Les polyèdres convexes aux faces régulières

par Sylvain Gagnon

Topologie Structurale ≠6, 1982

Ce dossier photographique, réalisé dans le cadre de l'unité en Topologie structurale à l'Université de Montréal, illustre tous les polyèdres convexes dénombrés à faces régulières (à l'exception des familles infinies que sont les prismes et les antiprismes, chacune d'entre elles étant représentée par un élément) par l'étude publiée sous le titre de *Convex Polyhedra with Regular Faces* du mathématicien russe Viktor A. Zalgaller.

Cette étude publiée en 1966 démontre mathématiquement les différents agencements de polygones réguliers aux sommets et parvient à fixer une limite à 92 polyèdres aux faces régulières, en plus de ceux déjà connus que sont les 5 polyèdres réguliers, les 13 semi-réguliers plus les deux familles infinies des prismes et des antiprismes.

La liste des collaborateurs de Zalgaller est longue: citons tout de même l'un des principaux, N.W. Johnson (U.S.A.) qui présente un tableau des polyèdres similaire à celui de Zalgaller, quoique plus complet sur certaines données; qui ont notamment servi dans l'élaboration du tableau présenté dans ce dossier. Soulignons que dans ce même tableau l'appellation des 92 polyèdres convexes autres que les réguliers et les semi-réguliers vient de la traduction de l'anglais et que ce travail fut facilité par le concours de la grammairienne de l'Université de Montréal, Madeleine Sauvé; mais qu'à cela ne tienne, la traduction n'en est pas pour autant officielle.

Pour ce qui est des polyèdres réguliers et semi-réguliers, le tableau respecte l'ordre d'énumération représenté dans le manuel du *Poly-kit* de Janos Baracs.

La présentation des polyèdres convexes par Zalgaller a ceci d'intéressant qu'elle nous indique la composition de chacun de ces polyèdres selon un groupe de 28 polyèdres

Structural Topology ≠6, 1982

Convex Polyhedra with Regular Faces

This collection of photographs, assembled in the context of project of the Structural Topology unit at the University of Montreal, illustrates all those convex polyhedra with regular faces (with the exception of two infinite families of such polyhedra, the prisms and antiprisms, each family here being represented by a single member) enumerated by the Soviet mathematician Viktor A. Zalgaller and published under the title *Convex Polyhedra with Regular Faces*.

His report, published in 1966, shows mathematically the different possible placements of regular polygons around a vertex, and proves that there are no more than 92 polyhedra with regular faces, in addition to those already known, that is the 5 regular polyhedra, the 13 semi-regular polyhedra, and the two infinite families of prisms and antiprisms.

The list of collaborators of Zalgaller is long: we mention only one of the principal collaborators, N.W. Johnson (U.S.A.) who published a table of polyhedra similar to that of Zalgaller, but more complete with respect to certain data, which has been important in the preparation of the table presented in this collection. We wish to point out that in our table, the nomenclature for the 92 convex polyhedra, other than the regular and semi-regular polyhedra, a task in which we were aided by the grammarian of the University of Montreal, Madeleine Sauvé; this does not mean, however that this translation is therefore official.

In so far as the regular and semi-regular polyhedra are concerned, our table respects the order of enumeration given in the manual for the *Poly-kit* of Janos Baracs.

The presentation of the convex polyhedra by Zalgaller is interesting in that it shows us the make up of each of the polyhedra in terms of a collection of 28 simple polyhedra simples aux faces régulières et de quelques prismes et antiprismes qui peuvent entrer dans la composition d'un ou de plusieurs polyèdres convexes aux faces régulières. Ces polyèdres simples sont pour quelques-uns des polyèdres entiers réguliers ou semi-réguliers. D'autres sont des polyèdres réguliers ou semi-réguliers qui sont diminués par l'ablation d'une, de deux ou trois calottes; et les calottes elles-mêmes figurent parmi ces polyèdres simples. Il ne reste que 8 cas exceptionnels ne découlant pas de polyèdres réguliers ou semi-réguliers et il s'agit de M_8, M_{20}, M_{21}, M_{22}, M_{23}, M_{24}, M_{25}, M_{28}.

J'ajouterai comme dernière précision que les polyèdres invertis, si différents de leur polyèdre premier, ne sont pas considérés topologiquement comme de nouveaux polyèdres.

Bibliographie

Johnson, N.W., *Convex Polyhedra with Regular Faces*, Canad. J. Math., 1966, 18(1): pp. 169-200.

Zalgaller, Viktor A., *Convex Polyhedra with Regular Faces*, Leningrad, Nanka Press, 1966; 2e édition, New York, Consultants Bureau, 1969, 95 pp.

with regular faces, and of certain prisms and antiprisms, which can enter into the composition of one or more convex polyhedra with regular faces. Among these simple polyhedra are the regular and semi-regular polyhedra. Others are the regular or semi-regular polyhedra, reduced by the removal of one, two or three calottes; these calottes themselves figure among the simple polyhedra. There then remain only 8 exceptional cases not arising from the regular and semi-regular polyhedra, namely M_8, M_{20}, M_{21}, M_{22}, M_{23}, M_{24}, M_{25}, M_{28}.

I add a final note of explanation that inverted polyhedra, if they differ from their original forms, are not considered topologically to be new polyhedra.

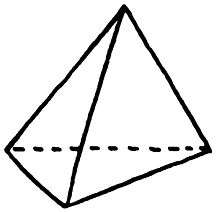
Bibliography

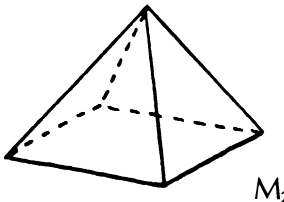
Johnson, N.W., *Convex Polyhedra with Regular Faces*, Canad. J. Math, 1966, 18(1): pp. 169-200.

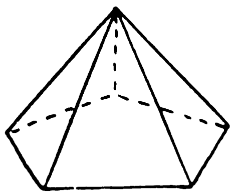
Zalgaller, Viktor A., *Convex Polyhedra with Regular Faces*, Leningrad, Nanka Press, 1966; 2nd edition, New York, Consultants Bureau, 1969, 95pp.

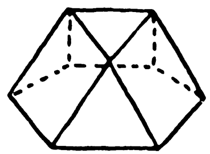
Les polyèdres simples et les prismes et antiprismes qui entrent dans la composition des polyèdres convexes aux faces régulières

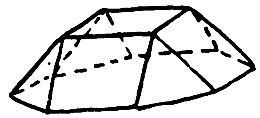
The simple polyhedra, the prisms and antiprisms which enter into the composition of convex polyhedra with regular faces

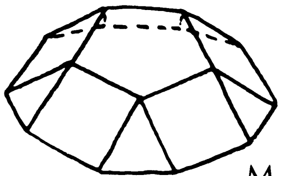

M_1

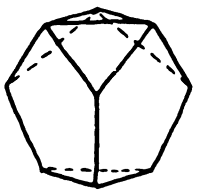

M_2

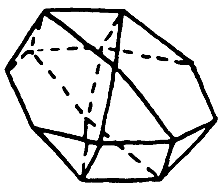

M_3

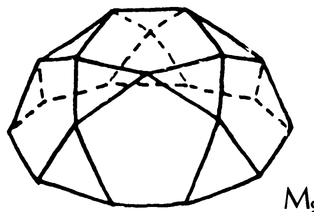

M_4

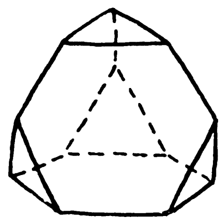

M_5

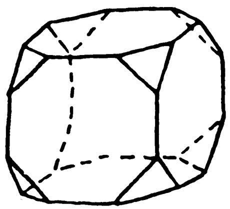

M_6

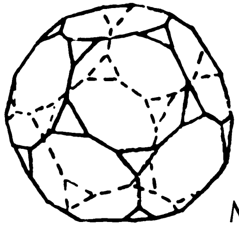

M_7

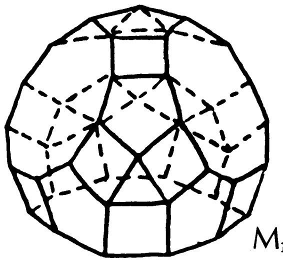

$\underline{M_8}$

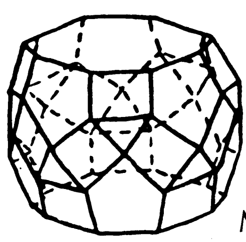

M_9

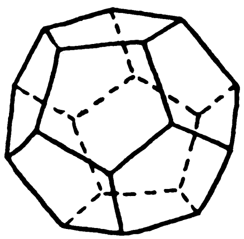

M_{10}


M_{11}


M_{12}

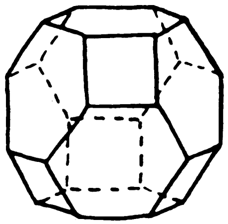

M_{13}


M_{14}

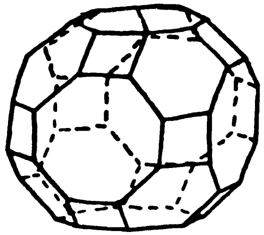

M_{15}

Les polyèdres simples et les prismes et antiprismes qui entrent dans la composition des polyèdres convexes aux faces régulières

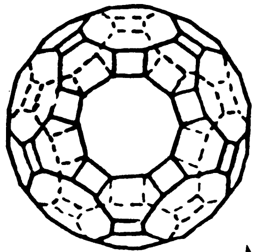
The simple polyhedra, the prisms and antiprisms which enter into the composition of convex polyhedra with regular faces



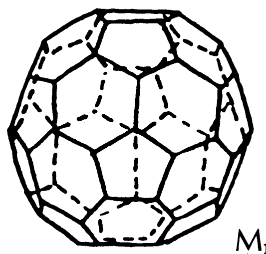
M_{16}



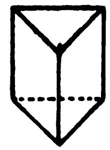
M_{17}



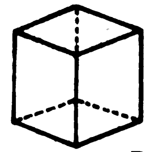
M_{18}



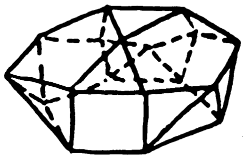
M_{19}



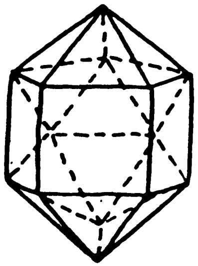
P_3



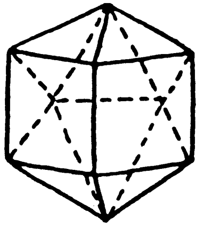
P_4



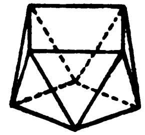
$\underline{M_{20}}$



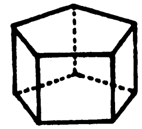
$\underline{M_{21}}$



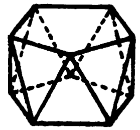
$\underline{M_{22}}$



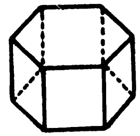
A_4



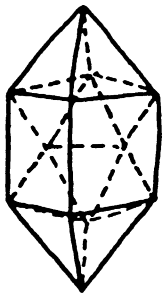
P_5



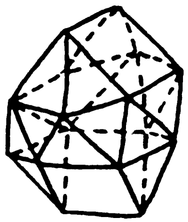
A_5



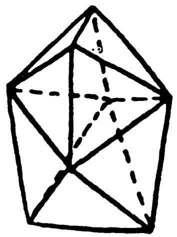
P_6



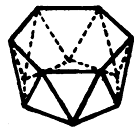
$\underline{M_{23}}$



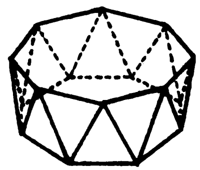
$\underline{M_{24}}$



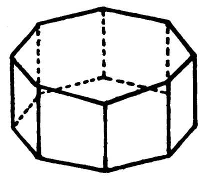
$\underline{M_{25}}$



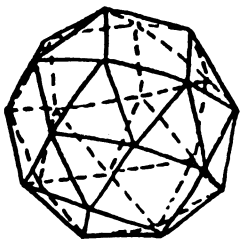
A_6



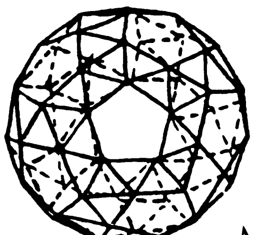
A_8



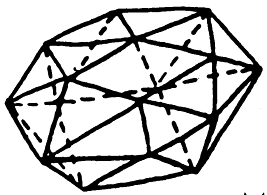
P_8



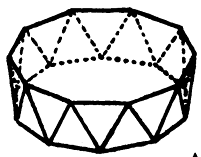
M_{26}



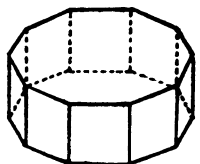
M_{27}



$\underline{M_{28}}$



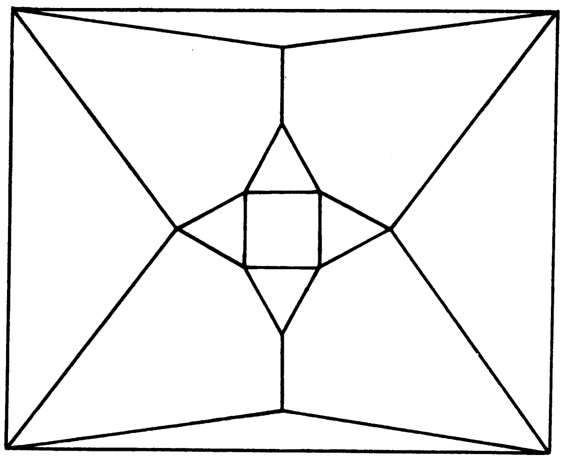
A_{10}



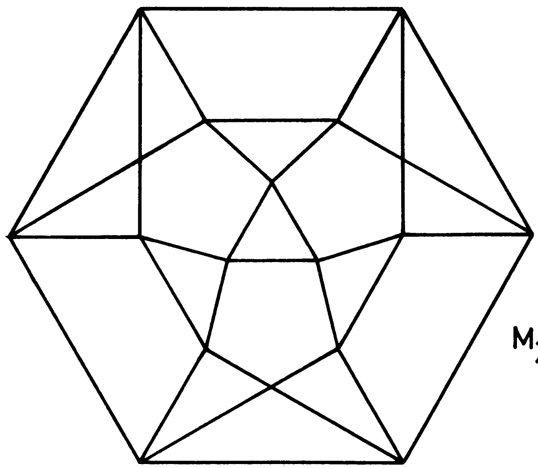
P_{10}

Diagrammes Schlegel des huit polyèdres spéciaux

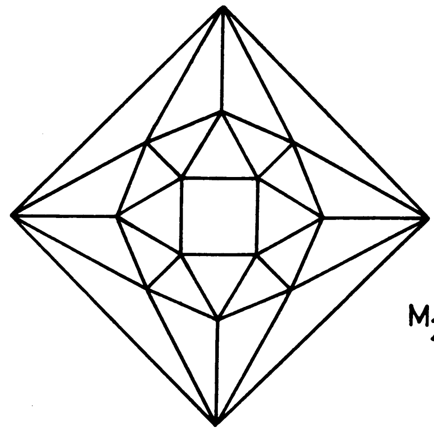
Schlegel diagrams of the eight special polyhedra



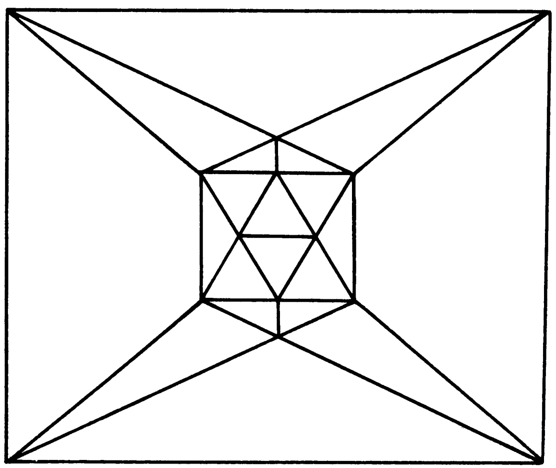
M_8



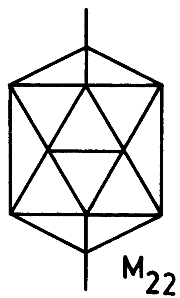
M_{20}



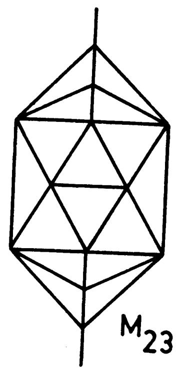
M_{28}



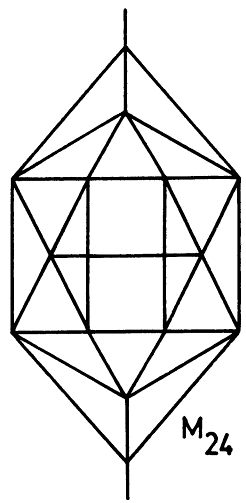
M_{21}



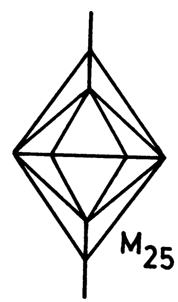
M_{22}



M_{23}



M_{24}



M_{25}

Polyèdres réguliers

N°	Nom		Composition	S	A	F 3	4	5	6	8	10
1 R	tétraèdre	tetrahedron	M_1	4	6	4					
2 R	octaèdre	octahedron	$2M_2 = A_3$	6	12	8					
3 R	cube	cube	P_4	8	12		6				
4 R	icosaèdre	icosahedron	$A_5 + 2M_3 = M_7 + 3M_3$	12	30	20					
5 R	dodécaèdre	dodecahedron	M_{15}	20	30			12			

Polyèdres semi-réguliers

N°	Nom		Composition	S	A	F 3	4	5	6	8	10
1 s-r	tétraèdre tronqué	truncated tetrahedron	M_{10}	12	18	4			4		
2 s-r	cube tronqué	truncated cube	M_{11}	24	36	8				6	
3 s-r	octaèdre tronqué	truncated octahedron	M_{16}	24	36		6		8		
4 s-r	cuboctaèdre	coboctahedron	$M_4 + \mathbf{M_4}$	12	24	8	6				
5 s-r	cuboctaèdre tronqué	truncated cuboctahedron	M_{17}	48	72		12		8	6	
6 s-r	rhombicuboctaèdre	rhombicuboctahedron	$M_5 + P_8 + M_5$	24	48	8	18				
7 s-r	cube adouci	snub cube	M_{26}	24	60	32	6				
8 s-r	dodécaèdre tronqué	truncated dodecahedron	M_{12}	60	90	20					12
9 s-r	icosaèdre tronqué	truncated icosahedron	M_{19}	60	90			12	20		
10 s-r	icosidodécaèdre	icosidodecahedron	$M_9 + \mathbf{M_9}$	30	60	20		12			
11 s-r	icosidodécaèdre tronqué	truncated icosidodecahedron	M_{18}	120	180		30		20		12
12 s-r	rhombicosidodécaèdre	rhombicosidodecahedron	$M_{14} + 2M_6 = M_{13} + 3M_6$	60	120	20	30	12			
13 s-r	dodécaèdre adouci	snub dodecahedron	M_{27}	60	150	80		12			
14 s-r	prismes	prisms	$2(n) + n(4)$	2n	3n		n				
15 s-r	antiprismes	antiprisms	$2(n) + 2n(3)$	2n	4n	2n					

Autres polyèdres convexes aux faces régulières

N° F	Nom		Composition	S	A	3	4	5	6	8	10
1	pyramide carrée	square pyramid	M_2	5	8	4	1				
2	pyramide pentagonale	pentagonal pyramid	M_3	6	10	5		1			
3	coupole triangulaire	triangular cupola	M_4	9	15	4	3		1		
4	coupole carrée	square cupola	M_5	12	20	4	5			1	
5	coupole pentagonale	pentagonal cupola	M_6	15	25	5	5	1			1
6	rotonde pentagonale	pentagonal rotunda	M_9	20	35	10		6			1
7	pyramide triangulaire allongée	elongated triangular pyramid	$M_1 + P_3$	7	12	4	3				
8	pyramide carrée allongée	elongated square pyramid	$M_2 + P_4$	9	16	4	5				
9	pyramide pentagonale allongée	elongated pentagonal pryamid	$M_3 + P_5$	10	20	5	5	1			
10	pyramide carrée gyroallongée	gyroelongated square pyramid	$M_2 + A_4$	9	20	12	1				
11	pyramide pentagonale gyroallongée	gyroelongated pentagonal pyramid	$M_3 + A_5 = M_7 + 2M_3$	11	25	15		1			
12	dipyramide triangulaire	triangular dipyramid	$2M_1$	5	9	6					
13	dipyramide pentagonale	pentagonal dipyramid	$2M_3$	7	15	10					
14	dipyramide triangulaire allongée	elongated triangular dipyramid	$M_1 + P_3 + M_1$	8	15	6	3				
15	dipyramide carrée allongée	elongated square dipyramid	$M_2 + P_4 + M_2$	10	20	8	4				
16	dipyramide pentagonale allongée	elongated pentagonal dipyramid	$M_3 + P_5 + M_3$	12	25	10	5				
17	dipyramide carrée gyroallongée	gyroelongated square dipyramid	$M_2 + A_4 + M_2$	10	24	16					
18	coupole triangulaire allongée	elongated triangular cupola	$M_4 + P_6$	15	27	4	9		1		
19	coupole carrée allongée	elongated square cupola	$M_5 + P_8$	20	36	4	13			1	
20	coupole pentagonale allongée	elongated pentagonal cupola	$M_6 + P_{10}$	25	45	5	15	1			1
21	rotonde pentagonale allongée	elongated pentagonal rotunda	$M_9 + P_{10}$	30	55	10	10	6			1
22	coupole triangulaire gyroallongée	gyroelongated triangular cupola	$M_4 + A_6$	15	33	16	3		1		
23	coupole carrée gyroallongée	gyroelongated square cupola	$M_5 + A_8$	20	44	20	5			1	
24	coupole pentagonale gyroallongée	gyroelongated pentagonal cupola	$M_6 + A_{10}$	25	55	25	5	1			1
25	rotonde pentagonale gyroallongée	gyroelongated pentagonal rotunda	$M_9 + A_{10}$	30	65	30	6				1
26	gyrobifastigium	gyrobifastigium	$P_3 + \mathbf{P}_3$	8	14	4	4				
27	orthobicoupole triangulaire	triangular orthobicupola	$2M_4$	12	24	8	6				
28	orthobicoupole carrée	square orthobicupola	$2M_5$	16	32	8	10				
29	gyrobicoupole carrée	square gyrobicupola	$M_5 + \mathbf{M}_5$	16	32	8	10				
30	orthobicoupole pentagonale	pentagonal orthobicupola	$2M_6$	20	40	10	10	2			
31	gyrobicoupole pentagonale	pentagonal gyrobicupola	$M_6 + \mathbf{M}_6$	20	40	10	10	2			
32	orthocoupole-rotonde pentagonale	pentagonal orthocupolarotunda	$M_6 + M_9$	25	50	15	5	7			
33	gyrocoupole-rodonde pentagonale	pentagonal gyrocupolarotunda	$M_6 + \mathbf{M}_9$	25	50	15	5	7			
34	orthobirotonde pentagonale	pentagonal orthobirotunda	$2M_9$	30	60	20		12			
35	orthobicoupole triangulaire allongée	elongated triangular orthobicupola	$M_4 + P_6 + M_4$	18	36	8	12				
36	gyrobicoupole triangulaire allongée	elongated triangular gyrobicupola	$M_4 + P_6 + \mathbf{M}_4$	18	36	8	12				
37	gyrobicoupole carrée allongée	elongated square gyrobicupola	$M_5 + P_8 + \mathbf{M}_5$	24	48	8	18				
38	orthobicoupole pentagonale allongée	elongated pentagonal orthobicupola	$M_6 + P_{10} + M_6$	30	60	10	20	2			
39	gyrobicoupole pentagonale allongée	elongated pentagonal gyrobicupola	$M_6 + P_{10} + \mathbf{M}_6$	30	60	10	20	2			
40	orthocoupole-rotonde pentagonale allongée	elongated pentagonal orthocupolarotunda	$M_6 + P_{10} + M_9$	35	70	15	15	7			
41	gyrocoupole-rotonde pentagonale allongée	elongated pentagonal gyrocupolarotunda	$M_6 + P_{10} + \mathbf{M}_9$	35	70	15	15	7			
42	orthobirotonde pentagonale allongée	elongated pentagonal orthobirotunda	$M_9 + P_{10} + \mathbf{M}_9$	40	80	20	10	12			
43	gyrobirotonde pentagonale allongée	elongated pentagonal gyrobirotunda	$M_9 + P_{10} + \mathbf{M}_8$	40	80	20	10	12			

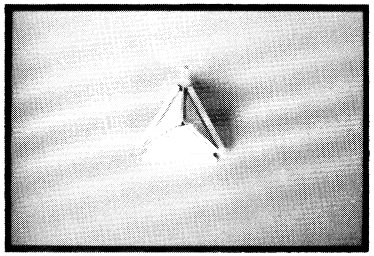
44	bicoupole triangulaire gyroallongée	gyroelongated triangular bicupola	$M_4 + A_6 + M_4$	18	42	20	6					
45	bicoupole carrée gyroallongée	gyroelongated square bicupola	$M_5 + A_8 + M_5$	24	56	24	10					
46	bicoupole pentagonale gyroallongée	gyroelongated pentagonal bicupola	$M_6 + A_{10} + M_6$	30	70	30	10	2				
47	coupole-rotonde pentagonale gyroallongée	gyroelongated pentagonal cupolarotunda	$M_6 + A_{10} + M_9$	35	80	35	5	7				
48	birotonde pentagonale gyroallongée	gyroelongated pentagonal birotunda	$M_9 + A_{10} + M_9$	40	90	40		12				
49	prisme triangulaire augmenté	augmented triangular prism	$P_3 + M_2$	7	13	6	2					
50	prisme triangulaire biaugmenté	biaugmented triangular prism	$P_3 + 2M_2$	8	17	10	1					
51	prisme triangulaire triaugmenté	triaugmented triangular prism	$P_3 + 3M_2$	9	21	14						
52	prisme pentagonal augmenté	augmented pentagonal prism	$P_5 + M_2$	11	19	4	4	2				
53	prisme pentagonal biaugmenté	biaugmented pentagonal prism	$P_5 + 2M_2$	12	23	8	3	2				
54	prisme hexagonal augmenté	augmented hexagonal prism	$P_6 + M_2$	13	22	4	5		2			
55	prisme hexagonal parabiaugmenté	parabiaugmented hexagonal prism	$M_2 + P_6 + M_2$	14	26	8	4		2			
56	prisme hexagonal métabiaugmenté	metabiaugmented hexagonal prism	$P_6 + 2M_2$	14	26	8	4		2			
57	prisme hexagonal triaugmenté	triaugmented hexagonal prism	$P_6 + 3M_2$	15	30	12	3		2			
58	dodécaèdre augmenté	augmented dodecahedron	$M_{15} + M_3$	21	35	5		11				
59	dodécaèdre parabiaugmenté	parabiaugmented dodecahedron	$M_3 + M_{15} + M_3$	22	40	10		10				
60	dodécaèdre métabiaugmenté	metabiaugmented dodecahedron	$M_{15} + 2M_3$	22	40	10		10				
61	dodécaèdre triaugmenté	triaugmented dodecahedron	$M_{15} + 3M_3$	23	45	15		9				
62	icosaèdre métabidiminué	metabidiminished icosahedron	$M_7 + M_3$	10	20	10		2				
63	icosaèdre tridiminué	tridiminished icosahedron	M_7	9	15	5		3				
64	icosaèdre tridiminué augmenté	augmented tridiminished icosahedron	$M_7 + M_1$	10	18	7		3				
65	tétraèdre tronqué augmenté	augmented truncated tetrahedron	$M_{10} + M_4$	15	27	8	3		3			
66	cube tronqué augmenté	augmented truncated cube	$M_{11} + M_5$	28	48	12	5			5		
67	cube tronqué biaugmenté	biaugmented truncated cube	$M_5 + M_{11} + M_5$	32	60	16	10			4		
68	dodécaèdre tronqué augmenté	augmented truncated dodecahedron	$M_6 + M_{12}$	65	105	25	5	1			11	
69	dodécaèdre tronqué parabiaugmenté	parabiaugmented truncated dodecahedron	$M_6 + M_{12} + M_6$	70	120	30	10	2			10	
70	dodécaèdre tronqué métabiaugmenté	metabiaugmented truncated dodecahedron	$M_{12} + 2M_6$	70	120	30	10	2			10	
71	dodécaèdre tronqué triaugmenté	triaugmented truncated dodecahedron	$M_{12} + 3M_6$	75	135	35	15	3			9	
72	rhombicosidodécaèdre circiné	gyrate rhombicosidodecahedron	$\mathbf{M}_6 + M_{14} + M_6 = \mathbf{M}_6 + M_{13} + 2M_6$	60	120	20	30	12				
73	rhombicosidodécaèdre parabicirciné	parabigyrate rhombicosidodecahedron	$\mathbf{M}_6 + M_{14} + \mathbf{M}_6$	60	120	20	30	12				
74	rhombicosidodécaèdre métabicirciné	metabigyrate rhombicosidodecahedron	$2\mathbf{M}_6 + M_{13} + M_6$	60	120	20	30	12				
75	rhombicosidodécaèdre tricirciné	trigyrate rhombicosidodecahedron	$3\mathbf{M}_6 + M_{13}$	60	120	20	30	12				
76	rhombicosidodécaèdre diminué	diminished rhombicosidodecahedron	$M_6 + M_{14} = 2M_6 + M_{13}$	55	105	15	25	11			1	
77	rhombicosidodécaèdre diminué paracirciné	paragyrate diminished rhombicosidodecahedron	$M_{14} + \mathbf{M}_6$	55	105	15	25	11			1	
78	rhombicosidodécaèdre diminué métacirciné	metagyrate diminished rhombicosidodecahedron	$M_{13} + M_6 + \mathbf{M}_6$	55	105	15	25	11			1	
79	rhombicosidodécaèdre diminué bicirciné	bigyrate diminished rhombicosidodecahedron	$M_{13} + 2\mathbf{M}_6$	55	105	15	25	11			1	
80	rhombicosidodécaèdre paradiminué	parabidiminished rhombicosidodecahedron	M_{14}	50	90	10	20	10			2	
81	rhombicosidodécaèdre métabidiminué	metabidiminished rhombicosidodecahedron	$M_{13} + M_6$	50	90	10	20	10			2	
82	rhombicosidodécaèdre bidiminué circiné	gyrate bidiminished rhombicosidodecahedron	$M_{13} + \mathbf{M}_6$	50	90	10	20	10			2	
83	rhombicosidodécaèdre tridiminué	tridiminished rhombicosidodecahedron	M_{13}	45	75	5	15	9			3	
84	disphénoïde adouci	snub disphenoid	M_{25}	8	18	12						
85	antiprisme carré adouci	snub square antiprism	M_{28}	16	40	24	2					
86	sphénocouronne	sphenocorona	M_{22}	10	22	12	2					
87	sphénocouronne augmentée	augmented sphenocorona	$M_{22} + M_2$	11	26	16	1					
88	sphénomégacouronne	sphenomegacorona	M_{23}	12	28	16	2					
89	hébésphénomégacouronne	hebesphenomegacorona	M_{21}	14	33	18	3					
90	disphénocingulum	disphenocingulum	M_{24}	16	38	20	4					
91	bilune-birotonde	bilunabirotunda	M_8	14	26	8	2	4				
92	hébésphénorotonde triangulaire	triangular hebesphenorotunda	M_{20}	18	36	13	3	3	1			

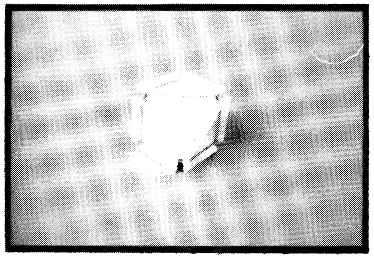
Les polyèdres convexes

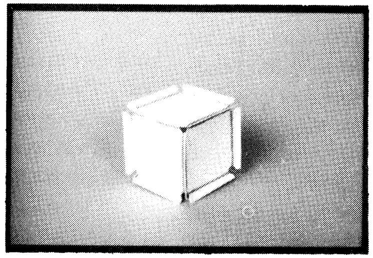
The convex polyhedra

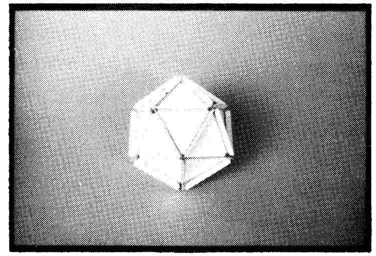
réguliers

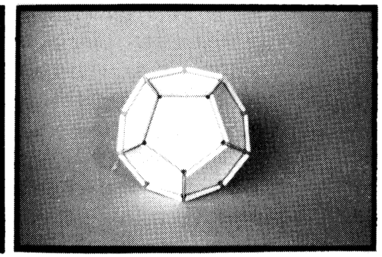
regular


1r


2r

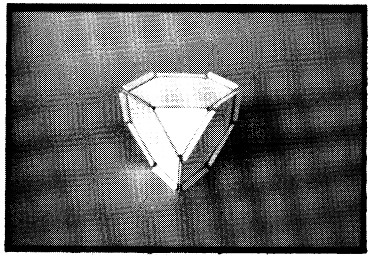

3r

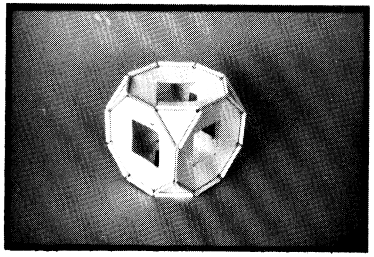

4r

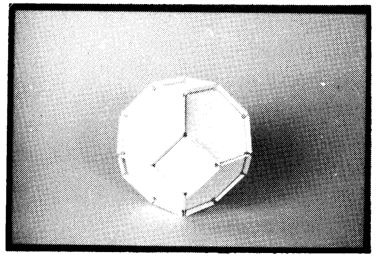

5r

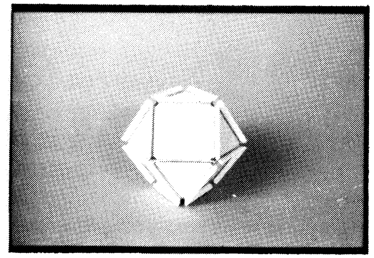
semi-réguliers

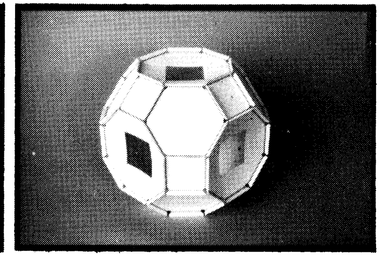
semi-regular

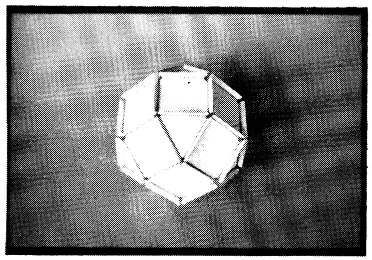

1s-r

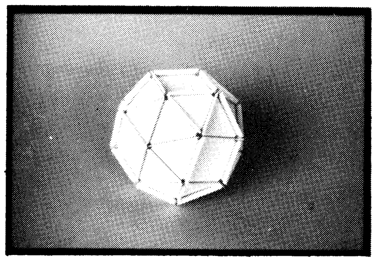

2s-r

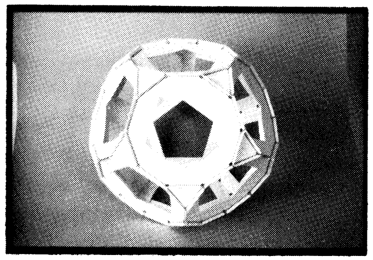

3s-r

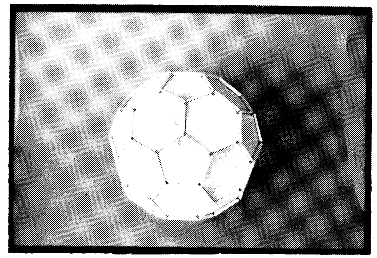

4s-r

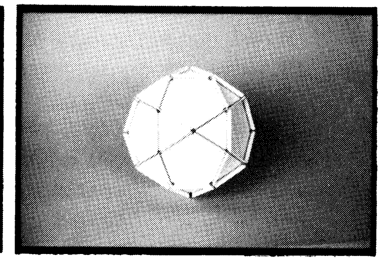

5s-r

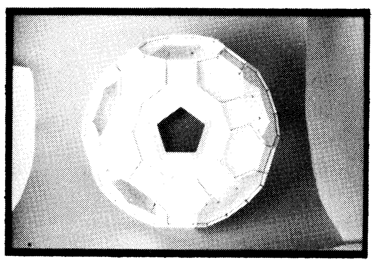

6s-r

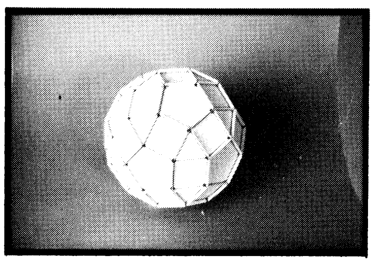

7s-r

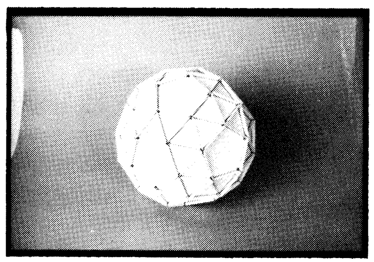

8s-r

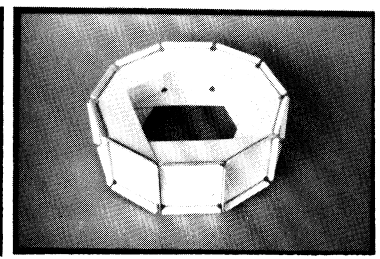

9s-r

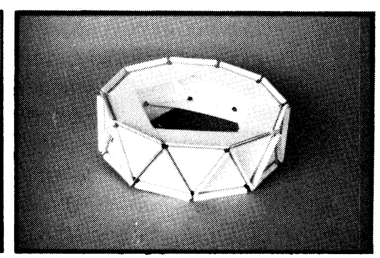

10s-r


11s-r


12s-r

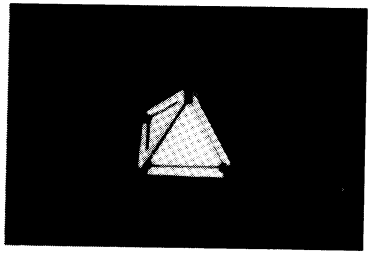

13s-r

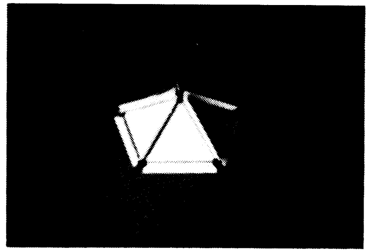

P_{10}

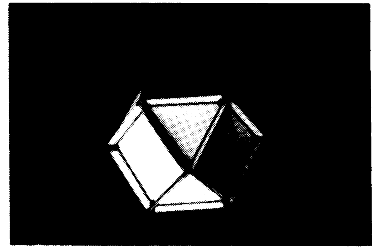

A_{10}

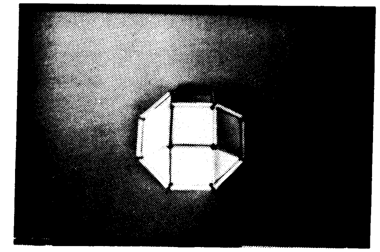
Les autres polyèdres convexes aux faces régulières

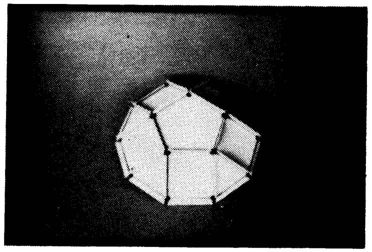
The other convex polyhedra with regular faces

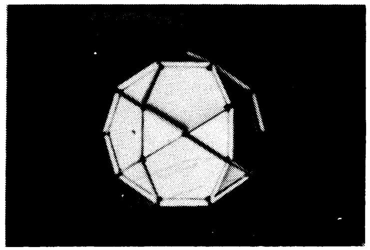

1

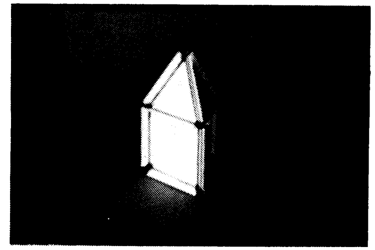

2

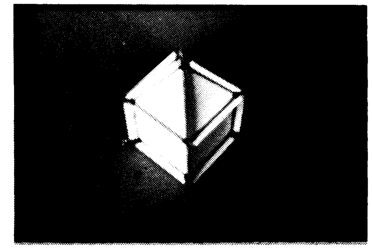

3

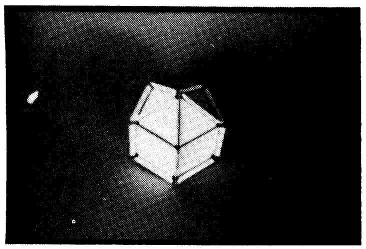

4

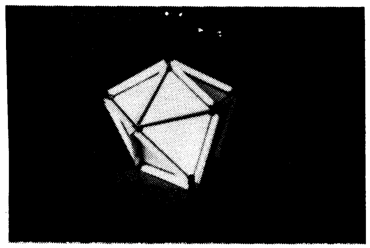

5

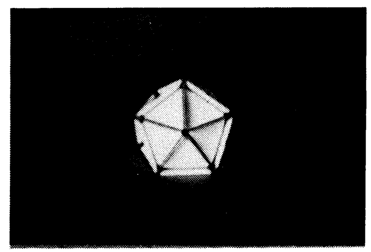

6

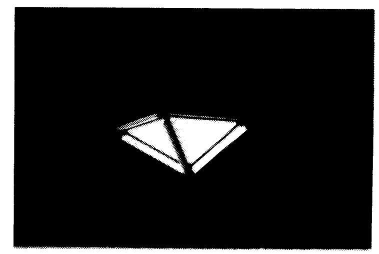

7

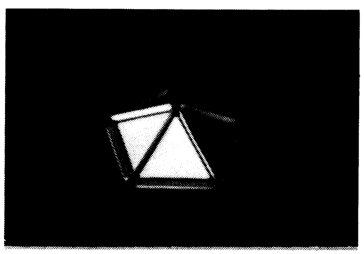

8

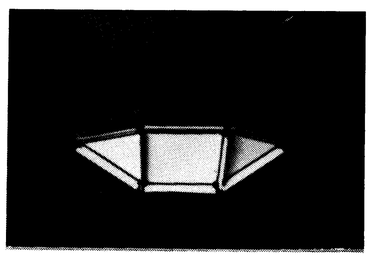

9

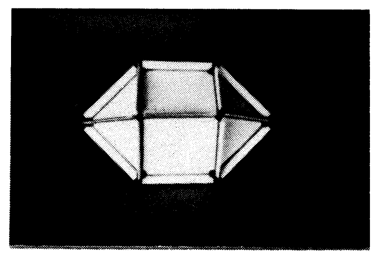

10

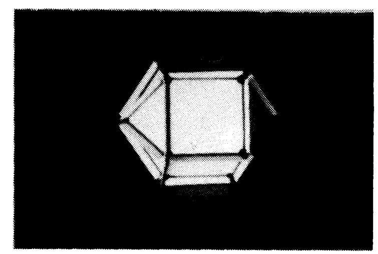

11

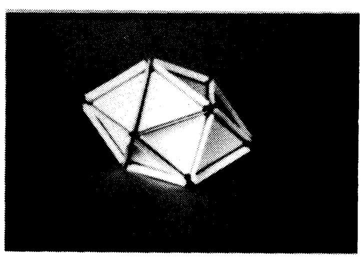

12

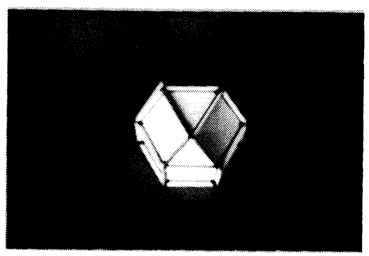

13

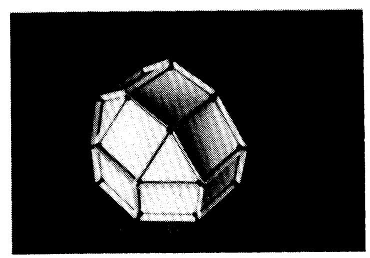

14

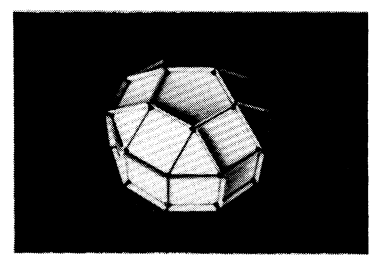

15


16


17

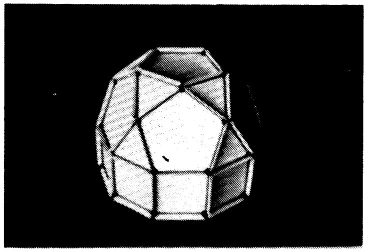

18

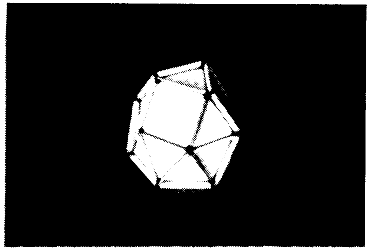

19

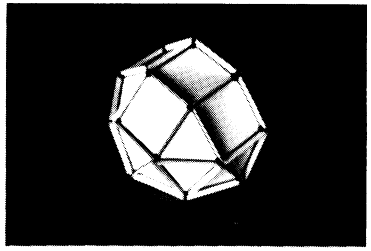

20

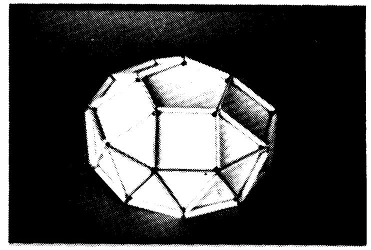
Les autres polyèdres convexes aux faces régulières

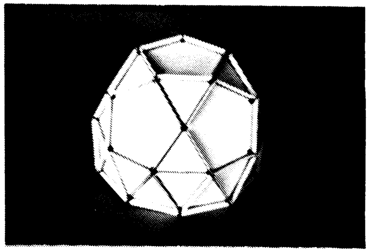
The other convex polyhedra with regular faces

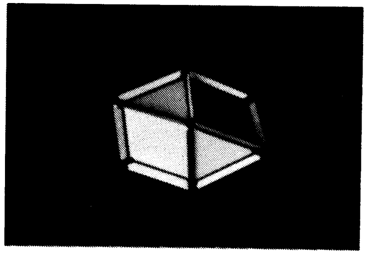

21

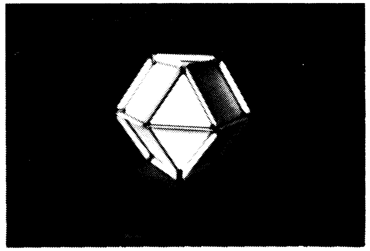

22

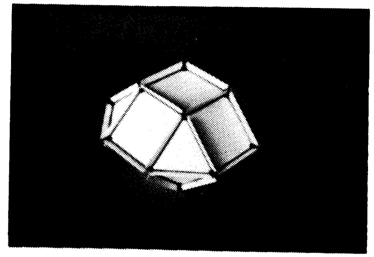

23

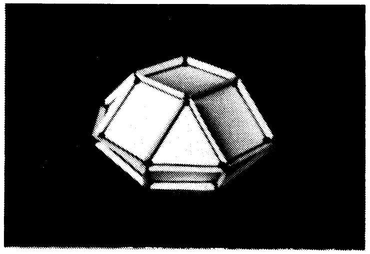

24

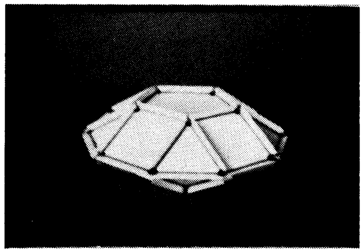

25

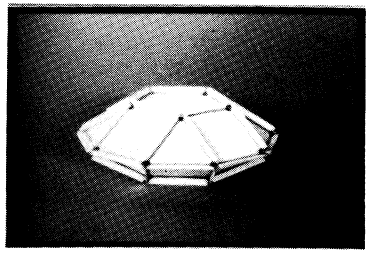

26

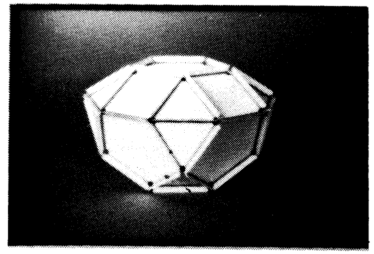

27

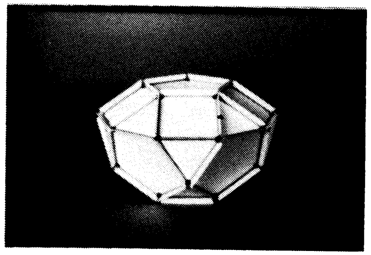

28

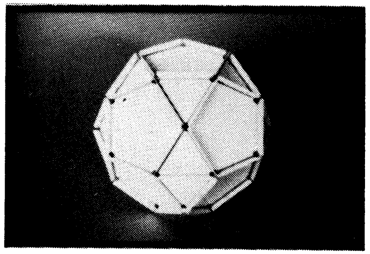

29

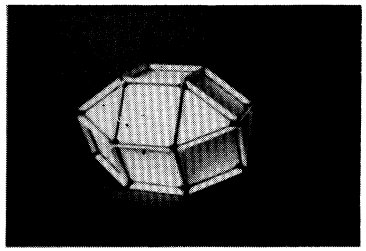

30

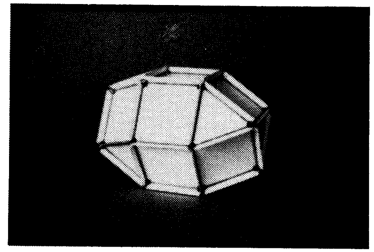

31

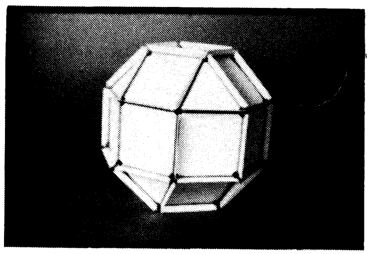

32

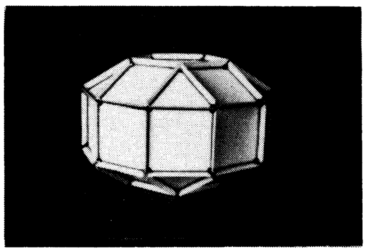

33

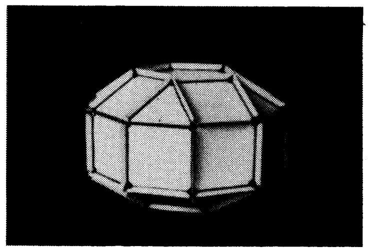

34

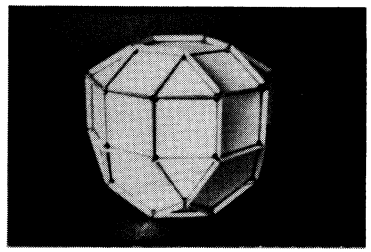

35


36


37

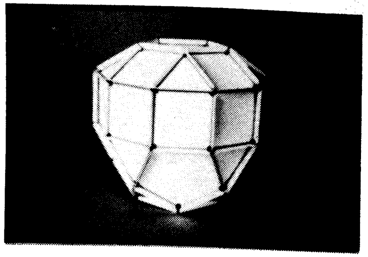

38

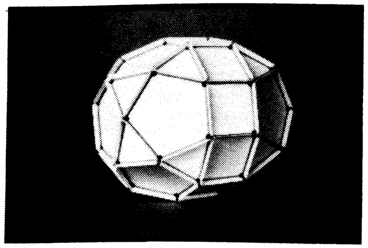

39

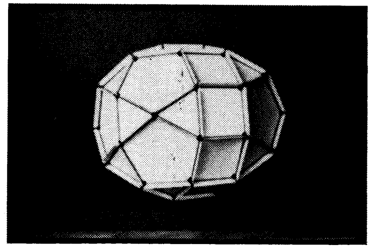

40

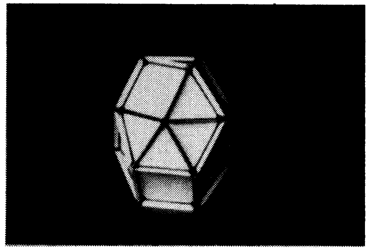
Les autres polyèdres convexes aux faces régulières

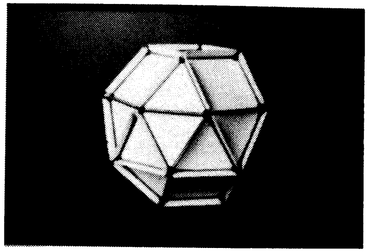
The other convex polyhedra with regular faces

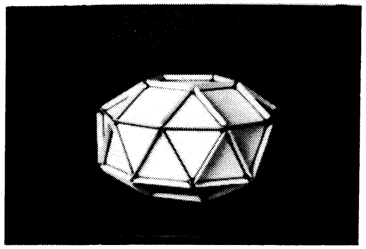

41

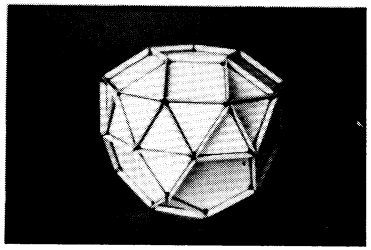

42

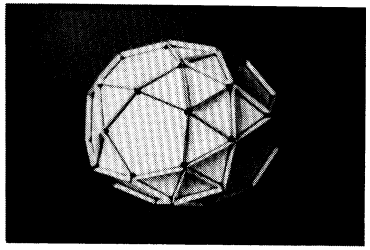

43

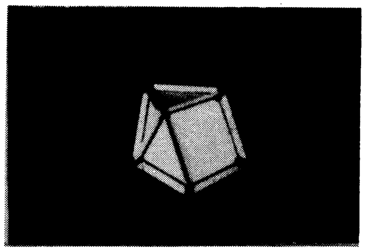

44

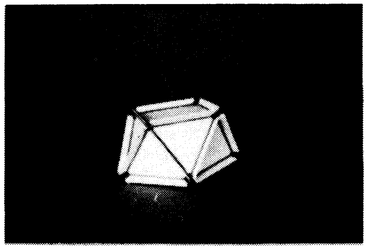

45

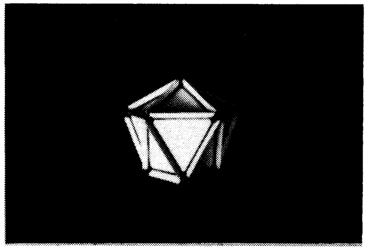

46

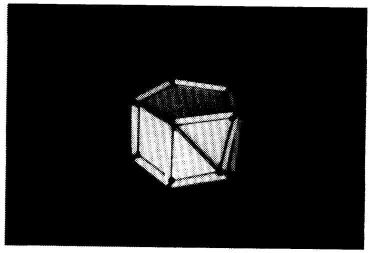

47

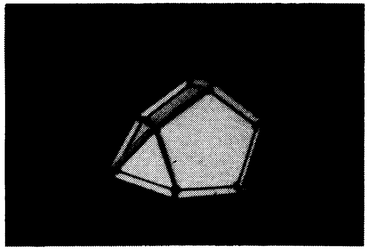

48

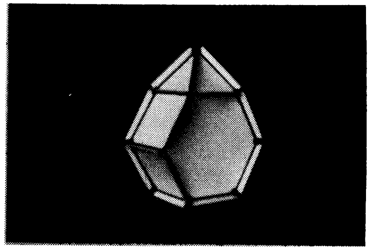

49

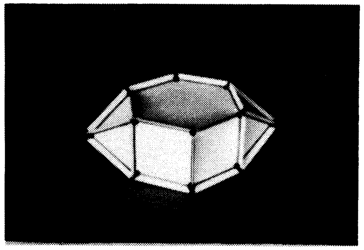

50

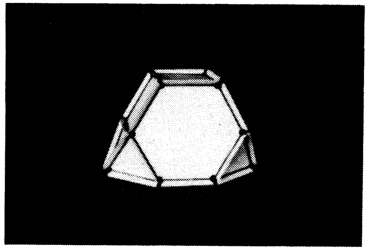

51

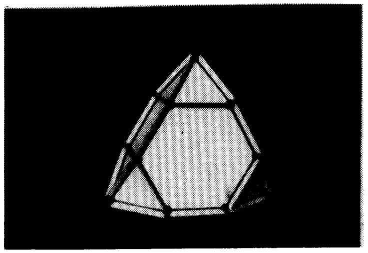

52

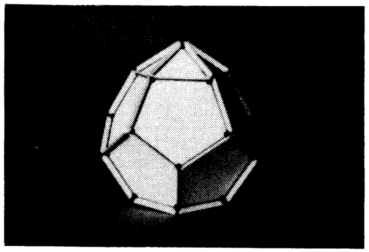

53

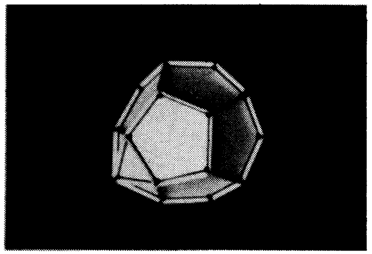

54

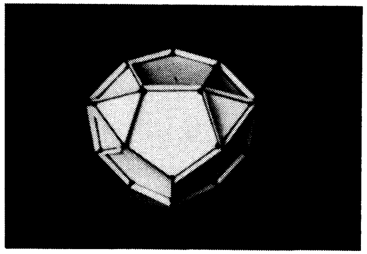

55


56


57

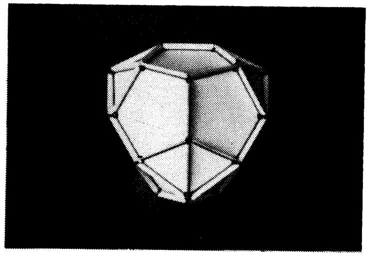

58

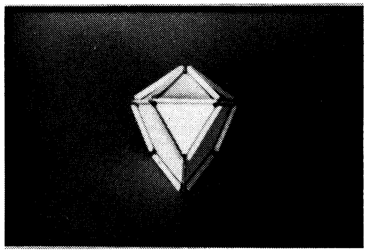

59

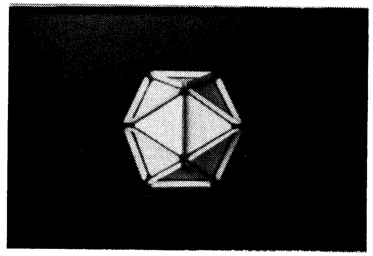

60

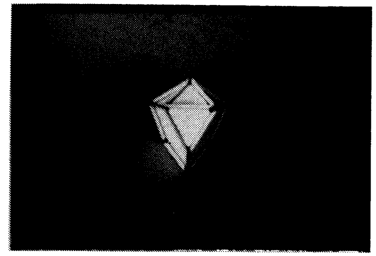
Les autres polyèdres convexes aux faces régulières

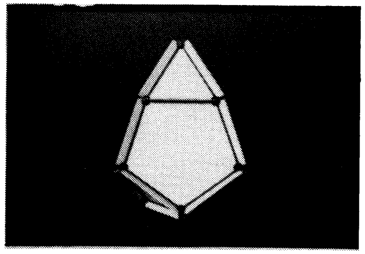
The other convex polyhedra with regular faces

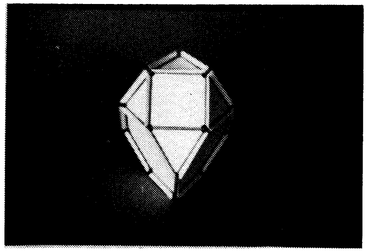

61

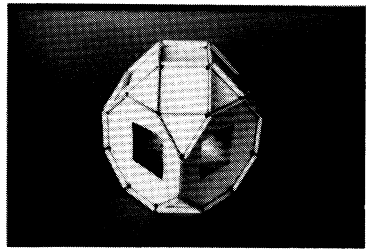

62a

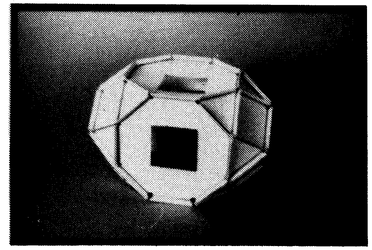

62b

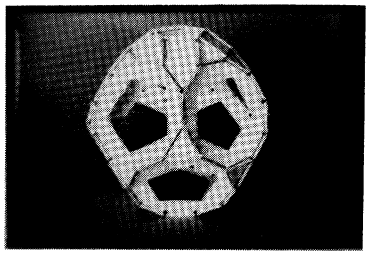

63

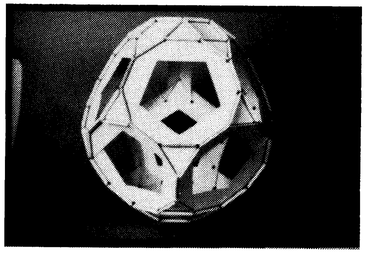

64

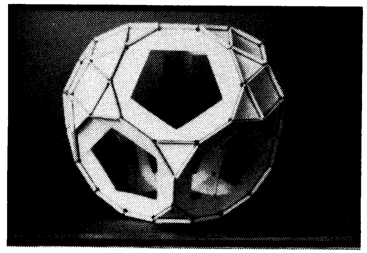

65

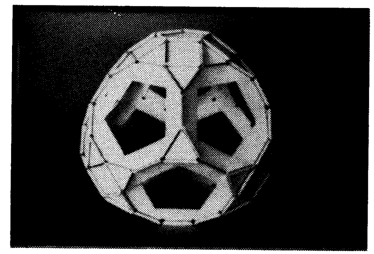

66

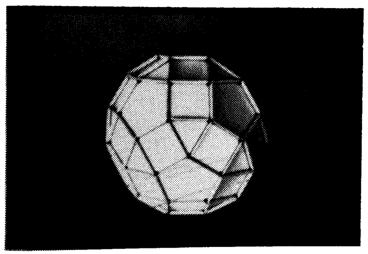

67

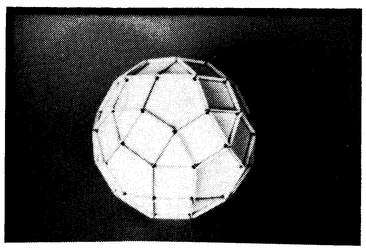

68

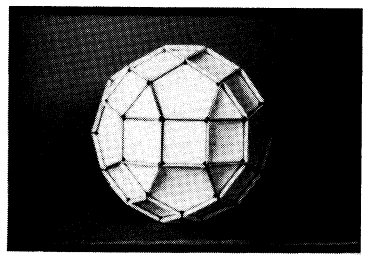

69

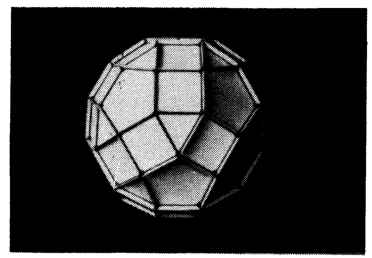

70

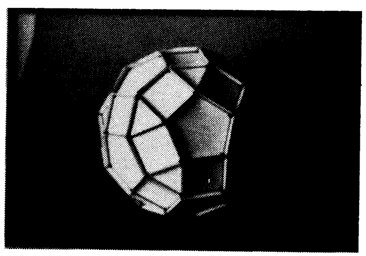

71

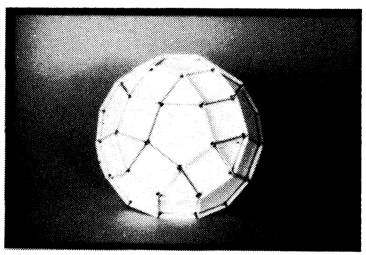

72

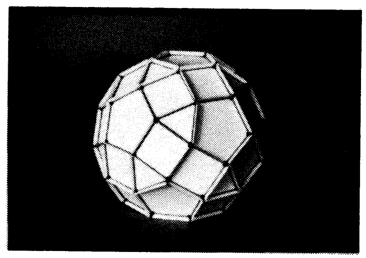

73

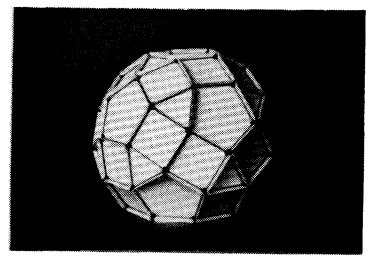

74


75


76

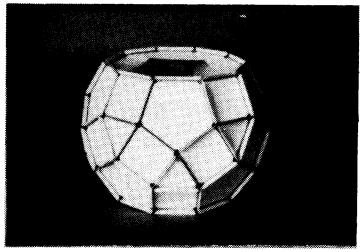

77

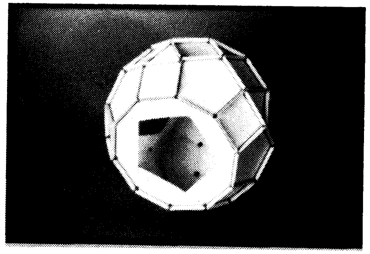

78

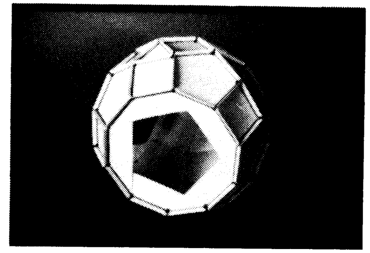

79

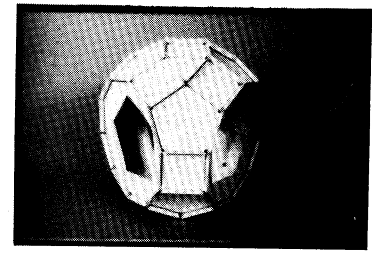
Les autres polyèdres convexes aux faces régulières

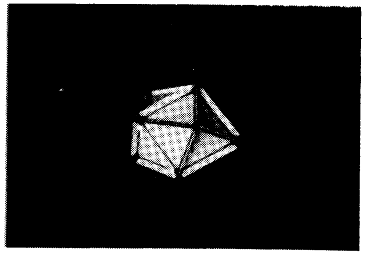
The other convex polyhedra with regular faces

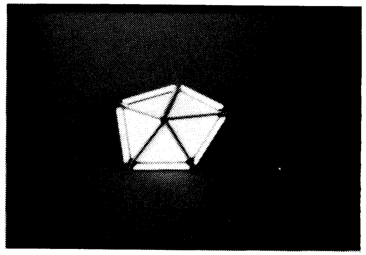

80

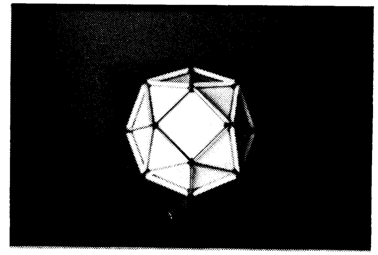

81

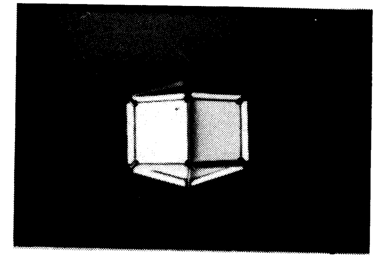

82

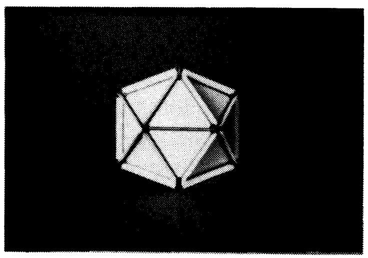

83

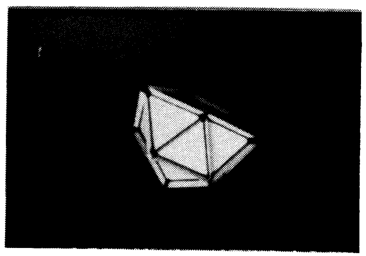

84a

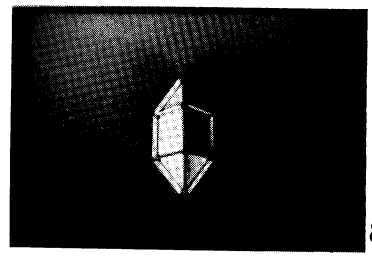

84b

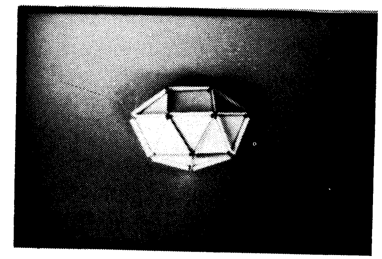

85

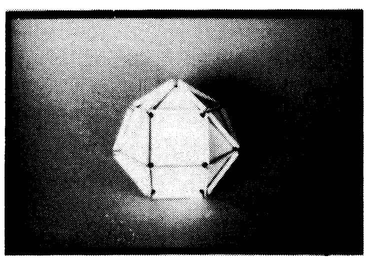

86a

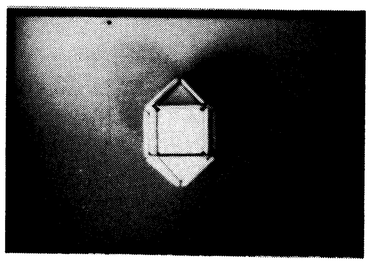

86b

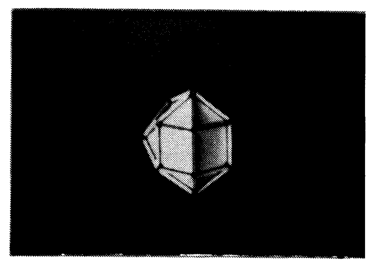

87

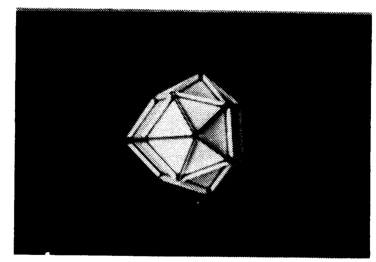

88a

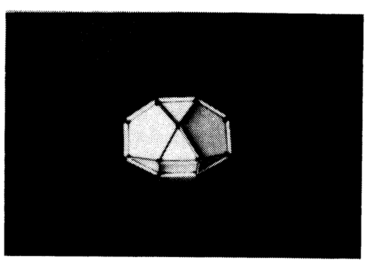

88b

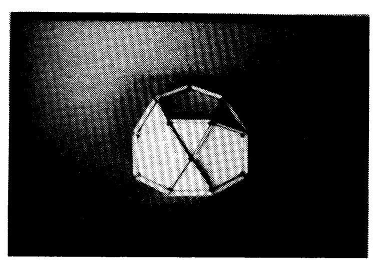

89a

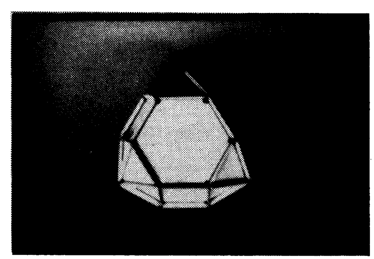

89b


90a


90b


91


92a


92b