

FE D'ERRADES

En el número 3 del volum 25 (2001), a la solució del problema 89 (pàgina 585) hi ha dues errades:

On diu:	Ha de dir:
$0(n-1)$	$0(n^{-1})$
$2(p\Pi)\operatorname{vec} V^{-1}$	$2(p+1)\operatorname{vec} V^{-1}$

SOLUCIÓ ALS PROBLEMES PROPOSATS AL VOLUM 25 N. 3

PROBLEMA N. 90

- 1) The median M satisfies $F(M) = 1/2$. Then $M = 1$ satisfies $F(1) = 1 - F(1)$, so $F(1) = 1/2$.
- 2) The covariance between X and $1/X$ must be negative. Hence

$$\begin{aligned}\operatorname{cov} \left(X, \frac{1}{X} \right) &= E \left(X \cdot \frac{1}{X} \right) - E(X) \cdot E \left(\frac{1}{X} \right) \\ &= 1 - \mu^2 < 0\end{aligned}$$

Thus $\mu > 1$.

- 3) The correlation coefficient between X and $1/X$ is positive:

$$\rho \left(X, \frac{1}{X} \right) = \frac{\mu^2 - 1}{\sigma^2} \leq 1.$$

Thus $\sigma^2 \geq \mu^2 - 1$.

- 4) Define the change $t = F(x)$. Then $F^{-1}(t) = x$ and, from $F(x) = 1 - F(1/x)$, $F^{-1}(1-t) = 1/x$. Thus

$$\begin{aligned}\int_0^1 \frac{F^{-1}(t)}{F^{-1}(1-t)} dt &= \int_R \frac{x}{1/x} dF(x) \\ &= \int_R x^2 dF(x) = \sigma^2 + \mu^2.\end{aligned}$$

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PROBLEMA N. 91

Let us find the covariance between $G(X)$ and $f'(X)/f(X)$ where

$$f(x) = F'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

Note that $f(x) = F(x)(1 - F(x))$. We have

$$\begin{aligned} E[f'(X)/f(X)] &= \int_R \frac{f'(x)}{f(x)} f(x) dx = \\ &= \int_R f'(x) dx = f(+\infty) - f(-\infty) = 0. \end{aligned}$$

Thus

$$\begin{aligned} \text{cov}(G(X), f'(X)/f(X)) &= \int_R G(x) f'(x) dx = \\ &= [G(x) f(x)]_{-\infty}^{+\infty} - \int_R G'(x) f(x) dx = \\ &= -E G'(X). \end{aligned}$$

Using the notation $f = F(1 - F)$, $f' = F(1 - F)(1 - 2F)$, the variance of $f'(X)/f(X)$ is

$$\begin{aligned} \int_R \left(\frac{f'(x)}{f(x)} \right)^2 f(x) dx &= \int_R (1 - 2F)^2 dF = \\ &= \int_0^1 (1 - 2t)^2 dt = \frac{1}{3}. \end{aligned}$$

Hence

$$(EG'(X))^2 \leq \text{var}(G(X)) \frac{1}{3}.$$

If $F = G$ then $G' = f$, $\text{var}(F(X)) = 1/12$ and

$$E f(X) = \int_R F(1 - F) dF = \frac{1}{6}.$$

Thus we obtain an equality.

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PROBLEMA PROPOSAT

PROBLEMA N. 92

Let $\mathbf{H} = \mathbf{I} - n^{-1}\mathbf{1}\mathbf{1}'$ the $n \times n$ centring matrix, $\mathbf{D} = (d_{ij})$ a $n \times n$ Euclidean distance matrix, and $\mathbf{A} = -1/2(d_{ij}^2)$. Define $\mathbf{B} = \mathbf{H}\mathbf{A}\mathbf{H}$. Then \mathbf{B} is the centred inner product matrix for \mathbf{D} .

- 1) Find the eigenvalues of \mathbf{H} .
- 2) Suppose $\mathbf{B} = \mathbf{H}$. Find the distance matrix \mathbf{D} .
- 3) If \mathbf{D} contains the distances between n objects, represent these objects by means of a dendrogram.

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