Master thesis of

Automatic control and Robotics

Control of Wind Turbines using Takagi-Sugeno Approach

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ETSEIB

Abstract

This thesis will investigate the use of the Takagi-Sugeno approach to the control design applied to the wind turbines. The wind turbine model will be transformed to the Takagi-Sugeno representation. From that, control strategies will be developed that will allow the wind turbine operate in case of faulty situations. The proposed solutions will be tested using a well-known wind turbine case study.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Wind energy world capacity

Nowadays, wind energy is world wild used, as an alternative to burning fossil fuels, it is plentiful, renewable, widely distributed, clean, produces no greenhouse gas emissions during operation, consumes no water, and uses little land. [1] The net effects on the environment are far less problematic than those of nonrenewable power sources.

As of 2015, Denmark generates 40% of its electric power from wind, and at least 83 other countries around the world are using wind power to supply their electric power grids [2]. In 2014, global wind power capacity expanded 16% to $369,553MW$ [3]. Moreover almost $55GW$ of wind power capacity was added during 2016, increasing the global total about 12% to nearly $487GW$ between 2000 and 2015 (See Figure 1.1), wind increased from 2.4% to 15.6% of total EU power capacity. Germany installed total of almost $50GW$. These installations reflected the grid connection of a large amount of offshore capacity that was constructed in 2015. Spain continued to rank second in the EU for total operating capacity ($23GW$) but add wind capacity less than $50MW$ in 2016. China added $23.4GW$ in 2016, for total installed capacity approaching $169GW$, and accounted for one-third of total global capacity by year's end [4].

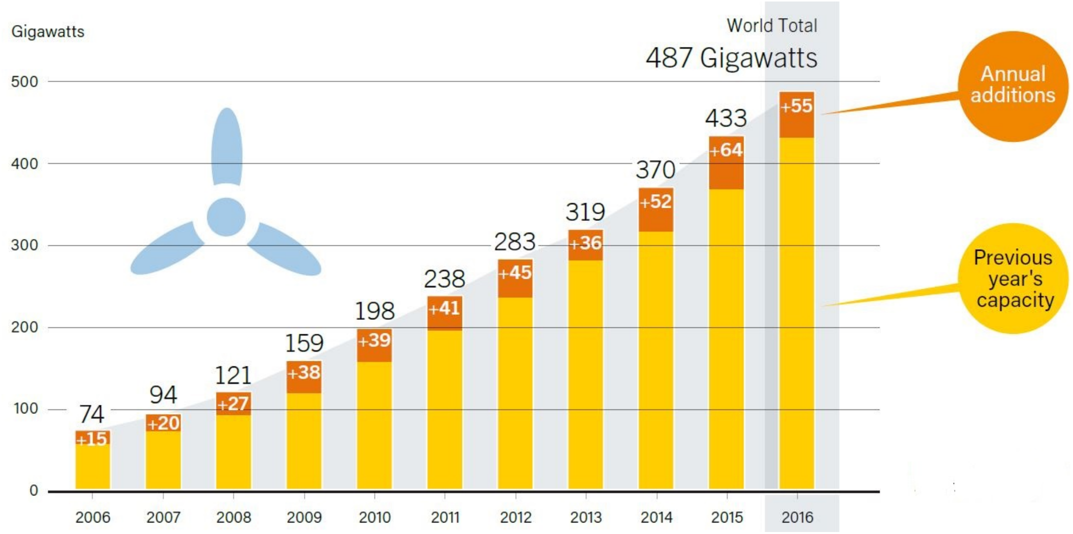

Figure 1.1: Wind Power Global Capacity and Annual Additions, 2006-2016. figure from [4]

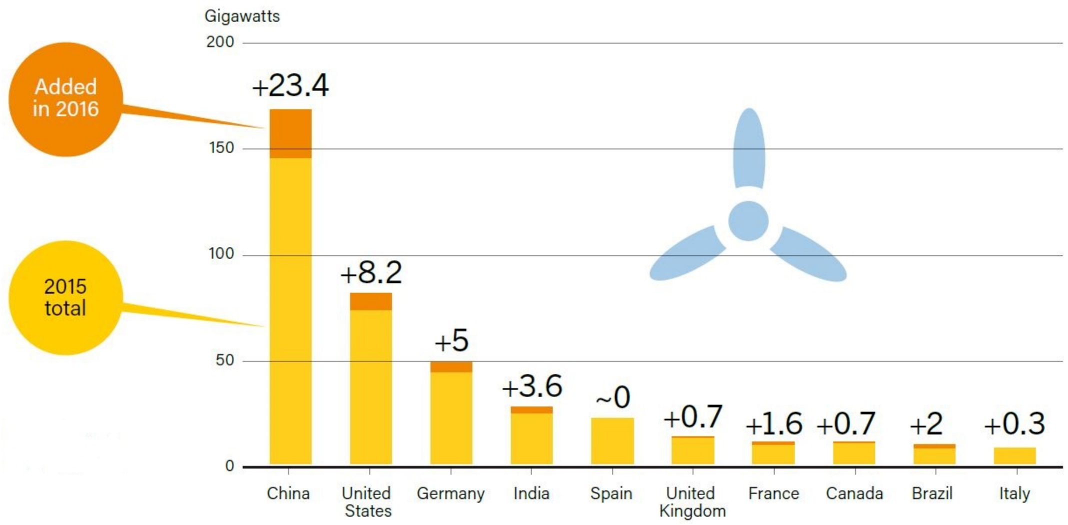

Figure 1.2: Wind Power Capacity and Additions, Top 10 Countries, 2016. figure from [4], Notes that Germany's additions are net of decommissioning and re-powering. "$\sim$ 0" denotes capacity additions of less than $50MW$.

1.2 Motivation

With the large capacity of wind turbines, control of wind turbine is important. And with rapidly growing popularity of fuzzy control systems in engineering applications, Tagaki-Sugerno approach has applied to many applications [5]:missiles [6], aircraft [7], energy production systems [8], robotic systems [9], active suspension of vehicles [10], engines [11] and fault tolerant control [12]. But there are very few people doing research on wind turbines, Sören Georg [24] [25] [26] [27]and Urs Giger [29], Xiaoxu Liu [28] etc. So this thesis will introduce the basics of Tagaki-Sugerno approach applied on wind turbine, Which is good way for a beginning understanding.

1.3 Objectives of project

As a size and flexible structures operating in uncertain environments, advanced control technology can improve their performance. For example, advanced controllers can help decrease the cost of wind energy by increasing turbine efficiency, and thus energy capture, and by reducing structural loading, which increases the lifetimes of the components and structures [15].

This project will focus on the usage of a fuzzy control technique, Tagaki-Sugerno (T-S) approach for the controller and observer design for a dynamic nonlinear wind turbine model. Both T-S controller and the T-S observer will be implemented and compared with the controller presented in [14]. The controller and observer gain will be obtained by using LMI [21].

All the simulations will be implemented using MATLAB and SIMULINK. The optimizer to be used is SeDuMi ($http://sedumi.ie.lehigh.edu/$).

1.4 Thesis structure

The structure of the main work is the following:

In Chapter 2, a set of wind turbine models are presented. It is divided in three parts, the first part will describe the wind turbine and its components. The second part presents its mathematics model of each components and transfer the systems to a state-space representation. The third part will compute the T-S model of the wind turbine.

Chapter 3 will present the state feedback control of the wind turbine. It is divided in three parts, the first part introduces the control structure. The second part presents the T-S controller for the wind turbine. The third part will present the state feedback control by using T-S observer.

Chapter 4 will make the comparison between the result with a PI controller and the T-S model and controller, and also the T-S observer based control.

Chapter 2

Wind Turbine Modeling

2.1 Wind turbine Basics

A wind turbine captures the wind kinematic energy and transforms it into mechanical energy (rotating shaft) first and then into electrical energy (generator). The main components of the horizontal-axis wind turbines (HAWT) in Figure 2.1 that are visible from the ground are the tower, nacelle, and rotor, as shown in Figure 2.2.

Figure 2.1: Wind turbine

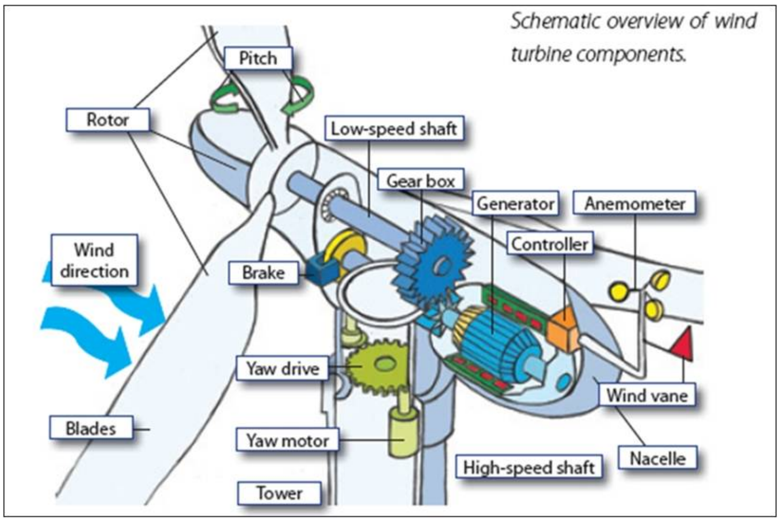

Figure 2.2: Wind turbine components. Figure from [15]

At first, the wind encounters the rotor on this upwind horizontal-axis turbine and rotates it. The low-speed shaft transfers energy to the gearbox, which steps up in speed and spins the high-speed shaft, which increases the speed and rotates the high-speed shaft. The high-speed shaft causes the generator to spin, producing electricity. In the figure, it is shown that the yaw-actuation mechanism, which is used to turn the nacelle so that the rotor faces into the wind [15].

2.2 Wind Turbine Modeling

In this thesis, the wind turbine model will be used is a three-bladed pitch-controlled variable-speed wind turbine with a nominal power of $4.8MW$ that is the one described in paper [14] The description of the model is presented in the following.

2.2.1 Aerodynamic model

The aerodynamics of the wind turbine is modeled as a torque acting on the blades, according to:

$$\tau_r(t) = \sum_{1 \leq i \leq 3} \frac{\rho \pi R^3 C_q(\lambda(t), \beta_i(t)) v_{w,i}(t)^2}{6} \tag{2.1}$$

where v_w is the wind speed, $\rho = 1.225kg/m^3$ is the air density, $R = 57.5m$ is the rotor radius, β_i is pitch position, and λ is the Tip Speed Ratio, defined as:

$$\lambda = \frac{\omega_r \cdot R}{v_w} \tag{2.2}$$

2.2.2 Pitch system model

For each blade, the hydraulic pitch system is modeled as a closed-loop transfer function between the pitch angle β_i and its reference $\beta_{i,ref}$, according to:

$$\frac{\beta_i(s)}{\beta_{i,ref}(s)} = \frac{\omega_n^2}{s^2 + 2 \cdot \xi\omega_n \cdot s + \omega_n^2} \tag{2.3}$$

which can be written as a differential equation:

$$\ddot{\beta}_i(t) = -2\xi\omega_n \cdot \dot{\beta}(t) - \omega_n^2\beta(t) + \omega_n^2\beta_{i,ref} \tag{2.4}$$

where $\xi = 0.6$ is the damping factor, and $\omega_n = 11.11rad/s$ is the natural frequency, and $i = 1, 2, 3$ for three blades.

2.2.3 Drive train model

The drive train is modeled by a two-mass model:

$$J_r\dot{\omega}_r(t) = \tau_r(t) - K_{dt}\theta_\Delta(t) - (B_{dt} + B_r)\omega_r(t) + \frac{B_{dt}}{N_g}\omega_g(t) \tag{2.5}$$

$$J_g\dot{\omega}_g(t) = \frac{\eta_{dt}K_{dt}}{N_g}\theta_\Delta(t) + \frac{\eta_{dt}B_{dt}}{N_g}\omega_r(t) - \left(\frac{\eta_{dt}B_{dt}}{N_g^2} + B_g\right)\omega_g(t) - \tau_g(t) \tag{2.6}$$

$$\dot{\theta}_\Delta(t) = \omega_r(t) - \frac{1}{N_g}\omega_g(t) \tag{2.7}$$

where $J_r = 55 \cdot 10^6 kg \cdot m^2$ is the moment of inertia of the low-speed shaft, $K_{dt} = 2.7 \cdot 10^9 Nm/rad$ is the torsion stiffness of the drive train, $B_{dt} = 775.49Nms/rad$ is the torsion damping coefficient of the drive train and $B_r = 7.11Nms/rad$, $B_g = 45.6Nms/rad$ is the viscous friction of the high-speed shaft, $N_g = 95$ is the gear ratio, $J_g = 390kg \cdot m^2$ is the moment of the inertia of the high-speed shaft, $\eta_{dt} = 0.97$ is the efficiency of the drive train, and $\theta_{\Delta}(t)$ is the torsion angle of the drive train.

2.2.4 Generator and converter model

The generator and converter dynamics can be modeled by a first transfer function

$$\frac{\tau_g(s)}{\tau_{g,ref}(s)} = \frac{\alpha_{gc}}{s + \alpha_{gc}} \tag{2.8}$$

The power produced by the generator is given by

$$P_g(t) = \eta_g \omega_g(t) \tau_g(t) \tag{2.9}$$

where $\alpha_{gc} = 50 rad/s$ is the generator and converter model parameter, $\eta_g = 0.98$ is the efficiency of the generator. Besides The generator torque τ_g is controlled by the reference $\tau_{g,ref}$. The dynamics can be approximated by a first order model with time constant t_g [16] .

$$\dot{\tau}_g(t) = -\frac{\tau_g(t)}{t_g} + \frac{\tau_{g,ref}(t)}{t_g} \tag{2.10}$$

where $t_g = 20 \cdot 10^{-3}$

2.3 PI control of wind turbine description

Figure 2.3 shows the different operating ranges of the wind turbine [14].

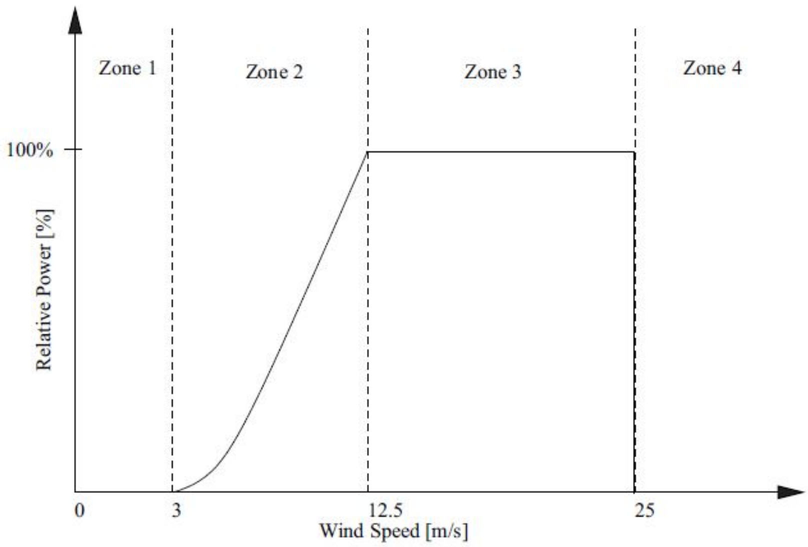

Figure 2.3: Illustration of the reference power curve for the wind turbine depending on the wind speed

The controller has two modes. Mode 1 corresponds to the wind zone 2 and mode 2 corresponds to the wind zone 3. Consider our wind data in Figure 2.4, at more or less time 2300s, the wind speed goes from zone 2 to zone 3. Hence, we can assume that from time 0 to 2300s, the PI controller is in mode 1, and after that it goes to mode 2 [14].

The control mode switches from mode 1 to 2 if

$$P_g[n] \geq P_r[n] \vee \omega_g[n] \geq \omega_{nom} \tag{2.11}$$

where $\omega_{nom} = 162rad/s$ is the nominal generator speed. The control mode switches from mode 2 to 1 if

$$\omega_g[n] < \omega_{nom} - \omega_{\Delta} \tag{2.12}$$

Control Mode 1:

$$\tau_{g,r}[n] = K_{opt} \cdot \left(\frac{\omega_g[n]}{N_g}\right)^2 \tag{2.13}$$

where

$$K_{opt} = \frac{1}{2}\rho A R^3 \frac{C_{Pmax}}{\lambda_{opt}^3} \tag{2.14}$$

where A is the area swept by the wind turbine blades, so we have $A = \pi R^2 = 1.0387 \times 10^4 m^2$, and λ_{opt} is the optimal value of λ, C_{Pmax} is the maximum value of the power coefficient.

Control Mode 2: In this mode, the major control actions are handled by the pitch system using a PI controller trying to keep $\omega_g[n]$ at ω_{nom}

$$\beta_r[n] = \beta_r[n-1] + K_p e[n] + (K_i \cdot T_s - K_p)e[n-1] \tag{2.15}$$

where $e[n] = \omega_g[n] - \omega_{nom}$, and the controller gain of the PI is $K_p = 4$ and $K_i = 1$. In this case, the converter reference is used to suppress fast disturbances by

$$\tau_{g,r}[n] = \frac{P_r[n]}{\eta_{gc} \cdot \omega_g[n]} \tag{2.16}$$

2.4 Data definition

The data of the system we are going to use are all described in the following table.

Parameter	value	unit
ρ	1.225	kg/m^3
R	57.5	m
ξ	0.6	$-$
ω_n	11.11	rad/s
J_r	$55 \cdot 10^6$	$kg \cdot m^2$
K_{dt}	$2.7 \cdot 10^9$	Nm/rad
B_{dt}	775.49	Nms/rad
B_r	7.11	Nms/rad
B_g	45.6	Nms/rad
N_g	95	$-$
J_g	390	$kg \cdot m^2$
η_{dt}	0.97	$-$
η_g	0.98	$-$
t_g	$20 \cdot 10^{-3}$	$-$

Table 2.1: Data of the system

And the wind data we are using is shown in the figure below,

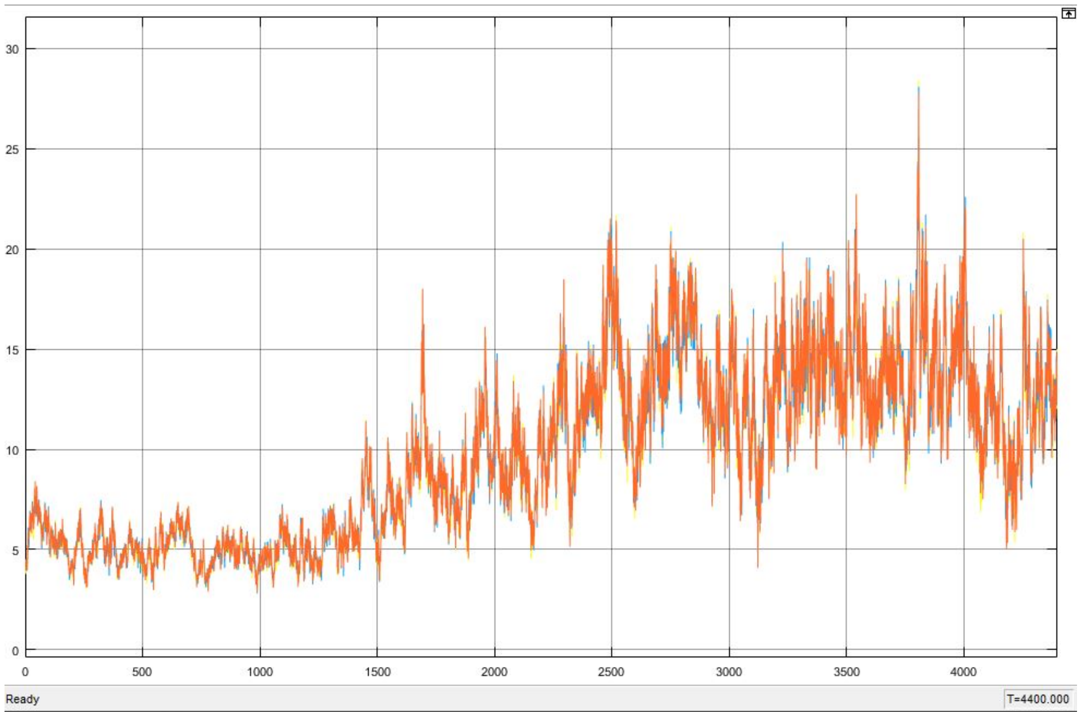

Figure 2.4: The wind speed

the reference of the inputs $\begin{bmatrix}\tau_{g,ref} & \beta_{1,ref} & \beta_{2,ref} & \beta_{3,ref}\end{bmatrix}^T$ are shown as follow, notice that the value of reference for each pitch angle to the blade.

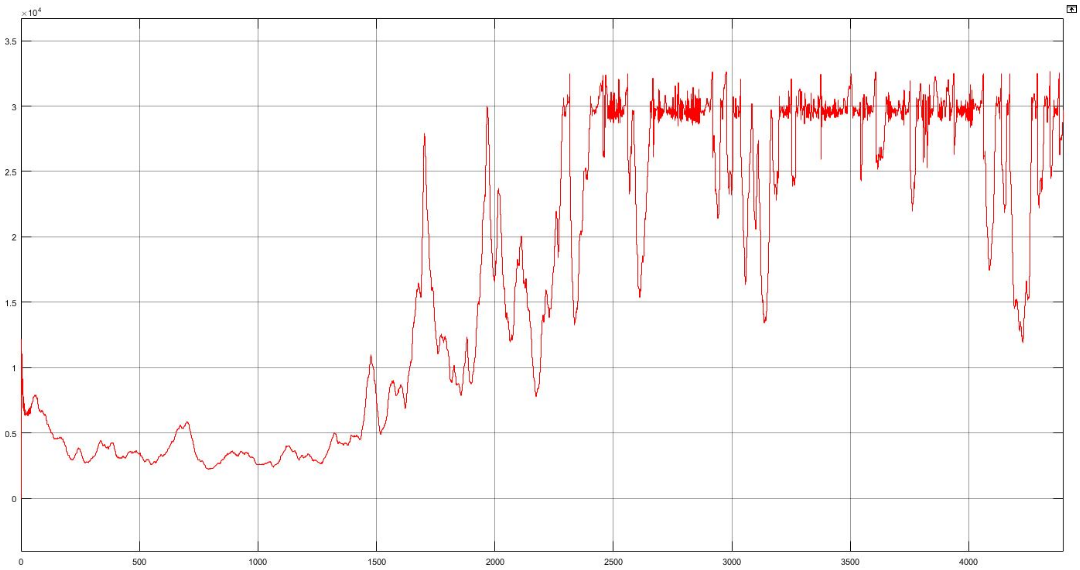

Figure 2.5: reference of the torque

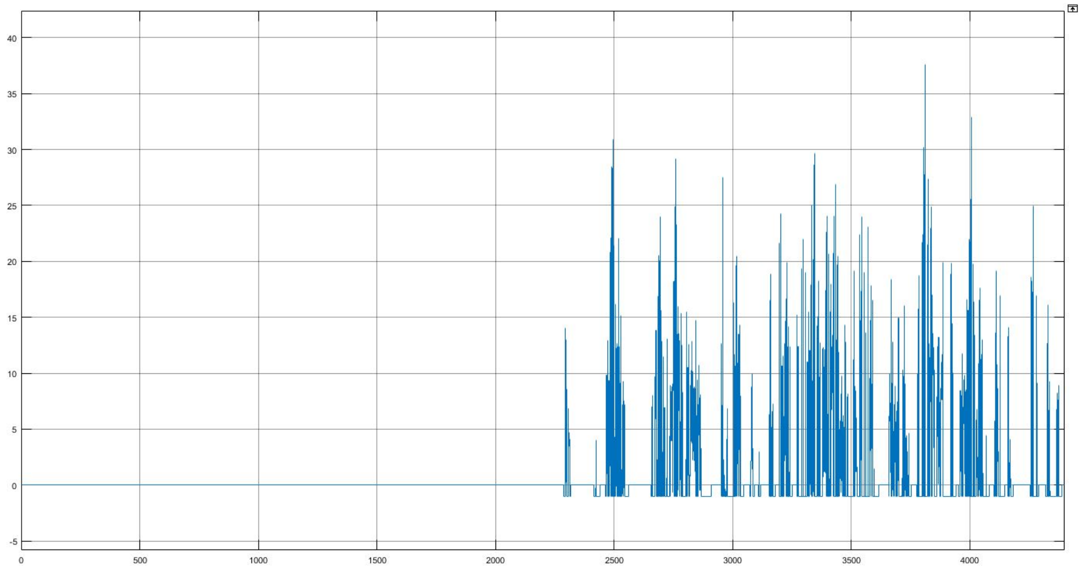

Figure 2.6: reference of the pitch angle

2.4.1 State space representation of the wind turbine

In order to use the Takagi-Sugeno Approach, first we need to transform our model into state-space representation. Defining the state and input vectors, as in [16]

$$x(t) = \begin{bmatrix} \omega_r & \omega_g & \theta_\Delta & \tau_g & \beta_1 & \dot{\beta}_1 & \beta_2 & \dot{\beta}_2 & \beta_3 & \dot{\beta}_3 \end{bmatrix}^T \tag{2.17}$$

$$u(t) = \begin{bmatrix} \tau_{g,ref} & \beta_{1,ref} & \beta_{2,ref} & \beta_{3,ref} \end{bmatrix}^T \tag{2.18}$$

the model of the wind turbine can be written into a state space embedding the non-linearities in the parameters

$$\dot{x} = Ax(t) + Bu(t) \tag{2.19}$$

$$y = Cx(t) \tag{2.20}$$

where

$$A = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_1(t) & 0 & z_2(t) & 0 & z_3(t) & 0 \\ \dfrac{\eta_{dt} B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt} K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.21}$$

where

$$z_1(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_1(t)) v_w(t)^2}{6 J_r \beta_1} \tag{2.22}$$

$$z_2(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_2(t)) v_w(t)^2}{6 J_r \beta_2} \tag{2.23}$$

$$z_3(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_3(t)) v_w(t)^2}{6 J_r \beta_3} \tag{2.24}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{t_g} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \omega_n^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_n^2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_n^2 \end{bmatrix} \tag{2.25}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.26}$$

2.5 Takagi-Sugeno Model

2.5.1 Takagi-Sugeno approach

To apply Takagi-Sugeno (T-S) model, here we are using the method which presented in Chapter 2 of the book [17]. The fuzzy model proposed by Takagi and Sugeno [18] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model.

The ith rules of the T-S fuzzy models are of the following form, where CFS and DFS denote the continuous fuzzy system and the discrete fuzzy system, respectively.

Model Rule i:

IF $z_1(t)$ is M_{i1}, $\ldots$ and $z_p(t)$ is M_{ip},

THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \qquad i = 1, 2, ..., r \tag{2.27}$$

Here, M_{ij} is the fuzzy set and r is the number of model rules; $x(t) \in R^n$ and $x(k) \in R^n$ are the state vectors, $u(t) \in R^m$ and $u(k) \in R^m$ are the input vectors, $y(t) \in R^q$ and $y(k) \in R^q$ are the output vectors, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$ and $C_i \in R^{q\times n}$, $z_1(t), ..., z_p(t)$ are known premise variables that may be functions of the state variables, external disturbances, and/or time.

Given a pair of $x(t), u(t)$, the final outputs of the fuzzy systems are inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} w_i(z(t))} \tag{2.28}$$

$$= \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \tag{2.29}$$

$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} \tag{2.30}$$

$$= \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \tag{2.31}$$

where

$$z(t) = [z_1(t) z_2(t) \dots z_p(t)] \tag{2.32}$$

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \tag{2.33}$$

$$h_i(t) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \tag{2.34}$$

for all t. The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Since

$$\begin{cases} \sum_{i=1}^{r} w_i(z(t)) > 0 \\ w_i(z(t)) \geq 0, i = 1, 2, ..., r. \end{cases} \tag{2.35}$$

we have

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) > 0 \\ h_i(z(t)) \geq 0, i = 1, 2, ..., r. \end{cases} \tag{2.36}$$

for all t.

2.5.2 Wind turbine Takagi-Sugeno model

From equation 2.29 to 2.34, we bound $z_1(t) \in [z_{1,min}, z_{1,max}]$, $z_2(t) \in [z_{2,min}, z_{2,max}]$, $z_3(t) \in [z_{3,min}, z_{3,max}]$

From the maximum and minimum values, $z_1(t)$, $z_2(t)$ and $z_3(t)$ can be represented by

$$z_1(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_1(t)) v_w(t)^2}{6 J_r \beta_1} = M_1(z_1(t)) \cdot z_{1,max} + M_2(z_1(t)) \cdot z_{1,min} \tag{2.37}$$

$$z_2(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_2(t)) v_w(t)^2}{6 J_r \beta_2} = N_1(z_2(t)) \cdot z_{2,max} + N_2(z_2(t)) \cdot z_{2,min} \tag{2.38}$$

$$z_3(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_3(t)) v_w(t)^2}{6 J_r \beta_3} = L_1(z_3(t)) \cdot z_{3,max} + L_2(z_3(t)) \cdot z_{3,min} \tag{2.39}$$

Therefore the membership functions can be calculated as

$$\begin{cases} M_1 = \dfrac{z_1 - z_{1,min}}{z_{1,max} - z_{1,min}} \\ M_2 = \dfrac{z_{1,max} - z_1}{z_{1,max} - z_{1,min}} \end{cases} \tag{2.40}$$

$$\begin{cases} N_1 = \dfrac{z_2 - z_{2,min}}{z_{2,max} - z_{2,min}} \\ N_2 = \dfrac{z_{2,max} - z_2}{z_{2,max} - z_{2,min}} \end{cases} \tag{2.41}$$

$$\begin{cases} L_1 = \dfrac{z_3 - z_{3,min}}{z_{3,max} - z_{3,min}} \\ L_2 = \dfrac{z_{3,max} - z_3}{z_{3,max} - z_{3,min}} \end{cases} \tag{2.42}$$

We name the membership functions "Positive", "Negative", respectively. Then, the nonlinear system is represented by the following fuzzy model.

Model Rule 1:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN $\dot{x}(t) = A_1x(t) + Bu(t)$

Model Rule 2:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN $\dot{x}(t) = A_2x(t) + Bu(t)$

Model Rule 3:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN $\dot{x}(t) = A_3x(t) + Bu(t)$

Model Rule 4:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN $\dot{x}(t) = A_4x(t) + Bu(t)$

Model Rule 5:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN $\dot{x}(t) = A_5x(t) + Bu(t)$

Model Rule 6:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN $\dot{x}(t) = A_6x(t) + Bu(t)$

Model Rule 7:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN $\dot{x}(t) = A_7x(t) + Bu(t)$

Model Rule 8:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN $\dot{x}(t) = A_8x(t) + Bu(t)$

For illustrative purposes, this can be represented by the following table, where "*Positive*" can be represented by "+" and "*Negative*" can be represented by "$-$".

Set	$z_1(t)$	$z_2(t)$	$z_3(t)$	A matrix
rule 1	$-$	$-$	$-$	A_1
rule 2	$+$	$-$	$-$	A_2
rule 3	$-$	$+$	$-$	A_3
rule 4	$+$	$+$	$-$	A_4
rule 5	$-$	$-$	$+$	A_5
rule 6	$+$	$-$	$+$	A_6
rule 7	$-$	$+$	$+$	A_7
rule 8	$+$	$+$	$+$	A_8

Table 2.2: Fuzzy model

Figure 2.7 to 2.9 shows the graphical representation of the membership functions.

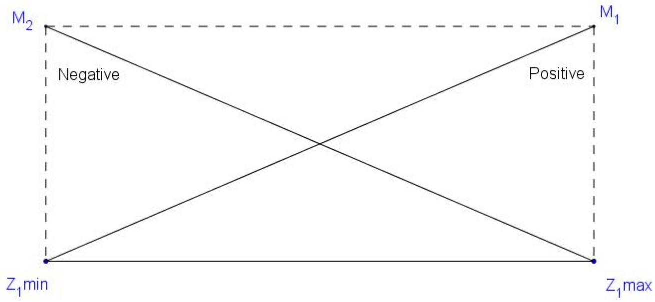

Figure 2.7: Membership Functions $M_1(z_1(t))$ and $M_2(z_1(t))$

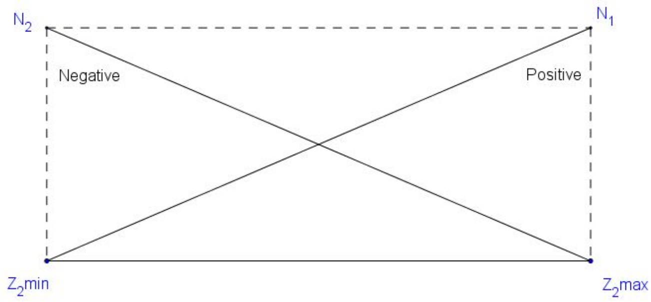

Figure 2.8: Membership Functions $N_1(z_2(t))$ and $N_2(z_2(t))$

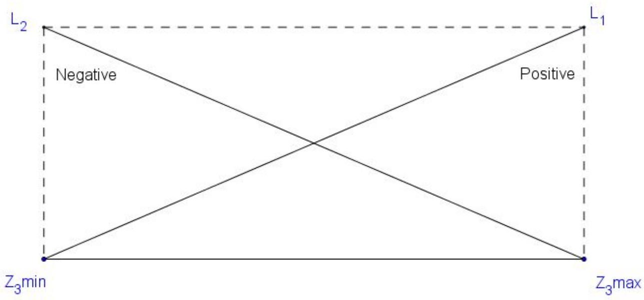

Figure 2.9: Membership Functions $L_1(z_3(t))$ and $L_2(z_3(t))$

Thus, the matrices of the local models are

$$A_1 = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,min} & 0 & z_{3,min} & 0 \\ \dfrac{\eta_{dt} B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt} K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \quad (2.43)$$

$$A_2 = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,min} & 0 & z_{3,min} & 0 \\ \dfrac{\eta_{dt} B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt} K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \quad (2.44)$$

$$A_3 = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,max} & 0 & z_{3,min} & 0 \\ \dfrac{\eta_{dt} B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt} K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \quad (2.45)$$

$$A_4 = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,max} & 0 & z_{3,min} & 0 \\ \dfrac{\eta_{dt}B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt}B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt}K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.46}$$

$$A_5 = \begin{bmatrix} -\dfrac{B_{dt}+B_r}{J_r} & \dfrac{B_{dt}}{N_g J_r} & -\dfrac{K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,min} & 0 & z_{3,max} & 0 \\ \dfrac{\eta_{dt}B_{dt}}{N_g J_g} & -\dfrac{\eta_{dt}B_{dt}}{N_g^2 J_g} - \dfrac{B_g}{J_g} & \dfrac{\eta_{dt}K_{dt}}{N_g J_g} & -\dfrac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\dfrac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.47}$$

$$A_6 = \begin{bmatrix} -\frac{B_{dt}+B_r}{J_r} & \frac{B_{dt}}{N_gJ_r} & -\frac{K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,min} & 0 & z_{3,max} & 0 \\ \frac{\eta_{dt}B_{dt}}{N_gJ_g} & -\frac{\eta_{dt}B_{dt}}{N_g^2J_g}-\frac{B_g}{J_g} & \frac{\eta_{dt}K_{dt}}{N_gJ_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.48}$$

$$A_7 = \begin{bmatrix} -\frac{B_{dt}+B_r}{J_r} & \frac{B_{dt}}{N_gJ_r} & -\frac{K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,max} & 0 & z_{3,max} & 0 \\ \frac{\eta_{dt}B_{dt}}{N_gJ_g} & -\frac{\eta_{dt}B_{dt}}{N_g^2J_g}-\frac{B_g}{J_g} & \frac{\eta_{dt}K_{dt}}{N_gJ_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.49}$$

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$$A_8 = \begin{bmatrix} -\frac{B_{dt}+B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & -\frac{K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,max} & 0 & z_{3,max} & 0 \\ \frac{\eta_{dt} B_{dt}}{N_g J_g} & -\frac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \frac{B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \tag{2.50}$$

The defuzzification is carried out as

$$\dot{x}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} M_i(Z_1(t)) N_j(Z_2(t)) L_k(Z_3(t)) \cdot A_l x(t) + Bu(t) \tag{2.51}$$

Chapter 3

State feedback control

3.1 Control of Wind Turbines

3.1.1 Design fuzzy controller

From the wind turbine TS model obtained in previous chapter, we are going to design a state feedback controller. Here we will use a design procedure called "parallel distributed compensation" (PDC) [20]. This model-based design procedure was proposed in [19].

In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy model (2.27), we construct the following fuzzy controller via the PDC:

Control Rule i:

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},

THEN $u(t) = -F_i x(t)$, $i = 1, 2, ..., r$.

where F_i is the feedback control gain, it can be described a fuzzy control rule.

The overall fuzzy controller is represented by

$$u(t) = -\frac{\sum_{i=1}^{r} w_i(z(t))F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t))F_i x(t) \tag{3.1}$$

Now to apply this procedure to our wind turbine case, we have.

Control Rule 1:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN $u(t) = -F_1x(t)$

Control Rule 2:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN $u(t) = -F_2x(t)$

Control Rule 3:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN $u(t) = -F_3x(t)$

Control Rule 4:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN $u(t) = -F_4x(t)$

Control Rule 5:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN $u(t) = -F_5x(t)$

Control Rule 6:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN $u(t) = -F_6x(t)$

Control Rule 7:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN $u(t) = -F_7x(t)$

Control Rule 8:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN $u(t) = -F_8x(t)$

Thus, we can design the feedback control law $u(t) = -F_ix(t)$ for each model, such that our system $\dot{x} = (A_i + BK_i)x(t)$ is asymptotically stable, where $K_i = -F_i$, therefore in our case, we have $B_1 = B_2 = \cdots = B_i = B$.

We can also present in following table.

Set	$z_1(t)$	$z_2(t)$	$z_3(t)$	A matrix	Control gain
rule 1	$-$	$-$	$-$	A_1	K_1
rule 2	$+$	$-$	$-$	A_2	K_2
rule 3	$-$	$+$	$-$	A_3	K_3
rule 4	$+$	$+$	$-$	A_4	K_4
rule 5	$-$	$-$	$+$	A_5	K_5
rule 6	$+$	$-$	$+$	A_6	K_6
rule 7	$-$	$+$	$+$	A_7	K_7
rule 8	$+$	$+$	$+$	A_8	K_8

Table 3.1: Fuzzy model with fuzzy control rule

The design is based on Lyapunov stability theory and LMI condition for stablility of T-S systems in book [21]. We have the LMI region stabilization problem in the case of $\mathbb{S}(\alpha, r, \theta)$ has a solution if and only if there exist a symmetric positive definite matrix P_i and a matrix W_i satisfying

$$A_i P_i + BW_i + PA_i^T + W_i^T B^T + 2\alpha P < 0 \tag{3.2}$$

$$\begin{bmatrix} -rP_i & qP_i + A_iP_i + BW_i \\ qP_i + P_iA_i^T + W_i^T B_i^T & -rP_i \end{bmatrix} < 0 \tag{3.3}$$

$$\begin{bmatrix} \big(A_iP_i + BW_i + PA_i^T + W_i^T B^T\big)sin\theta & \big(A_iP_i + BW_i - (PA_i^T + W_i^T B^T)\big)cos\theta \\ \big(PA_i^T + W_i^T B^T - (A_iP_i + BW_i)\big)cos\theta & \big(A_iP_i + BW_i + PA_i^T + W_i^T B^T\big)sin\theta \end{bmatrix} < 0 \tag{3.4}$$

In this case, the solution to our problem is given by

$$K_i = W_i P_i^{-1} \tag{3.5}$$

where α is the minimum speed of the response, r is the maximum speed of the response, and θ is the overshoot. The LMI region $\mathbb{S}$ is shown in the following figure [21].

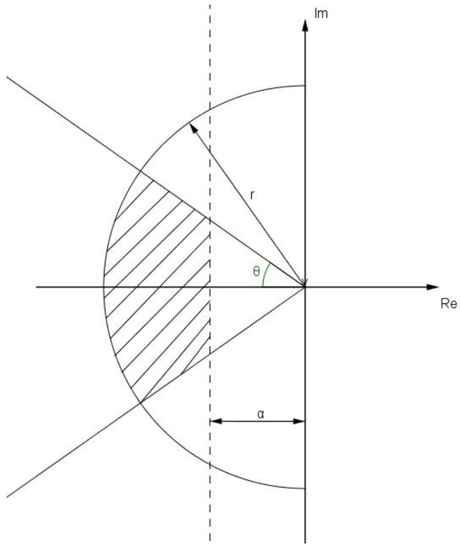

Figure 3.1: LMI region $\mathbb{S}(\alpha, r, \theta)$

All the poles should be inside the shadow region.

3.1.2 Observer design

For designing the observer, book [17] has presented the methodologies for designing the T-S fuzzy observer. In linear system theory, one of the most important results on observer design is the so-called separation principle, which means that the controller and observer design can be carried out separately without compromising the stability of the overall closed-loop system. As this point, we can design the observer based on LMIs. As in all observer designs, fuzzy observers [22] [23] are required to satisfy

$$\lim_{t \to \infty} \big(x(t) - \hat{x}(t)\big) = 0 \tag{3.6}$$

where $\hat{x}(t)$ denotes the state vector estimated by a fuzzy observer. This condition guarantees that the steady-state error between $x(t)$ and $\hat{x}(t)$ converges to 0. As in the case of controller design, the PDC concept is employed to arrive at the following fuzzy observer structures:

Observer Rule i

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}

THEN

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_i\hat{x}(t), i = 1, 2, ..., r$$

where L_i is the observer gain. For our wind turbine case, we have fuzzy observer law is given by (notice that in our case $B_i = B_1 = ... = B_8$ and $C_i = C_1 = ... = C_8$).

Observer Rule 1:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN

$$\dot{\hat{x}}(t) = A_1\hat{x}(t) + B_1u(t) + L_1(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_1\hat{x}(t)$$

Observer Rule 2:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"

THEN

$$\dot{\hat{x}}(t) = A_2\hat{x}(t) + B_2u(t) + L_2(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_2\hat{x}(t)$$

Observer Rule 3:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN

$$\dot{\hat{x}}(t) = A_3\hat{x}(t) + B_3u(t) + L_3(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_3\hat{x}(t)$$

Observer Rule 4:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"

THEN

$$\dot{\hat{x}}(t) = A_4\hat{x}(t) + B_4u(t) + L_4(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_4\hat{x}(t)$$

Observer Rule 5:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_5\hat{x}(t) + B_5u(t) + L_5(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_5\hat{x}(t)\end{aligned}$$

Observer Rule 6:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_6\hat{x}(t) + B_6u(t) + L_6(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_6\hat{x}(t)\end{aligned}$$

Observer Rule 7:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_7\hat{x}(t) + B_7u(t) + L_7(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_7\hat{x}(t)\end{aligned}$$

Observer Rule 8:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_8\hat{x}(t) + B_8u(t) + L_8(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_8\hat{x}(t)\end{aligned}$$

For a better understanding,, this can be represented by the following table, where "*Positive*" can be represented by "+" and "*Negative*" can be represented by "$-$".

Set	$z_1(t)$	$z_2(t)$	$z_3(t)$	A matrix	Observer gain
rule 1	$-$	$-$	$-$	A_1	L_1
rule 2	$+$	$-$	$-$	A_2	L_2
rule 3	$-$	$+$	$-$	A_3	L_3
rule 4	$+$	$+$	$-$	A_4	L_4
rule 5	$-$	$-$	$+$	A_5	L_5
rule 6	$+$	$-$	$+$	A_6	L_6
rule 7	$-$	$+$	$+$	A_7	L_7
rule 8	$+$	$+$	$+$	A_8	L_8

Table 3.2: Fuzzy model with fuzzy observer rule

Now in order to obtain the observer gain L_i, for a full-order state observers design following the LMIs condition [21]. It has a solution if and only if there exist a symmetric positive definite matrix P_i and a matrix W_i satisfying

$$A_i^T P + C^T W_i + (A_i^T P + C^T W_i)^T + 2\lambda P < 0 \tag{3.7}$$

$$\begin{bmatrix} -rP_i & qP_i + A_i^T P_i + C^T W_i \\ (qP_i + A_i^T P_i + C^T W_i)^T & -rP_i \end{bmatrix} < 0 \tag{3.8}$$

$$\begin{bmatrix} \big(A_i^T P_i + C^T W_i + (A_i^T P + C^T W_i)^T\big) sin\theta & \big(A_i^T P_i + C^T W_i - (A_i^T P + C^T W_i)^T\big) cos\theta \\ \big(-(A_i^T P_i + C^T W_i) + (A_i^T P + C^T W_i)^T\big) cos\theta & \big(A_i^T P_i + C^T W_i + (A_i^T P + C^T W_i)^T\big) sin\theta \end{bmatrix} < 0 \tag{3.9}$$

In this case, the solution to our problem is given by

$$L_i = P_i^{-1} W_i \tag{3.10}$$

Similarly, the poles of the observer should be in the shadow area in Figure 3.1 [24] [25].

3.2 Obtaining the state feedback controller

To implement the observer using the methodology introduced in Subsection 3.1.1, the following LMIs parameter are considered: $r = 50$, $q = 0$, $\alpha = 0.5$ and $\theta = \pi/6$, applied to our Wind Turbine case study. The resulting closed loop poles are presented in Figure 3.2.

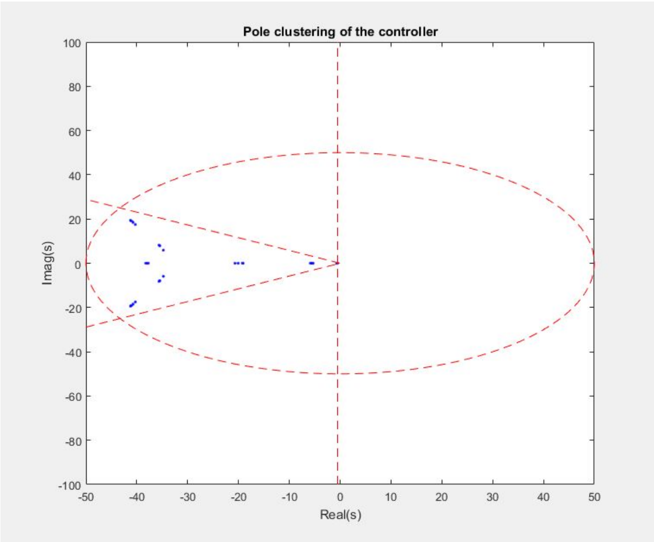

Figure 3.2: Poles of the controller

From that figure, we can see that all the poles are located in the shadow region presented in Figure 3.1.

3.2.1 Control structure of Wind Turbines

The considered control structure can be represent by the following diagram

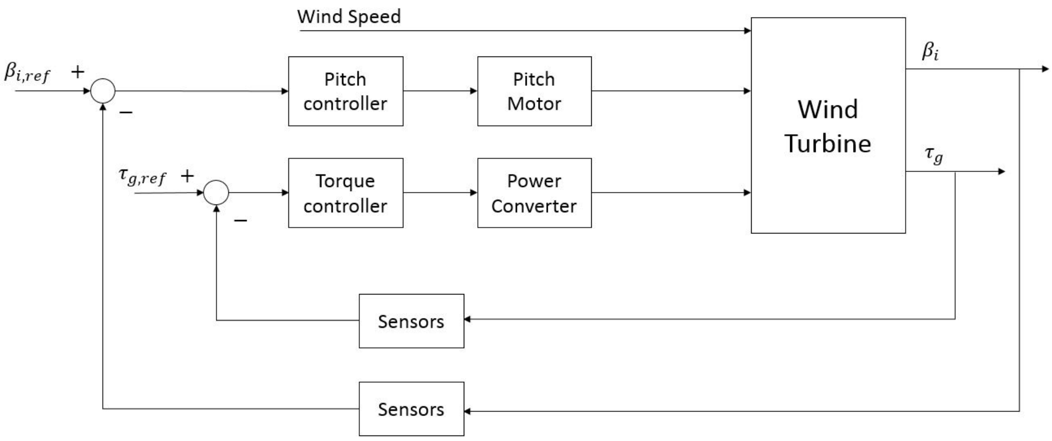

Figure 3.3: Wind turbine control feedback loops

The designed control is tested in SIMULINK, leading to the results presented in Figure 3.4.

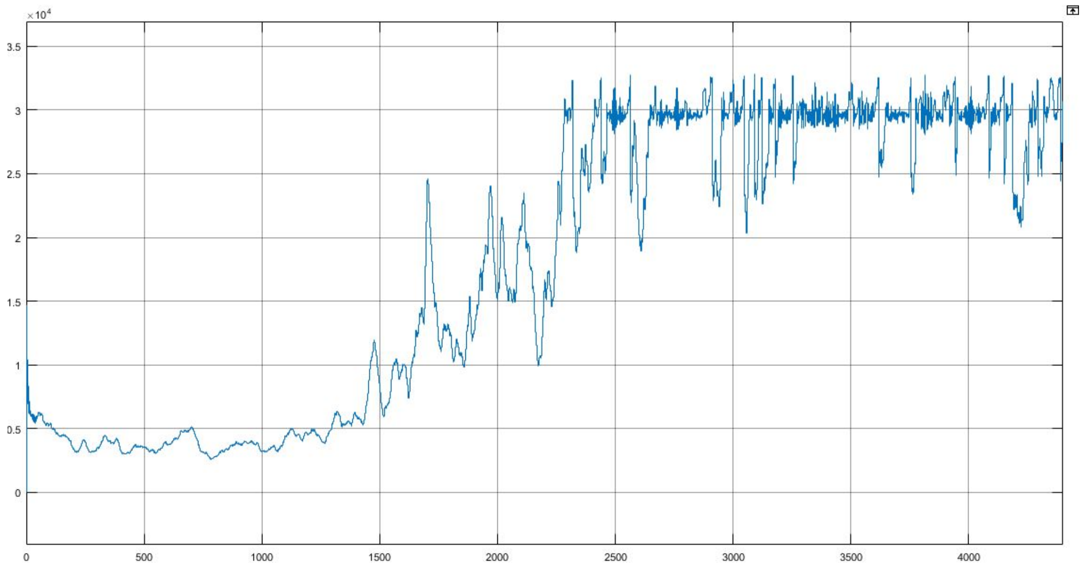

Figure 3.4: Controlled torque

In this figure, we can see that the output torque.

Similarly, we have the output of the pitch angle,

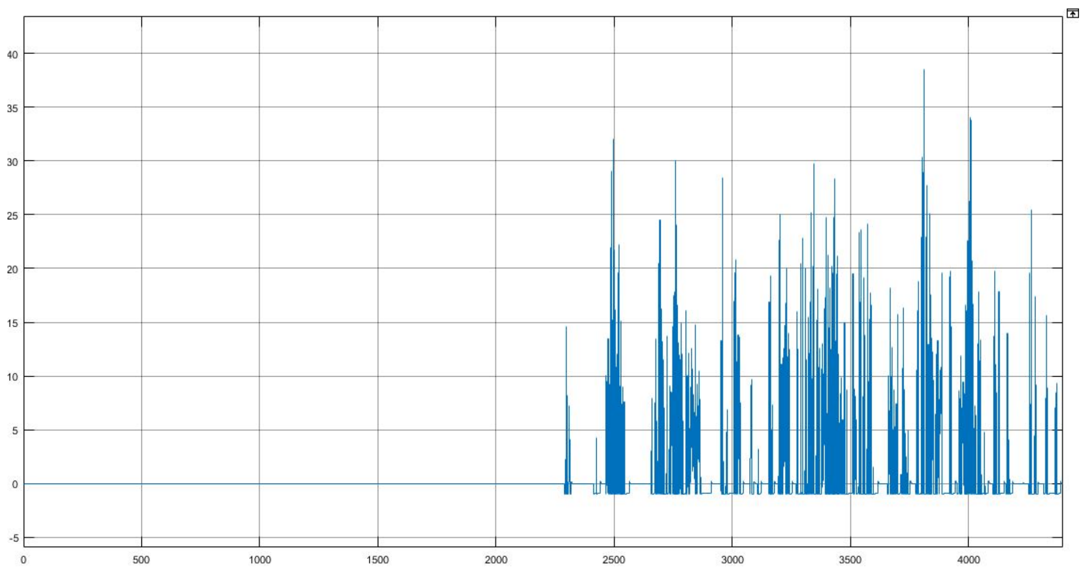

Figure 3.5: Controlled pitch angle

3.3 Obtaining the observer

To implement the observer using the methodology introduced in Subsection 3.1.2, the following LMI parameter are considered: $r = 500$, $q = 0$, $\alpha = 50$ and $\theta = \pi/3$, applied to our Wind turbine case study, The resulting observer poles are presented in Figure 3.6.

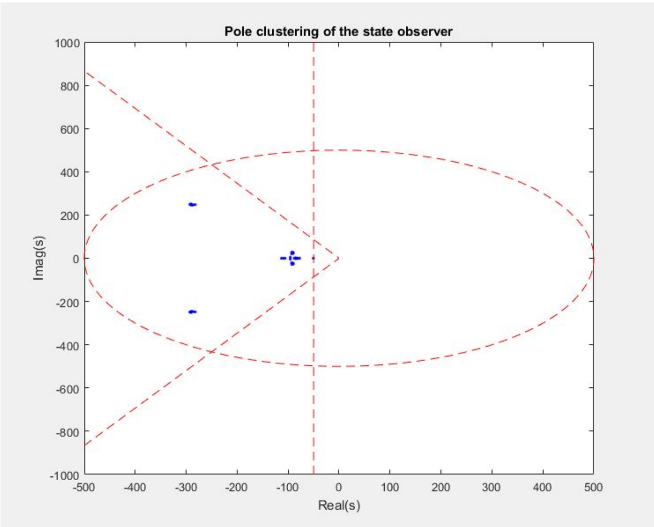

Figure 3.6: Poles of the observer

From the figure, we can see that all the poles are located in the shadow region that described in figure 3.1.

3.3.1 Observer based control

The observer based estimation scheme considered is presented in Figure 3.7.

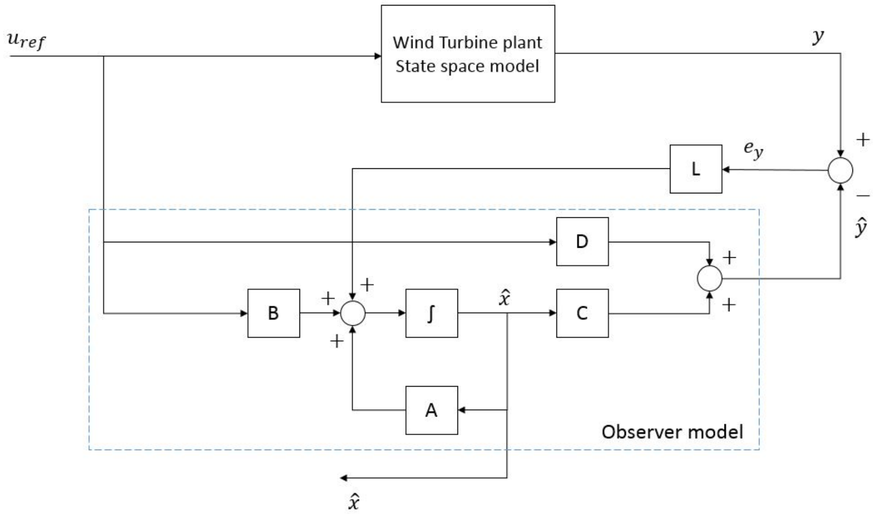

Figure 3.7: Closed-loop estimation by using the observer

This observer schemes is integrated with the state feedback controller designed previously and implemented in SIMULINK, leading to the following result.

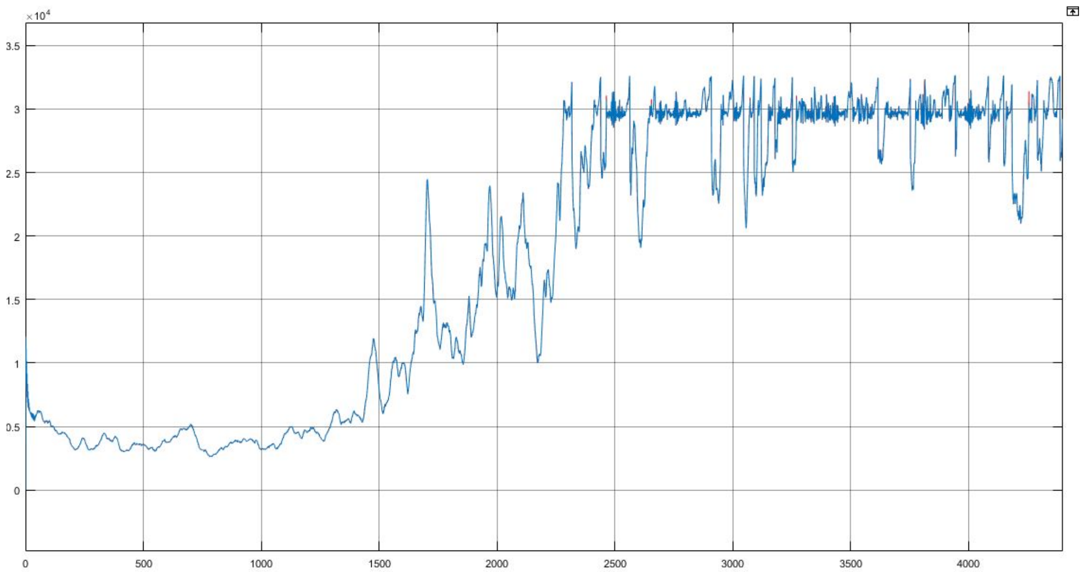

Figure 3.8: Torque estimated by the observer

In this figure, the estimated torque (in blue) match the reference (in red) very well, we can make a zoom in to see the details.

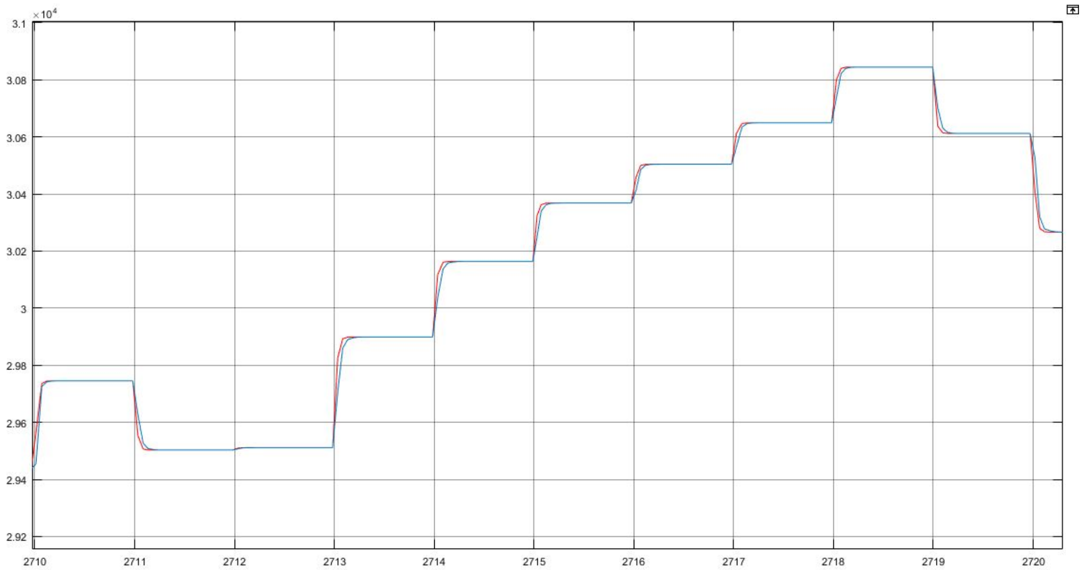

Figure 3.9: Zoom in of the torque estimated by the observer

Also, we can see the estimated pitch angle in Figure 3.10,

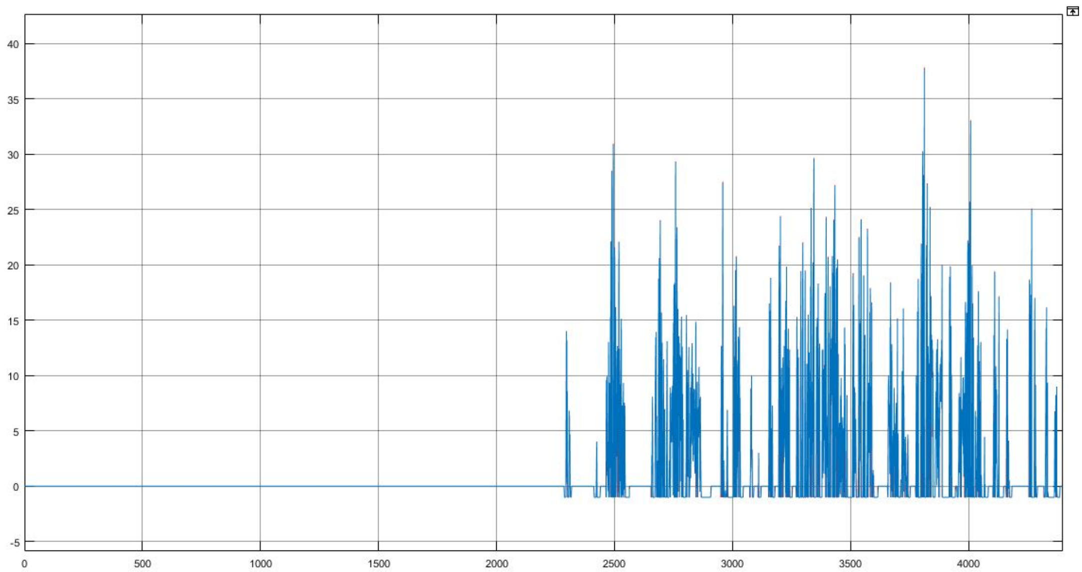

Figure 3.10: Pitch angle generated by the observer

3.3.2 State feedback using observer

Based on the model in Figure 3.7, we can use the state feedback controller for it.

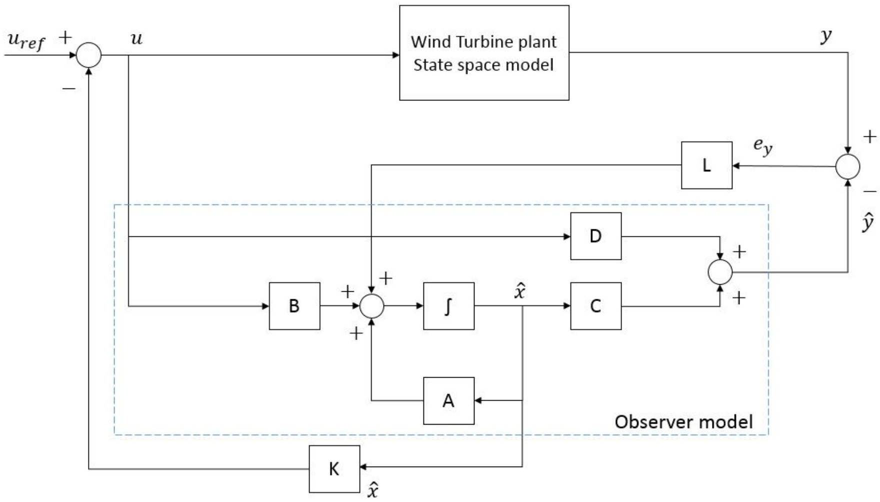

Figure 3.11: State feedback using the observer

Testing this observer with the controller in SIMULINK, we have the following result.

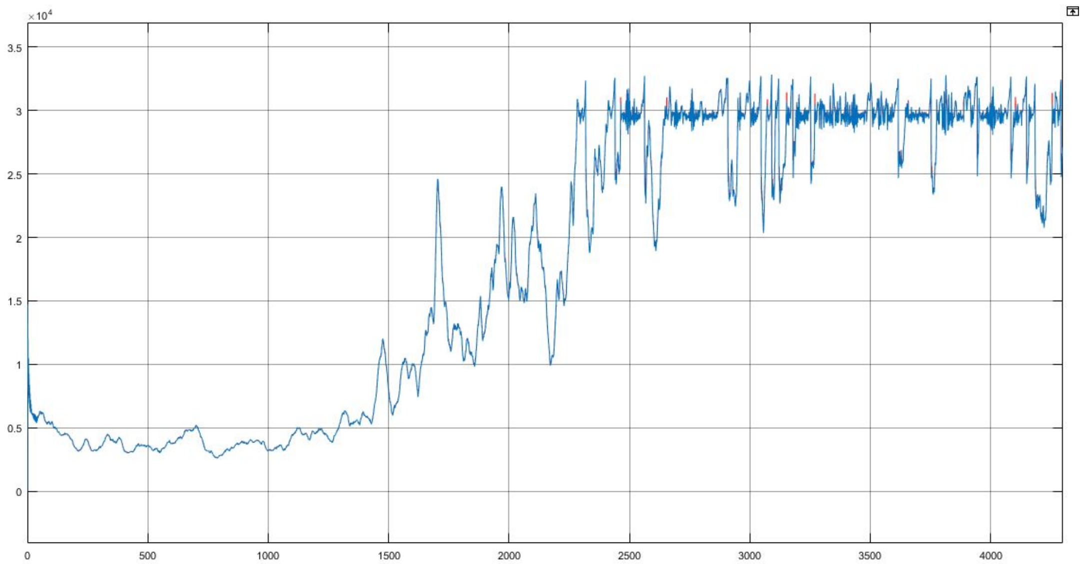

Figure 3.12: Controlled torque obtained by state feedback using the observer

Figure 3.13 shows the pitch angle

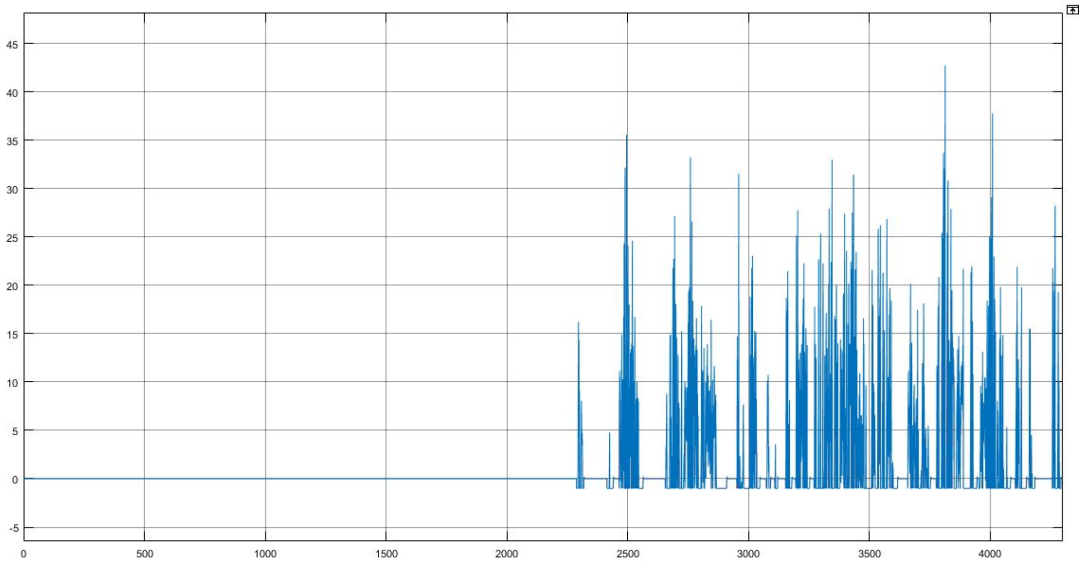

Figure 3.13: Controlled pitch angle obtained by state feedback using the observer

Chapter 4

Comparison with PI controller

4.1 T-S controller

Now we can compare the generated torque and pitch angle with the result we obtained from state feedback T-S controller.

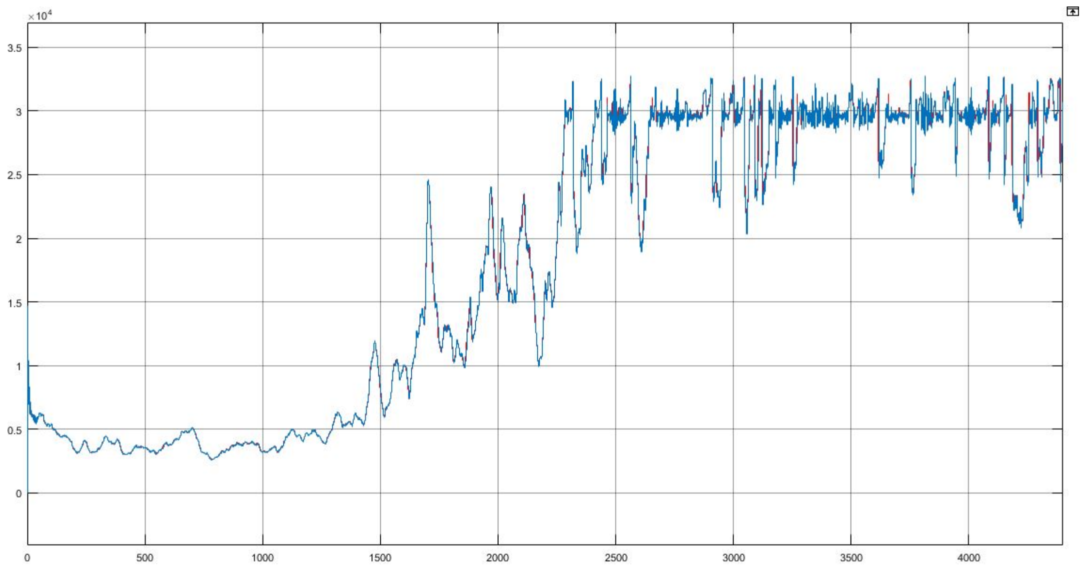

Figure 4.1: Output torque generated by state feedback T-S controller and PI controller

In this figure, the torque generated by state feedback T-S controller (blue) is almost match the torque generated by PI controller (red).

We can make a zoom in of mode 1 part. We can see in the figure below.

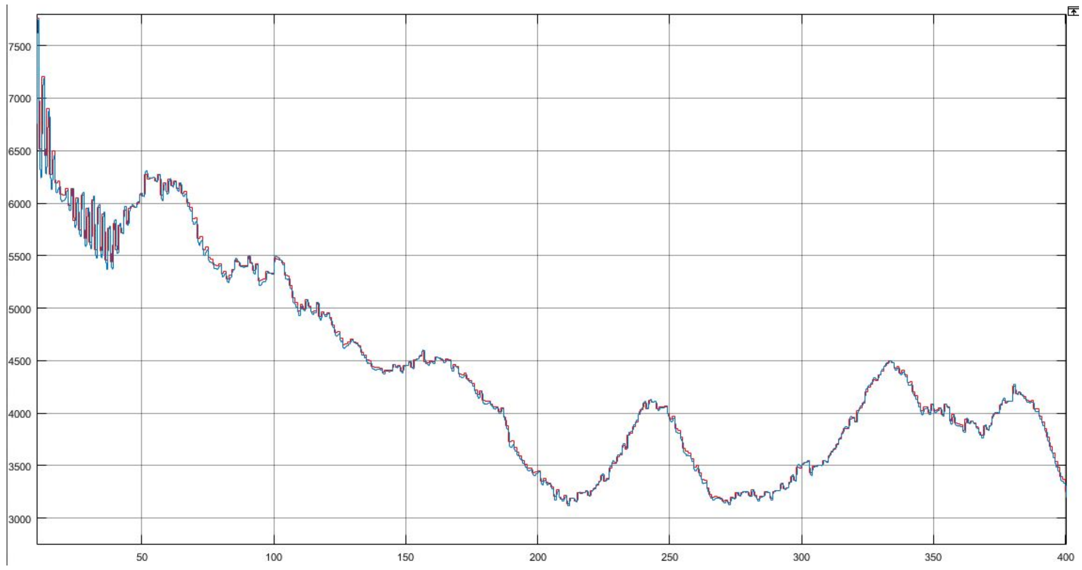

Figure 4.2: Output torque generated by state feedback T-S controller and PI controller in time 0 to 400s

Theoretically, in this part the two curves should be the same, because in mode 1, system does not has state feedback. We can see that there are small difference between two curves, a possible reason on this maybe is the error of the simulation between differential equation and the state-space model.

Then we can make zoom in on mode 2, we can see the figure below.

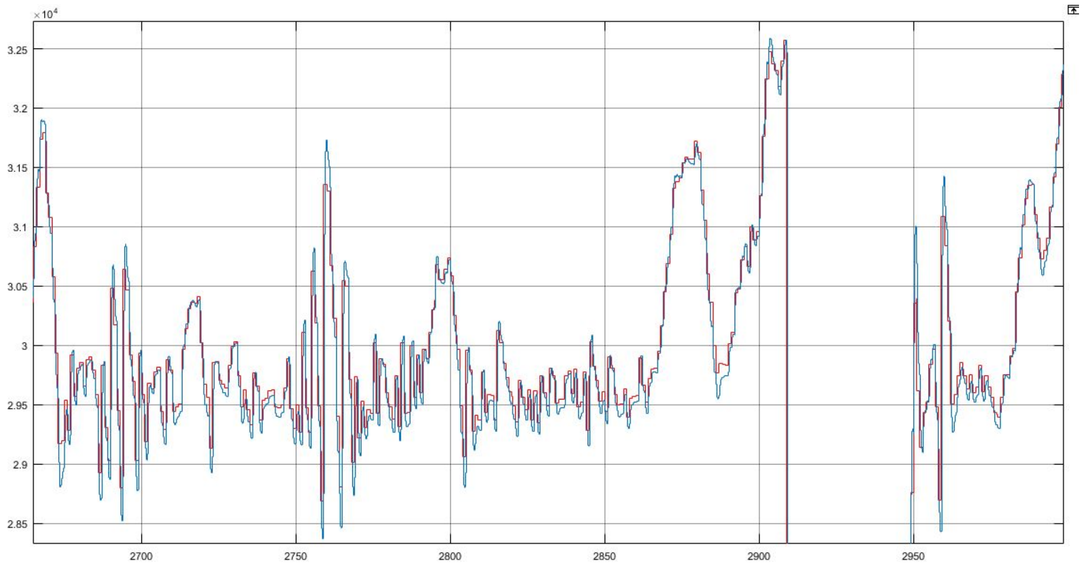

Figure 4.3: Output torque generated by state feedback T-S controller and PI controller in time 2600s to 3000s

In this part, the difference becomes larger. The state feedback of PI (mode 2) starts to work. And the torque under the T-S controller (blue) has a little overshoot.

Additionally, we can see the pitch angle.

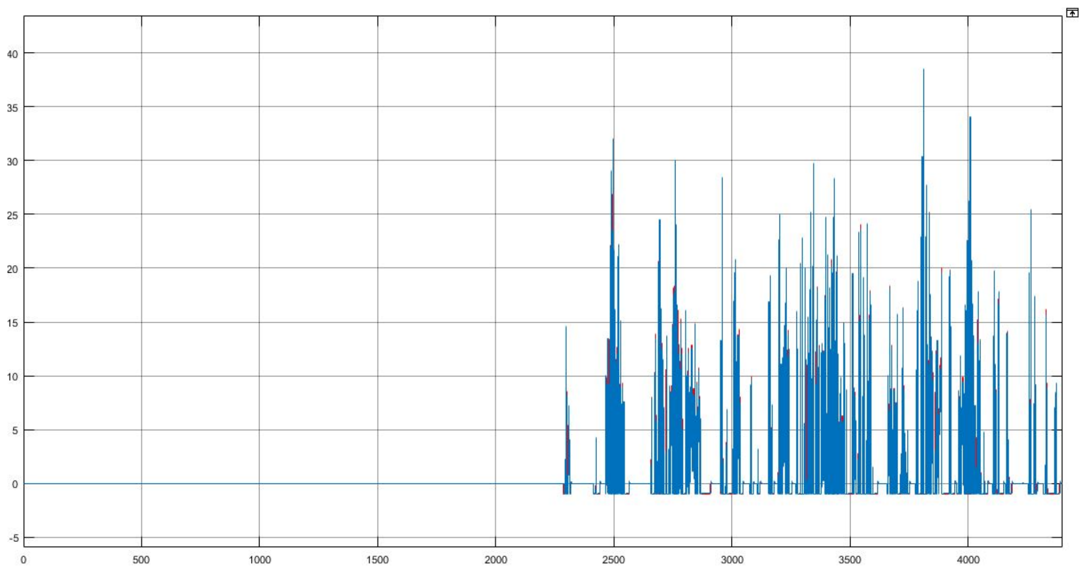

Figure 4.4: Output pitch angle generated by T-S controller and PI controller

In Figure 4.4, we can see the pitch angle of the state feedback T-S controller (blue) is almost match the pitch angle generated by PI controller (red).

Also we can make a zoom in of this result. We can see that at zone 2, there is no turning on the blade, the pitch angle is 0. So we can see the detail from 2600s.

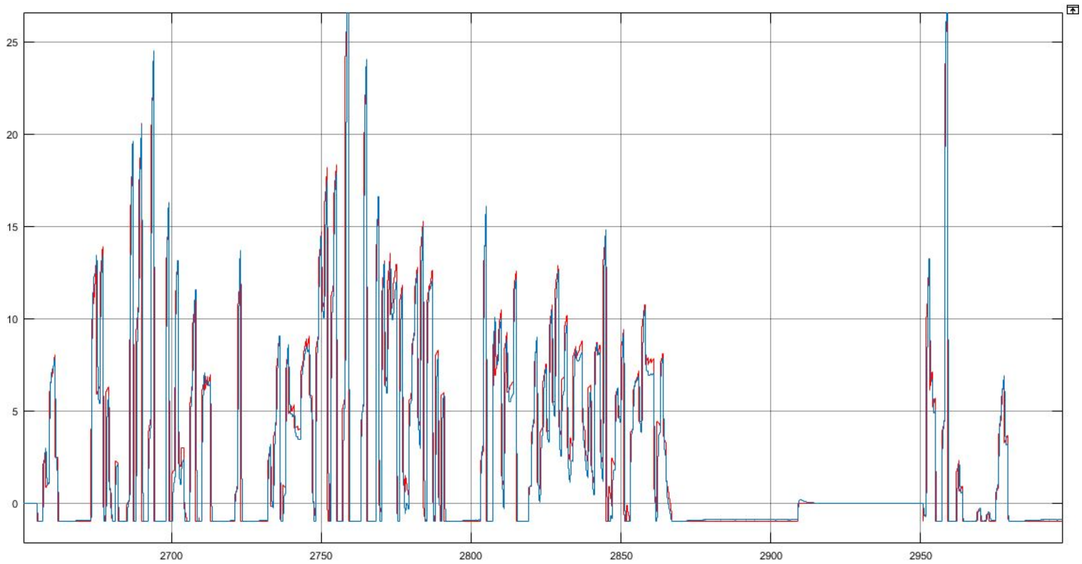

Figure 4.5: Output pitch angle generated by T-S controller and PI controller in time 2600s to 3000s

In Figure 4.5 we can see that there are small overshoot.

4.2 T-S observer based control

Similarly, we can also compare the result with T-S observer based control.

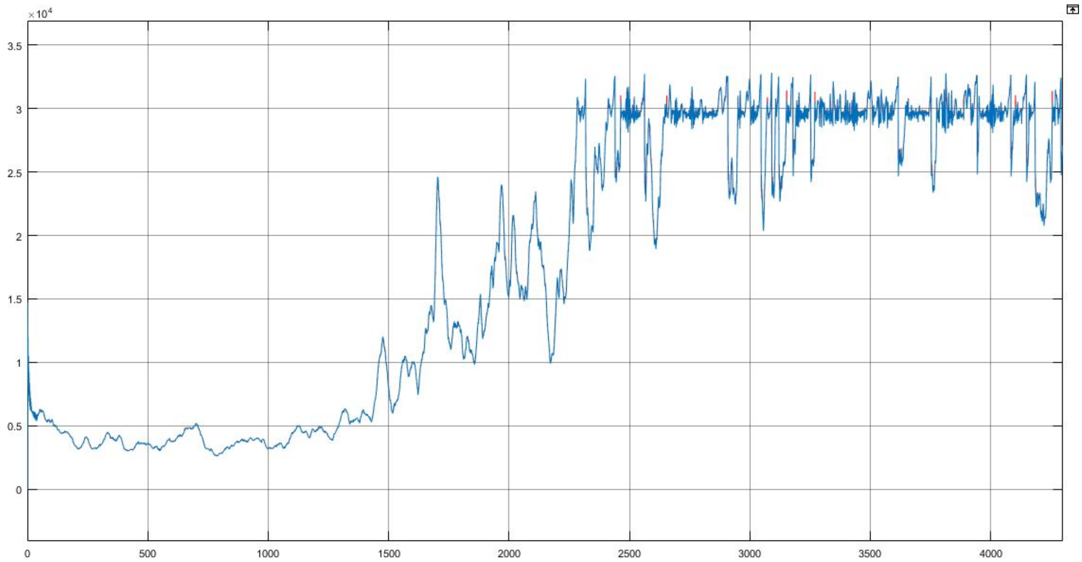

Figure 4.6: Output torque generated by T-S observer based state feedback T-S controller and PI controller

The result looks similar with the previous in Figure 4.1, we can also make a zoom in of each mode. Firstly, we can see the mode 1 part in the figure below.

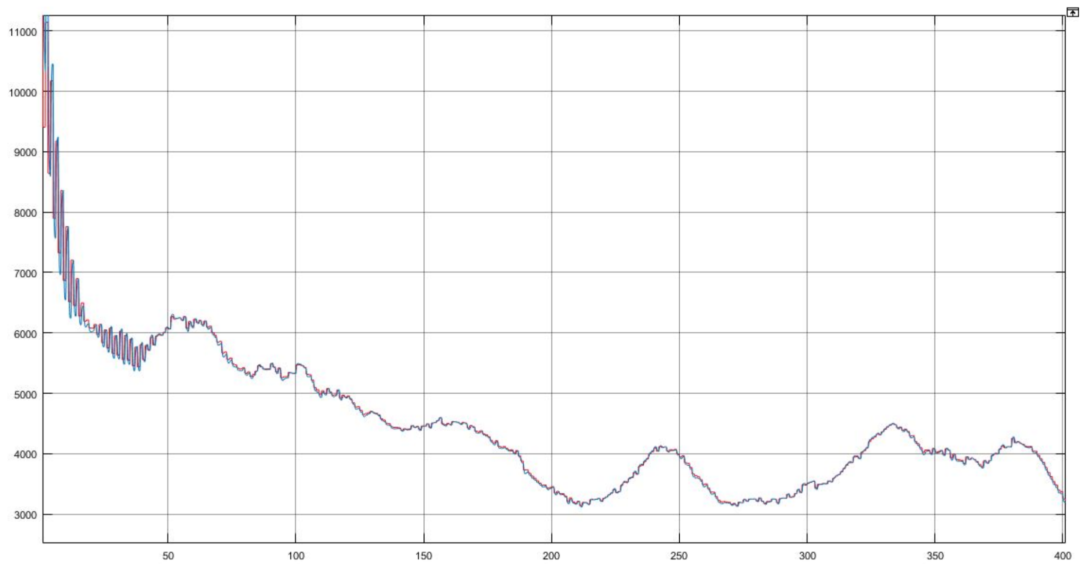

Figure 4.7: Output torque generated by T-S observer based state feedback T-S controller and PI controller from time 0 to 400s

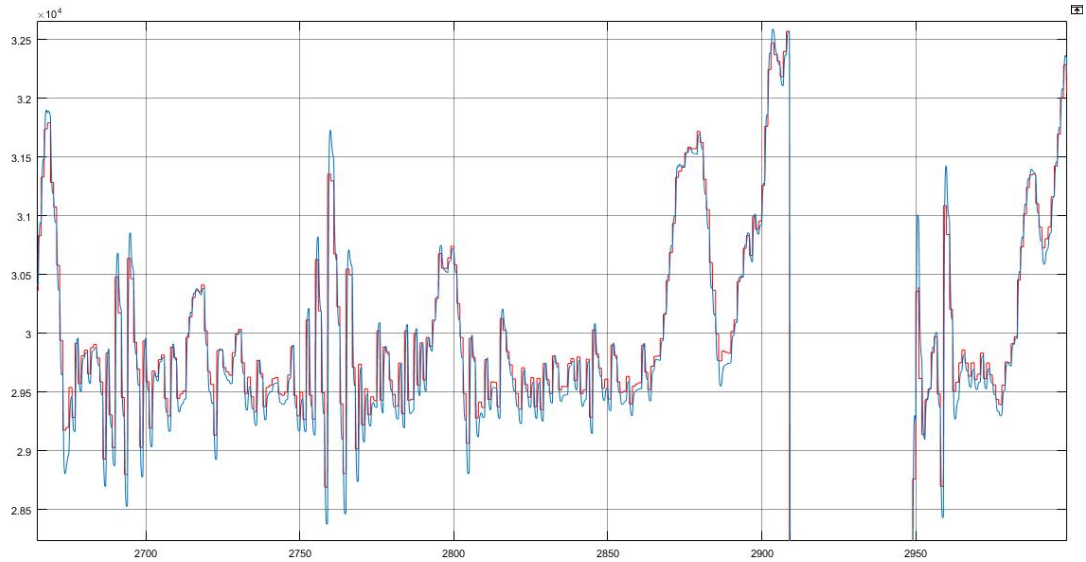

Figure 4.8: Output torque generated by T-S observer based state feedback T-S controller and PI controller from time 2600s to 3000s

Comparing Figures 4.2 and 4.3, there is no significant improvement, the overshoot is more or less the same, also the setting time, but the curve becomes more smooth.

Also we can take a look for the pitch angle.

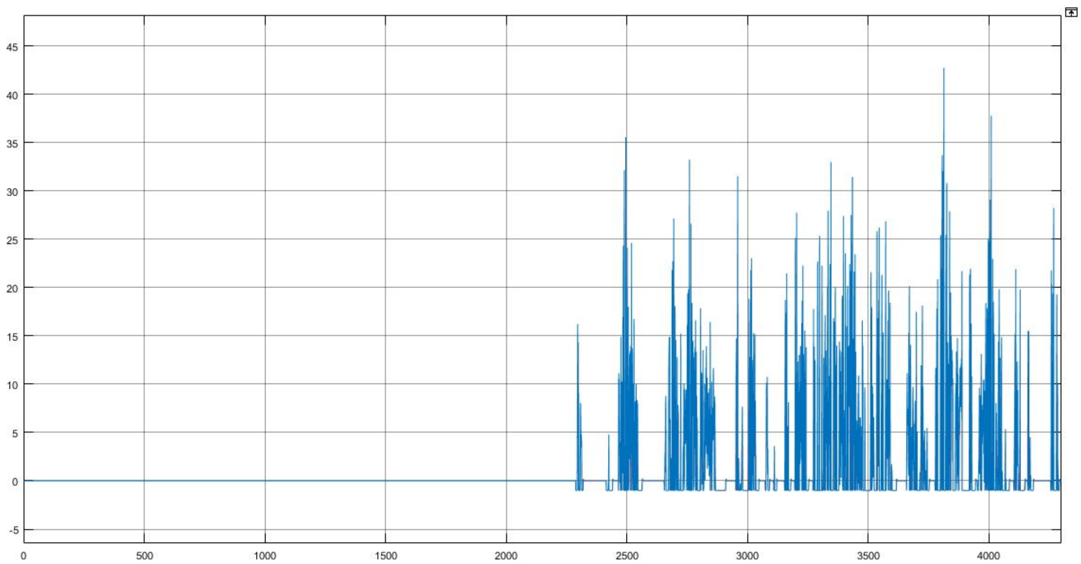

Figure 4.9: Output pitch angle generated by T-S observer based state feedback T-S controller and PI controller

For pitch angle there is a significant improvement, we can see the detail from a zoom in.

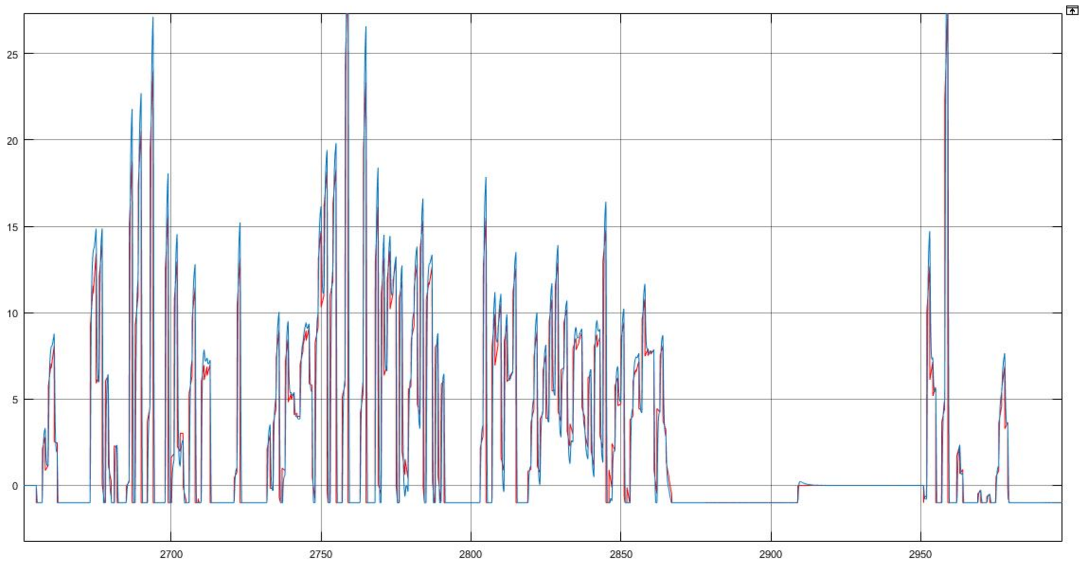

Figure 4.10: Output pitch angle generated by T-S observer based state feedback T-S controller and PI controller in time 2600s to 3000s

We can see that the overshoot is smaller than the previous 4.5.

Chapter 5

Conclusions

5.1 Work Summery

In this thesis, a horizontal-axis wind turbine (HAWT) has modeled into a state-space representation and transformed into a Takagi-Sugeno (T-S) model structure. The T-S model exactly represents the nonlinear model as a weighted combination of linear models.

Then a state feedback control schemes for wind turbines were investigated based on a Takagi-Sugeno controller and Takagi-Sugeno observer. The controller and observer were obtained by using LMIs, where the constrains are based on Lyapunov stability theory and LMI region $\mathbb{S}(\alpha, r, \theta)$ stabilization [21]. In this part, choosing the suitable parameter (α, r, θ) is very important. They can directly influence the controller performance, α is the minimum speed of the response, r is the maximum speed of the response, and θ is the overshoot. These parameter can not set as much as possible, otherwise it will obtain positive poles or the poles are out of the LMI region $\mathbb{S}$.

By tested on T-S wind turbine model. The wind speed we are using include low speed and high speed, which means that it include Zone 2 and Zone 3 (See Figure 2.3 and 2.4). The performance is very well, with only the T-S controller, the outputs keep reaching the reference and no too much overshoot, then the observer based state feedback control were tested, the performance is similar like the previous, approximately same overshoot, same setting time, but more smooth, where the performance is similar with the PI controller in [14].

For a conclusion, we can say that Takagi-Sugeno approach is a good way for presenting the nonlinear system of wind turbine. The T-S controller can give a very good performance under a suitable LMI condition. The T-S observer estimate the states very perfect. For wind turbine case study, T-S approach can be a powerful tool for the future research.

5.2 Future work

This T-S model can be improved, for decreasing the error.

The performance of the controller can be improved, and also it can apply by other control methodology on T-S model, for example sliding model control, H_∞ control, MPC, etc.

For the simulation the $4.8MW$ HAWT by using SIMULINK, this T-S model can be embeded in the benchmark model [14], and replace the controller Mode 2 by T-S controller. Then see if there are better performance.

Additionally this work can be tested on The FAST (Fatigue, Aerodynamics, Structures, and Turbulence) Code, it should be more accurate for wind turbine case study.

Furthermore, this can be a starting point for FDI (Fault detection and isolation) and FTC (Fault Tolerant Control) concepts, because now the accidents on wind turbine are getting increase. The following figure shows the accidents up to May 2017.

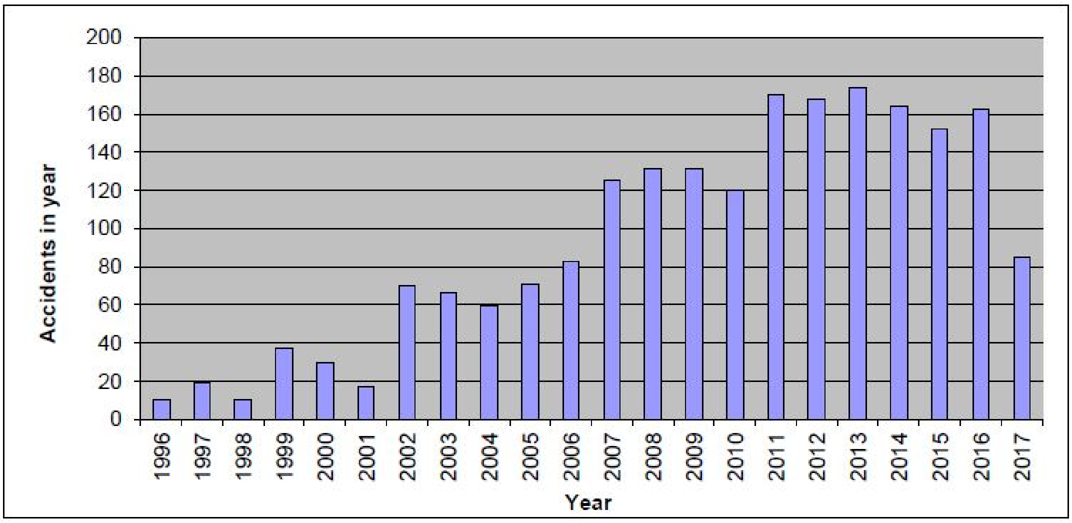

Figure 5.1: Wind turbine accidents in year, up to 31 of May 2017.Figure from [13]

Many cases can cause the wind turbine accident, blade failure, fire, structural failure, Ice throw, transport, environmental damage (including bird deaths) and other miscellaneous (Component or mechanical failure, lack of maintenance, electrical failure, Construction and construction support accidents, lightning strikes). In these accidents, poor quality control can cause a portion of structural failure.

Year	Before 2000	2000	2001	2002	2003	2004	2005	2006
Number of Accidents	15	9	3	9	7	4	7	9
Year	2007	2008	2009	2010	2011	2012	2013	2014
Number of Accidents	13	9	16	9	13	10	14	13
Year	2015	2016	2017					
Number of Accidents	12	11	6					

Table 5.1: Structural failure of wind turbine up to 31 May 2017

For decrease this kind of accident, keeping the wind turbine works in a normal and stable status seems very important, especially FDI and FTC technique.

Appendix

MATLAB code for TS controller design

Notice that the Aerodynamics data is required, which it contains λ, β, C_q and C_p (See in Section 2.2).

```
% TS model for controller desgin
clear all; clc; close all;

load AeroDynamics.mat
[ANGLE,LAMBDA] = meshgrid(Angle,Lambda);
ANGLE = ANGLE(:,11:end);
LAMBDA = LAMBDA(:,11:end);
Cq = Cq(:,11:end);
CqLAMBDA = Cq./ANGLE;
% wind turbine parameter
omega_n=11.11; xi=0.6; rho=1.225; R=57.5; J_r=55e6; B_dt=775.49; B_g=45.6;
B_r=7.11; N_g=95; K_dt=2.7e9; eta_dt=0.97; J_g=390; vwmax = 25; tau_g = 20e-3;

thetamin = rho*pi*R^3*vwmax^2*min(min(CqLAMBDA))/(6*J_r);
thetamax = rho*pi*R^3*vwmax^2*max(max(CqLAMBDA))/(6*J_r);

thetarange = [thetamin thetamin thetamin ;
              thetamax thetamax thetamax]';

amatcaixa = pvec('box',thetarange);
caixavertex = polydec(amatcaixa);

nx = 10; ny = 6;
Avertex = zeros(nx,nx,size(caixavertex,2));
ATvertex = zeros(nx,nx,size(caixavertex,2));
Cvertex = zeros(ny,nx,size(caixavertex,2));
CTvertex = zeros(nx,ny,size(caixavertex,2));

a11 = -(B_dt+B_r)/J_r; a12 = B_dt/(N_g*J_r); a13 = -K_dt/J_r;
a21 = eta_dt*B_dt/(N_g*J_g); a22 = -(eta_dt*B_dt/(N_g^2*J_g)+B_g/J_g); a23 = eta_dt*K_dt/(N_g*J_g);
```

```
a24 = -1/J_g; a32 = -1/N_g; a44 = -1/tau_g; b41 = 1/tau_g; a88 = -2*xi*omega_n;
a65 = -omega_n^2; a66 = -2*xi*omega_n; b62 = omega_n^2; a87 = -omega_n^2;
b83 = omega_n^2; a109 = -omega_n^2; a1010 = -2*xi*omega_n; b104 = omega_n^2;

for k=1:size(caixavertex,2)
    Avertex(:,:,k) = [a11 a12 a13 0 caixavertex(1,k) 0 caixavertex(2,k) 0 caixavertex(3,k) 0 ;
                      a21 a22 a23 a24 0 0 0 0 0 0 ;
                      1 a32 0 0 0 0 0 0 0 0 ;
                      0 0 0 a44 0 0 0 0 0 0 ;
                      0 0 0 0 0 1 0 0 0 0 ;
                      0 0 0 0 a65 a66 0 0 0 0 ;
                      0 0 0 0 0 0 0 1 0 0 ;
                      0 0 0 0 0 0 a87 a88 0 0 ;
                      0 0 0 0 0 0 0 0 0 1 ;
                      0 0 0 0 0 0 0 0 a109 a1010];
    ATvertex(:,:,k)= Avertex(:,:,k)';
    Bvertex(:,:,k) = [0 0 0 b41 0 0 0 0 0 0 ;
                      0 0 0 0 0 b62 0 0 0 0 ;
                      0 0 0 0 0 0 0 b83 0 0 ;
                      0 0 0 0 0 0 0 0 0 b104]';
    BTvertex(:,:,k) = Bvertex(:,:,k)';
    Cvertex(:,:,k) = [1 0 0 0 0 0 0 0 0 0 ;
                      0 1 0 0 0 0 0 0 0 0 ;
                      0 0 0 1 0 0 0 0 0 0 ;
                      0 0 0 0 1 0 0 0 0 0 ;
                      0 0 0 0 0 0 1 0 0 0 ;
                      0 0 0 0 0 0 0 0 1 0];
    CTvertex(:,:,k)= Cvertex(:,:,k)';

end


vertices = size(Avertex,3); % 8

%% DESIGN OF THE OBSERVER

rL = 50; % r
qL = 0;  % q
lambdaL = 0.5; % alpha
thetaL = pi/6; % theta
Kvertex = zeros(4,nx,vertices); % (4,10,8)
PolesK = zeros(nx,vertices); % (10,8)

XL = sdpvar(nx); % P
W = cell(vertices,1); % W
for k=1:vertices
    W{k} = sdpvar(4,nx);
end
```

```

clear F
tic
F = [XL>0];



% LMI condition D-stability
for ii = 1:vertices
    F = [F, Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)+2*lambdaL*XL<0];
    F = [F, [-rL*XL qL*XL+Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii};...
        qL*XL+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii) -rL*XL]<0];
    F = [F, [sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))...
        cos(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}-(XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)));...
        cos(thetaL)*(-(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii})+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))...
        sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))]<0];
end
sdpoptions = sdpsettings('showprogress',1,'solver','sedumi','sedumi.eps',1e-10,'sedumi.maxiter',300);
diagnosticsL = solvesdp(F,[],sdpoptions);
temp = double(XL); clear XL;
XL = double(temp);
for k=1:vertices
    W{k} = double(W{k});
    Kvertex(:,:,k) = W{k}*inv(XL);
    PolesK(:,k) = eig(Avertex(:,:,k)+Bvertex(:,:,k)*Kvertex(:,:,k));
end

toc
%%
display('The LMIs for designing the state feedback controller are:')
if diagnosticsL.problem == 0
 disp('Feasible')
elseif diagnosticsL.problem == 1
 disp('Infeasible')
else
 disp('Something else happened')
end

eigtest = zeros(3*vertices+1,1);
k = 1;
eigtest(k) = max(eig(XL));
for ii = 1:vertices
    k = k+1; eigtest(k) = max(eig(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*
```

```
        BTvertex(:,:,ii)+2*lambdaL*XL));
    k = k+1; eigtest(k) = max(eig([-rL*XL Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii} ; ...
        XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii) -rL*XL]));
    k = k+1; eigtest(k) = max(eig([sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)) ...
        cos(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}-(XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))) ; ...
        cos(thetaL)*(-(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii})+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)) ...
        sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))]));
end
%%
clear W

figure(1);
plot(real(PolesK),imag(PolesK),'.b'); title('Pole clustering of the controller');
hold on;
plot([-lambdaL -lambdaL],[-2*rL 2*rL],'r--',-rL*cos(linspace(0,2*pi,200)),rL*sin(linspace(0,2*pi,200)),'r--',[qL-rL 0 qL-rL],[(-qL+rL)*tan(-thetaL) 0 (-qL+rL)*tan(thetaL)],'r--')
xlabel('Real(s)'); ylabel('Imag(s)');
figure(2);
if(eigtest(1)>0)
    plot(eigtest(2:end));
else
    plot(eigtest);
end
title('Eigenvalues test for the design of the controller');

save datacontroller.mat thetarange Kvertex PolesK Avertex Bvertex
```

MATLAB code for TS observer design

```
% TS model for observer desgin
clear all; clc; close all;

load AeroDynamics.mat
[ANGLE,LAMBDA] = meshgrid(Angle,Lambda);
ANGLE = ANGLE(:,11:end);
LAMBDA = LAMBDA(:,11:end);
Cq = Cq(:,11:end);
CqLAMBDA = Cq./ANGLE;
% wind turbine parameter
omega_n=11.11; xi=0.6; rho=1.225; R=57.5; J_r=55e6; B_dt=775.49; B_g=45.6;
B_r=7.11; N_g=95; K_dt=2.7e9; eta_dt=0.97; J_g=390; vwmax = 25; tau_g = 20e-3;

```

```
thetamin = rho*pi*R^3*vwmax^2*min(min(CqLAMBDA))/(6*J_r);
thetamax = rho*pi*R^3*vwmax^2*max(max(CqLAMBDA))/(6*J_r);

thetarange = [thetamin thetamin thetamin ;
              thetamax thetamax thetamax]';

amatcaixa = pvec('box',thetarange);
caixavertex = polydec(amatcaixa);

nx = 10; ny = 6;
Avertex = zeros(nx,nx,size(caixavertex,2));
ATvertex = zeros(nx,nx,size(caixavertex,2));
Cvertex = zeros(ny,nx,size(caixavertex,2));
CTvertex = zeros(nx,ny,size(caixavertex,2));

obsv_UNFAULTY = zeros(size(caixavertex,2),1);
obsv_LOSS1 = zeros(size(caixavertex,2),1);
obsv_LOSS2 = zeros(size(caixavertex,2),1);
obsv_LOSS3 = zeros(size(caixavertex,2),1);
obsv_LOSS4 = zeros(size(caixavertex,2),1);
obsv_LOSS5 = zeros(size(caixavertex,2),1);
obsv_LOSS6 = zeros(size(caixavertex,2),1);

a11 = -(B_dt+B_r)/J_r; a12 = B_dt/(N_g*J_r); a13 = -K_dt/J_r;
a21 = eta_dt*B_dt/(N_g*J_g); a22 = -(eta_dt*B_dt/(N_g^2*J_g)+B_g/J_g); a23 = eta_dt*K_dt/(N_g*J_g);
a24 = -1/J_g; a32 = -1/N_g; a44 = -1/tau_g; b41 = 1/tau_g; a88 = -2*xi*omega_n;
a65 = -omega_n^2; a66 = -2*xi*omega_n; b62 = omega_n^2; a87 = -omega_n^2;
b83 = omega_n^2; a109 = -omega_n^2; a1010 = -2*xi*omega_n; b104 = omega_n^2;

for k=1:size(caixavertex,2)
    Avertex(:,:,k) = [a11 a12 a13 0 caixavertex(1,k) 0 caixavertex(2,k) 0 caixavertex(3,k) 0 ;
                      a21 a22 a23 a24 0 0 0 0 0 0 ;
                      1 a32 0 0 0 0 0 0 0 0 ;
                      0 0 0 a44 0 0 0 0 0 0 ;
                      0 0 0 0 0 1 0 0 0 0 ;
                      0 0 0 0 a65 a66 0 0 0 0 ;
                      0 0 0 0 0 0 0 1 0 0 ;
                      0 0 0 0 0 0 a87 a88 0 0 ;
                      0 0 0 0 0 0 0 0 0 1 ;
                      0 0 0 0 0 0 0 0 a109 a1010];
    ATvertex(:,:,k)= Avertex(:,:,k)';
    Bvertex(:,:,k) = [0 0 0 b41 0 0 0 0 0 0 ;
                      0 0 0 0 0 b62 0 0 0 0 ;
                      0 0 0 0 0 0 0 b83 0 0 ;
                      0 0 0 0 0 0 0 0 0 b104]';
    Cvertex(:,:,k) = [1 0 0 0 0 0 0 0 0 0 ;
                      0 1 0 0 0 0 0 0 0 0 ;
                      0 0 0 1 0 0 0 0 0 0 ;
```

```
                      0 0 0 0 1 0 0 0 0 0 ;
                      0 0 0 0 0 0 1 0 0 0 ;
                      0 0 0 0 0 0 0 0 1 0];
    CTvertex(:,:,k)= Cvertex(:,:,k)';

    obsv_UNFAULTY(k) = rank(obsv(Avertex(:,:,k),Cvertex(:,:,k)));
    obsv_LOSS1(k) = rank(obsv(Avertex(:,:,k),Cvertex([2 3 4 5 6],:,k)));
    obsv_LOSS2(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 3 4 5 6],:,k)));
    obsv_LOSS3(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 4 5 6],:,k)));
    obsv_LOSS4(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 5 6],:,k)));
    obsv_LOSS5(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 4 6],:,k)));
    obsv_LOSS6(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 4 5],:,k)));
end

    A = [a11 a12 a13 0 ; a21 a22 a23 a24 ; 1 a32 0 0 ; 0 0 0 a44];
    C = [1 0 0 0 ; 0 1 0 0 ; 0 0 0 1];
    obsv_reduced1 = rank(obsv(A,C));
    A = [0 1 ; a65 a66];
    C = [1 0];
    obsv_reduced2 = rank(obsv(A,C));

vertices = size(Avertex,3);

%% DESIGN OF THE OBSERVER

rL = 500;
qL = 0;
lambdaL = 50;
thetaL = pi/3;
Lvertex = zeros(nx,ny,vertices);
PolesL = zeros(nx,vertices);

XL = sdpvar(nx);
W = cell(vertices,1);
for k=1:vertices
    W{k} = sdpvar(ny,nx);
end

clear F
tic
F = [XL>0];
for ii = 1:vertices
    F = [F, ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})'+2*lambdaL*XL<0];
    F = [F, [-rL*XL qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii} ; ...
        (qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})' -rL*XL]<0];
    F = [F, [sin(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ...
```

```
        cos(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}-(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ; ...
        cos(thetaL)*(-(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ...
        sin(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})')]<0];
end
sdpoptions = sdpsettings('showprogress',1,'solver','sedumi','sedumi.eps',1e-10,'sedumi.maxiter',300);
diagnosticsL = solvesdp(F,[],sdpoptions);
temp = double(XL); clear XL;
XL = double(temp);
for k=1:vertices
    W{k} = double(W{k});
    Lvertex(:,:,k) = (W{k}/XL)';
    PolesL(:,k) = eig(Avertex(:,:,k)+Lvertex(:,:,k)*Cvertex(:,:,k));
end
toc

display('The LMIs for designing the state observer are:')
if diagnosticsL.problem == 0
 disp('Feasible')
elseif diagnosticsL.problem == 1
 disp('Infeasible')
else
 disp('Something else happened')
end

eigtest = zeros(3*vertices+1,1);
k = 1;
eigtest(k) = max(eig(XL));
for ii = 1:vertices
    k = k+1; eigtest(k) = max(eig(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})'+2*lambdaL*XL));
    k = k+1; eigtest(k) = max(eig([-rL*XL qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii} ; ...
        (qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})' -rL*XL]));
    k = k+1; eigtest(k) = max(eig([sin(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ...
        cos(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}-(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ; ...
        cos(thetaL)*(-(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})') ...
        sin(thetaL)*(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})')]));
end
%%
clear W

figure(1);
```

```
plot(real(PolesL),imag(PolesL),'.b'); title('Pole clustering of the state observer');
hold on;
plot([-lambdaL -lambdaL],[-2*rL 2*rL],'r--',-rL*cos(linspace(0,2*pi,200)),rL*sin(linspace(0,2*pi,200)),'r--'
     ,[qL-rL 0 qL-rL],[(-qL+rL)*tan(-thetaL) 0 (-qL+rL)*tan(thetaL)],'r--')
xlabel('Real(s)'); ylabel('Imag(s)');
figure(2);
if(eigtest(1)>0)
    plot(eigtest(2:end));
else
    plot(eigtest);
end
title('Eigenvalues test for the design of the state observer');

save dataObserver.mat thetarange Lvertex
```

The Matrix A of Wind Turbine T-S model

$$A_1 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & -0.0304 & 0 & -0.0304 & 0 & -0.0304 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & 0.0873 & 0 & -0.0304 & 0 & -0.0304 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & -0.0304 & 0 & 0.0873 & 0 & -0.0304 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & 0.0873 & 0 & 0.0873 & 0 & -0.0304 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & -0.0304 & 0 & -0.0304 & 0 & 0.0873 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & 0.0873 & 0 & -0.0304 & 0 & 0.0873 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & -0.0304 & 0 & 0.0873 & 0 & 0.0873 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

$$A_8 = \begin{bmatrix} -1.4229\times10^{-5} & 1.4842\times10^{-7} & -49.0909 & 0 & 0.0873 & 0 & 0.0873 & 0 & 0.0873 & 0 \\ 0.0203 & -0.1171 & 7.0688\times10^{4} & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \end{bmatrix}$$

The Matrix B of Wind Turbine T-S model

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 123.4321 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 123.4321 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 123.4321 \end{bmatrix}$$

The Controller Gain of Wind Turbine T-S model

$$K_1 = \begin{bmatrix} -6.6081 \times 10^5 & -235.4576 & -235.5610 & -235.5351 \\ 7.5821 \times 10^3 & 2.5520 & 2.5532 & 2.5529 \\ 3.8556 \times 10^7 & -878.9696 & -877.3409 & -877.2067 \\ -0.5291 & -1.4289 \times 10^{-5} & -1.4377 \times 10^{-5} & -1.4365 \times 10^{-5} \\ 152.3250 & -9.1843 & -0.1076 & -0.1081 \\ -14.8008 & -0.4547 & 6.7634 \times 10^{-4} & 6.7468 \times 10^{-4} \\ 152.2279 & -0.1079 & -9.1843 & -0.1080 \\ -14.8008 & 6.7544 \times 10^{-4} & -0.4547 & 6.7469 \times 10^{-4} \\ 152.2559 & -0.1080 & -0.1081 & -9.1843 \\ -14.7992 & 6.7379 \times 10^{-4} & 6.7472 \times 10^{-4} & -0.4547 \end{bmatrix}^T$$

$$K_2 = \begin{bmatrix} -6.4775\times 10^{5} & 301.8732 & -256.2100 & -256.1775 \\ 7.4389\times 10^{3} & -3.6483 & 2.7718 & 2.7714 \\ 3.8140\times 10^{7} & 4.5556\times 10^{3} & -1.0690\times 10^{3} & -1.0690\times 10^{3} \\ -0.5117 & -3.5147\times 10^{-5} & -1.2809\times 10^{-5} & -1.2787\times 10^{-5} \\ -3.0018\times 10^{3} & -9.5430 & -0.6342 & -0.6342 \\ -18.7857 & -0.4638 & -0.0128 & -0.0128 \\ 204.1472 & 0.0521 & -9.2606 & -0.1845 \\ -13.7177 & -0.0016 & -0.4553 & 3.1784\times 10^{-5} \\ 204.1747 & 0.0521 & -0.1844 & -9.2607 \\ -13.7181 & -0.0016 & 3.3548\times 10^{-5} & -0.4553 \end{bmatrix}^T$$

$$K_3 = \begin{bmatrix} -6.4775\times 10^{5} & -256.1874 & 301.99112 & -256.2039 \\ 7.4390\times 10^{3} & -2.7715 & -3.6496 & 2.7717 \\ 3.8140\times 10^{7} & -1.0698\times 10^{3} & 4.5540\times 10^{3} & -1.0695\times 10^{3} \\ -0.5117 & -1.2776\times 10^{-5} & -3.5065\times 10^{-5} & -1.2784\times 10^{-5} \\ 204.1122 & -9.2606 & 0.0525 & -0.1884 \\ -13.7188 & -0.4553 & -0.0016 & 3.3218\times 10^{-5} \\ -3.0018\times 10^{3} & -0.6341 & -9.5431 & -0.6342 \\ -18.7858 & -0.0128 & -0.4638 & -0.0128 \\ 204.1072 & -0.1844 & 0.0524 & -9.2607 \\ -13.7179 & 3.1949\times 10^{-5} & -0.0016 & -0.4553 \end{bmatrix}^T$$

$$K_4 = \begin{bmatrix} -6.5441\times 10^{5} & 289.2313 & 289.1852 & -287.8942 \\ 7.5187\times 10^{3} & -3.4788 & -3.4783 & 3.1342 \\ 3.7992\times 10^{7} & 3.8168\times 10^{3} & 3.8172\times 10^{3} & -1.2454\times 10^{3} \\ -0.5088 & -1.6466\times 10^{-5} & -1.6499\times 10^{-5} & -1.7288\times 10^{-5} \\ -2.9996\times 10^{3} & -9.3801 & -0.3039 & -0.6121 \\ -19.3131 & -0.4603 & -0.0050 & -0.0127 \\ -2.9996\times 10^{3} & -0.3039 & -9.3803 & -0.6120 \\ -19.3130 & -0.0050 & -0.4603 & -0.0127 \\ 202.1078 & -0.0012 & -0.0012 & -9.3055 \\ -13.4734 & -0.0030 & -0.0030 & -0.4554 \end{bmatrix}^T$$

$$K_5 = \begin{bmatrix} -6.4779 \times 10^5 & -256.2578 & -256.3059 & 302.0925 \\ 7.4394 \times 10^3 & 2.7724 & 2.7730 & -3.6509 \\ 3.8141 \times 10^7 & -1.0689 \times 10^3 & -1.0687 \times 10^3 & 4.5528 \times 10^3 \\ -0.5117 & -1.2831 \times 10^{-5} & -1.2861 \times 10^{-5} & -3.4975 \times 10^{-5} \\ 204.0164 & -9.2607 & -0.1843 & 0.0524 \\ -13.7205 & -0.4553 & 3.7125 \times 10^{-5} & -0.0016 \\ 203.9850 & -0.1843 & -9.2606 & 0.0524 \\ -13.7193 & 3.4116 \times 10^{-5} & -0.4553 & -0.0016 \\ -3.0021^3 & -0.6342 & -0.6343 & -9.5429 \\ -18.7911 & -0.0128 & -0.0128 & -0.4638 \end{bmatrix}^T$$

$$K_6 = \begin{bmatrix} -6.5431 \times 10^5 & -289.1592 & -287.8585 & 289.1112 \\ 7.5176 \times 10^3 & -3.4779 & 3.1338 & -3.4774 \\ 3.7991 \times 10^7 & 3.8180 \times 10^3 & -1.2461 \times 10^3 & 3.8189 \times 10^3 \\ -0.5087 & -1.6545 \times 10^{-5} & -1.7249 \times 10^{-5} & -1.6584 \times 10^{-5} \\ 2.9994 \times 10^3 & -9.3803 & -0.6119 & -0.3043 \\ -19.3082 & -0.4603 & -0.0127 & -0.0050 \\ 202.2692 & -0.0012 & -9.3054 & -0.0013 \\ -13.4725 & -0.0030 & -0.4554 & -0.0030 \\ -2.9994^3 & -0.3042 & -0.6120 & -9.3806 \\ -19.3067 & -0.0050 & -0.0127 & -0.4603 \end{bmatrix}^T$$

$$K_7 = \begin{bmatrix} -6.5417 \times 10^5 & -287.7359 & 288.9450 & 288.9411 \\ 7.5159 \times 10^3 & 3.1324 & -3.4755 & -3.4754 \\ 3.7991 \times 10^7 & -1.2472 \times 10^3 & 3.8202 \times 10^3 & 3.8206 \times 10^3 \\ -0.5086 & -1.7175 \times 10^{-5} & -1.6681 \times 10^{-5} & -1.6689 \times 10^{-5} \\ 202.6746 & -9.3050 & -0.0018 & -0.0020 \\ -13.4656 & -0.4554 & -0.0030 & -0.0030 \\ -2.9997 \times 10^3 & -0.6120 & -9.3800 & -0.3038 \\ -19.3152 & -0.0127 & -0.4603 & -0.0050 \\ -2.9997^3 & -0.6121 & -0.3037 & -9.3800 \\ -19.3137 & -0.0127 & -0.0050 & -0.4603 \end{bmatrix}^T$$

$$K_8 = \begin{bmatrix} -6.6936\times10^5 & 283.3040 & 283.2523 & 283.2774 \\ 7.6992\times10^3 & -3.3850 & -3.3844 & -3.3847 \\ 3.7898\times10^7 & 3.0947\times10^3 & 3.0946\times10^3 & 3.0947\times10^3 \\ -0.5117 & 1.6220\times10^{-6} & 1.6020\times10^{-6} & 1.6185\times10^{-6} \\ -2.9959\times10^3 & -9.2665 & -0.1902 & -0.1903 \\ -19.7544 & -0.4582 & -0.0028 & -0.0028 \\ -2.9959\times10^3 & -0.1902 & -9.2667 & -0.1903 \\ -19.7535 & -0.0028 & -0.4582 & -0.0028 \\ -2.9959^3 & -0.1903 & -0.1904 & -9.2665 \\ -19.7543 & -0.0028 & -0.0028 & -0.4582 \end{bmatrix}^T$$

The Observer Gain of Wind Turbine T-S model

$$L_1 = \begin{bmatrix} -110.1902 & -7.6203 & -7.9445 & 3.4867 & -7.9037 & 4.4180 \\ -1.6351\times10^3 & -547.8591 & -476.0504 & 104.0550 & -310.4509 & 66.7191 \\ -8.6364 & -2.0073 & -1.7948 & 0.7026 & -1.6 & 0.4723 \\ 0.3972 & 0.0098 & -0.5319 & 0.0950 & 0.0702 & -0.2058 \\ 4.4347 & -0.6755 & -46.4516 & -177.8392 & -3.5263 & -0.0018 \\ 173.5854 & -53.9899 & -2.9764\times10^3 & -6.6615\times10^3 & -182.9723 & -29.7156 \\ -6.5259 & 0.4094 & 12.6860 & -1.8113 & -168.4083 & -0.4079 \\ -148.0912 & 48.1471 & 831.9713 & -198.4434 & -6.4346\times10^3 & 68.5189 \\ 5.8482 & 0.9137 & 1.7098 & -0.1267 & 2.3324 & -172.2188 \\ 258.1297 & 36.5226 & 83.2163 & -3.9799 & 144.8436 & -6.86\times10^3 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -109.6385 & -7.8222 & -6.1151 & 6.9819 & -7.0975 & 3.9843 \\ -1.6174\times10^3 & -548.5564 & -342.1237 & 291.6040 & -260.0630 & 42.1194 \\ -8.5620 & -2.0151 & -1.3090 & 1.4244 & -1.4161 & 0.3755 \\ 0.3836 & 0.0144 & -0.4801 & 0.0834 & 0.0731 & -0.2005 \\ -2.7396 & -1.2344 & -3.7865 & -182.0787 & 0.0469 & 0.3345 \\ -282.5246 & -75.4089 & -257.3216 & -6.8845\times10^3 & 39.4311 & -15.9691 \\ -0.7699 & 0.1088 & -16.2923 & 5.4132 & -172.9995 & -1.3064 \\ 233.0251 & 39.3652 & -1.0074\times10^3 & 211.0560 & -6.6884\times10^3 & 18.8557 \\ 5.3453 & 0.7321 & -2.9040 & 1.3258 & 0.4436 & -168.9640 \\ 233.7646 & 27.8699 & -218.2892 & 93.2598 & 48.6677 & -6.7379\times10^3 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -111.1250 & -8.8792 & -8.3852 & -0.0866 & 0.2484 & 8.0118 \\ -1.7058 \times 10^3 & -550.5897 & -455.7737 & -72.7657 & 114.5116 & 242.2445 \\ -8.8884 & -2.0611 & -1.7441 & 0.0235 & 0.0269 & 1.1479 \\ 0.3936 & 0.0148 & -0.4821 & 0.0851 & 0.0596 & -0.2124 \\ 3.4701 & 0.8554 & -16.8637 & -185.9632 & 0.0267 & 0.3237 \\ 137.1636 & 16.0830 & -1.0974 \times 10^3 & -7.1771 \times 10^3 & 41.7604 & 13.1356 \\ -2.1533 & -2.3220 & -1.4028 & 0.6235 & -169.3548 & -2.5547 \\ 91.3365 & -61.7626 & -54.7692 & -65.0241 & -6.5388 \times 10^3 & -72.9122 \\ 4.8400 & -0.4825 & 3.2434 & -1.0478 & 1.1018 & -170.4604 \\ 164.0856 & -26.2823 & 183.05044 & -63.0097 & 111.1238 & -6.8330 \times 10^3 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} -105.4526 & -7.9962 & -0.2105 & -0.6779 & -0.4861 & 4.4681 \\ -1.3389 \times 10^3 & -553.2440 & -37.3995 & -125.0797 & 74.1895 & 75.8004 \\ -7.5244 & -2.0432 & -0.1249 & -0.1555 & -0.1283 & 0.5032 \\ 0.3867 & 0.0110 & -0.5257 & 0.1028 & 0.0603 & -0.2070 \\ -0.3065 & -0.0667 & 4.6760 & -178.4511 & 6.4975 & -1.7869 \\ -79.6761 & -34.8378 & 262.1044 & -6.6427 \times 10^3 & 432.0387 & -145.0900 \\ -4.1902 & -2.9425 & 20.8248 & 3.8026 & -168.8737 & 1.5010 \\ -83.4520 & -91.3365 & 1.3566 \times 10^3 & 106.8090 & -6.4836 \times 10^3 & 173.4844 \\ 4.7563 & 1.0182 & -7.6826 & -0.7090 & 0.8058 & -170.5892 \\ 218.6711 & 40.1636 & -503.3398 & -46.2311 & 61.2126 & -6.7996 \times 10^3 \end{bmatrix}$$

$$L_5 = \begin{bmatrix} -104.7293 & -6.4917 & 24.0376 & 7.0548 & 2.4146 & 3.9324 \\ -1.3555 \times 10^3 & -543.2514 & 1.2564 \times 10^3 & 302.6711 & 223.6370 & 41.8943 \\ -7.5587 & -1.9626 & 4.8024 & 1.4568 & 0.4466 & 0.3750 \\ 0.3870 & 0.0353 & -0.4843 & 0.0799 & 0.0468 & -0.2117 \\ 1.6193 & -2.1645 & 3.9421 & -180.3373 & -0.6355 & -0.2503 \\ 40.0312 & -120.0279 & 304.7652 & -6.8308 \times 10^3 & 23.5521 & -52.1217 \\ -2.6792 & -1.8969 & 3.9508 & 2.5382 & -169.4048 & -0.4300 \\ 57.2773 & -48.7051 & 93.2176 & 36.1387 & -6.5357 \times 10^3 & 68.0529 \\ 3.2066 & 0.4190 & 14.3290 & -0.4677 & -0.6618 & -171.8501 \\ 100.0895 & 12.1342 & 1.0014 \times 10^3 & -33.9605 & -8.1962 & -6.9250 \times 10^3 \end{bmatrix}$$

$$
L_6 = \begin{bmatrix}
-111.6844 & -7.8952 & -6.2935 & -3.8477 & -4.9570 & 7.27096 \\
-1.7188 \times 10^3 & -549.0893 & -332.3252 & -275.2274 & -161.1051 & 227.0177 \\
-8.9500 & -2.0231 & -1.2835 & -0.7522 & -1.0234 & 1.0784 \\
0.3991 & 0.0113 & -0.4464 & 0.1040 & 0.0612 & -0.2174 \\
3.9544 & 1.5892 & -16.9282 & -183.5102 & -1.8489 & 1.0338 \\
179.6454 & 38.8406 & -1.0875 \times 10^3 & -7.0217 \times 10^3 & -79.1611 & 31.6258 \\
-6.0559 & -0.2800 & -5.5666 & -1.4296 & -167.0782 & 1.1970 \\
-114.8826 & 21.9083 & -341.4645 & -133.5394 & -6.3844 \times 10^3 & 144.0781 \\
4.3025 & 0.3519 & 14.3059 & 1.6999 & 0.6438 & -172.1566 \\
157.9751 & 12.4279 & 915.3956 & 70.4329 & 61.2577 & -6.9344 \times 10^3
\end{bmatrix}
$$

$$
L_7 = \begin{bmatrix}
-108.2040 & -7.5045 & -9.7605 & 7.7550 & -1.4068 & 7.7646 \\
-1.5263 \times 10^3 & -546.7569 & -534.3626 & 327.4700 & 45.2978 & 253.0217 \\
-8.2192 & -2.0028 & -2.0338 & 1.5653 & -0.2470 & 1.1782 \\
0.3890 & 0.0088 & -0.4474 & 0.0768 & 0.0656 & -0.2123 \\
-0.1912 & -1.3068 & -3.7625 & -183.4477 & 4.2944 & -0.0096 \\
-99.7616 & -80.7704 & -265.9834 & -6.9850 \times 10^3 & 301.3019 & -18.5448 \\
-3.7186 & -0.1513 & 16.9524 & 4.2745 & -166.8024 & -0.8920 \\
-11.7978 & 34.1723 & 1.1577 \times 10^3 94.7052 & -6.3681 \times 10^3 & 14.3286 & \\
4.7414 & 0.3204 & 10.3315 & -0.3334 & 1.1943 & -171.7243 \\
200.1382 & 10.6690 & 651.1185 & 1.0255 & 106.8290 & -6.8470 \times 10^3
\end{bmatrix}
$$

$$
L_8 = \begin{bmatrix}
-109.0576 & -7.4757 & -3.3499 & 0.0981 & -9.1298 & 3.3808 \\
-1.5833 \times 10^3 & -546.7527 & -209.0207 & -74.8028 & -362.9247 & 18.1625 \\
-8.4310 & -2.0021 & -0.7810 & 0.0192 & -1.8085 & 0.2815 \\
0.3930 & 0.0090 & -0.4829 & 0.0879 & 0.0743 & -0.2084 \\
-0.2199 & 0.3541 & 4.3510 & -186.0633 & -0.7371 & -0.3030 \\
-95.8112 & -12.5396 & 261.7076 & -7.1860 \times 10^3 & -6.8450 & -59.0251 \\
-2.2270 & 0.7792 & 12.3329 & 0.5793 & -168.2786 & 0.1182 \\
122.1867 & 69.0153 & 820.7416 & -35.2306 & -6.3964 \times 10^3 & 98.3385 \\
5.1005 & 1.0760 & 0.9720 & 0.2994 & 1.8858 & -171.3168 \\
226.8578 & 41.5065 & 49.0076 & 2.6549 & 107.9132 & -6.8893 \times 10^3
\end{bmatrix}
$$

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