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A local stability condition for dc grids with constant power loads ⋆

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Abstract: Currently, there are an increasing number of power electronics converters in electrical grids, performing the most diverse tasks, but most of them, work as constant power loads (CPLs). This work presents a sufficient condition for the local stability of dc linear time-invariant circuits with constant power loads for all the possible equilibria (depending on the drained power) of the systems. The condition is shown as a method with successive steps that should be met. Its main step is expressed as a linear matrix inequality test which is important for easiness of verification reasons. The method is illustrated with two examples: a single-port RLC circuit connected to a CPL and a two-port linear dc circuit connected to two CPLs.

Keywords: Local stability, dc LTI circuits, constant power loads, frequency domain methods, linear matrix inequalities.

1. INTRODUCTION

The current trend in power grids is to increase the presence of electronic power converters due to their versatility to transform, condition and accumulate electrical energy. However, most of these devices behave as constant power loads. This type of non-linear loads can compromise the stability of the electrical system since, incrementally, they behave as negative resistive elements. So, this characteristic puts at risk the quality of the supply and the integrity of the electrical system. For these reasons, the development of new and, if possible, easy to check conditions for the stability of grids with CPLs is required.

The stability of electrical circuits with constant power loads connected to them has been studied using different approaches in the literature, i.e. Middlebrook (1976), Belkhayat et al. (1995b), Belkhayat et al. (1995a), Sanchez et al. (2014). See, also, the recent survey by Singh et al. (2017) for a state of the art in the behaviour and typical effects of CPLs, stability criteria and compensation techniques.

In this work, we present a new frequency domain method applied to the linearization of dc grids feeding CPLs around the equilibrium. This method is based on the properties of negative imaginary (NI) systems, e.g. Petersen and Lanzon (2010), and it establishes a sufficient condition for the stability of the system for all the possible equilibria that come from sweeping the power of the CPLs. To illustrate the method we apply it to two examples: one port and a two port linear time invariant (LTI) dc electrical circuits connected to CPLs. Due to the method is based on linear matrix inequalities (LMI) conditions its numerical solution can be computed efficiently by means of convex programming.

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The paper is organized as follows. Section 2 shows some previously published results on NI systems that will be used subsequently. Sections 3 and 4 describe and formalize the problem and present the sufficient condition for the stability, respectively, Section 5 shows two examples: a one-port dc RLC circuit connected to a CPL and a two-port dc RLC with two CPLs. Finally, Section 6 summarizes the contributions and propose some future extensions.

2. PREVIOUS RESULTS

Some previous results that characterize NI systems in the frequency domain and in the state space are reproduced in this section to facilitate the understanding of the subsequent developments.

First, the definition of an NI transfer function matrix and an NI LTI system.

Definition 1. (Petersen and Lanzon (2010)). The square transfer function matrix $R(s)$ is *negative imaginary* if the following conditions are satisfied:

1) All the poles of $R(s)$ lie in the OLHP.
2) for all $\omega \geq 0$,

$$j[R(j\omega) - R^*(j\omega)] \geq 0.$$

A linear time-invariant system is NI if its transfer function matrix is NI.

The NI property can be checked, from a state-space description of the system, using the Lemma 2. Besides, Corollary 3 allows checking, incrementally, if the system is strictly negative imaginary (SNI). It is worth to remark that the main condition to check in Lemma 2 is described by an LMI and, then, it is a convex problem.

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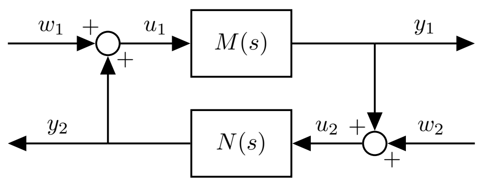

Fig. 1. A positive feedback interconnection.

Lemma 2. (Petersen and Lanzon (2010)).
Consider the minimal state-space system

$$\dot{x} = Ax + Bu, \quad (1)$$
$$y = Cx + Du, \quad (2)$$

Where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{m\times m}$. The system (1), (2) is NI if and only if

1) A has no eigenvalues on the imaginary axis,
2) $D = D^\top$, and
3) there exists a matrix $Y = Y^\top > 0$, $Y \in \mathbb{R}^{n\times n}$, such that

$$AY + YA^\top \leq 0, \quad \textit{and} \quad B + AYC^\top = 0.$$

Corollary 3. (Petersen and Lanzon (2010)). If the minimal state-space system (1), (2), satisfying conditions (1), (2), and (3) from Lemma 2, additionally meets that

4) The transfer function matrix $M(s) = C(sI-A)^{-1}B + D$ is such that $M(s) - M(-s)^\top$ has no transmission zeros on the imaginary axis except possibly at $s = 0$,

then the system is *strictly negative imaginary* (SNI).

Finally, next theorem guarantees the stability of the positive feedback interconnection of Fig. 1.

Theorem 4. (Song et al. (2011)). Given that $M(s)$ is NI and $N(s)$ is SNI, and suppose that $M(\infty)N(\infty) = 0$ and $N(\infty) \geq 0$. Then, the positive-feedback interconnection of these two systems illustrated in Fig. 1 is internally stable if and only if the maximun eigenvalue of the matrix $M(0)N(0)$, denoted by $\bar{\lambda}(M(0)N(0))$, satisfies

$$\bar{\lambda}(M(0)N(0)) < 1. \quad (3)$$

3. PROBLEM STATEMENT

The system of Fig. 2 consists of a linear dc circuit Σ, including dc voltage and current sources, with m CPLs connected to it. These loads do not behave like conventional impedances, instead, their characteristic curves, in the voltage-current plane, are first and third quadrant hyperbolas. Thus, their incremental impedance is negative. The goal is to find an easy to check sufficient condition to determine if the whole system is locally stable for all the possible values of power in the CPLs for which an equilibrium exists in the system. In order to do that, in this Section, the system is defined and, after that, its linearization is calculated.

In the considered networks, see Fig. 2, Σ is a dc LTI RLC network consisting of an arbitrary interconnection of resistors (R), inductors (L), capacitors (C), current sources (I_s), voltage sources (E_s) and LTI magnetic couplings[1]. Assume that Σ has a well-defined minimal order state-

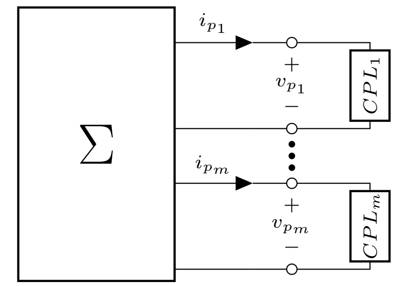

Fig. 2. LTI RLC network including sources connected to m CPLs (m-port case).

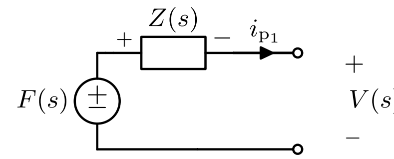

Fig. 3. Thevenin equivalent of the circuit Σ in Fig. 2.

space realization, formulated in terms of the d-inductor currents and q-capacitor voltages, as

$$\begin{aligned} \dot{x} &= Ax + Bi + f, \\ v &= Cx + Di, \\ v_{p_i} i_{p_i} &= P_i > 0, \qquad i = 1, ..., m. \end{aligned} \quad (4)$$

where $i = [i_{p_1}, i_{p_2}, ..., i_{p_m}]$ is the input vector, $v = [v_{p_1}, v_{p_2}, ..., v_{p_m}]$ is the output vector, f is a vector of dc voltage and/or current constant sources, P_i are the constant powers absorbed for each port CPL, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$ are constant matrices with $n = d + q$.

The port input-output representation of Σ in the Laplace domain is

$$V(s) = G(s)\, I(s) + F(s), \quad (5)$$

where s is the Laplace variable, $V(s) = \mathcal{L}\{v(t)\}, I(s) = \mathcal{L}\{i(t)\}$. $G(s) = C(sI - A)^{-1}B + D$ and $F(s) = C(sI - A)^{-1}f$ are rational matrices with real coefficients.

The Thevenin equivalent circuit of Σ, see Fig. 3, is

$$V(s) = -Z(s)\, I(s) + F(s). \quad (6)$$

Matching (5) with (6) we clearly identify $G(s)$ as the equivalent multiport impedance with a negative sign $G(s) = -Z(s)$, and the equivalent multiport voltage source as $F(s)$.

Assuming a given set of fixed CPL powers $P_e := \text{col}\{P_{e_1}, P_{e_2}, ..., P_{e_m}\}$, if the equilibrium of system (4) exists, a set of port voltages $v_e := \text{col}\{v_{e_1}, v_{e_2}, ..., v_{e_m}\}$ and a set of port currents $i_e := \text{col}\{i_{e_1}, i_{e_2}, ..., i_{e_m}\}$ are obtained. Then, the third equation in (4) can be linearized around this equilibrum resulting

$$\Delta i = -K\Delta v + \gamma \Delta P. \quad (7)$$

where $\Delta i := i - i_e$, $\Delta v := v - v_e$ and $\Delta P := P - P_e$ are the incremental vectors of port variables, $K := \text{diag}\{\frac{P_{e_1}}{v_{e_1}^2}, \frac{P_{e_2}}{v_{e_2}^2}, ..., \frac{P_{e_m}}{v_{e_m}^2}\}$ is a constant diagonal gain matrix, and $\gamma := \text{diag}\{\frac{1}{v_{e_1}}, \frac{1}{v_{e_2}}, ..., \frac{1}{v_{e_m}}\}$ is the gain of incremental power disturbances. Obviously, in equation (7) $-K$ represents the negative incremental admitance of the CPLs. Applying Laplace transform to (7), we obtain the linear

[1] For the sake of brevity, the time arguments, $t \in \mathbb{R}$, of the circuit variables are not included unless necessary.

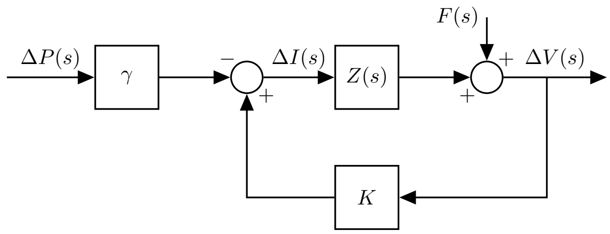

Fig. 4. Block diagram of interconnected system around the equilibrium.

closed-loop system, see Fig. 4, that describes the dynamics around the equilibrium point,

$$\Delta V(s) = Z(s)(K\Delta V(s) - \gamma \Delta P) + F(s). \qquad (8)$$

4. STABILITY CHECKING METHOD

This section presents a method, based on some frequency domain properties of the circuit Σ, to obtain a sufficient local stability condition for the whole system.

The block diagram for equation (8), see Fig. 4, shows the positive-feedback interconnection of Z and K. Then, using Theorem 4, we can guarantee the stability of the system around the equilibrium if we meet that K is NI, using Lemma 2, and $Z(s)$ is SNI, using Corollary 3. So, the stability of the system depends on Z and K NI characteristic.

4.1 Checking if K is NI

From Definition 1, condition 1) holds due to K is constant and condition 2)

$$j[K(j\omega) - K^*(j\omega)] = j[K - K^*] = 0, \quad \forall\omega.$$

also holds. So, K is NI.

4.2 Checking if Z(s) is SNI

$Z(s)$ is the impedance matrix representation of an LTI network, composed of passive elements including resistive ones, hence $Z(s)$ has no poles with Re(s) ≥ 0. Therefore, condition 1) of Lemma 2 is fulfilled.

Depends on network characteristics if condition 2) of Lemma 2 ($D = D^\top$) holds but, assuming that $Z(s)$ is a stricly proper transfer function matrix ($Z(\infty) = D = 0$), this condition is fulfilled. Furthermore, assumptions

$$Z(\infty)K(\infty) = 0 \quad \text{and} \quad Z(\infty) \geq 0 \qquad (9)$$

in Theorem 4 are also satisfied.

Remark 5. Considering a more realistic dynamical behavior for the CPLs, e.g. a first-order dynamics (finite bandwidth CPLs), in equation (7) the diagonal gain matrix can be substituted by

$$K'(s) := \mathrm{diag}\{\frac{P_{e_1}}{v_{e_1}^2}\frac{1}{\tau_1 s+1}, \frac{P_{e_2}}{v_{e_2}^2}\frac{1}{\tau_2 s+1}, ..., \frac{P_{e_m}}{v_{e_m}^2}\frac{1}{\tau_m s+1}\} \qquad (10)$$

where $\tau_i > 0$ and, then, $K'(s)$ is exponentially stable. With this assumption $K'(\infty) = 0$ and the method can broaden for circuits with biproper impedance matrix $Z(s)$. In this case, it is only necessary to check [2] $D = D^\top$, $Z(\infty) > 0$ since $K'(s)$ is NI.

[2] Assuming that the circuit Σ is reciprocal implies that $D = D^\top$ as $Z(s)$ is symmetric.

Besides, to verify if $Z(s)$ is SNI is necessary to check condition 3) of Lemma 2. As this condition is an LMI, checking its feasibility is a convex programming problem that can be computed efficiently. This allows even to sweep over selected parameters of the circuit Σ to find a parameter set that results in the feasibility of the LMI condition.

Finally, last condition to check for the SNI property of $Z(s)$ is condition 4) of Corollary 3. Computing the transmission zeros numerically is an easy task and condition 4) of Corollary 3 is always meet except in the boundary of the LMI feasibility region of condition 3) of Lemma 2. Anyway, if condition 4) fails to meet, it can be always enforced by adding a small offset $\delta > 0$ in the LMI inequality,

$$AY + YA^\top \leq \delta I$$

of Lemma 2.

4.3 Sufficient local stability condition

If all the previous conditions are meet it can be applied the Theorem 4 to assure the stability of the whole system if the inequality in equation (3) holds. It is worth to remark that this inequality only depends on the gain, that is a function of the power and the port voltages at the equilibrium, and the resistive parameters of the circuit Σ.

To sum up, if $Z(s)$ is a strictly proper transfer function matrix with resistive elements and the CPLs are ideal then the method consist of three steps:

1) Check the feasibility of the LMI condition in Lemma 2. In this stage, if possible, some parameters of $Z(s)$ can be changed to enforce the feasibility of the condition.
2) Check if $Z(s) - Z(-s)^\top$ has no transmission zeros on the imaginary axis except at $s = 0$.
3) Check if condition in equation (3) of Theorem 4 holds.

If all the checks hold then the whole system will be locally stable.

5. EXAMPLES

The stability checking method proposed in Section 4 is illustrated with two examples: a single-port RLC circuit connected to a CPL and a two-port linear dc circuit with two CPLs.

5.1 Single-port circuit example

In this example, the method is applied to the one-port RLC network of Fig. 5. A minimal order state-space realization of the circuit is defined by the matrices

$$A = \begin{bmatrix} -RL^{-1} & -L^{-1} \\ C_1{}^{-1} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -C_1^{-1} \end{bmatrix}, C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D = [\,0\,]. \qquad (11)$$

Thus, from Section 3, $Z(s) = -[C(sI - A)^{-1}B + D]$ and, then,

$$Z(s) = \frac{Ls + R}{C_1 L s^2 + C_1 R s + 1}.$$

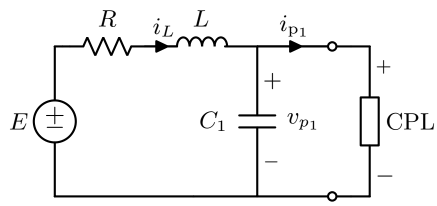

Fig. 5. One-port RLC network connected to a CPL.

Solving analitically condition 3) of Lemma 2, using the criterion from (Horn and Johnson (1990)[Corollary 7.1.5 and Theorem 7.2.5]), we obtain

$$R \geq \sqrt{\frac{L}{C_1}}. \tag{12}$$

that implies the NI property of $Z(s)$.

To test if $Z(s)$ is SNI we have to check condition 4) of corollary 3 obtaining

$$Z(s) - Z(-s)^\top = \frac{2s(C_1 L^2 s^2 + L - C_1 R^2)}{C_1^2 L^2 s^4 - C_1^2 R^2 s^2 + 2C_1 L s^2 + 1}.$$

The transmission zeros are $s = 0$ and

$$s = \pm\frac{\sqrt{C_1 R^2 - L}}{\sqrt{C_1} L}.$$

However, if inequality (12) is fulfilled, the last two zeros are always real. Accordingly, $Z(s)$ is SNI if and only if inequality (12) holds.

Now, from Theorem 4, the maximum allowable gain, K_{st}, to have a stable closed-loop can be computed as

$$\bar{\lambda}(Z(0)K_{st}) < 1 \Rightarrow K_{st} := \frac{1}{Z(0)}. \tag{13}$$

This result is strongly related to the existence of equilibria in this system, as shown previously in (Arocas-Pérez and Griñó (2016)). From the Thevenin's equation (6) and the CPL equation in (4) follows that, in the equilibria,

$$v_e^2 - F(0)v_e + Z(0)P_e = 0. \tag{14}$$

Enforcing a positive or null discriminant in (14) the power limit for the existence of equilibria is $P_{eq} := \frac{F(0)^2}{4Z(0)}$ and, for this value, the port voltage is $v_{eq} := \frac{F(0)}{2}$. Therefore, the maximum gain is

$$K_{eq} = \frac{P_{eq}}{v_{eq}^2} = \frac{1}{Z(0)}. \tag{15}$$

Hence, from (13) and (15), the limit of existence of equilibria and the limit of stability for the whole system have the same value. Moreover, the open-loop transfer function of the system $L(s) = (-Z(s))K$, see Fig. 4, in the limit of existence of equilibria K_{eq}, takes the value

$$L(0)_{eq} = (-Z(0))\frac{1}{Z(0)} = -1 \tag{16}$$

Considering that the Nyquist plot of a positive-feedback interconnection of systems, involves a sign change, that is, a 180° rotation, Theorem 4 and equation (16) are equivalent. So, for the one-port electrical network Σ, connected to a CPL, we have that the limit of existence of equilibria and the limit of stability of the system occurs when the open-loop transfer function polar plot at $\omega = 0$ (equivalently, the eigenvalue of the open-loop system at $\omega = 0$) departs from -1. Using the parameters of Table 1 and solving for the equilibrium point for an initial power value $P_0 = 3000\,\mathrm{W}$ results $v_0 = 16.58\,\mathrm{V}$ and the Nyquist plot in Fig. 6 shows that the closed-loop system is stable. However, $-L(s) = Z(s)K$ is not NI (think that it should be rotated 180^o). If the power of the CPL is progressively increased, at some power value the polar plot will encircle the -1 point and the closed-loop system, according to the Nyquist criterion, will be unstable.

Table 1. Single-port RLC network parameters.

E (V)	R (Ω)	L (H)	C_1 (F)
24	0.041	7.83e-6	3e-3

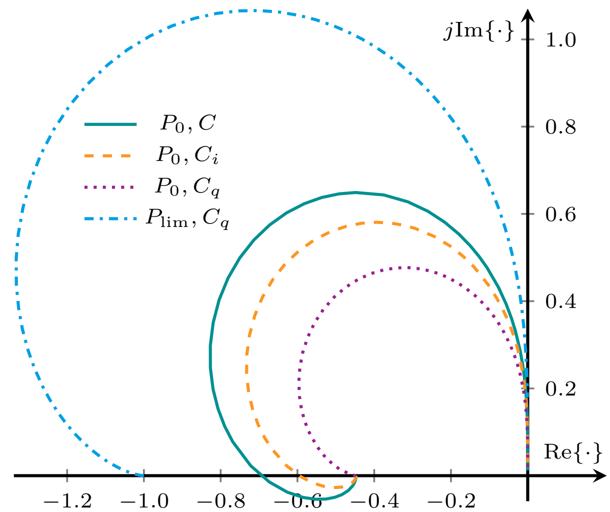

Fig. 6. Nyquist diagram of $L(s)$ for different capacitor values: a) (turquoise) $P_0 = 3000\,\mathrm{W}$, $v_0 = 16.58\,\mathrm{V}$, $C = 3\,\mathrm{mF}$, b) (orange) P_0, v_0, $C_i = 3.5\,\mathrm{mF}$, c) (violet) P_0, v_0, $C_q = 4.66\,\mathrm{mF}$ and d) (cyan) $P_{\mathrm{lim}} = 3512.2\,\mathrm{W}$, $v_{\mathrm{lim}} = 12\,\mathrm{V}$, C_q.

By placing an additional capacitor C' at the port, the capacitance can be increased to the minimum value $C_q = C_1 + C'$ that makes $-L(s)$ NI. In this way, the power can be raised to the limit of existence of equilibria maintaining the closed-loop stability. Note that, in the limit case, $L(0)$ is at the -1 point, see the curve for $P_{lim} = 3512.2\mathrm{W}, v_{lim} = 12\mathrm{V}, C_q = 4.66\mathrm{mF}$ in Fig. 6.

Solving the feasibility problem [3] in condition 3) of Lemma 2, the minimal value of the capacitor at the port results $C_q = 4.658$ mF that agree with the discussion in the previous paragraph. Using the analytical result of equation (12) the capacitance value is $C_q = 4.6579\,\mathrm{mF}$. As it can be seen, the values obtained by different methods agree except for numerical errors.

Fig. 7 shows the closed-loop stability and instability regions, as well as the region where the NI property holds, for a load power sweep against the capacitance value at the circuit port. The figure also shows that the NI property does not depend on the load, since it only depends on the circuit parameters (condition (12)). Another important fact that can be deduced from the graph is that, for power values below the limit of existence of equilibria, $P < P_{eq}$, the capacitance value at the port can be decreased below

[3] The LMI feasibility problems are solved using the software CVX (Grant and Boyd (2015)).

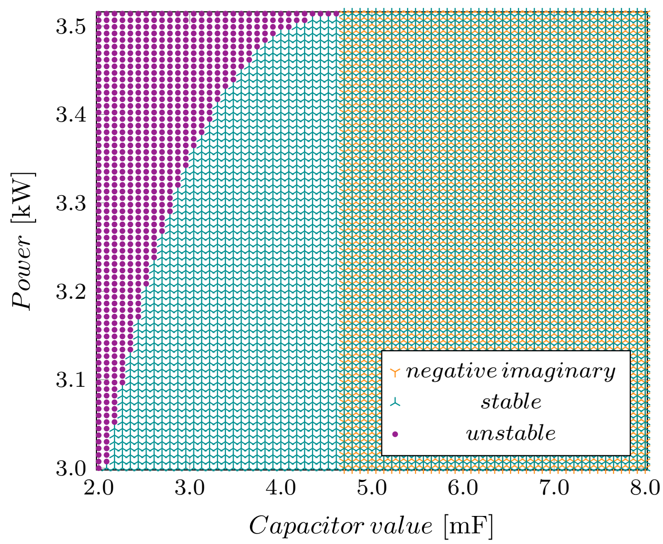

Fig. 7. Stability, instability and NI property of the one-port network for different values of power and capacitance.

C_q maintaining the stability of the system. Therefore, the criterion based on NI property is a sufficient, but not necessary, condition for stability.

5.2 Two-port circuit example

As an example of a multi-port case, the two-port circuit ($m = 2$) in Fig. 8, is selected. The system state-space matrices are

$$A = \begin{bmatrix} -R_1 L_1^{-1} & 0 & -L_1^{-1} & 0 \\ 0 & -R_2 L_2^{-1} & L_2^{-1} & -L_2^{-1} \\ C_1^{-1} & -C_1^{-1} & 0 & 0 \\ 0 & C_2^{-1} & 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -C_1^{-1} & 0 \\ 0 & C_2^{-1} \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (17)$$

With the circuit parameters of Table 2, $Z(s)$ does not meet the NI property. As we know that this property does not depend on loads power (that fix the operation point) additional capacitors, (C_1', C_2'), are connected to the ports. If $C_{q_1} = C_1 + C_1'$, $C_{q_2} = C_2 + C_2'$ are the minimum values of the capacitors that enforce $Z(s)$ to be NI then the closed-loop stability of the system is guaranteed for all the values of powers for which equilibria exist.

Table 2. Parameters for the circuit in Fig. 8

$R_1 = 0.04\ \Omega$	$L_1 = 78.0\ \mu$H	$C_1 = 2.0$ mF	$E = 24.0$ V
$R_2 = 0.06\ \Omega$	$L_2 = 98.0\ \mu$H	$C_2 = 1.0$ mF	

Following the steps at the end of Section 4 the procedure is:

1) Feasibility condition 3) of Lemma 2 is tested numerically for a grid of values of (C_{q_1}, C_{q_2}). Among the values that give $Z(s)$ NI we choose a pair that meets our design requirements ($C_{q_1} = 89$ mF, $C_{q_2} = 88$ mF).
2) For these values the transmission zeros of $Z(s) - Z(-s)^\top$ are obtained resulting that they have no transmission zeros on the imaginary axis except for $s = 0$. So, the circuit is SNI.
3) The last step is to find conditions, within the limits of existence of the equilibria, satisfying inequality (3) for the gain parameters that depend on the power of the loads. However, finding solutions for the set of quadratic equations of the system at equilibrium is not a trivial issue. In Proposition 3 of Barabanov et al. (2016) a necessary and sufficient condition, in terms of an LMI, for the existence of equilibria is given for the $m = 2$ case. This result is used to find the boundary for the existence of equilibria in terms of powers P_1, P_2. We choose an equilibrium point ($P_1 = 2860$ W, $P_2 = 533.6$ W) in a close vicinity of the boundary to show the method. The corresponding port voltages are $v_1 = 13.12$ V, $v_2 = 9.88$ V. With all these values the eigenvalues at $\omega = 0$ are
$$\lambda_1(Z(0)K) = 0.2199, \quad \lambda_2(Z(0)K) = 0.9913.$$
So, inequality (3) holds. Then, as K is NI and $Z(s)$ is SNI, the closed-loop stability of the system is assured.

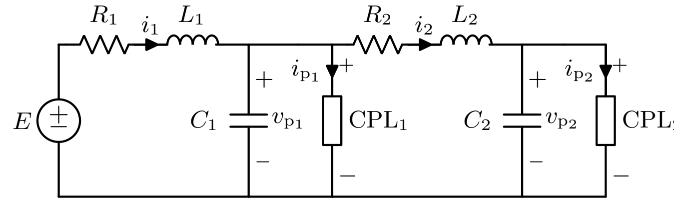

Fig. 8. Two-port linear circuit with CPLs.

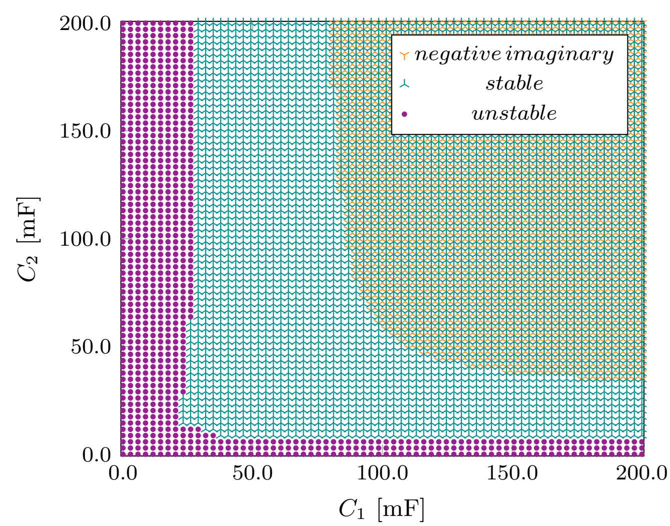

Fig. 9. Stability, instability and NI regions for a particular equilibrium point with respect to capacitor values at the ports.

Fig. 9 shows, for the equilibrium point ($P_1 = 2860$ W, $P_2 = 533.6$ W, $v_1 = 13.12$ V, $v_2 = 9.88$ V), the closed-loop stability and instability regions and the NI property region sweeping the capacitor values (C_1, C_2). Clearly, the NI approach gives a sufficient condition for the stability of the closed-loop system.

These results can be supported by an alternative way doing a graphical check, see Fig. 10, of the characteristic loci for $Z(j\omega)K$. As it can be seen, both characteristic

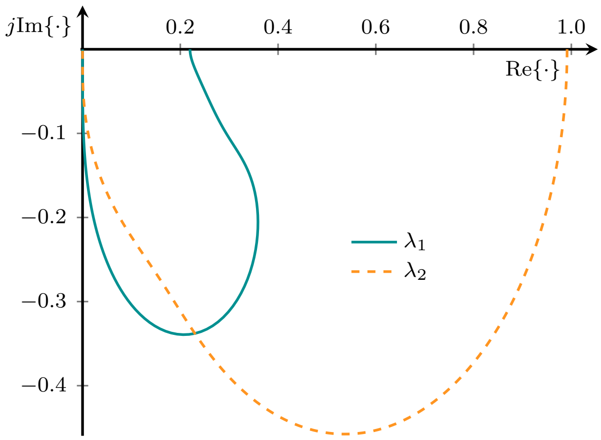

Fig. 10. Characteristic loci of $Z(s)K$ $\forall\omega > 0$ ($C_{q_1} =$ 89 mF, $C_{q_2} = 88$ mF, $P_1 = 2860$ W, $P_2 = 533.6$ W, $v_1 = 13.12$ V, $v_2 = 9.88$ V).

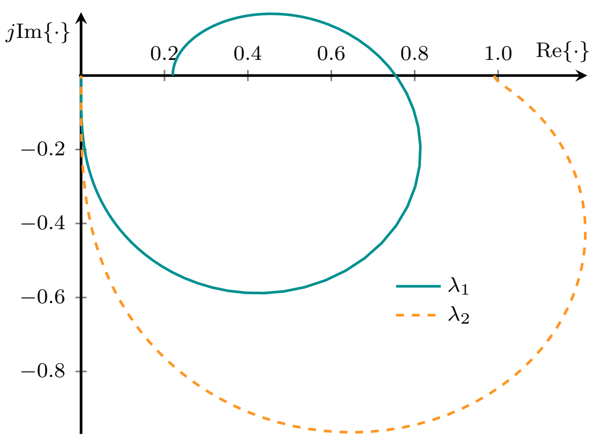

Fig. 11. Characteristic loci of $Z(s)K$ $\forall\omega > 0$ ($C_{q_1} =$ 37 mF, $C_{q_2} = 12$ mF, $P_1 = 2860$ W, $P_2 = 533.6$ W, $v_1 = 13.12$ V, $v_2 = 9.88$ V).

functions [4] (λ_1, λ_2) satisfy the Nyquist stability criterion (note that, for positive feedback, the critical point is $1 + j0$). Additionally, we realize that both functions are SNI.

From this graphical approach, two important facts for the multi-port cases, $m > 1$, are derived:

1) NI property is *sufficient* but not *necessary* for closed-loop system stability. The application of the LMI from Lemma 2 to a circuit transfer function matrix forces all characteristic functions of the matrix to hold the NI property. For example, in Fig. 10, it can be seen that for closed-loop stability it would not be necessary to have an NI λ_1, it suffices an NI λ_2 as it is shown in Fig. 11 for a lower value of the capacitors. The latter case gives a stable closed-loop system although not all the characteristic functions fulfill the NI property.
2) The limit of existence of equilibria is achieved when the absolute value of the maximum NI characteristic function $\bar{\lambda}_{NI}$ at $\omega = 0$ for the open-loop system $L = -Z(s)K$, is equal to 1. That is, $\left|\bar{\lambda}_{NI}(L(0))\right| = 1$.

[4] As defined in McFarlane (1977).

6. CONCLUSION

This work presents a sufficient condition for the local stability of dc LTI circuits with CPLs for all the possible equilibria (depending on the drained power) of the systems. This condition is shown as a method with successive steps that should be met. The method is illustrated with two examples: a single-port dc LTI RLC circuit with a constant voltage source connected to an ideal CPL and a two-port dc LTI circuit with a single constant voltage source connected to two CPLs.

Current ongoing research addresses the stability problem of dc LTI circuits with CPLs from a large signal point of view and it tries to quantify the size of the domain of attraction of the equilibria.

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