OPTIMAL COLLAPSE SIMULATOR FOR THREE-DIMENSIONAL FRAMES

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Resum: *In this work a limit analysis for 3D structures software package is presented. The goal is to obtain for a certain structure the load factor λ that applied to the external loads induces collapse to the structure. The static theorem of limit analysis is the theoretical basis for the Structural Collapse Simulator (SCS), that is finding a stress distribution in equilibrium that does not violate yield criteria anywhere. The limit analysis is developed and written as a Linear Programming Problem, which consists of the maximization of the collapse load factor subject to equilibrium and yield criteria.* The Structural Collapse Simulator has been applied to several types of structures to assess its capabilities on world applications.

1. INTRODUCCIÓ

Limit analysis has been an increasingly and widely used tool for structure designing and soil mechanics analysis since its initial developments. The problem aimed to be solved by means of limit analysis consists of finding the minimum multiple of the load distribution in a solid subject that drives to the complete collapse of the body, assuming a plastic behavior of the subject, i.e. elastic range is left. In this project we have applied the limit analysis to finding conditions of failure of statically loaded 3D and 2D-structures of ductile materials, particularly steel. Continuous beams and frames of steel can carry loads considerably greater than the ones that cause to reach the elastic limit of the material. In general, when loading increases plastic yield is attained in some elements of the structure, which implies the partial loss of its bearing capacities. If the process of loading does not cease it may incur the physical failure of the structure, when the load has reached a certain value called collapse load (see Figure 1.a). Above this factor, small loading increases may result in much larger permanent deformations than the ones experienced before. The so-called plastic methods attempt to estimate the collapse load factor, and hence provide both knowledge of its bearing capacity and a better use of materials in the design process.
Taking into account progresses in limit analysis and linear programming (LP) in this project we have developed a computational tool that compute a lower, λ_{lw}, and upper bound, λ_{up}, of the load factor, λ, that applied to the external loads induces collapse to the structure. It is important to point out that this analysis can be applied to 2D and 3D structures. First, global equilibrium equations are considered in matrix notation, bearing in mind geometric restrictions and kinematic constraints. One of the new approaches to the problem is the yield condition linearization. Yield surface of standard beam cross-sections is explicitly written,

and it is adaptively approximated in a manner that every element of the structure can have its yield surface differently approximated if desired (see Figure 1.b). Besides that, this approximation yields to lower and upper bounds of the exact collapse load whether the yield surface (always convex) is approximated inwards or outwards. The second major innovation is the possibility of considering Uniform Distributed Loads (UDL), and an adaptive procedure is sought as well. Combining both the yield conditions, uniform distributed load and adaptively the bound gap can be arbitrarily reduced, and therefore a more precise collapse load factor can be found.

Figure 1. (a) Collapse of a frame structure. (b) Approximation to a yield surface performed by standard codes

2. EXEMPLES

To illustrate and assess the capabilities of the SCS we present two examples. The goal of the first example is to show the robustness of the SCS. To this end, we consider a 2D analysis of a three-storey building where the influence of the approximation of the yield curve and the UDL are considered. A schematic representation of the frame is in Figure 1. Geometric data is l_1 = 7.5 m, l_2 = 4.5 m for horizontal spans and h_1 = 3.55 m, h_2 = 4.5 m, h_3 = 3.3 m for vertical spans. The vertical loads correspond to use loads, with a value of p = 65 kN/m, whereas the horizontal ones are wind loads, with values of q_1 = 5 kN/m, q_2 = 3 kN/m. All loads have already been increased by the appropriate load increase coefficients, which can be found in the normative of steel structures.

This frame has been studied by continuously subdividing the elements bearing UDL, and at each step analysis of the convergence will be carried out varying the number of lines used to approximate the yield curve. Figure 2 summarizes the results of the convergence analysis. We denote by Si, i = 0, 1, 2, 4 the number of subdivisions applied to the elements of the structure. For each one of these cases, we have increased the number of lines used to approximate the yield curve, nl= 4, 8, 16, 20. Figure 2 shows the upper and lower bounds of the load factor., and the bound gap for all the subdivision. We realize that SCS performs precise computations that enable the user to know a lower and an upper bound to the exact collapse load factor. The convergence analysis shows that to achieve an accurate result both techniques have to be used simultaneously. Subdividing the beam elements bearing UDL is necessary in order to describe the frame more accurately, ensuring to detect the collapse mechanism and thus reducing the bound gap. Furthermore, approximating the yield curve using more lines guarantees convergence of the upper and lower bounds, but is useless without the element subdivision. The proof of this latter is the fact that the bounds remain stagnant despite using more lines, therefore the need of subdividing the UDL elements arises. Consequently, the combination of these two techniques is necessary to obtain satisfying results, ensuring success of SCS.

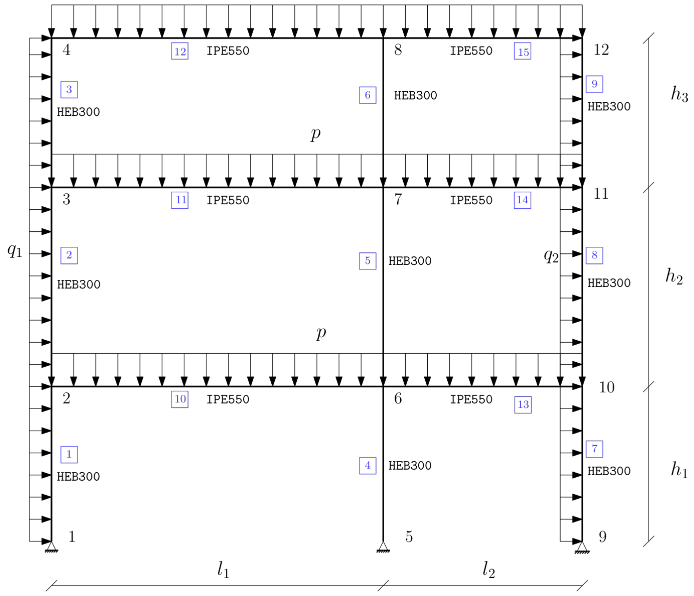

Figure 2. Schematic representation of the three-storey building.

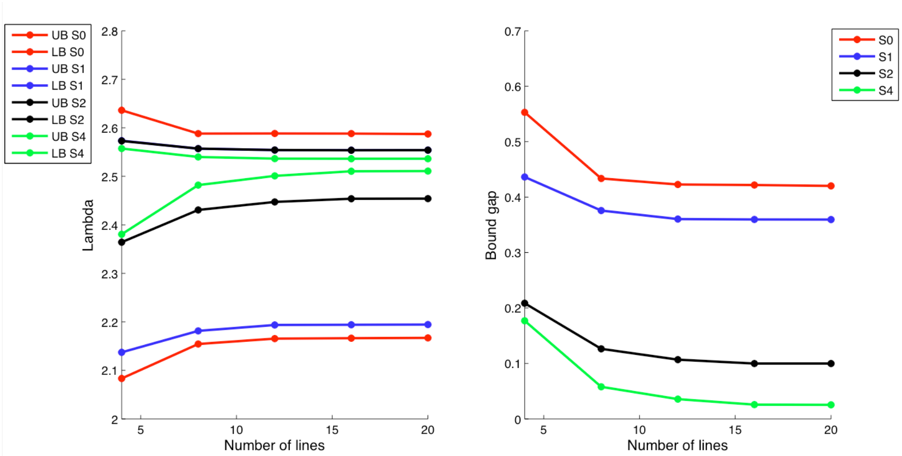

Figure 3. (a) Convergence of the upper and lower bounds of the load factor l versus the number of lines used to approximate the yield curve. (b) Convergence of the bound gap versus the number of lines used to approximate the yield curve.

The goal of the second example is to illustrate the advantages of employing SCS in comparison to linear analysis methods. We consider a trussed 3D tower. The geometrical description of the tower is presented in see Figure 3. The used beams have square 100 × 100 mm cross-sections. The plastic values are NT = −2 NC, where NC = $5 \cdot 10^5$ kN. In this analysis we apply horizontal loads to the trussed structure. Since the structure is trussed the failure of a section occurs whenever the axial stress reaches the plastic limit. Hence, the normative criterion equals the real behavior of the yield curve.

Linear analysis detects the first element to exceed the elastic limit (either traction or compression). In this case, due to problem symmetries various elements attain plastic flow at the same time. The collapse load found using SAAP2000 is F_{lin} = 216440 kN. Nevertheless, the result obtained is not the true mechanism of collapse, as the structure can bear more loading without collapsing.

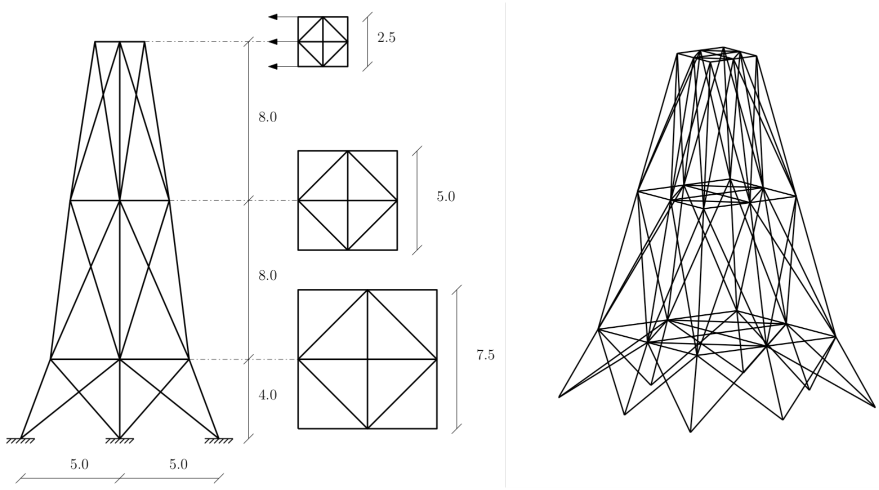

Figure 3. Schematic representation and 3D view of a steel tower.

However, computing this trussed tower with SCS leads to more realistic results. A distribution of elements that generate the collapse mechanism can be obtained. In Figure 2 the elements that fail are depicted, in red the elements that fail for traction and in blue for compression. Figure 4 shows that only the upper part of the structure fails, and therefore only this part collapses.

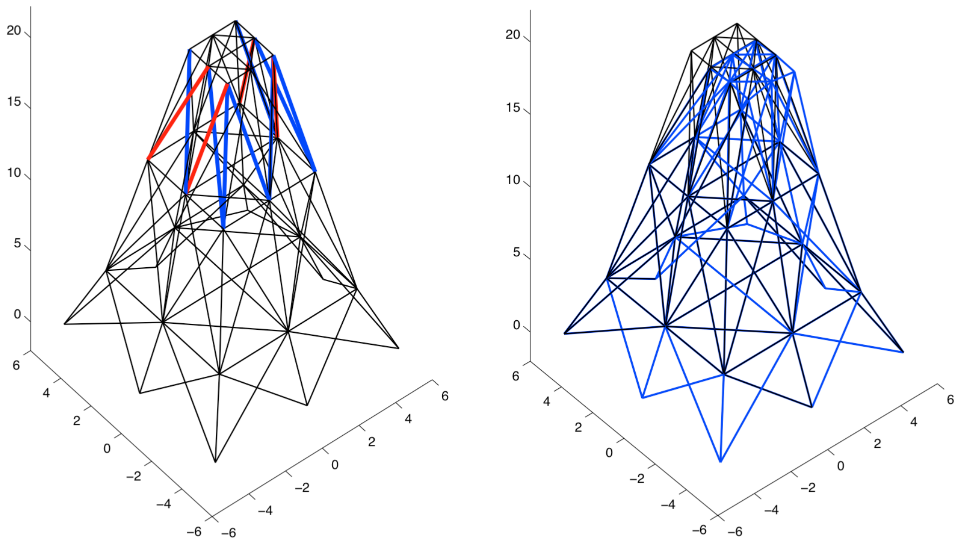

Figure 4. Right. Collapse mechanism: elements that fail due to traction (red) and compression (blue). Left. Amplified deformed structure (blue).

The collapse load obtained by SCS is F_{real} = 30,0075.7 kN, which can be related to the one obtained with the linear analysis similarly to what done before. It is important to point out that F_{real}/F_{lin} = 1.4 Therefore, the proposed methods, SCS, detects that structure can bear 40% more loading, besides providing the actual collapse mechanism.

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