost of energy production increases. The next task for extending this work will be devoted to tackling uncertainty of renewable energy sources.

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