PROBLEMES PROPOSATS

PROBLEMA N. 93

Let the $(m \times m)$ random matrix S follow a central Wishart distribution with scale matrix Σ and n degrees of freedom. Find the expected value of $(\mathrm{tr}S)^{-1}S$, exact or approximate.

Heinz Neudecker
Cesaro
The Netherlands

PROBLEMA N. 94

Find the expected value of $SAS^{-1}BS^2$ when $S \sim W_m(\Omega, n)$.

Heinz Neudecker
Cesaro
The Netherlands

PROBLEMA N. 95

Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be iid observations from continuous distribution functions F and G, respectively. To test the hypothesis $F = G$ the following statistic can be used

$$B(m,n) = \frac{mn}{(m+n)} \int (F_m(x) - G_n(x))^2 \, dx,$$

where F_m and G_n are the corresponding empirical distribution functions. The hypothesis may be rejected for large values of $B(m,n)$. Now suppose

$$F(x) = \frac{1}{1+e^{-x}} \qquad -\infty < x < +\infty,$$

the logistic distribution. Prove that if $F = G$ then

$$B(m,n) \xrightarrow[m,n \to \infty]{} \sum_{i=1}^{\infty} \frac{Z_i^2}{i(i+1)}$$

where $Z_1, Z_2, \ldots$ are iid $N(0,1)$ random variables and the convergence is in distribution.

C. M. Cuadras
Universitat de Barcelona

PROBLEMA N. 96

Let $\mathbf{U} = (u_{ij})$ be a $n \times n$ ultrametric distance matrix, i.e.,

$$u_{ij} \leqslant \max\{u_{ik}, u_{jk}\} \qquad i,j,k = 1,\ldots,n,$$

where $n \geqslant 3$ and $u_{ij} > 0$ for $i \neq j$. It is well-known that $\mathbf{U}$ is a Euclidean distance matrix. Thus, if $\mathbf{H} = \mathbf{I} - n^{-1}\mathbf{1}\mathbf{1}'$ is the $n \times n$ centring matrix and $\mathbf{A} = -\frac{1}{2}(u_{ij}^2)$, then the centred inner product matrix $\mathbf{B} = \mathbf{HAH}$ has non-negative eigenvalues

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{n-1} > \lambda_n = 0.$$

Prove that the smallest positive eigenvalue is given by

$$\lambda_{n-1} = \min\left\{\frac{1}{2}u_{ij}^2 \,|\, i \neq j = 1,\ldots,n\right\}.$$

C. M. Cuadras
Universitat de Barcelona

PROBLEMA N. 97

Let $F(x)$ be a continuous cumulative distribution function. Define

$$K(x) = F(x)\big(1 - F(x)\big).$$

Find $F(x)$ in terms of $K(x)$.

C. M. Cuadras
Universitat de Barcelona

PROBLEMA N. 98

Let $a_i > 0$, $i = 1,\ldots,n$, be n positive integer numbers. Use the well-known property that the likelihood ratio lies between 0 and 1, to prove the inequality

$$a_1^{a_1} \cdots a_n^{a_n} \geqslant \left(\frac{a_1 + \cdots + a_n}{n}\right)^{a_1+\cdots+a_n}$$

with equality if $a_1 = \cdots = a_n$.

C. M. Cuadras
Universitat de Barcelona