

Assembly of Customized Food Pantries in a Food Bank by Fuzzy Optimization

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Abstract:

Purpose: The contribution of this research is to propose a new problem of linear-mixed programming model (LMPM) for the allocation-packing of multiple pantries personalized for Food Banks (FB) considering the opinion of the Decision Maker (DM) in the selection of the best solution.

Design/methodology/approach: A food allocation-packing system is modeled as a mixed integer problem (MIP) and a fuzzy mixed integer linear problem (FMILP). 250 families and 100 products were considered. The solutions were found using Lingo 13® (for both deterministic and fuzzy model). To select a good solution in the fuzzy model, this research adapted an interactive method proposed in the literature. The relevance of this modification is that the opinion of a decision maker (DM) is included and considered.

Findings: The results for the deterministic and fuzzy model are compared in terms of their accomplishment of the restrictions (mainly nutritional and logistic) and the time needed to achieve a solution.

Research limitations/implications: This paper was done considering quantity, weight and volume restrictions so that the pantry will contain a variety of products; it is not considered how the products will be stored into the pantry.

Practical implications: This research proposes an alternative food management system at a food bank. The proposed system organizes the content of customized food pantries by the bias of a food allocation model.

Social implications: Our paper analyzes a Food Bank (FB) in México. With this proposal, food will be distributed to families in poverty considering their particular nutritional needs.

Originality/value: The main contribution of this article lies in the proposal of a new model of mixed integer linear problem (MILP) for the allocation=packing of food, solved with fuzzy possibilistic programming that simultaneously considers nutritional and logistic restrictions applied to a type of organization that has been little studied in the literature and where the opinion of Decision Maker (DM) is very important in the operational decisions involved in the Food Supply Chain (FSC) of a Food Bank (FB).

Keywords: humanitarian logistics, diet and food packing problems, fuzzy mixed-integer linear programming, perishable and nonperishable food, food bank

1. Introduction

This research is motivated by a problem in a food supply chain (FSC) of a food bank (FB). This food bank receives donated food, classifies the products, and separates food in good condition, which is the one that will enter the food bank. The food received in poor condition will be discarded. Different communities are attended every day, so after calculating the food required for the community to be attended, products are assigned to the communities. Then, these products are sent and the community leader receives and divides them equally. Each family will then receive the same number of products.

After several studies, the FB decided to operate differently. It was realized that different families have different nutritional needs, this is, the nutritional needs of a family formed by two adults and two children are different from the ones of a family formed by two seniors, for example. The new system will operate as follows: instead of delivering food and divided it equally, the food will be delivered in customized pantries. To fulfill the pantries, it will be taken into account the nutritional needs of the family that will receive it. Considering that the beneficiated families are used to receive the same number of products, the customized pantries must have certain volume and weight constraints, so that the families perceive that they receive an equal amount of products. The benefit of this system is that the families will receive

products according to their nutritional needs and in this way, the food bank will better accomplish its mission.

This new system, arises the problem of attributing food to customized pantries, taking into account the nutritional needs for the family that will receive it. When analyzing the problem, we found that it shares characteristics from both the diet problem (DP) and the food packing problem. In the diet problem, the aim is to determine the most economical combination of foodstuffs, in such a way that satisfies the minimum or maximum nutritional requirements. The food packing problem seeks to place together objects as densely as possible.

From the characteristics of the problem, we can identify several uncertainties. The pantries are composed of both packed products (can, box, etc.) and bulk products (fruits, vegetables, etc.). Consider for example, packed products for which the number of calories and weight is considered a constant, but bulk products (an apple) have an uncertain number of calories and weight. It was decided then to model and solve the problem as a possibilistic fuzzy model.

The contribution of this research lays in proposing a mixed integer linear programming (MILP) problem for the allocation-packing of multiple personalized food pantries for FB, considering the decision maker (DM) opinion in the selection of the best solution and simultaneously integrate parameters from nutritional and logistic restrictions in an environment of uncertainty through a possibilistic fuzzy programming model. Some related problems (allocation of limited resources in a FSC, diet problem (DP), food-packing problem (FPP)) consider some of the elements of our model but consider them independently and few jobs use fuzzy models. The uncertainty of the model parameters is represented by *triangular* fuzzy numbers and linear membership functions are used to jointly measure the degree of satisfaction of DM in meeting the target value and other model performance parameters established by the organization. The rest of his section comments on the reviewed literature, oriented toward fuzzy models and problems of food supply chain (FSC), the diet problem, and the food packing problem using this kind of models.

It is very difficult to dispose of complete information (data, values) in real-world problems. This is the reason why some times information needs to be calculated by approximation. Sahinidis (2004) mentions that stochastic programming and fuzzy programming are methods for optimization under uncertainty. In the first, uncertainty is modeled through continuous or discrete probability functions, while the second considers the random parameters as fuzzy numbers and constraints are analyzed as fuzzy sets. Although Zimmermann (2001) made the area of fuzzy mathematical programming (FMP) known, many of their contributions are based in the research of Bellman and Zadeh (1970).

Rommelfanger (1996) and Baykasoglu and Göcken (2008) perform a classification of problems with FPM and Sahinidis (2004) mentions two types: *flexible* (Equation 1) and *possibilistic* (Equation 2). The former deals with right-side uncertainties while the latter recognizes uncertainties in the objective function coefficients and in the constraint coefficients. In both cases, a membership function represents the degree of satisfaction of the constraints, the expectations of the decision maker (DM) on the level of the objective function and the range of uncertainty of the coefficients.

$$\widetilde{Max} c^t x, \text{ s.t. } Ax \lesseqgtr b, x \geq 0 \quad (1)$$

$$\widetilde{Max} c^t x, \text{ s.t. } \tilde{A}x \leq \tilde{b}, x \geq 0 \quad (2)$$

Although in Mula, Pedro, Díaz-Madroñero and Vicens (2010) and Wong and Lai (2011) introduced a summary of fuzzy sets, both applications and theory and FMP in the planning of production-transport in supply chains, it is observed that although the studies integrate fuzzy parameters in their models (demand, cost, capacity, availability), do not include nutritional restrictions related to food management (nutritional groups, perishable or non-perishable products, nutritional food supply).

Rong, Akkerman and Grunow (2011), Borghi, Guirardello and Cardozo (2009) and Cai, Chen, Xiao and Xu (2008) analyze the problem of handling-food distribution in FSC; they use linear programming (LP) and focus on logistic constraints without integrating a nutritional approach in their models. In the context of an agro-food supply chain (ASC), in Ahumada and Villalobos (2009), they carry out a literature review of the application of planning models in this type of chains; one of their conclusions indicates that the investigations have focused mainly on management of nonperishable products and although models based on LP have been studied, many of them focus on the problem of production and/or food harvest without integrating nutritional restrictions with logistics.

The DP was raised in 1941, but solved for the first time using linear programming (LP) by Stigler; however, there is little literature on its fuzzy approach (Vergara, Rodríguez & Saavedra, 2006). DP has been addressed with individual and integrated methods, although FMP is one of the areas with the greatest potential for future research (Rahman, Ang & Ramli, 2010). The review of these models indicates that the authors focus on nutritional aspects without considering the logistical limitations associated with the management and distribution of the food, for example: multiple allocations, capacity constraints and/or stock availability restrictions.

There is little literature found where DP is analyzed using FMP. Vergara et al. (2006) analyze the problem of the design of diets for poultry farms, where diets including fuzzy parameters are designed through an information system. Cadenas, Pelta, Pelta and Verdegay (2004) developed a software that allows determining the amount of each type of input that should be included in a beef diet that meets nutritional requirements at the lowest cost. The fuzzy parameter included in the model is the energy

contribution in food. SalooKolayi, Yansari and Nasserri (2010) analyzed the problem of formulating a diet for cows using fuzzy optimization. In all these cases, the FMP is used to include some fuzzy parameters and to solve exclusively the PD modeled in Rahman et al. (2010). However, these models are limited to food selection (X_i) for designing a diet but do not include food allocation (X_{ji}) subject to product availability or other important logistical constraints on food handling.

Mahalik and Nambiar (2010) and Brody, Bugusu, Han, Sand and McHugh (2008) mention that it has been found that the food packing problem (FPP) has been studied with focus on food preservation rather than as an optimization problem. The packing problem from an optimization approach involves placing objects together (within a container) as densely as possible. In this context, there have been few studies (Karuno, Nagamochi & Wang, 2007; Karuno, Nagamochi & Wang, 2010; Imahori, Karuno & Yoshimoto, 2010) of the FPP using mathematical modeling: they focus mainly on food selection to minimize the total weight of articles (there is a weight limit) and give priority in the selection of products with longer waiting times in hoppers, use binary decision variables, the restrictions focus on the weight of the products and do not take into account characteristics and nutritional restrictions for the product packaging in containers.

The results of our research will provide a FB with a system of food container allocation based on the availability and dimensional-nutritional characteristics of foods and on the energy requirements of the families served daily.

The rest of the document is organized as follows: Section 2 present the fuzzy model and the fuzzy mathematical programming, Section 3 present the results considering 100 types of food and 250 families. Finally, Section 4 concludes.

2. Model Presentation

2.1. The Fuzzy Model

The model proposed in this research (Figure 1) generates a selection-allocation of the type and amount of food that should be delivered daily to families with different characteristics. Foods can be classified according to their nutritional group in: vegetables, fruits, grains, dairy, meats, oils (United States Department of Agriculture, 2011) and their energy input measured in Kilocalories (Kcal.), depends on the type of food. The minimum energy requirements of each family are determined according to the number of members and characteristics: age, sex, physical activity, weight and height (United States Department of Agriculture, 2011).

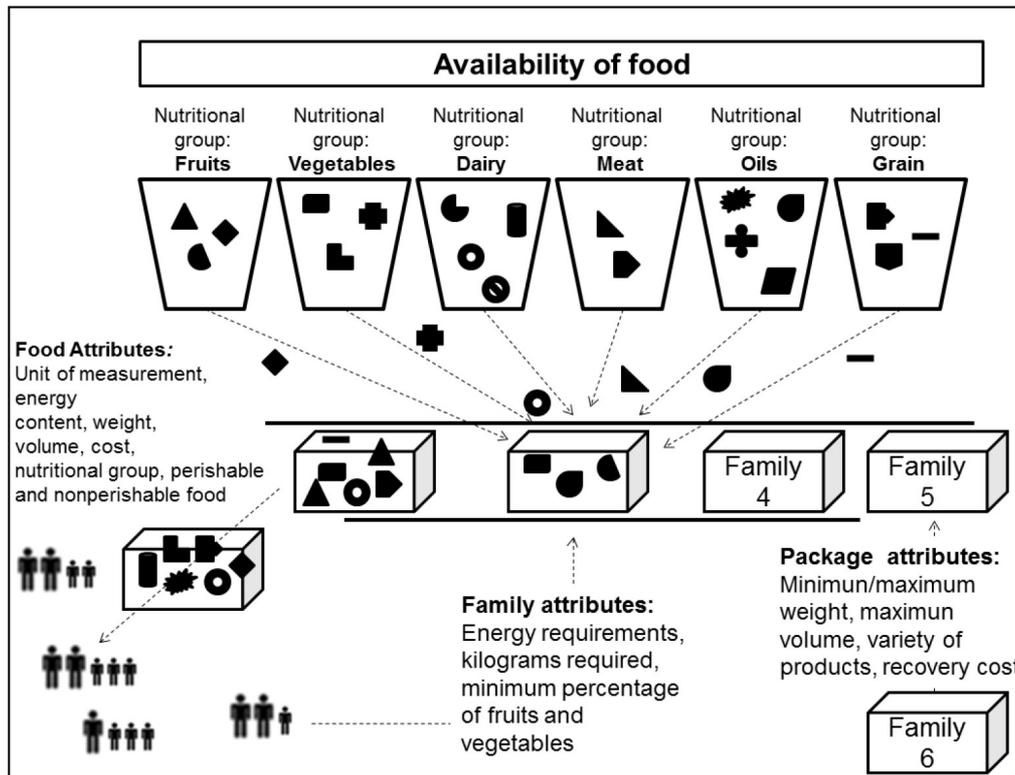


Figure 1. Food allocation-packing system to customize food pantries
(Cuevas-Ortuño & Gomez-Padilla, 2013)

The decision variables included in the model are:

X_{js} = Kilograms of the j food of the nutritional group i that is send to the s family.

Z_{iks} = Number of units of k food of the nutritional group i that is send to the s family.

ZF_{ils} = Number of fuzzy units of the l food of the nutritional group i that is send to the s family.

Parameters for the mathematical programming model (fuzzy parameters are shown with a tilde) are:

$\widetilde{APNK}_{i,j}$ = Energy content (kilocalories) per kilogram of food j of the nutritional group i .

β_{ij} = Available kilograms in stock of the food j of the nutritional group i .

Y_{ij} = Maximum allowed quantity (kilograms) of food j of the nutritional group i per food pantry.

$\widetilde{VK}_{i,j}$ = Volume (cm^3) per kilogram of food j in the nutritional group i .

CK_{ij} = Cost per kilogram of food j of the nutritional group i .

TA_{ij} = Type of food j of the nutritional group $i = \{1: \text{perishable food}, 0: \text{nonperishable food}\}$.

$\widetilde{APNPF}_{i,l}$ = Energy content (kilocalories) of fuzzy units of food l of the nutritional group i .

β_{il} = Available fuzzy units in the stock of food l of the nutritional group i .

$Y_{i,l}$ = Maximum allowed quantity (fuzzy units) of food l of the nutritional group i for food pantry

$\widetilde{VPF}_{i,l}$ = Volume (cm³) of fuzzy unit of food l of the nutritional group i .

$\widetilde{P}_{i,l}$ = Weight (kilograms) of fuzzy unit of food l of the nutritional group i .

$CP_{i,l}$ = Cost of the unit of food l of the nutritional group i .

$TA_{i,l}$ = Kind of food l of the nutritional group $i = \{1: \text{perishable food}, 0: \text{nonperishable food}\}$

$\widetilde{APNP}_{i,k}$ = Energy content (kilocalories) of unit of food k of the nutritional group i .

$\beta_{i,k}$ = Available pieces in the stock of food k in the nutritional group i .

$Y_{i,k}$ = Maximum allowed quantity (of units) of food k of the nutritional group i per food pantry.

$VP_{i,k}$ = Volume (cm³) of piece of food k of the nutritional group i .

$P_{i,k}$ = Weight (kilograms) per piece of the food k of the nutritional group i .

$CP_{i,k}$ = Cost per unit of food k of the nutritional group i .

$TA_{i,k}$ = Kind of food k of the nutritional group $i = \{1: \text{perishable food}, 0: \text{nonperishable food}\}$.

\widetilde{MR}_s = Minimum energetic requirement (Kcalories) for each s family.

Cre = Recovery cost of each food pantry. Cost established by the FB.

Φ = Minimum percentage of fruits and vegetables (in weight) that each food pantry should contain.

\widetilde{Ctr} = Approximate volume (cm³) of each proposed container for sending their food pantry to each family.

\widetilde{PMin} = Minimum approximated weight of the assigned food per container.

\widetilde{PMax} = Maximum approximated weight of the assigned food per container.

The *objective function* (Equation 3) maximizes the quantity of energy content (kilocalories) that is sent to all the families served in a day.

$$\text{Maximize } Y = \sum_{i=1}^6 \sum_{j=1}^n \sum_s^n [\widetilde{APNK}_{i,j}] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1}^n \sum_s^n [\widetilde{APNP}_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1}^n \sum_s^n [\widetilde{APNPF}_{i,l}] [ZF_{i,l,s}] \quad (3)$$

Supply constraints (Equations 4-6), assures that the assigned goods are available in stock.

$$\sum_s^n X_{i,j,s} \leq \beta_{i,j}, \forall i,j \quad (4)$$

$$\sum_s^n Z_{i,k,s} \leq \beta_{i,k}, \forall i,k \quad (5)$$

$$\sum_s^n ZF_{i,l,s} \leq \beta_{i,l}, \forall i,l \quad (6)$$

Minimum weight constraint (Equation 7), ensures that each pantry assigned to a family contains a minimum amount of food within the container.

$$\sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [\widetilde{P}_{i,l}] [ZF_{i,l,s}] \geq \widetilde{PMin}_s, \forall s \in S \quad (7)$$

Maximum weight constraint (Equation 8), ensures that each pantry assigned to a family contains maximum food within the container.

$$\sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [\widetilde{P}_{i,l}] [ZF_{i,l,s}] \leq \widetilde{PMax}_s, \forall s \in S \quad (8)$$

Volume constraint (Equation 9), allows the total volume of food selected for each custom pantry to not exceed the maximum volume of the container.

$$\sum_{i=1}^6 \sum_{j=1} [\widetilde{VK}_{i,j}] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1} [VP_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [\widetilde{VP}_{i,l}] [ZF_{i,l,s}] \leq \widetilde{Ctr}, \forall s \in S \quad (9)$$

Maximum amount of product (Equations 10-12) is included to ensure a greater variety of products in each pantry by not allowing a maximum amount of each type of food per container to be exceeded.

$$X_{i,j,s} \leq \gamma_{i,j}, \forall i,j,s \quad (10)$$

$$Z_{i,k,s} \leq \gamma_{i,k}, \forall i,k,s \quad (11)$$

$$ZF_{i,l,s} \leq \gamma_{i,l}, \forall i,l,s \quad (12)$$

Nutritional constraint (Equation 13), ensures that each food pantry sent to a family contains the minimum energy requirements.

$$\sum_{i=1}^6 \sum_{j=1} [APNK_{i,j}] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1} [APNP_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [APNPF_{i,l}] [ZF_{i,l,s}] \geq \widetilde{MR}_s, \forall s \in S \quad (13)$$

Nutritional group constraint (Equation 14), allows the food bank to set the minimum percentage (α) of fruits and vegetables (in weight) that each food pantry will contain.

$$\sum_{j=1} X_{1,j,s} + \sum_{j=1} X_{2,j,s} + \sum_{k=1} [P_{1,k}] [Z_{1,k,s}] + \sum_{k=1} [P_{2,k}] [Z_{2,k,s}] + \sum_{i=1}^2 \sum_{l=1} [\widetilde{P}_{i,l}] [\widetilde{ZF}_{i,l,s}] - \left[\left(\frac{\varphi_s}{100} \right) * \left[\sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [\widetilde{P}_{i,l}] [ZF_{i,l,s}] \right] \right] \geq 0, \forall s \in S \quad (14)$$

Food pantry cost constraint (Equation 15), ensures that the cost of products included in a food parcel do not exceed the recovery cost***.

$$\sum_{i=1}^6 \sum_{j=1} [CK_{i,j}] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1} [CP_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} [CPF_{i,l}] [ZF_{i,l,s}] \leq Crec, \forall s \in S \quad (15)$$

Non-negativity constraints (Equations 16-18).

$$X_{i,j,s} \geq 0 \tag{16}$$

$$Z_{i,k,s} = \{0,1,2,3,\dots,q\} \tag{17}$$

$$ZF_{i,l,s} = \{0,1,2,3,\dots,t\} \tag{18}$$

***The *recovery cost* (C_{rec}) is a payment (≤ 100 mexican pesos) that has to be made by the beneficiary (family) for each pantry received. This payment helps Food Bank to purchase non-perishable items that are not donated and have to be purchased.

2.2. Fuzzy Mathematical Programming Approach

The fuzzy model presented in the previous section incorporates fuzzy constraints (technological coefficients and right-hand coefficients) as well as fuzzy coefficients of the objective function. Under this perspective, several papers have been published to propose methods to solve these kinds of problems. In this paper, the solution method used was proposed by Jimenez, Arenas, Bilbao and Rodríguez (2007) and Peidro, Mula, Jiménez and Botella (2010).

According to Peidro et al. (2010), a linear programming problem with fuzzy parameters is defined as:

$$\text{Minimize } z = \tilde{c}^t x$$

Subject to

$$X \in N(\tilde{A}, \tilde{b}) = \{x \in R^n | \tilde{a}_i x \geq \tilde{b}_i, i = 1, \dots, m, x \geq 0\} \tag{19}$$

Where, $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_n)$, $\tilde{A} = [\tilde{a}_{i,j}] m \times n$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m)$ are considered as fuzzy parameters and they are included in the objective function as well as in the constraints. These fuzzy parameters are represented by fuzzy numbers using a possibility distribution. $X = (X_1, X_2, \dots, X_n)$ is the crisp decision vector. A fuzzy set A of a universe Ω is characterized by its membership function $\mu_A : \Omega \rightarrow [0,1]$. Where $r = \mu_A(x)$; $x \in \Omega$, is the degree of membership from x to A . It is then posed:

Minimize $EV(\tilde{c})x$, subject to (Equation 20)

$$[(1 - \alpha)E_2^{ai} + \alpha E_1^{ai}]x \geq \alpha E_2^{bi} + (1 - \alpha)E_1^{bi}, i = 1, \dots, m, x \geq 0, \alpha \in [0,1] \tag{20}$$

Where, Peidro et al. (2010) showed that, for a fuzzy number, the expected value denoted $EV(\tilde{c})$, is half of its expected interval and $0 \leq \alpha \leq 1$ is a cut-off value that can be parametrically established by the DM, thus, in Peidro et al. (2010) indicates that to transform the problem (Equation 19) into the crisp equivalent parametric linear programming problem defined in (Equation 20), we can use an approach described by Jimenez (1996), where α represents the degree in which the constraints are fulfilled (at least); that is, the feasibility degree of a decision x is represented by α .

$$EV(\tilde{c}) = \left[\frac{E_1^c + E_2^c}{2} \right] \tag{21}$$

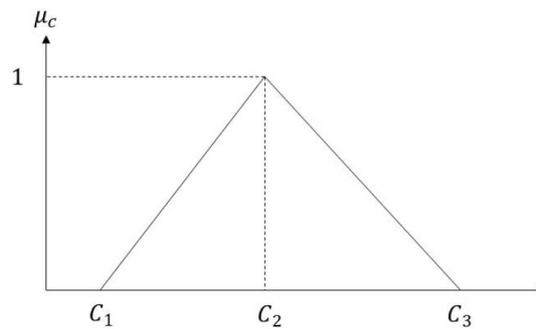


Figure 2. Triangular fuzzy number, c

From Jimenez et al. (2007), the expected range and value of a fuzzy number, when it is *triangular* (Figure 2), may be calculated by (Equation 22):

$$EI(\tilde{c}) = \left[\frac{1}{2} (c_1 + c_2), \frac{1}{2} (c_2 + c_3) \right]; EV(\tilde{c}) = \left[\frac{c_1 + 2c_2 + c_3}{4} \right] \tag{22}$$

If (Equation 20) was a constraint of the type less than or equal, \leq , according (Peidro et al., 2010) it could be transformed into the following constraint equivalent crisp:

$$\left[(1 - \alpha)E_1^{ai} + \alpha E_2^{ai} \right] x \leq \alpha E_1^{bi} + (1 - \alpha)E_2^{bi}, i = 1, \dots, m, x \geq 0, \alpha \in [0,1] \tag{23}$$

As a result, we can obtain an auxiliary crisp mixed-integer linear programming model (MILP) fitting the approach of (Equation 22) to the FMILP model defined in Section 2.1:

$$\begin{aligned} & \text{Maximize } Y = \\ & \sum_{i=1}^6 \sum_{j=1} \sum_S^n \left[\frac{APNK_{i,j,1} + 2APNK_{i,j,2} + APNK_{i,j,3}}{4} \right] [X_{i,j,s}] + \\ & \sum_{i=1}^6 \sum_{k=1} \sum_S^n \left[\frac{APNP_{i,k,1} + 2APNP_{i,k,2} + APNP_{i,k,3}}{4} \right] [Z_{i,k,s}] + \\ & \sum_{i=1}^2 \sum_{l=1} \sum_S^n \left[\frac{APNPF_{i,l,1} + 2APNPF_{i,l,2} + APNPF_{i,l,3}}{4} \right] [ZF_{i,l,s}] \end{aligned} \tag{24}$$

Subject to:

$$\begin{aligned} & \sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \\ & \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{P_{i,l2} + P_{i,l3}}{2} \right) + \alpha \left(\frac{P_{i,l1} + P_{i,l2}}{2} \right) \right] [ZF_{i,l,s}] \geq \alpha \left(\frac{PMin_{s2} + PMin_{s3}}{2} \right) + (1 - \\ & \alpha) \left(\frac{PMin_{s1} + Ps2}{2} \right), \forall s \in S \end{aligned} \quad (25)$$

$$\begin{aligned} & \sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \\ & \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{P_{i,l1} + P_{i,l2}}{2} \right) + \alpha \left(\frac{P_{i,l2} + P_{i,l3}}{2} \right) \right] [ZF_{i,l,s}] \leq \alpha \left(\frac{PMin_{s1} + PMin_{s2}}{2} \right) + (1 - \\ & \alpha) \left(\frac{PMin_{s2} + Ps3}{2} \right), \forall s \in S \end{aligned} \quad (26)$$

$$\begin{aligned} & \sum_{i=1}^6 \sum_{j=1} \left[(1 - \alpha) \left(\frac{VK_{i,j1} + VK_{i,j2}}{2} \right) + \alpha \left(\frac{VK_{i,j2} + VK_{i,j3}}{2} \right) \right] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1} [VP_{i,k}] [Z_{i,k,s}] + \\ & \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{VPP_{i,l1} + VPP_{i,l2}}{2} \right) + \alpha \left(\frac{VPP_{i,l2} + VPP_{i,l3}}{2} \right) \right] [ZF_{i,l,s}] \leq \alpha \left(\frac{Ctr_{s1} + Ctr_{s2}}{2} \right) + (1 - \\ & \alpha) \left(\frac{Ctr_{s2} + Ctr_{s3}}{2} \right), \forall s \in S \end{aligned} \quad (27)$$

$$\begin{aligned} & \sum_{i=1}^6 \sum_{j=1} \left[(1 - \alpha) \left(\frac{APNK_{i,j2} + APNK_{i,j3}}{2} \right) + \alpha \left(\frac{APNK_{i,j1} + APNK_{i,j2}}{2} \right) \right] [X_{i,j,s}] + \sum_{i=1}^6 \sum_{k=1} \left[(1 - \right. \\ & \alpha) \left(\frac{APNP_{i,k2} + APNP_{i,k3}}{2} \right) + \alpha \left(\frac{APNP_{i,k1} + APNP_{i,k2}}{2} \right) \left. \right] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{APNPF_{i,l2} + APNPF_{i,l3}}{2} \right) + \right. \\ & \left. \alpha \left(\frac{APNPF_{i,l1} + APNPF_{i,l2}}{2} \right) \right] [ZF_{i,l,s}] \geq (1 - \alpha) \left(\frac{MR_{s1} + MR_{s2}}{2} \right) + \alpha \left(\frac{MR_{s2} + MR_{s3}}{2} \right), \forall s \in S \end{aligned} \quad (28)$$

$$\begin{aligned} & \sum_{j=1} X_{1,j,s} + \sum_{j=1} X_{2,j,s} + \sum_{k=1} [P_{1,k}] [Z_{1,k,s}] + \sum_{k=1} [P_{2,k}] [Z_{2,k,s}] + \\ & \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{P_{i,l2} + P_{i,l3}}{2} \right) + \alpha \left(\frac{P_{i,l1} + P_{i,l2}}{2} \right) \right] [ZF_{i,l,s}] - \left[\left(\frac{\varphi_s}{100} \right) * \left[\sum_{i=1}^6 \sum_{j=1} X_{i,j,s} + \right. \right. \\ & \left. \left. \sum_{i=1}^6 \sum_{k=1} [P_{i,k}] [Z_{i,k,s}] + \sum_{i=1}^2 \sum_{l=1} \left[(1 - \alpha) \left(\frac{P_{i,l2} + P_{i,l3}}{2} \right) + \alpha \left(\frac{P_{i,l1} + P_{i,l2}}{2} \right) \right] [ZF_{i,l,s}] \right] \right] \geq 0, \forall s \in S \end{aligned} \quad (29)$$

The non-fuzzy constraints (Equation 4-6), (Equation 10-12) and (Equation 15) are included in the model as in the original way.

Jimenez et al. (2007) showed that in order to get a decision vector that complies with the expectations of the DM, two conflicting factors should be evaluated: the feasibility degree (α) and the reaching for an acceptable value for the objective function (Y). To solve the problem, α need to be established parametrically to obtain the value of the objective function for each of the $\alpha \in [0,1]$. The result is a fuzzy set and the DM would have to decide which pair (α, Y) considers optimal if it wants to get a crisp solution. After knowing the information given for the different $(Y_0) (\alpha_k)$, the DM must specify a goal \tilde{G} and its tolerance interval G . Therefore, if $Y \geq G$, the TM will be completely satisfied, but if $Y \leq G$ its degree of satisfaction will be null. The goal is expressed by means of a fuzzy set G where its membership function Pedro et al. (2010) is as shown in (Equation 30).

$$\mu_{\tilde{G}}(Z) = \begin{cases} 1, & \text{if } Z \geq G \\ \alpha \in [0,1] \text{ increasing on } G - t \leq Z \leq G \\ 0, & \text{if } Z \leq G - t \end{cases} \quad (30)$$

To obtain a solution (balanced) between the *satisfaction degree* the *target value has* and the *feasibility degree* α , from Jimenez et al. (2007), we calculate:

$$\omega_{\alpha} = \mu_{\tilde{D}}(X^*) = \alpha_k * K_{\tilde{G}}(\tilde{Z}^0(\alpha_k)) \quad (31)$$

To establish a decision vector that considers the expectations of DM, its level of satisfaction is evaluated in the search for an acceptable value for different performance parameters previously defined by DM. The DM shall specify an aspiration level G and its tolerance interval t , for the numerical value obtained in each evaluation parameter. In the case of a “*greater is better*” the level of satisfaction of the DM will be expressed by a fuzzy set G . The membership function is (Equation 30) and in the case of “*minor is better*” as in (Equation 32):

$$\mu_{\tilde{G}}(Z) = \begin{cases} 1, & \text{if } Z \leq G \\ \alpha \in [0,1] \text{ decreasing on } G \leq Z \leq G + t \\ 0, & \text{if } Z \geq G + t \end{cases} \quad (32)$$

From Peidro et al. (2010), the degree $[\lambda_i (i = 1, 2, 3)]$ to which the corresponding fuzzy aspiration levels (previous parameters) are satisfied and the weight $w_i (i = 1, 2, 3)$ can be assigned by the DM to indicate the importance level of the parameter with respect to all others. The global satisfaction degree is obtained as:

$$\Psi = \sum_{i=1}^3 w_i \lambda_i = w_1 \lambda_1 + w_2 \lambda_2 + w_3 \lambda_3; \text{ where } \sum_{i=1}^3 w_i = 1 \quad (33)$$

According to (Peidro et al., 2010), to find a balanced solution between the satisfaction degree of the objective value (ω_{α}) and the satisfaction degree of the global performance in the parameters (ω_{Ψ}) that is calculated by the DM through the membership function (Equation 31), a recommendation for a final decision is obtained by means of a joint acceptance index K with the two degrees of acceptance:

$$K = \begin{cases} \beta \cdot \omega_{\alpha} + (1 - \beta) \cdot \omega_{\Psi}, & \text{if } \omega_{\alpha} \neq 0 \text{ and } \omega_{\Psi} \neq 0 \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

Where, DM assigns the relative importance: $\beta \in [0, 1]$ between the satisfaction and the feasibility.

3. Results

3.1. Application to a FB

The proposed model has been evaluated considering 100 types of food. Table 1 presents in detail the number of food of each type from the six nutritional groups considered. For example, from the nutritional group of vegetables, there are 12 types of vegetables measured in kilograms, 5 types of vegetables measured with deterministic units and 17 vegetables measured in fuzzy units.

Nutritional group (<i>i</i>)	Types of unit of measurement	Types of food (<i>j</i>)
Vegetables	Kilograms	12
	Deterministic units	5
	Fuzzy units	17
Fruits	Kilograms	2
	Deterministic units	4
	Fuzzy units	11
Grains	Kilograms	2
	Deterministic units	14
Diary	Kilograms	1
	Deterministic units	24
Meat	Kilograms	2
	Deterministic units	2
Oils	Kilograms	2
	Deterministic units	2

Table 1. Types of food considered in experiments

The number of families included in the simulation is 250. Figure 3 presents the information about the composition of the families. We can see in the histogram that most of the families have 5 members, but there are also families composed by 2 members as well as families of 10 members.

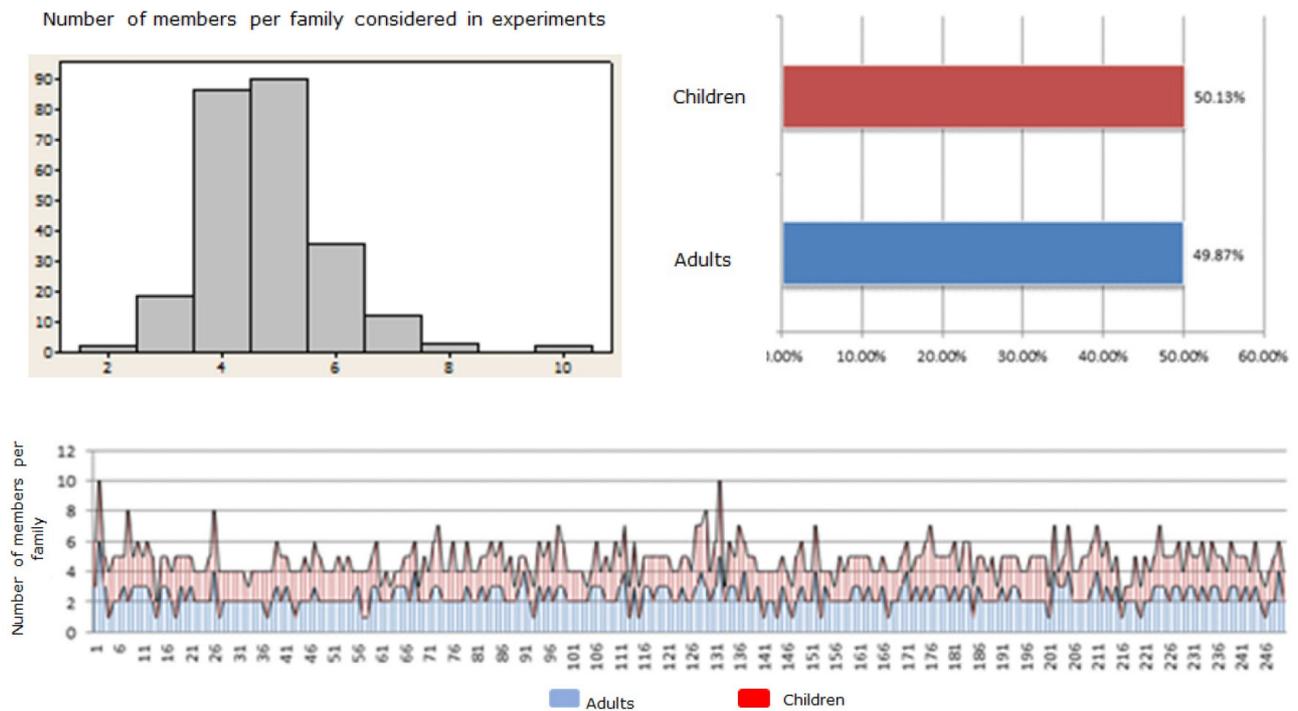


Figure 3. Number of members per family considered in experiments

In Figure 3 we can also see that there are about the same number of children and adults.

Other assumptions of the model are: $C_{rec} = \$100$, $\varphi_s = 30\%$, $\widetilde{PMin} \cong 30$, $\widetilde{PMax} \cong 40$, $\widetilde{Ctr} \cong 103,587 \text{ cm}^3$. Triangular fuzzy numbers were defined by the decision maker (DM) as deviation percentages of the crisp value. These percentages vary from 3 to 20% depending on the parameter. The decision variables $Z_{i,k,s}$ and ZF_{ils} are considered as *integer* because they are going to be handled as *units*, meanwhile $X_{i,j,s}$ is considered as *continue* when it is handled as *kilograms*. It was considered a maximum CPU time of solution of 300 seconds. Therefore, the model is solved as a mixed-integer linear programming model (MILP).

3.2. Implementation and Solution

The architecture model used for the implementation and resolution of the model described above is illustrated in Figure 4. The experiments of the model were performed integrating the optimization software Lingo 13® and a spreadsheet utilized to input and output data of the model. A computer with an Intel® Core 2 Duo processor and 4.0 GB of RAM memory was used.

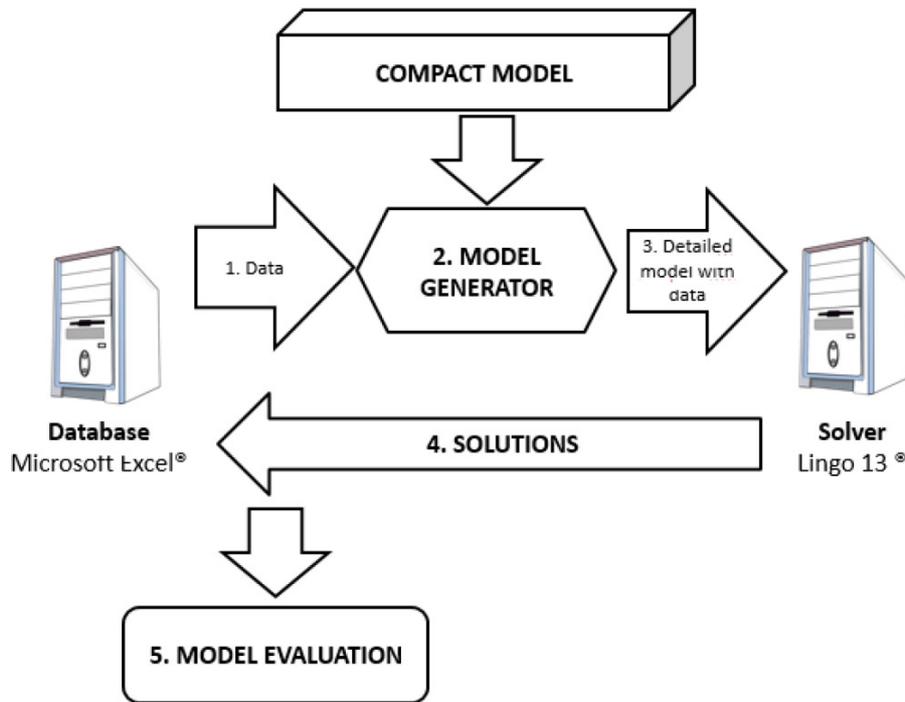


Figure 4. Diagram of computational experiments (adapted from Mula et al. 2010)

3.3. Discussion of Results

In this section, we compare the results obtained with the fuzzy model against the deterministic model. It is intended to analyze the possible improvements offered by the fuzzy model which incorporates the uncertainties present in a FB supply chain. Table 2 presents a comparison between the computational efficiency of the deterministic model and the fuzzy model. The analysis indicates that the number of variables and decisions is the same, because according to Peidro et al. (2010) one of the advantages of this method is not to increase the number of variables or restrictions in the fuzzy model. The only exception is for the case of the equality type constraints that should be transformed into two equivalent constraints for the fuzzy model if it was necessary.

Model	Iterations*	Variables	Integers	Constraints	CPU Time (seconds)
Deterministic	71,192	25,000	19,750	26,601	78
Fuzzy	80,642	25,000	19,750	26,601	246.27

Table 2. Efficiency of the computational experiments

Figure 5 shows the optimal solution (kilocalories) obtained from the deterministic model and from the FMILP for each cut $\alpha \in [0, 1]$. We can see that for $\alpha = 0.5$, $\alpha = 0.9$ and $\alpha = 1$, the kilocalories allocated are the same for the deterministic and fuzzy model.

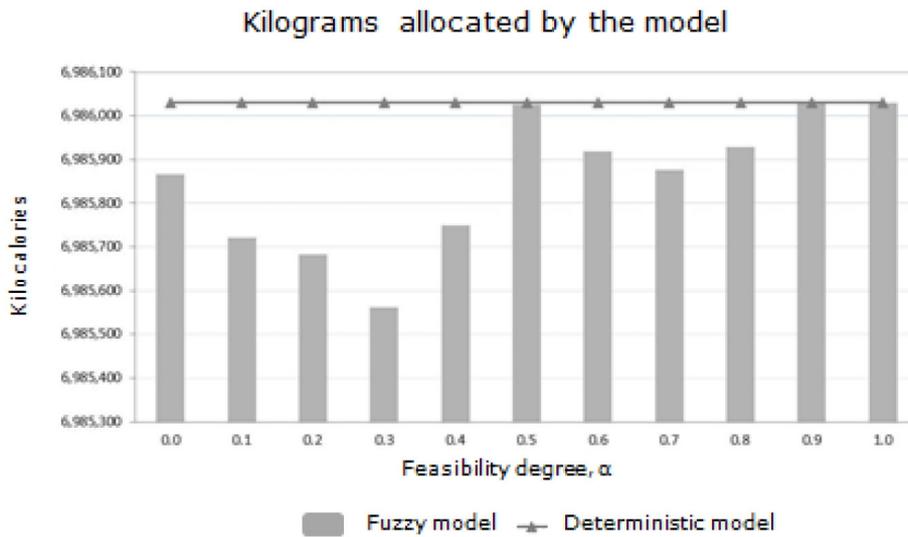


Figure 5. Variation of food allocation (kilocalories) for each α

Then, based on (Equation 30) and Figure 6, the DM specifies an aspiration level $G(G = 6,985,929.58 \text{ Kcal.})$ and its tolerance threshold $t(G - t = 6,985,866.0 \text{ Kcal.})$, to calculate $\omega_\alpha = \alpha_k * K_{\tilde{G}}(\tilde{Z}^0(\alpha_k))$ in Table 3. The analysis indicates the best cut- $\alpha = 1.0$ solution considering the target value.

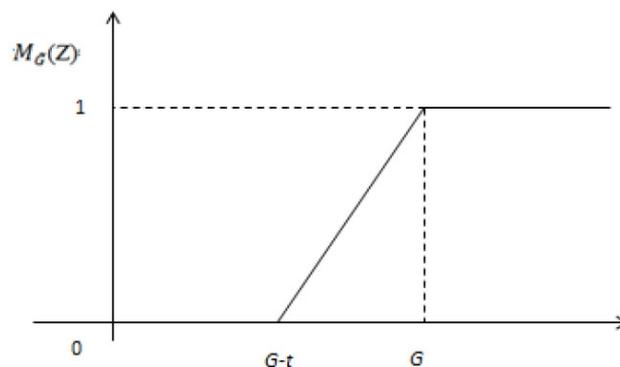


Figure 6. Variation of the food allocation performed by the model for each α .

Feasibility degree (α)	Objective Value (Kilocalories)	$K_{\bar{c}}(\bar{Z}^0(\alpha_k))$	Acceptation degree (α) ω_α
0	6,985,866.30	0.0048	0.0000
0.1	6,985,721.47	0.0000	0.0000
0.2	6,985,683.05	0.0000	0.0000
0.3	6,985,562.61	0.0000	0.0000
0.4	6,985,748.75	0.0000	0.0000
0.5	6,986,025.25	1.0000	0.5000
0.6	6,985,919.54	0.8421	0.5053
0.7	6,985,875.03	0.1578	0.1105
0.8	6,985,929.58	1.0000	0.8000
0.9	6,986,030.47	1.0000	0.9000
1	6,986,029.87	1.0000	1.000

Table 3. Satisfaction degree of the DM in the fulfillment of objective value and α

Three other parameters have been considered key by DM to include them in the evaluation of results of the model: average percentage of vegetables and fruits (δ_1), average food variety (δ_2), average percentage of nonperishable food (δ_3); All indicators are considered per family-container. The weight w_i is defined by the DM for each parameter. The satisfaction degree ψ (Equation 33) is calculated with the following weights assigned by DM: $\psi = \sum_{i=1}^3 w_i \lambda_i = 0.2\lambda_1 + 0.5\lambda_2 + 0.3\lambda_3$

Satisfaction degrees (λ_i where calculated according to (30) for δ_1 ($G = 77.7\%$, $G - t = 77.1\%$) y δ_2 ($G = 21.224$, $G - t = 20.688$), and (32) is used for δ_3 ($G = 10.2\%$, $G + t = 10.0\%$) and can be established by the DM.

Table 4 summarizes the value of the satisfaction degree ψ per cut- α .

From the previous analysis, the best solution by indicator would be: $\delta_1 = 0.1$, $\delta_2 = 0.9$ and $\delta_3 = 0.7$. Finally, the satisfaction degree of acceptance, ω_ψ , is calculated based on a fuzzy set in which its function of membership (Equation 30) represents the degree of acceptance of the DM for the parameters δ_1 and δ_2 and (Equation 32) for the parameter δ_3 . To obtain a recommendation for a final decision, a joint acceptance index of the set K according to (Equation 34) is calculated. Table 5 summarizes the K values for each α .

Feasibility degree (α)	Objective Value (Kilocalories)	$\delta_{1(\%)}$	δ_2	$\delta_{3(\%)}$	λ_1	λ_2	λ_3	ψ
0	6,985,866.30	77.6183%	20.87	10.1566%	0.8411	0.3358	0.2491	0.4109
0.1	6,985,721.47	77.7132%	20.98	10.1906%	1.0000	0.5373	0.0384	0.4802
0.2	6,985,683.05	77.5229%	20.73	10.1968%	0.6810	0.0821	0.0000	0.1772
0.3	6,985,562.61	77.5780%	20.82	10.1436%	0.7732	0.2463	0.3296	0.3767
0.4	6,985,748.75	77.6053%	20.84	10.0956%	0.8192	0.2836	0.6277	0.4939
0.5	6,986,025.25	77.3687%	20.69	10.0404%	0.4224	0.0000	0.9696	0.3754
0.6	6,985,919.54	77.4969%	20.79	10.0830%	0.6373	0.1866	0.7057	0.4325
0.7	6,985,876.03	77.4640%	20.74	10.0355%	0.5822	0.0970	1.0000	0.4650
0.8	6,985,929.58	77.4323%	20.74	10.1032%	0.5291	0.0896	0.5803	0.3247
0.9	6,986,030.47	77.2167%	21.22	10.1454%	0.1676	1.0000	0.3186	0.6291
1	6,986,029.87	77.1167%	21.12	10.1529%	0.0000	0.8060	0.2721	0.4846
Deterministic model		77.5683%	20.95	10.11%	0.7570	0.4851	0.5224	

Table 4. Calculation of satisfaction levels by indicator for each α

Feasibility degree (α)	Objective Value (Kilocalories)	Acceptation degree (α) ω_α	Acceptation degree (ψ), ω_ψ	$\beta = 0.5$ Joint acceptance index, K
0	6,985,866.30	0.0000	0.4059	0.20293
0.1	6,985,721.47	0.0000	0.8678	0.43389
0.2	6,985,683.05	0.0000	0.0000	0.00000
0.3	6,985,562.61	0.0000	0.1778	0.08891
0.4	6,985,748.75	0.0000	0.9595	0.47974
0.5	6,986,025.25	0.5000	0.1691	0.33454
0.6	6,985,919.54	0.5053	0.5497	0.52748
0.7	6,985,876.03	0.1105	0.7663	0.43840
0.8	6,985,929.58	0.8000	0.0000	0.40000
0.9	6,986,030.47	0.9000	1.0000	0.95000
1	6,986,029.87	1.0000	0.8975	0.94874

Table 5. Calculation of the joint satisfaction index (K) for each α

Considering the results presented in Table 5, it is observed that, for a neutral decision ($\beta = 0.5$) between the satisfaction degree ω_α and ω_ψ , the best selection for the DM should be the solution of the model (3-18) obtained with $\alpha = 0.9$.

Figure 7 shows a comparative analysis of nutritional and logistic characteristics in pantries configured by the deterministic model and the fuzzy model ($\alpha = 0.9$) selected as the best solution by DM.

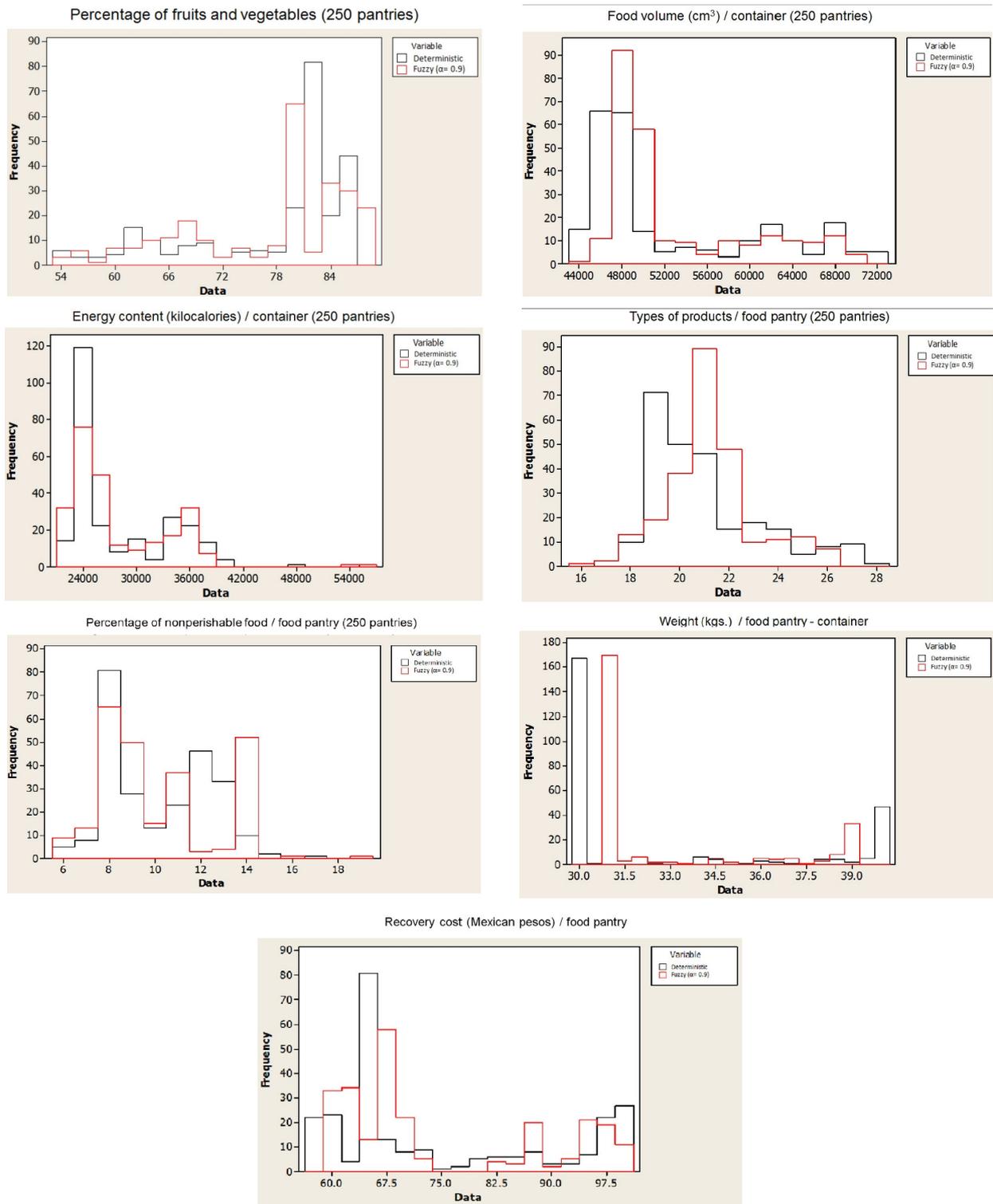


Figure 7. Comparative analysis between deterministic model and fuzzy model ($\alpha = 0.9$) per family

The previous analysis indicates a better performance of the fuzzy model ($\alpha = 0.9$) than the deterministic in several nutritional parameters (food variety by pantry-container) and logistic (container pantry weight and volume).

Finally, Figure 8 shows a comparative analysis by family of the characteristics (nutritional and logistic) of the pantries configured by both models.

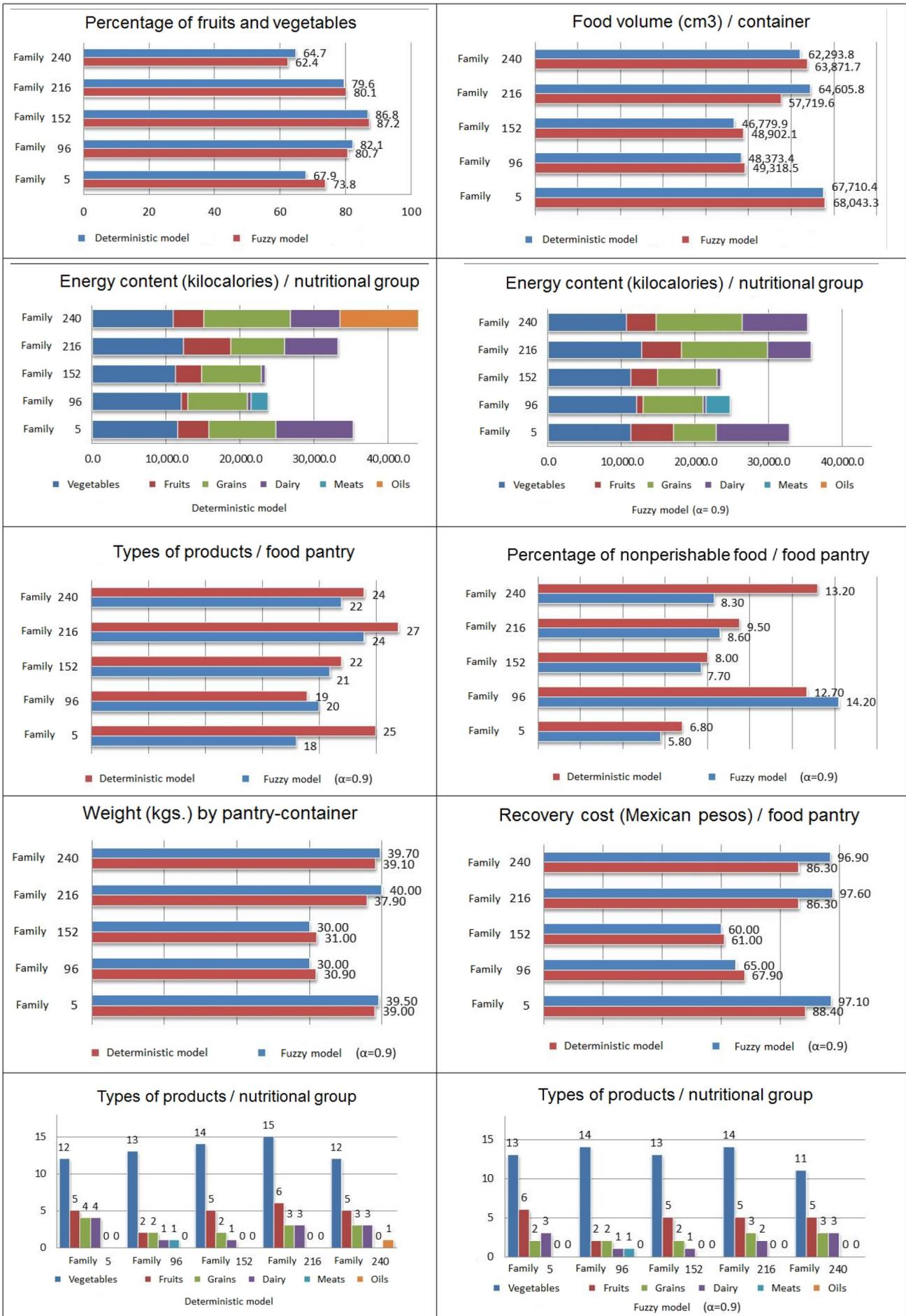


Figure 8. Comparative analysis between deterministic model and fuzzy model ($\alpha = 0.9$) per family

We can see from Figure 8 that there is not a big difference between the results for the deterministic and the fuzzy model in terms of percentage of fruits and vegetables in the pantry, the volume and the weight per pantry.

4. Conclusions

This article proposes a new model of MILP that allows to make food allocations for customized pantries. The originality of the model lies in the approach and solution of a new problem that simultaneously integrates parameters and nutritional and logistic restrictions that have been studied in problems separately. By means of our model, the DM in a FB will be able to realize food allocations considering restrictions related to the handling of foods in a SC (food availability, cost of pantry, amount and weight demanded of food, preservation of food by means of containers, etc.) and nutritional (energy supply of food, energy requirement of families, quantity of perishable product, minimum quantity required by nutritional group). Since some information about the process and the characteristics of some foods cannot be accurately known, our research proposes a fuzzy programming model in which is possible to include this uncertainty in the information of some parameters of the model. This article further considers a method of selecting the best solution found that integrates the methods proposed by Jimenez et al. (2007) and Peidro et al. (2010). Through this methodology, the DM in the FB can select the best solution based on the global satisfaction degree of parameters defined by the same organization.

The computational performance of the deterministic model and the fuzzy model was analyzed through the application to a real case in a FB of México, showing a good performance in solution time and obtained results. The main contribution of this article lies in the proposal of a new model for the allocation-food packaging resolved with fuzzy possibilistic programming that simultaneously considers nutritional and logistic restrictions applied to a type of organization that has been little studied in the literature Gopakumar, Koli, Srihari, Sundarma and Wang (2008), Nguyen, Godbole, Kalkundri and Lam (2009), Okore-Hanson, Winbush, Davis and Jian (2012), Sengul, Ivy and Uzsoy (2016) and Cuevas-Ortuño and Gomez-Padilla (2013) and where the opinion of DM is very important in the operational decisions involved in the FSC of a FB.

From the comparison between deterministic model and fuzzy model, in terms of iterations, the fuzzy model has slightly more, and in terms of time, the deterministic model needs about 1/3 of the time the fuzzy model needs.

We found that for with $\alpha = 0.9$, the fuzzy model ($\alpha = 0.9$) gives better results than the deterministic model in several nutritional parameters (food variety by pantry-container) and logistic (container pantry weight and volume).

Finally, further research can be oriented to use other fuzzy mathematical programming approaches, and also to analyze the performance of the model with a higher number of served families.

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