RESEARCH GROUP ON STOCHASTIC PROCESSES

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The research group on Stochastic processes started his activities by the end of the seventies at the Universitat de Barcelona. During the past twenty years, the group has developed into several teams in three universities in Catalonia –Universitat de Barcelona, Universitat Autònoma de Barcelona and Universitat Pompeu Fabra. These teams are being funded by the main research programmes from the Spanish government, the European Union and the Catalan autonomous government. The American Mathematical Society database MathSciNet includes 259 publications authored by members of this group since 1976.

The research interests of the group are in Stochastic Analysis; that is, the study of stochastic processes and their related analytical tools with the purpose of mathematical random modelling of nonsmooth evolution dynamics. Since 1990, the range of topics studied by the group goes from Malliavin calculus and anticipating stochastic calculus, with applications to a broad variety of problems, to stochastic differential and stochastic partial differential equations. For the basis of the first two topics and an overwiew of some of their applications, we refer to the text books (Nualart, 1995 and Nualart, 1998).

Malliavin calculus is an infinite dimensional differential calculus on the Wiener space. It has been introduced by P. Malliavin in a seminal work in 1976 to better understand the interplay between probabilistic and deterministic problems in analysis and differential geometry. The first application of Malliavin calculus has been to derive a criterion to decide whether the law of a random vector is absolutely continuous with respect to Lebesgue measure and if the density is a smooth function. By applying this result to diffusion processes one can ensure hypoellipticity for a class of differential operators.

The paper by Florit and Nualart (1995) provides a new version of the criterium and it is applied to an unsmooth functional of the Wiener process. The papers by Caballero, Fernández and Nualart (1995), Rovira and Sanz-Solé (1996), Rovira and Sanz-Solé (1997), Millet and Sanz-Solé (1999) and Márquez-Carreras, Mellouk and Sarrà (2001) show different examples of new applications of the criterion to anticipative stochastic differential equations and stochastic partial differential equations.

The techniques of Malliavin calculus have led to many research directions. We first mention some applications of one of its main ingredients –the integration by parts formula– to the study of local time in different frameworks (see for instance Nualart

and Wschebor (1991), Imkeller and Nualart (1993) and Imkeller, and Nualart (1994)) or as the startpoint of the development of Poisson-based Malliavin Calculus (see Nualart and Vives (1995)).

In the mid eighties Malliavin calculus has been at the origin of the development of the anticipating stochastic calculus through the work of many researchers –Nualart, Pardoux, Zakai, among others. An extension of this calculus to multiparameter stochastic processes is given in Jolis, and Sanz-Solé (1990), Solé and Utzet (1991), Delgado, and Sanz-Solé (1995) (see also Delgado and Sanz-Solé (1992), Jolis and Sanz-Solé (1992) for additional contributions to the essentials).

Besides the mathematical interest of getting rid of adaptedness of the integrands in the Itô stochastic calculus, anticipating stochastic calculus has other solid motivations, like the analysis of stochastic differential systems with an anticipating initial condition or with boundary conditions. Both situations lead in a natural way to anticipating schemes which can be formulated by means of different types of anticipating stochastic integrals, basically the Skorohod and the Stratonovich integral. The papers Nualart, and Pardoux (1992), Buckdahn and Nualart (1994), provide a small sample of such examples. Numerical approximations schemes for these type of equations are presented in Ahn, and Kohatsu-Higa (1995) and Ferrante, Kohatsu-Higa and Sanz-Solé (1996).

An important question in connection with the properties of the solution of boundary value stochastic problems is the validity of the Markov field property. Some techniques of Malliavin calculus have been successfully applied to analyze this issue. A representative sample of tools and results can be found in Nualart and Pardoux (1991), Alabert, Ferrante, and Nualart (1995), Ferrante and Nualart (1995) and Alabert and Marmolejo (2001).

Anticipating stochastic calculus is also an appropiate tool to give a rigourous mild type formulation to stochastic evolution equations defined through differential operators with random coefficients. This approach has been undertaken by León and Nualart (1998) and Alòs, León and Nualart (1999).

Malliavin calculus, initially developped for the Wiener process, is also valid for an arbitrary Gaussian process. In particular one can consider Gaussian processes given by integration of either singular or regular deterministic kernels with respect to the Wiener process. This idea has been implemented by Alòs, Mazet and Nualart (2001) to develop a stochastic calculus with respect to some classes of Gaussian processes including in particular the fractional Brownian motion (see also Alòs, Mazet and Nualart (2000)). This is a challenging new field with applications to mathematical financial models.

Perhaps the most essential tool of Itô stochastic calculus is the change of variable formula, usually known as the Itô formula. In its classical version this formula gives the

martingale decomposition of a twice differentiable function of a semimartingale. Using Malliavin calculus it is possible to extend the formula to particular cases of semimartingales, such as diffusion processes, by relaxing the requirements on the function. Some achievements in this direction have been published in Bardina and Jolis (1997), Moret and Nualart (2000) and Bardina and Jolis (2002).

Probabilists have been led to study stochastic partial differential equations motivated by problems coming from physics, engineering, biology, chemistry, etc. This is quite a young subject; problems like existence and uniqueness of solution, the search for the appropriate functional analytic framework, numerical approximations, are still not closed. Some important contributions on these issues can be found in Nualart and Rozovskii (1997), Gyöngy and Rovira (1999), Gyöngy, and Nualart (1999), Gyöngy and Rovira (2000), Nualart and Viens (2000), Alabert and Gyöngy (2001), Gyöngy and Nualart (1995) (see also Nualart and Pardoux (1992) and Nualart and Tindel (1995)).

The law of the solution of stochastic equations gives relevant quantitative information. Besides its absolutely continuity an important issue is to know a characterization of the topological support of the law. Since the solution of a stochastic partial differential equation is a random stochastic process with values in an infinite-dimensional space, the question is not easy to handle. An approach based on approximations of the equation by smoothing the noise has been used by Millet and Sanz-Solé (1994), Bally, Millet and Sanz-Solé (1995), Gyöngy, Nualart and Sanz-Solé (1995) and Millet and Sanz-Solé (2000) to study hyperbolic and parabolic type equations.

The analysis of the difference between the stochastic and the deterministic model can be approached by perturbing the driving noise of the stochastic equation by a deterministic parameter and then studying different limit problems when this parameter tends to zero. Problems of this type include large deviations estimates, logarithmic estimates and Taylor expansions of the density. Malliavin calculus also enter here as a natural tool. Our contributions go from rather general settings, such as in Márquez-Carreras and Sanz-Solé (1999), to the more concrete models of anticipating type studied in Millet, Nualart and Sanz-Solé (1992), delay equations in Ferrante, Rovira and Sanz-Solé (2000) and the heat equation in Kohatsu-Higa, Márquez-Carreras and Sanz-Solé (2001), Rovira and Tindel (2001).

The preceding overwiew aims to give a sketch of the main currents of our research along the last decade, based on published work; it is by no means complete. The day-by-day activity is being posted at our URL site http://www.orfeu.mat.ub.es/~gaesto.

Stochastic Analysis has also lead to a rapid development of the area of Mathematical Finance. The group of Stochastic Analysis in Barcelona has also been interested in these developments and has studied problems related to the study of stock markets generated by Lévy processes (see León, Solé, Utzet and Vives (2002), Nualart and Schoutens (2001)) and fractional Brownian motion (see Alòs, Mazet and Nualart (2000), Alòs,

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Mazet and Nualart (2001)), as well as the study of asymmetric information problems (see León, Navarro and Nualart (2002), Corcuera, Imkeller, Kohatsu-Higa and Nualart (2002)).

On the area of numerical simulations the applications of Malliavin Calculus have also lead to a rapid development of new numerical methods to the approximations of sensitivity quantities in Finance (see Fournié, Lasry, Lebuchoux, Lions and Touzi (1999), Bermin, Kohatsu-Higa and Montero (2003) and Kohatsu-Higa and Pettersson (2002)).

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