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ENGINYERIES INDUSTRIALS

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

UPCGRAU

Control and Automation in Chemical Engineering

Problems

Josep M. Guadayol
José L. Medina

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Index

1. General considerations...11

Introduction...11
1.1 Transfer functions and block diagrams 11
Problem 1.1.1 11
Problem 1.1.2 13
Problem 1.1.3 15
1.2 Dynamic study of processes and control systems...16
Problem 1.2.1 16
Problem 1.2.2 18
Problem 1.2.3 19
Problem 1.2.4 22
Problem 1.2.5 24
Problem 1.2.6 25
Problem 1.2.7 25
Problem 1.2.8 28
Problem 1.2.9 29
Problem 1.2.10 31

2. Feedback systems 35

Introduction...35
2.1 Systems response, the characteristic equation and stability..............................36
Problem 2.1.1 36
Problem 2.1.2 38
Problem 2.1.3 39
Problem 2.1.4 40
Problem 2.1.5 42
Problem 2.1.6 45
Problem 2.1.7 48
Problem 2.1.8 50
Problem 2.1.9 51

Problem 2.1.10 52
Problem 2.1.11 54
Problem 2.1.12 57
Problem 2.1.13 60
Problem 2.1.14 64
2.2 Study of feedback control systems applying the frequency response 66
Problem 2.2.1 66
Problem 2.2.2 67
Problem 2.2.3 69
Problem 2.2.4 70
Problem 2.2.5 72
Problem 2.2.6 73
Problem 2.2.7 74
Problem 2.2.8 77
Problem 2.2.9 78
Problem 2.2.10 80
2.3 Applying several techniques to the study of feedback systems 83
Problem 2.3.1 83
Problem 2.3.2 88
Problem 2.3.3 92
Problem 2.3.4 96
Problem 2.3.5 105
Problem 2.3.6 106

3. Feedforward and other Control Systems 113

Introduction 113
3.1 Systems based on feedforward control 113
Problem 3.1.1 113
Problem 3.1.2 118
Problem 3.1.3 119
Problem 3.1.4 121
Problem 3.1.5 124
Problem 3.1.6 127
Problem 3.1.7 128
3.2 Other control systems 130
Problem 3.2.1 130
Problem 3.2.2 131
Problem 3.2.3 132
Problem 3.2.4 135
Problem 3.2.5 137

4. Automation of Discrete Processes 141

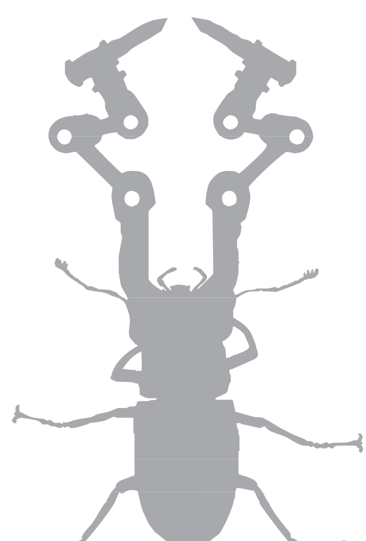
Introduction 141
4.1 Combinational and sequential pneumatic systems 142

Problem 4.1.1 142
Problem 4.1.2 142
Problem 4.1.3 143
Problem 4.1.4 144
Problem 4.1.5 147
Problem 4.1.6 149
Problem 4.1.7 151
Problem 4.1.8 152
Problem 4.1.9 153
Problem 4.1.10 155
4.2 Programming a basic discrete process in Ladder and SFC....................158
Problem 4.2.1 159
Problem 4.2.2 160
Problem 4.2.3 160
Problem 4.2.4 161
Problem 4.2.5 162
Problem 4.2.6 163
Problem 4.2.7 165
Problem 4.2.8 168
Problem 4.2.9 169
Problem 4.2.10 170
Problem 4.2.11 172
4.3 Programming TIMERS and COUNTERS....................176
Problem 4.3.1 178
Problem 4.3.2 181
Problem 4.3.3 183
Problem 4.3.4 188
Problem 4.3.5 191
Problem 4.3.6 195

5. Automation and control of continuous processes with PLC 201

Introduction....................201
5.1 Syntax for the numeric instructions....................203
5.2 Basic control of continuous processes....................205
Problem 5.2.1 205
Problem 5.2.2 208
Problem 5.2.3 211
Problem 5.2.4 212
Problem 5.2.5 215
Problem 5.2.6 218
Problem 5.2.7 221
5.3 Control of continuous processes with PID function 224
Problem 5.3.1 226

Problem 5.3.2 227
Problem 5.3.3 230
Problem 5.3.4 232
Problem 5.3.5 233
Problem 5.3.6 234
Problem 5.3.7 235
Problem 5.3.8 237



→ 1

General considerations

Introduction

This chapter refreshes the reader the concepts of Process Control which will be basic below. Once the reader has solved the problems in this chapter, he should be able to have a good grasp of block diagrams. The reader is assumed to be familiar with the concept of feedback control. Nevertheless this concept is only used in the study of the block diagrams; its dynamic behaviour is not studied in this chapter.

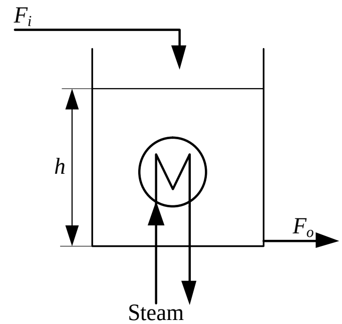
The reader should have sufficient knowledge on the Laplace transform and the inverse Laplace transform, as well as on partial-fractions expansion used in this chapter to generate the corresponding time function. The reader is reminded that changes in the process cannot be graphed solely based on the Laplace transform: the time function is also required.

Use of the Final-Value Theorem and Initial-Value Theorem sometimes obviates calculation of the inverse Laplace transform; therefore, these theorems should be applied whenever possible.

1.1 Transfer functions and block diagrams

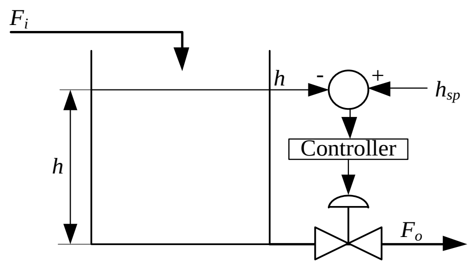
Problem 1.1.1

The figure below shows a tank whose liquid level *(a)* and temperature *(b)* must be controlled. Design a block diagram of a feedback control system for each variable to be controlled. For each system, explain every element that has been included (e.g. manipulated variables).

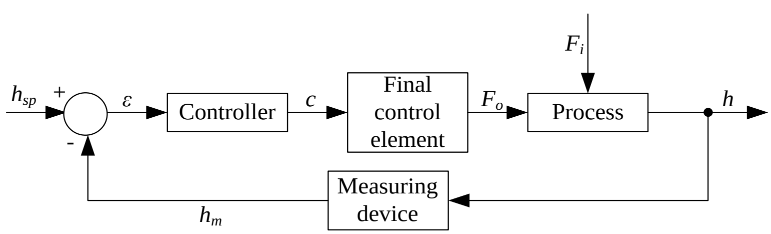


Solution

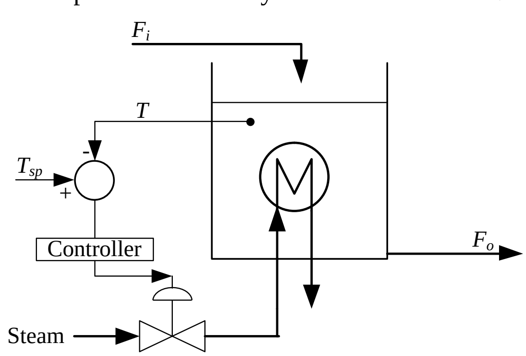
a) A schematic of the level control system is shown below:



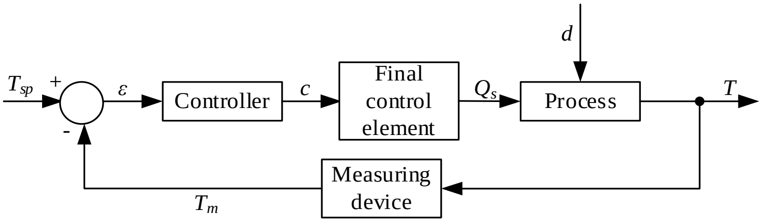
The block diagram can be constructed from the above schematic:



b) A schematic of the temperature control system is shown below:



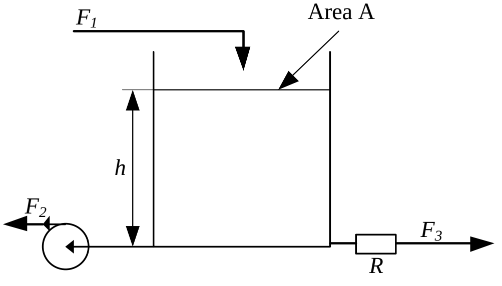
The block diagram can be constructed from the above schematic:



Note: As observed, the two block diagrams share the same general structure, but differ in their respective elements.

Problem 1.1.2

The figure below shows a tank that contains one input (F1) and two outputs (F2 and F3). The flow rate for F2 is kept constant by a constant-displacement pump. Assume that the effluent flow rate for F3 is related linearly to the hydrostatic pressure of the liquid level, through the resistance $F_3 = \dfrac{h}{R}$. Calculate the transfer function between the level and the input flow rate, using deviation variables. Then, classify the process and calculate the values of its characteristic parameters.



Solution

To obtain the transfer function, an overall mass balance must be applied to the entire system:

Σ(inputs)- Σ(outpts) = accumulation

$$F_1 - F_2 - F_3 = A\frac{dh}{dt}$$

Substituting $F_3 = \dfrac{h}{R}$ (according to the given equation) in the previous equation gives:

$$F_1 - F_2 - \frac{h}{R} = A\frac{dh}{dt}$$

The system is assumed to be in steady state, since F_2 is kept constant:

$$F_{1,s} - F_2 - \frac{h_s}{R} = 0$$

Subtracting the two previous equations gives the deviation variables:

$$(F_1 - F_{1,s}) - (F_2 - F_2) - \left(\frac{h}{R} - \frac{h_s}{R}\right) = A\frac{dh}{dt}$$

Where the deviation variables are defined as:

$$F_1 - F_{1,s} = F_1^{'}$$
$$h - h_s = h^{'}$$

Substituting the deviation variables into the above equation gives:

$$F_1^{'} - \frac{h^{'}}{R} = A\frac{dh^{'}}{dt}$$

Rearranging this expression gives:

$$RF_1^{'} - h' = AR\frac{dh'}{dt}$$

$$AR\frac{dh^{'}}{dt} + h^{'} = RF_1^{'}$$

Applying the Laplace transform to the above equation gives:

$$ARs\overline{h}'(s) + \overline{h}'(s) = R\overline{F_1^{'}}(s)$$
$$\overline{h}^{'}(s)\left[ARs + 1\right] = R\overline{F_1^{'}}(s)$$

Rearranging this expression gives:

$$\frac{\overline{h}^{'}(s)}{\overline{F_1^{'}}(s)} = \frac{R}{ARs + 1}$$

This expression matches a first-order transfer function.

The parameters that characterize the process are:

$$\tau_p = AR$$
$$K_p = R$$

Problem 1.1.3

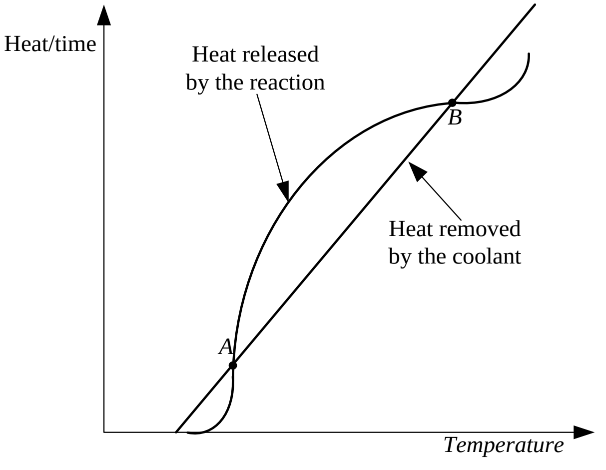
The graph below shows two plots for an exothermic reaction occurring inside a CSTR reactor; the released heat as a function of the reactor's internal temperature (sigmoid curve), and the heat removed by a coolant that flows through a jacket around the reactor as a function of the reactor's internal temperature (straight line).

Explain whether points *A* and *B* are stable or unstable during each of the two following circumstances:

a) When the reactor's feed temperature increases.

b) When the reactor's feed temperature decreases.

Note: the feed temperature must be considered as a disturbance that affects the whole process.



Solution

a) Point *A*: When the feed temperature increases, the heat released by the reaction is *greater* than the heat removed by the coolant; therefore, the reactor's internal temperature will increase shifting towards point *B*. Thus, point *A* is unstable.

Point *B*: When the feed temperature increases, the heat released by the reaction is *less* than the heat removed by the coolant; therefore, the reactor's internal temperature will return to point *B*. Thus, point *B* is stable.

b) Point *A*: When the feed temperature decreases, the heat released by the reaction is *less* than the heat removed by the coolant; therefore, the reactor's internal temperature will decrease shifting towards the left-hand side of the plot. Thus, point *A* is unstable.

Point *B*: When the feed temperature decreases the heat released by the reaction is *greater* than the heat removed by the coolant; therefore, the reactor's internal temperature will return to point *B*. Thus, point *B* is stable.

1.2 Dynamic study of processes and control systems

Problem 1.2.1

The behaviour of a process can be expressed using the following differential equation:

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=2f(t)$$

where *y(t)* and *f(t)* are the system's output and input respectively.

a) Calculate the transfer function and its dynamic characteristics (natural period, damping factor and gain) assuming that the initial conditions are null.

b) Calculate the time response when a unit step-change is applied and then check the above assumption on the initial conditions.

Solution

a) Applying the Laplace transform to the differential equation that describes the process and assuming that the initial conditions are null, gives:

$$s^2\overline{y}(s)+3s\overline{y}(s)+2\overline{y}(s)=2\overline{f}(s)$$

$$(s^2+3s+2)\overline{y}(s)=2\overline{f}(s)$$

$$\frac{\overline{y}(s)}{\overline{f}(s)}=\frac{2}{s^2+3s+2}$$

Rearranging the previous expression gives:

$$\frac{\overline{y}(s)}{\overline{f}(s)}=\frac{1}{\frac{1}{2}s^2+\frac{3}{2}s+1}$$

The expression shows a second-order transfer function with the following characteristics:

$$K_p=1$$

$$\tau^2=\frac{1}{2}\ \rightarrow\ \tau=0.707$$

$$2\zeta\tau=\frac{3}{2}\rightarrow\ \zeta=1.06$$

b) When a unit step input $\rightarrow \overline{f}(s)=\frac{1}{s}$ is applied, the response value will be:

$$\bar{y}(s) = \frac{1}{s}\frac{2}{(s^2+3s+2)}$$

To obtain the time response, a partial-fractions expansion must be done, this case deals with three real and different roots:

$$\frac{1}{s}\frac{2}{(s^2+3s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

where *A*, *B* and *C* are the unknown constants to be evaluated

$$\begin{aligned} 2 &= A(s+1)(s+2) + Bs(s+2) + Cs(s+1) \\ 2 &= As^2 + 3As + 2A + Bs^2 + 2Bs + Cs^2 + Cs \end{aligned}$$

The coefficients are identified as:

$$\begin{cases} 0 = A + B + C \\ 0 = 3A + 2B + C \\ 2 = 2A \end{cases}$$

Solving the system, gives the following values:

$$\begin{aligned} A &= 1 \\ B &= -2 \\ C &= 1 \end{aligned}$$

Therefore, the response stated in Laplace terms is:

$$\bar{y}(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

Consequently, the inverse Laplace transform is easily found:

$$y(t) = \mathcal{L}^{-1}\left[\bar{y}(s)\right] = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}\right] = \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{2}{s+1}\right] + \mathcal{L}^{-1}\frac{1}{s+2}$$

The time response is:

$$y(t) = 1 - 2e^{-t} + e^{-2t}$$

The assumption on the initial conditions can now be checked if it is correct. Computing the values of the time response and of its first derivative gives:

$$y(0) = 1 - 2e^{0} + e^{0} = 0$$
$$y'(t) = 2e^{-t} - 2e^{-t}$$
$$y'(0) = 2e^{0} - 2e^{0} = 0$$

Problem 1.2.2

The following two first-order transfer functions are available:

$$G_1(s) = \frac{3}{2s+1}$$
$$G_2(s) = \frac{5}{3s+1}$$

and a step input is applied to each one:

a) Which of these functions will reach 80% of its final value first?

b) Calculate the final value, in each case.

c) Determine the characteristics of the transfer function that results from placing these two functions in series.

d) Based on the new characteristics outlined in the previous section, determine the response when a step input is applied to the new series system.

Solution

a) The first function to reach 80% of its final value will be $G_1(s)$ because its time constant is smaller than that of $G_2(s)$.

b) The final response value of each transfer function will be:

$Kp_1 = 3A$, for $G_1(s)$

$Kp_2 = 5A$, for $G_2(s)$

where A is the step input amplitude.

c) The new overall transfer function $G_o(s)$ is obtained by rearranging the previous functions in series:

$$G_o(s) = G_1(s)G_2(s)$$

and then substituting the values of each transfer function:

$$G_o(s) = \frac{15}{(2s+1)(3s+1)}$$

This provides a second-order transfer function:

$$G_o(s) = \frac{15}{6s^2 + 5s + 1}$$

The following expressions can be inferred from the equation above:

$$\tau^2 = 6$$
$$\tau = \sqrt{6}$$
$$2\zeta\tau = 5$$

Therefore, the value of ζ will be:

$$\zeta = \frac{5}{2\sqrt{6}} = 1.02$$

and, also $\rightarrow K_p = 15$

d) If a step input is applied to the function $G_o(s)$, the response obtained will be critically damped, because $\zeta \cong 1$.

Problem 1.2.3

The following two first-order transfer functions must be studied:

$$G_1(s) = \frac{2}{(s+2)}$$

$$G_2(s) = \frac{3}{(s+4)}$$

a) Imagine that a step input is applied (amplitude = 3) to each function. Determine which function will reach 20% of its final value first and then calculate this value.

b) Let $G_1(s)$ and $G_2(s)$ be arranged in series. Explain the characteristics of the new transfer function and the shape of the response when a unit step input is applied to it.

c) Calculate the expression of the time response when the input $y(t) = t$ is applied to the transfer function obtained above.

Solution

a) First, the transfer function must be transformed into functions from which the first-order parameters (τ_p and K_p) can be obtained.

For $G_1(s)$:

$$G_1(s) = \frac{2}{s+2} = \frac{\frac{2}{2}}{\frac{s}{2}+\frac{2}{2}} = \frac{1}{0.5s+1}$$

The following expression can be obtained from the above expression:

$$\tau_{p1} = 0.5$$
$$K_{p1} = 1$$

For $G_2(s)$:

$$G_2(s) = \frac{3}{s+4} = \frac{\frac{3}{4}}{\frac{s}{4}+\frac{4}{4}} = \frac{0.75}{0.25s+1}$$

The following values can be obtained from the above expression:

$$\tau_{p2} = 0.25$$
$$K_{p2} = 0.75$$

G_2 will reach 20% of its final value first, because its time constant is smaller than that of G_1.

The final response values are calculated by applying the Final-Value Theorem, which obviates calculation of the inverse Laplace transform.

For $G_1(s)$:

$$\lim_{t\to\infty} y_1(t) = \lim_{s\to 0}\left[s\left(\frac{3}{s}\right)\frac{1}{(0.5s+1)}\right] = 3$$

For $G_2(s)$:

$$\lim_{t\to\infty} y_2(t) = \lim_{s\to 0}\left[s\left(\frac{3}{s}\right)\frac{0.75}{(0.2s+1)}\right] = 2.25$$

b) Rearranging the two functions in series gives:

$$G(s) = G_1(s)G_2(s)$$

The overall transfer function is the product of these two functions:

$$G(s) = \frac{2}{(s+2)}\frac{3}{(s+4)} = \frac{6}{s^2+6s+8}$$

The response is second-order and its shape can be studied provided that the value of ζ is known. Comparing the expression obtained above to the general expression for the second-order transfer function:

$$G(s) = \frac{\frac{6}{8}}{\frac{s^2}{8}+\frac{6}{8}s+\frac{8}{8}} = \frac{0.75}{0.125s^2+0.75s+1}$$

The following expressions can be derived from the above expression:

$$\tau^2 = 0.125 \rightarrow \tau = 0.353$$

$$K_p = 0.75$$

$$2\zeta\tau = 0{,}75 \rightarrow \zeta = 1.06$$

The value of ζ indicates that the response is overdamped. In fact, it is almost critically damped because its value is very close to 1.

c) The input $f(t) = t$ is $\overline{f}(s) = \frac{1}{s^2}$ in the Laplace transform expression. This input is applied to the previously obtained function to give:

$$\overline{y}(s) = \frac{6}{(s^2+s+8)}\frac{1}{s^2} \rightarrow \overline{y}(s) = \frac{6}{s^2(s^2+6s+8)}$$

The roots of the previous functions are:

$$s^2 = 0 \rightarrow s = 0 \text{ (double)}$$

This indicates two real and equal roots:

$$s^2+6s+8 = 0 \rightarrow s = -2,\ s = -4$$

This indicates two real and different roots.

The above expression can be expanded into partial fractions as follows:

$$\frac{6}{s^2(s^2+6s+8)} = \frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+2}+\frac{D}{s+4}$$

$$6 = A(s+2)(s+4) + Bs(s+2)(s+4) + Cs^2(s+4) + Ds^2(s+2)$$

Operating and grouping the terms of the previous expression gives:

$$6 = As^2 + 6As + 8A + Bs^3 + 6Bs^2 + 8Bs + Cs^3 + 4Cs^2 + Ds^3 + 2Ds^2$$

$$\begin{cases} 0 = B + C + D \\ 0 = A + 6B + 4C + 2D \\ 0 = 6A + 8B \\ 6 = 8A \end{cases}$$

This gives the following results:

$$\begin{aligned} A &= 0.75 \\ B &= 0.56 \\ C &= 0.75 \\ D &= -0.18 \end{aligned}$$

Substituting the above values into above equation gives:

$$\bar{y}(s) = \frac{0.75}{s^2} - \frac{0.56}{s} + \frac{0.75}{s+2} - \frac{0.18}{s+4}$$

The response time is:

$$y(t) = \mathcal{L}^{-1}\left[\bar{y}(s)\right] = 0.75t - 0.56 + 0.75e^{-2t} - 0.18e^{-4t}$$

Problem 1.2.4

Consider a first-order process with a time constant of 0.8 minutes and a unit steady-state gain. Assuming that the input is *f(t)* = *t* (ramp function), calculate the response in the time domain.

Solution

The process transfer function is:

$$G(s) = \frac{1}{0.8s + 1}$$

The process, its input and its response may be represented with the following block diagram:

$$\bar{f}(s) \longrightarrow \boxed{\frac{1}{0.8s+1}} \longrightarrow \bar{y}(s)$$

According to the problem statement *f(t)* $= t \rightarrow \bar{f}(s) = \dfrac{1}{s^2}$ *;* therefore:

$$\bar{y}(s) = G(s)\bar{f}(s) = \left(\frac{1}{0.8s+1}\right)\frac{1}{s^2}$$

Expansion into partial fractions yields:

$$\bar{y}(s) = \frac{1}{s^2(0.8s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{0.8s+1}$$

Operating and simplifying gives:

$$1 = A(0.8s+1) + Bs(0.8s+1) + Cs^2$$
$$1 = 0.8As + A + 0.8Bs^2 + Bs + Cs^2$$

Computing the constants gives:

$$\begin{cases} 0 = 0.8B + C \\ 0 = 0.8A + B \\ 1 = A \end{cases}$$

The values of the above system are:

$$A = 1$$
$$B = -0.8$$
$$C = 0.64$$

Therefore:

$$\bar{y}(s) = \frac{1}{s^2} - \frac{0.8}{s} + \frac{0.64}{0.8s+1}$$

The time response is obtained from the above equation:

$$y(t) = \mathcal{L}^{-1}\left[\bar{y}(s)\right]$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2} - \frac{0.8}{s} + \frac{0.64}{0.8s+1}\right] = \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] - \mathcal{L}^{-1}\left[\frac{0.8}{s}\right] + \frac{0.64}{0.8}\mathcal{L}^{-1}\left[\frac{1}{s+1.25}\right]$$

$$y(t) = t - 0.8 + 0.8e^{-1.25t}$$

Problem 1.2.5

A process is represented by the transfer function shown below:

$$G(s)=\frac{\overline{y}(s)}{\overline{f}(s)}=\frac{K}{s}$$

A unit step input is applied to this function.

a) Obtain the time response.

b) Plot the input and the output against time, and justify the shape of the output plot.

Solution

a) The input Laplace transform is:

$$\overline{f}(s)=\frac{1}{s}$$

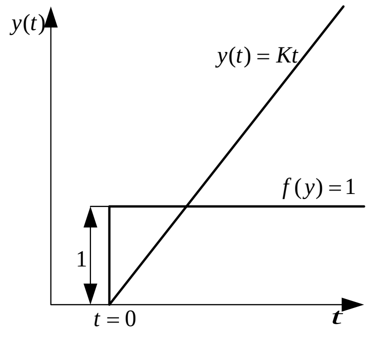
The output is given by:

$$\overline{y}(s)=G(s)\overline{f}(s)=\left(\frac{K}{s}\right)\left(\frac{1}{s}\right)=\frac{K}{s^2}$$

The inverse Laplace transform can now be found. The following result is obtained:

$$y(t)=Kt$$

b) The graph is shown below



The output falls to a system which has a transfer function which is a pure integrator. Moreover, the output is not bounded: it tends to infinite.

The *K* value is the slope of the straight line which represents the output. If *K* increases, the output is more vertical and develops faster than if small *K* values are taken in account.

Problem 1.2.6

A unit step input is applied to a first-order process. Calculate the time (in function of the time constant) that the process takes to reach 98% of the final value.

Solution

The first-order system response to a unit step input is:

$$\bar{y}(s) = \left(\frac{K_p}{\tau_p s + 1}\right)\left(\frac{1}{s}\right)$$

The response final value will be:

$$y(t)(t \to \infty) = \lim_{s \to 0}\left[s\ \left(\frac{1}{s}\right)\left(\frac{K_p}{(\tau_p s + 1)}\right)\right] = K_p$$

The time response is:

$$y(t) = K_p(1 - e^{-\frac{t}{\tau_p}})$$

Substituting the value of $y(t) = 0.98 K_p$ into the above expression:

$$0.98 K_p = K_p(1 - e^{-\frac{t}{\tau_p}})$$

The value of *t* can be obtained from the above expression:

$$t = 3.91\ \tau_p$$

Problem 1.2.7

A tank with an area of 1.5 m^2 has, in steady state, a level of 1.2 m when the input and output flow rates are both 5 m^3/h. The input flow rate suddenly varies, decreasing in value to 4 m^3/h. Assuming that $F_0 = \beta h_s^{\frac{1}{2}}$ and that the differential equation (expressed in deviation variables) which describes the process is:

$$A\frac{dh^{'}}{dt}+\frac{\beta}{2\sqrt{h_s}}h^{'}=F_i^{'} \qquad (*)$$

calculate:

a) The τ_p and K_p values without applying any formula used for a specific situation.

b) The transfer function that connects the level to the input flow rate (in deviation variables).

c) The time expression of the level.

d) The level when considering the new steady state.

Solution

a) Multiplying the expression which describes the process by $2\frac{\sqrt{h_s}}{\beta}$, yields:

$$\frac{2\sqrt{h_s}}{\beta}A\frac{dh^{'}}{dt}+h^{'}=\frac{2\sqrt{h_s}}{\beta}F_i^{'}$$

The β value can be calculated from the expression $F_0=\beta\sqrt{h_s}$, which gives:

$$\beta=\frac{F_0}{\sqrt{h_s}}=\frac{5}{\sqrt{1.2}}=4.56$$

The coefficient of $\frac{dh^{'}}{dt}$ is:

$$\tau_p=\frac{2\sqrt{h_s}}{\beta}A=\frac{2\sqrt{1.2}}{4.45}(1.5)=0.72$$

The coefficient of *F'* is:

$$K_p=\frac{2\sqrt{h_s}}{\beta}=\frac{2\sqrt{1.2}}{4.56}=0.48$$

b) The value of $\frac{\overline{h^{'}}(s)}{\overline{F_i^{'}}(s)}$ must be calculated; it can be obtained by substituting the above values into the equation given in the statement:

$$0.48\frac{dh^{'}}{dt}+h^{'}=0.72F_i^{'}$$

Applying the Laplace transform to the above expression and taking into account the deviation variables gives:

$$0.48s\overline{h'}(s)+\overline{h'}(s)=0.48\overline{F_i'}(s)$$

Grouping the terms of the above expression gives the following expression:

$$\frac{\overline{h'}(s)}{\overline{F_i'}(s)}=\frac{0.48}{0.72s+1}$$

c) The sudden input can be considered to be unit step input; therefore, taking into account that the input is negative (in terms of deviation variables), its value is $\overline{F_i'}=-\frac{1}{s}$:

$$\overline{h'}(s)=\frac{0.48}{0.72s+1}\left(-\frac{1}{s}\right)=\frac{-0.48}{(0.72s+1)\ s}=\frac{\frac{-0.48}{0.72}}{\left(s+\frac{1}{0.72}\right)s}=\frac{-0.67}{(s+1.39)\ s}=\frac{A}{s}+\frac{B}{s+1.39}$$

Operating:

$$-0.67=As+1.39A+Bs$$

The coefficients are identified as:

$$A=-0.482$$
$$B=0.482$$

The expression is:

$$\overline{h'}(s)=\frac{-0.482}{s}+\frac{0.482}{s+1.39}$$

Calculating the inverse Laplace transform gives:

$$h'(t)=-0.482+0.482e^{-1.39t}=0.482\left(e^{-1.39t}-1\right)$$

d) The level (in terms of deviation variables) in the new steady state (time approaches infinity) is calculated as:

$$h'(t)(t\to\infty)=0.482(e^{-\infty}-1)=-0.482$$

The final level is calculated as:

$$h'(t)=h(t)-h_s(t)\Rightarrow h(t)=h'(t)+h_s(t)=-0.482+1.2=0.718\text{ m}$$

Problem 1.2.8

A process transfer function has been studied whose result is the expression $G(s)=\dfrac{4}{2s+2}$. Calculate the following when the input *f (t)* = *t* is applied to the function:

a) The value of the time response when the elapsed time is very long (time approaches infinity).

b) The expression of the time response value.

c) The response value after five time units have elapsed.

Solution

a) The response value when $t \to \infty$ can be calculated by applying the Final-Value Theorem:

$$\mathcal{L}^{-1}\left[f(t)\right]=\left[\frac{1}{t}\right]=\frac{1}{s^2}$$

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0} s\left(\frac{1}{s^2}\right)\frac{4}{(2s+2)}=\infty$$

b) To calculate the time response, the partial fraction expression must first be found:

$$\bar{y}(s)=\frac{1}{s^2}\frac{4}{(2s+2)}=\frac{4}{s^2(2s+2)}$$

This leads to the following expression:

$$\frac{4}{s^2(2s+2)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{2s+2}$$

Solving the previous equality:

$$4=A(2s+2)+Bs(2s+2)+Cs^2$$

And identifying the coefficients:

$$\begin{cases} 2B+C=0 \\ 2A=4 \\ 2A+2B=0 \end{cases}$$

The system solution is:

$$C = 4$$
$$B = \text{-}2$$
$$A = 2$$

The time response can be obtained by substituting these values into equation after substituting the coefficients:

$$y(t) = \mathcal{L}^{-1}\left[\bar{y}(s)\right] = \mathcal{L}^{-1}\left[\frac{2}{s^2} - \frac{2}{s} + \frac{4}{2s+2}\right] = \mathcal{L}^{-1}\left[\frac{2}{s^2}\right] - \mathcal{L}^{-1}\left[\frac{2}{s}\right] + \mathcal{L}^{-1}\left[\frac{2}{s+1}\right]$$

This gives the following expression:

$$y(t) = 2t - 2 + 2e^{-t} = 2(t - 1 + e^{-t})$$

c) The response value, once five units of time have elapsed, is calculated by substituting the following expressions into the above equation:

$$y(t) = 2\left(t - 1 + e^{-t}\right)$$

$$\text{if } t = 5 \rightarrow y(t) = 2\left(5 + e^{-5} - 1\right) = 8.01$$

Problem 1.2.9

Consider a pneumatically operated control valve with the following values:

Diaphragm area= 600 cm^2

Stem mass = 1.8 kg

k = 1,000 N/cm

c = 40 Ns/cm

a) Obtain the transfer function relating the stem displacement to the pressure variation and indicate whether or not the system is damped.

b) Calculate the stem displacement when the pressure suddenly increases from 4 to 8 N/cm^2.

c) Calculate the value of *c* required to obtain a critically damped response (assuming that the other parameter values remain constant).

Solution

a) Applying some simplifications, it can be considered that the transfer function relating the stem displacement $\bar{x}(s)$ to the pressure variation $\bar{p}(s)$ in a control valve is in accordance with a second-order function, like the below expression:

$$\frac{\bar{x}(s)}{\bar{p}(s)} = \frac{\frac{A}{k}}{\frac{m}{k}s^2 + \frac{c}{k}s + 1}$$

Substituting the formula values into the above expression gives:

$$\frac{\bar{x}(s)}{\bar{p}(s)} = \frac{\frac{600}{1.000}}{\left(\frac{1.8}{1.000}\right)s^2 + \left(\frac{40}{1.000}\right)s + 1} = \frac{0,6}{(0.0018)s^2 + (0.04)s + 1}$$

When comparing the result to a second-order transfer function:

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\tau\zeta s + 1}$$

Now, comparing the two expressions enables calculation of the second-order transfer function parameters values:

$$0.0018 = \tau^2 \Rightarrow \tau = 0.042$$

$$2\zeta\tau = 0.04 \Rightarrow \zeta = \frac{0.04}{2(0.042)} = 0.476$$

Since $\zeta < 1$, the process is underdamped which indicates that the system response will oscillate.

b) The pressure value is shown by $\bar{p}(s) = \frac{4}{s}$, which, expressed as a Laplace transform, is the expression of the input variation applied to the valve. The output will be:

$$\bar{x}(s) = \left(\frac{4}{s}\right)\frac{0.6}{\left((0.0018)s^2 + (0.04)s + 1\right)}$$

If one considers that the value to be calculated is the $x(t)$ value when $t \to \infty$, then the Final-Value Theorem can be applied, and therefore, calculation of the inverse Laplace transform in not required:

$$x(t) = \lim_{s\to 0}\left[s\bar{x}(s)\right] = \lim_{s\to 0} s\left(\frac{4}{s}\right)\frac{006}{\left((0.0018)s^2 + (0.04)s + 1\right)} = 2.4 \text{ cm}$$

c) If the response must be critically damped $\Rightarrow \zeta = 1$, the value of τ does not vary:

$$\frac{c}{k} = 2\zeta\tau \Rightarrow c = 2(1)(0.042)(1000) = 84 \;\; Ns/cm$$

Problem 1.2.10

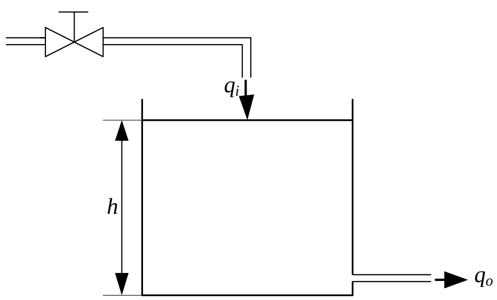
A tank input valve is partially opened and closed by a two-position control system: when the valve is in the minimum flow rate position, fluid enters the tank at 0.001 m^3/s, and when it is in the maximum flow rate position, fluid enters the tank at 0.003 m^3/s.

The level effect on the output flow rate is constant and can be considered negligible (it is equal to 0.002 m^3/s). The area of the tank is 2 m^2. The process delay is 10 seconds. The neutral zone of the system is equivalent to ±0.005 m of the level change.

a) Determine the level change velocity if the two positions of the controller are studied.

b) Plot the level against time and justify the results.

c) Determine the period and the level oscillation.



Solution

a) Applying a mass balance to the system, the built-up velocity in the tank can be calculated:

$$q_a = q_i - q_o$$

The change in level velocity is given by:

$$\frac{dh}{dt} = \frac{q_a}{A} = \frac{q_i - q_o}{2} \text{ m/s}$$

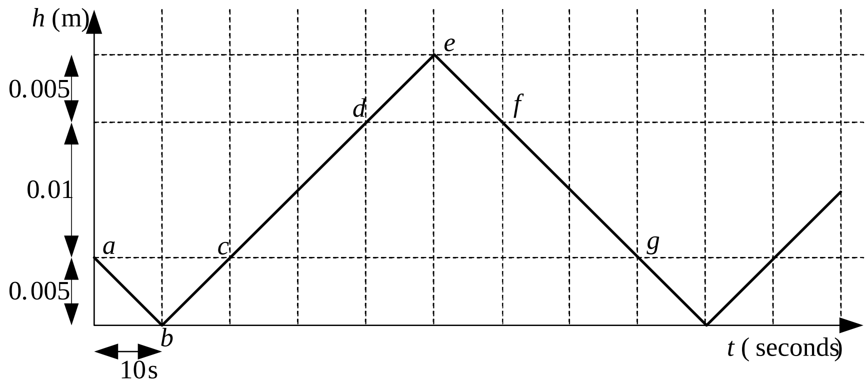
When the valve is in the maximum flow rate position:

$$\frac{0.003-0.002}{2}=0.0005 \text{ m/s}$$

When the valve is in the minimum flow rate position:

$$\frac{0.001-0.002}{2}=-0.0005 \text{ m/s}$$

b) The plot must incorporate all of the premises followed by the system:



$$t_b-t_a=t_e-t_d=t_f-t_e=10 \text{ s}$$

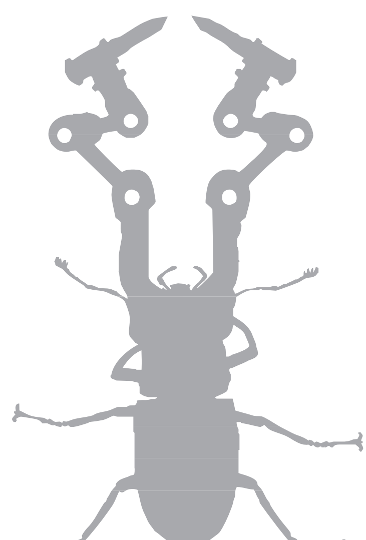
$$t_g-t_f=t_d-t_c=20 \text{ s}$$

$$h_c-h_b=h_e-h_d=h_f-h_e=(0.0005)(10)=0.005 \text{ m}$$

$$h_g-h_f=h_d-h_c=(0.0005)(20)=0.01 \text{ m}$$

c) The plot reveals a period of 80 seconds. The amplitude is:

$$\text{amplitude}=0.005+0.01+0.005=0.02 \text{ m}$$



→2

Feedback systems

Introduction

The knowledge of the feedback control systems is essential in Chemical Engineering because these systems are the most used in this field. This chapter presents the mathematics-based rules and guidelines to study their characteristics; therefore, the reader must have a good grasp of them.

The study of the response of a feedback system is focused assuming that there are two inputs: *set point* and *disturbance*. In this chapter to simplify the way of solving the problems and make the applications more accessible it has been considered that, in certain situations, only one of these is variable. When both inputs are variable the overall response has been obtained by applying the *superposition principle.*

The *time response* is always the most required response for understanding the behaviour of the processes; therefore, the inverse Laplace transform must be often applied. Furthermore, the *Final-Value Theorem* and the *Initial-Value Theorem* may be applied in order to simplify the calculation of the time response for cases in which only the values supplied by these theorems are sought.

The *controller*, considered to be the basic element of feedback systems, is studied as a member of a control system. It is also studied because it is responsible for changes in the system response.

Stability studies are performed by the *Routh-Hurwitz criterion* using the *characteristic equation*. Furthermore, stability will be studied by the *Bode criterion*, using the frequency response. It is very interesting to extract some conclusions of the information obtained from the two different methods; this justifies the complexity when applying the last one of the cited methods. In addition, after Rout-Hurwitz criterion some examples about *Root Locus Analysis* are included.

In many cases, *Bode diagrams* are only plotted in their asymptotic approach; this simplifies deeply the graphs; in addition, this enables the visualization of the evolutions of *amplitude ratio (AR)* and of *phase lag (PL)*. Sometimes, when necessary, one must resort to mathematical solutions; this enables exact knowledge of the studied values. It is very important to extract some design conclusions from the Bode diagrams, due to the complete information that they provide. Tuning controllers is the last application using the frequency response.

In this chapter, as in the remaining ones, solving the problems requires some concepts that are known from previous chapters. Including these problems here enables comparison of different techniques and offers a global view of the topic being studied.

2.1 Systems response, the characteristic equation and stability

Problem 2.1.1

Consider a control system that has the following elements:

$$G_C(s) = K_c$$

$$G_f(s) = \frac{1}{(s+20)}$$

$$G_p(s) = \frac{(s+40)}{s(s+10)}$$

$$G_m(s) = 1$$

$$G_d(s) = \frac{1}{s(s+10)}$$

a) Calculate the values of K_c at which the system will be stable.

b) Calculate the offset when the disturbance is a unit step change and the set point remains constant. Explain the results obtained in the following two scenarios:

b.1 Using the transfer functions of the problem statement.

b.2 When $G_C(s) = K_C\left(1+\dfrac{3}{s}\right)$, transfer function that matches with a proportional-integral (PI) controller and the rest of the transfer functions remain the same.

Solution

a) The characteristic equation comprises the transfer functions given above:

$$1+G_c(s)G_f(s)G_p(s)G_m(s) = 0 \rightarrow 1+K_c\frac{1}{(s+20)}\frac{(s+40)}{s\ (s+10)}(1) = 0$$

Rearranging the above equation gives:

$$s^3+30s^2+200s+K_c(s+40) = 0 \rightarrow s^3+30s^2+(200+K_c)\ s+40K_c = 0$$

Forming the Routh array gives:

s³	1	$200+K_c$
s²	30	$40K_c$
s¹	$200-K_c/3$	0
s⁰	$40K_c$	

Applying the stability condition to the results obtained above gives:

$$40K_c > 0 \rightarrow K_c > 0 \text{ and } 200 - \frac{K_c}{3} > 0 \rightarrow 600 > K_c$$

Therefore, the system will be stable when the following overall condition is met:

$$0 < K_c < 600$$

b.1) When the set point remains constant, the closed loop response will be:

$$\bar{y}(s) = \frac{G_d(s)}{1+G_c(s)G_f(s)G_p(s)G_m(s)}\bar{d}(s)$$

$$\bar{y}(s) = \frac{\dfrac{1}{s\ (s+10)}}{1+K_c\dfrac{1}{(s+20)}\dfrac{(s+40)}{s\ (s+10)}(1)}\left(\frac{1}{s}\right)$$

$$\bar{y}(s) = \frac{\dfrac{1}{s(s+10)}}{\dfrac{s^3+30s^2+200s+K_cs+40K_c}{(s+20)\ s\ (s+10)}}\left(\frac{1}{s}\right)$$

Simplifying the above equation gives:

$$\bar{y}(s) = \frac{(s+20)}{s^3+30s^2+(200+K_c)\ s+40K_c}\left(\frac{1}{s}\right)$$

Applying the concept of offset:

offset=(new set point)-(ultimate value of the response)

Applying the Final-Value Theorem to obtain the ultimate value of the response gives:

$$\underset{t\to\infty}{y(t)} = \lim_{s\to 0}\left[s\frac{(s+20)}{s^3+30s^2+(200+K_c)s+40K_c}\left(\frac{1}{s}\right)\right] = \frac{20}{40K_c} \to \underset{t\to\infty}{y(t)} = \frac{1}{2K_c}$$

$$\text{Offset} = 0 - \frac{1}{2K_c} = -\frac{1}{2K_c}$$

As expected, the system has offset because it has a proportional (P) controller.

b.2) The value of $\underset{t\to\infty}{y(t)}$ is calculated applying the Final-Value Theorem:

$$\bar{y}(s) = \frac{G_d(s)}{1+G_c(s)G_f(s)G_p(s)G_m(s)}\bar{d}(s)$$

$$\bar{y}(s) = \frac{\dfrac{1}{s\ (s+10)}}{1+K_c\left(\dfrac{s+3}{s}\right)\dfrac{1}{(s+20)}\dfrac{(s+40)}{s\ (s+10)}(1)}\left(\frac{1}{s}\right)$$

$$\underset{t\to\infty}{y(t)} = \lim_{s\to 0}\left[s\frac{\dfrac{1}{s\ (s+10)}}{1+K_c\left(\dfrac{s+3}{s}\right)\dfrac{1}{(s+20)}\dfrac{(s+40)}{s\ (s+10)}(1)}\left(\frac{1}{s}\right)\right]$$

$$\underset{t\to\infty}{y(t)} = \lim_{s\to 0}\left[\frac{s\ (s+20)}{s^4+30s^3+200s^2+K_c\left(s^2+43s+120\right)}\right] = \frac{0}{120K_c} = 0$$

The value of the offset is calculated as follows:

$$\text{Offset} = 0 - 0 = 0$$

As expected, the system has no offset because it has a PI controller that eliminates any offset.

Problem 2.1.2

Using the Routh-Hurwitz criterion, determine the stability of the feedback control system whose characteristic equation is:

$$s^4 + s^3 + s^2 + s + K = 0$$

Solution

The following Routh array is formed to study the stability:

s^4	1	1	K
s^3	1	1	0
s^2	$0 \rightarrow \varepsilon$	K	
s^1	$\frac{\varepsilon - K}{\varepsilon}$	0	
s^0	K		

When a zero is found in the first column, it must be substituted by ε>o and it can go on applying the Routh criterion.

The following conditions must be accomplished to have a stable control system:

$$K > 0 \ and \ \frac{(\varepsilon) - K}{(\varepsilon)} > 0 \ \text{ and; therefore, } 1 - \frac{K}{(\varepsilon)} > 0 \ \Rightarrow \ \frac{K}{(\varepsilon)} < 1$$

The condition $\frac{K}{\varepsilon} < 1$ can never be met because K and ε are assumed to be positive. Thus, the control system will be unstable for any value of K.

Problem 2.1.3

Using the Routh-Hurwitz criterion, determine the stability of a feedback control system whose characteristic equation is:

$$s^4 + 3s^3 + 4s^2 + 3s + 3 = 0$$

Solution

Firstly, the Routh array is formed:

s^4	1	4	3
s^3	3	3	0
s^2	3	3	0
s^1	0(6)	0	0
s^0	3		

The s^1 row contains only zeros that do not allow us to go on applying the Routh criterion. Thus, the first derivative of the immediately previous row must be calculated. This gives:

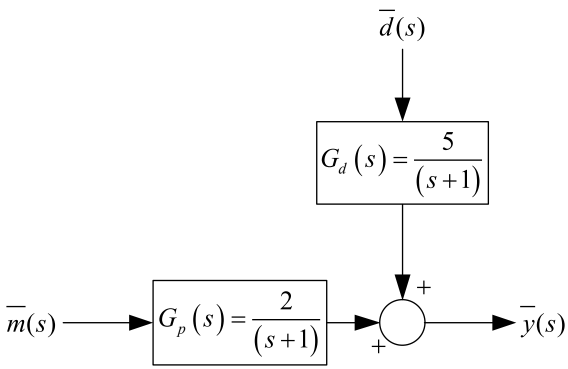
$$\frac{d}{ds}(3s^2+3)=6s+0$$

These values must be substituted into the corresponding row (in parenthesis).

According to the Routh array, the system is stable; because there are not sign changes in the first column (all the elements are positive).

Problem 2.1.4

Consider a process whose behaviour can be expressed by the following transfer functions (placed in its block diagram):



A feedback control loop is formed by adding a proportional controller. The remaining elements are described by the following transfer functions:

$$G_c(s)=K_c$$
$$G_m(s)=1$$
$$G_f(s)=\frac{1}{(s+1)}$$

a) Calculate the order of the response when only the set point changes.

b) Calculate the offset value when a unit step input is applied to the set point.

c) Determine the stability of the feedback control system according to the K_c values when the transfer function $G_m(s)$ changes to $G_m(s)=\frac{1}{(3s+1)}$ and the other transfer functions remain the same.

Solution

a) The system characteristic equation, considering only the set point changes, will be:

$$\bar{y}(s)=\frac{G_c(s)G_p(s)G_f(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\bar{y}_{sp}(s)\rightarrow \bar{y}(s)=\frac{K_c\dfrac{1}{(s+1)}\dfrac{2}{(s+1)}}{1+K_c\dfrac{1}{(s+1)}\dfrac{2}{(s+1)}}\bar{y}_{sp}(s)$$

Rearranging and simplifying the above equation gives:

$$\bar{y}(s)=\frac{2K_c}{s^2+2s+1+2K_c}\bar{y}_{sp}(s)$$

The above equation describes the response when only the set point changes. As observed, it is a second-order response.

c) Offset is defined as:

offset=(new set point)-(ultimate value of the response)

Applying the above definition gives:

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0}\left[s\frac{2K_c}{s^2+2s+1+K_c}\left(\frac{1}{s}\right)\right]=\frac{2K_c}{1+2K_c}$$

Applying the Final-Value Theorem, and substituting into the offset definition:

$$\text{Offset}=1-\frac{2K_c}{1+2K_c}=\frac{1}{1+2K_c}$$

c) To determine the stability of the control system, the equation $1+G_{OL}(s)=0$ must be found.

Therefore, substituting the statement values gives:

$$1+K_c\frac{1}{(s+1)}\frac{2}{(s+1)}\frac{1}{(3s+1)}=0$$

Rearranging to obtain the characteristic equation gives:

$$3s^3+7s^2+5s+1+2K_c=0$$

Applying the Routh array gives:

s^3	3	5
s^2	7	$1+2K_c$
s^1	$\dfrac{-6K_c+32}{7}$	0
s^0	$1+2K_c$	

Imposing the stability condition (i.e. that all the elements in the first column must be positive) gives:

$$-6K_c+32>0 \Rightarrow K_c<\frac{16}{3}$$

and

$$1+2K_c>0 \Rightarrow K_c>-\frac{1}{2}$$

For the system to be stable, both conditions must be met:

$$-\frac{1}{2}<K_c<\frac{16}{3}$$

Problem 2.1.5

Consider a feedback control system that is made up of elements whose transfer functions are:

$$G_c(s)=K_c$$

$$G_f(s)=\frac{2}{(s+1)}$$

$$G_p(s)=\frac{2}{(s+2)(s+3)}$$

$$G_m(s)=1$$

$$G_d(s)=\frac{3}{(s+1)}$$

The following relation is also assumed:

$$\overline{y}(s)=G_p(s)\overline{m}(s)+G_d(s)\overline{d}(s)$$

a) Plot the block diagram of the feedback process.

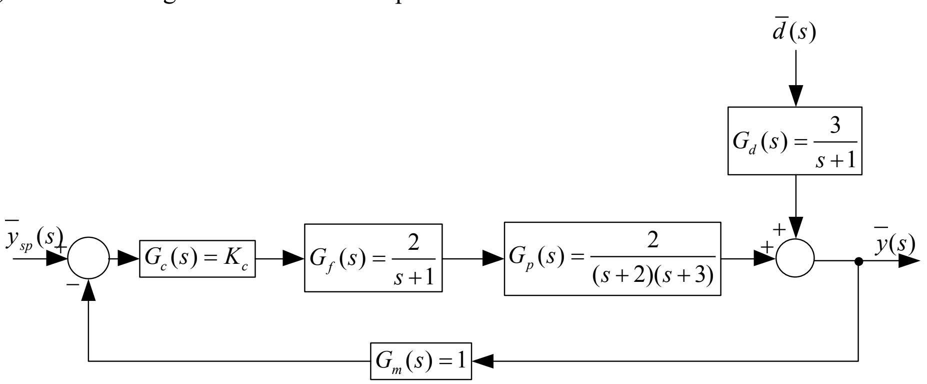
b) Determine the stability of the control system for different K_c values.

c) Determine the offset when the set point suddenly changes (unit step input), assuming that the disturbance does not change.

d) Determine the offset when the disturbance suddenly changes (unit step input), assuming that the set point does not change.

Solution

a) The block diagram of the feedback process is:



b) Substituting the values of the transfer functions into the characteristic equation gives:

$$1 + G_p(s)G_c(s)G_f(s)G_m(s) = 0 \rightarrow 1 + K_c \frac{2}{(s+1)} \frac{2}{(s+2)(s+3)} 1 = 0$$

Rearranging the above equation gives:

$$(s+1)(s+2) + 4K_c = 0$$

Grouping according to the s degree gives:

$$s^3 + 6s^2 + 11s + (6 + 4K_c) = 0$$

Forming the Routh array gives:

s^3	1	11
s^2	6	$6+4K_c$
s^1	$10-\frac{2}{3}K_c$	0
s^0	$6+4K_c$	

Applying the stability conditions:

$$10-\frac{2}{3}K_c>0 \Rightarrow K_c<15$$

$$6+4K_c>0 \Rightarrow K_c>-\frac{3}{2}$$

The above equations indicate that the system will be stable for all K_c values inside the following interval:

$$-\frac{3}{2}<K_c<15$$

c) Substituting the values of transfer functions into the characteristic equation and considering the statement conditions gives:

$$\bar{y}(s)=\frac{K_c\frac{2}{(s+1)}\frac{2}{(s+2)(s+3)}}{1+K_c\frac{2}{(s+1)}\frac{2}{(s+2)(s+3)}}\left(\frac{1}{s}\right)$$

Rearranging the above equation gives:

$$\bar{y}(s)=\frac{4K_c}{s^3+6s^2+11s+6+4K_c}\frac{1}{s}$$

Applying the Final-Value Theorem gives:

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0}\left[s\frac{4K_c}{s^3+6s^2+11s+6+4K_c}\left(\frac{1}{s}\right)\right]=\lim_{t\to\infty} y(t)=\frac{4K_c}{6+4K_c}$$

Substituting the above result into the offset definition gives:

$$\text{Offset}=1-\frac{4K_c}{6+4K_c} \quad \Rightarrow \text{Offset}=\frac{3}{3+2K_c}$$

d) Substituting the values of transfer functions into the characteristic equation and considering the statement conditions gives:

$$\bar{y}(s)=\frac{G_d(s)}{1+G_c(s)G_f(s)G_m(s)G_p}\left(\frac{1}{s}\right)\rightarrow \bar{y}(s)=\frac{\frac{3}{(s+1)}}{1+K_c\frac{2}{(s+1)}\frac{2}{(s+2)(s+3)}}\left(\frac{1}{s}\right)$$

Rearranging and simplifying gives:

$$\bar{y}(s)=\frac{3\left(s^2+5s+6\right)}{s^3+6s^2+11s+6+4K_c}\left(\frac{1}{s}\right)$$

Applying the Final-Value Theorem gives:

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0}\left[s\frac{3\left(s^2+5s+6\right)}{s^3+6s^2+11s+6+4K_c}\left(\frac{1}{s}\right)\right]=\frac{18}{6+4K_c}$$

Substituting the above result into the offset definition gives:

$$\text{Offset}=0-\frac{18}{6+4K_c}=-\frac{9}{3+2K_c}$$

Problem 2.1.6

Consider a feedback control system that is made up of elements whose transfer functions are:

$$G_p(s)=\frac{6}{0.1s+1}$$
$$G_c(s)=K_c\left(1+\frac{1}{\tau_I s}\right)$$
$$G_f(s)=1$$
$$G_m(s)=\frac{K}{\tau s+1}$$

a) Calculate a pair of values of K_c and τ_I that would enable the closed loop response to be stable when $K = 1$ and $\tau = 1$.

b) Determine the influence of K and τ on the stability of closed loop response when $\tau_I = 1$ and $K_c = 1$.

c) Calculate the values of K_c required for obtaining a closed loop response that remains stable when the controller is proportional assuming $\tau = 1$ and $K = 1$.

Solution

a) Substituting the transfer functions values into the characteristic equation gives:

$$1+G_p(s)G_c(s)G_f(s)G_m(s)=0 \rightarrow 1+\left(\frac{6}{0.1s+1}\right)K_c\left(1+\frac{1}{\tau_I s}\right)(1)\left(\frac{K}{\tau s+1}\right)=0$$

Rearranging the above equation gives:

$$\left(0.1s+1\right)\left(\tau s+1\right)\tau_I s+6KK_c(\tau_I s+1)=0$$

Substituting the statement values of K and τ into the above expression gives:

$$\left(0.1s+1\right)\left(s+1\right)\tau_I s+6K_c(\tau_I s+1)=0$$

Rearranging the above expression gives:

$$(0.1)\tau_I s^3+(1.1)\tau_I s^2+\tau_I s+6\tau_I K_c s+6K_c=0$$

The above equation is grouped as follows:

$$(0.1)\tau_I s^3+(1.1)\tau_I s^2+\left(\tau_I+6\tau_I K_c\right)\ s+6K_c=0$$

Forming the Routh array gives:

s^3	$0.1\,\tau_I$	$\tau_I+6\tau_I K_c$
s^2	$1.1\,\tau_I$	$6K_c$
s^1	$\tau_I(1+6K_c)-(0.545)K_c$	0
s^0	$6K_c$	

The following conditions must be met:

$$0.1\tau_I>0 \rightarrow \tau_I>0$$
$$6K_c>0 \rightarrow K_c>0$$
$$\tau_I(1+6K_c)-(0.545)K_c>0$$

These conditions are met when $\tau_I=1$ and $K_c=1$.

b) Proceeding as in the previous section gives:

$$1+G_p(s)G_c(s)G_f(s)G_m(s)=0 \rightarrow 1+\left(\frac{6}{0.1s+1}\right)K_c\left(1+\frac{1}{\tau_I s}\right)(1)\left(\frac{K}{\tau s+1}\right)=0$$

Rearranging the above equation gives:

$$\left(0.1s+1\right)\left(\tau s+1\right)\tau_I s+6KK_c(\tau_I s+1)=0$$

Substituting K_c=1 and $\tau_I=0.1$ into the above equation gives:

$$\left(0.1s+1\right)\left(\tau s+1\right)(0.1s)+6K(0.1s+1)=0$$

Operating and grouping gives:

$$(0.01)\tau s^3+(0.01+0,1\tau)s^2+\left(0.1+0.6K\right)s+6K=0$$

Forming the Routh array gives:

s^3	0.01τ	$0.1+0.6K$
s^2	$0.01+0,1\tau$	$6K$
s^1	$0.1+0.6K-\dfrac{0.06K\tau}{0.01+0.1\tau}$	0
s^0	$6K$	

The system will be stable when the following conditions are met:

$$0.1+0.6K-\frac{0.06K\tau}{0.01+0.1\tau}>0 \text{ and } 6K>0$$

K can have any positive value because:

$$0.001+0.01\tau+0.006K+0.06K\tau-0.06K\tau>0\rightarrow 0.001+0.01\tau+0.006K>0$$

The above expression is true for any positive value of K and assuming that $\tau>0$.

Considering:

$$0.01+0.1\tau>0$$

The following equation can be inferred:

$$0.1\tau>-0.01\Rightarrow \tau>-0.1$$

Therefore, the system will be stable, for any positive value of K and τ.

c) Repeating the above operation gives:

$$1+G_p(s)G_c(s)G_f(s)G_m(s)=0\rightarrow 1+\left(\frac{6}{0.1s+1}\right)K_c\left(\frac{1}{s+1}\right)=0$$

Rearranging the above expression:

$$(0.1s+1)(s+1)+6K_c=0 \rightarrow 0.1s^2+1.1s+6K_c+1=0$$

Forming the Routh array:

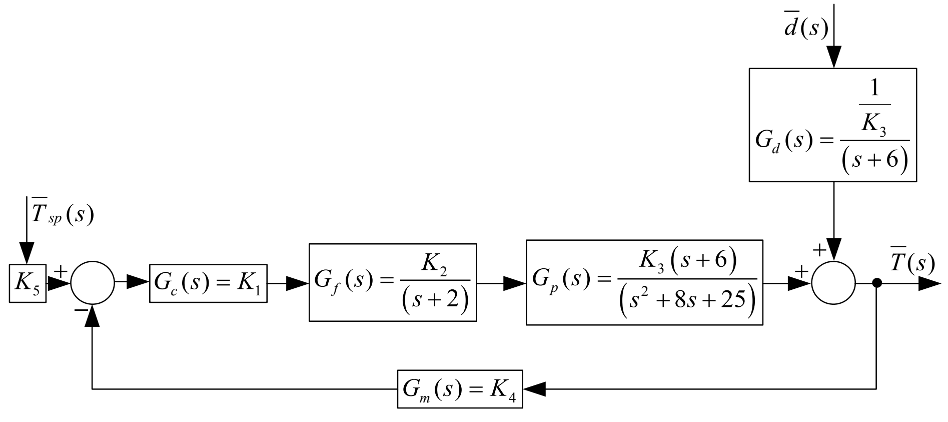
s^2	0.1	$6K_c+1$
s^1	1.1	0
s^0	$6K_c+1$	

The system will be stable when:

$$6K_c+1>0 \Rightarrow K_c>-1/6$$

Problem 2.1.7

Consider the pneumatic temperature-control system depicted by the following block diagram:



a) Obtain the output equation in function of the inputs $\bar{T}_{sp}(s)$ and $\bar{d}(s)$. Justify the result obtained.

b) Determine the stability of the control system for $K=K_1K_2K_3K_4$.

c) *c-1)* Discuss the variation in stability when the value of K_5 increases by two times.

c-2) Discuss the variation in stability when the value of K_5 decreases by half.

c-3)-Discuss whether the system has any offset.

Solution

a) The overall equation of the feedback output is:

$$\bar{T}(s) = \frac{G_c(s)G_p(s)G_f(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\bar{T}_{sp}(s) + \frac{G_d(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\bar{d}(s)$$

Substituting the values of the transfer functions from the block diagram into the above equation gives:

$$\bar{T}(s) = K_5 \frac{K_1 \dfrac{K_2}{(s+2)} \dfrac{K_3(s+6)}{(s^2+8s+25)}}{1+K_1K_4 \dfrac{K_2}{(s+2)} \dfrac{K_3(s+6)}{((s^2+8s+25)}}\bar{T}_{sp}(s) + \frac{\dfrac{1}{K_3(s+6)}}{1+K_1K_4 \dfrac{K_2}{(s+2)} \dfrac{K_3(s+6)}{(s^2+8s+25)}}\bar{d}(s)$$

Rearranging and simplifying the above equation gives the response value:

$$\bar{T}(s) = \frac{K_1K_2K_3K_5(s+6)}{(s+2)(s^2+8s+25)+K_1K_2K_3K_4(s+6)}\bar{T}_{sp}(s) +$$
$$+\frac{(s+2)}{(s+2)(s^2+8s+25)+K_1K_2K_3K_4(s+6)}\bar{d}(s)$$

b) The characteristic equation of the control system is:

$$(s+2)(s^2+8s+25)+K_1K_2K_3K_4(s+6) = 0$$

Rearranging and simplifying the above equation gives:

$$s^3+10s^2+41s+50+K(s+6) = 0$$

The above equation is grouped as follows:

$$s^3+10s^2+(41+K)s+(50+6K) = 0$$

To determine the stability of the system the Routh array is formed:

s^3	1	$41+K$
s^2	10	$50+6K$
s^1	$36+4K/10$	0
s^0	$50+6K$	

The system will be stable when the following conditions are met:

$$36+\frac{4}{10}K>0\Rightarrow 360+4K>0\Rightarrow K>-90$$

and

$$50+6K>0\Rightarrow 6K>-50\Rightarrow K>-8.3$$

Thus, the system will be stable at $K > -8.3$. This implies that the system will be stable for all positive values of K.

c) c-1) Stability does not vary when K_5 doubles, because K_5 does not influence the stability.

c-2) Stability does not vary when K_2 decreases by half because the control system is always stable for positive values of K.

c-3) The control system has offset because it contains a proportional controller (this type of controller always generates offset).

Problem 2.1.8

Consider a process represented by the transfer function shown below:

$$G_p(s)=\frac{K_p}{\tau^2 s^2+2\zeta\tau s+1}$$

Consider also a control system comprising the following elements whose transfer functions are:

$$G_c(s)=K_c \text{ and } G_m(s)=G_f(s)=1$$

Determine the effects of the proportional controller on the controlled system, according to the response order and the characteristics of its parameters, when the disturbance remains constant.

Solution

The system response will be:

$$\overline{y}(s)=\frac{G_c(s)G_p(s)G_f(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\overline{y}_{sp}(s)+\frac{G_d(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\overline{d}(s)$$

Substituting the values of the elements into the above equation gives:

$$\bar{y}(s)=\frac{K_c\left(\dfrac{K_p}{\tau^2s^2+2\zeta\tau s+1}\right)}{1+K_c\left(\dfrac{K_p}{\tau^2s^2+2\zeta\tau s+1}\right)}\bar{y}_{sp}(s)$$

Rearranging and simplifying the above equation gives:

$$\bar{y}(s)=\frac{K_cK_p}{\tau^2s^2+2\zeta\tau\ s+1+K_cK_p}\bar{y}_{sp}(s)$$

The system type is determined by dividing by $1+K_cK_p$ which gives:

$$\bar{y}(s)=\frac{\dfrac{K_cK_p}{1+K_cK_p}}{\dfrac{\tau^2}{1+K_cK_p}s^2+\dfrac{2\zeta\tau}{1+K_cK_p}s+1}\bar{y}_{sp}(s)$$

τ' and ζ' are calculated as:

$$\tau'=\frac{\tau}{\sqrt{1+K_cK_p}}$$

$$\zeta'=\frac{\zeta}{\sqrt{1+K_cK_p}}$$

The response is:

$$\bar{y}(s)=\frac{K'}{\left(\tau'\right)^2s^2+2\zeta'\tau's+1}\bar{y}_{sp}(s)$$

Thus, the response remains second-order. The value of ζ decreases; therefore, the overdamped process can become an underdamped process. Thus, it will show an oscillatory response when a step input is applied.

Problem 2.1.9

A proportional derivative (PD) controller is applied to the first-order process shown below:

$$G_p(s)=\frac{10}{(s+1)}$$

The PD controller parameters are:

$$G_c(s)=K_c\tau_Ds,\quad \text{with } K_c=5\ \ i\ \ \tau_D=0.1$$

The remaining control system elements are given by:

$$G_m(s) = G_f(s) = 1$$

Assuming that the disturbance remains constant,

a) Determine the response of the controlled system when the set point varies and indicate its order.
b) Discuss the velocity of the response.

Solution

a) The system response is:

$$\bar{y}(s) = \frac{G_c(s)G_p(s)G_f(s)}{1 + G_c(s)G_p(s)G_f(s)G_m} \bar{y}_{sp}(s)$$

Substituting the values of K_c and τ_D into the above equation gives:

$$G_c(s) = (5)(0.1)s = 0.5s$$

Substituting the values of $G_m(s)$, $G_c(s)$, $G_p(s)$ and $G_f(s)$ into the system response equation gives:

$$\bar{y}(s) = \frac{(0.5s)\dfrac{10}{(s+1)}}{1 + (0.5s)\dfrac{10}{(s+1)}(1)} \bar{y}_{sp}(s) = \frac{\dfrac{5s}{(s+1)}}{1 + \dfrac{5s}{(s+1)}} \bar{y}_{sp}(s)$$

Rearranging and simplifying the above equation gives:

$$\bar{y}(s) = \frac{5s}{6s+1} \bar{y}_{sp}(s)$$

Thus, a first-order response is obtained.

b) Before the derivative control was installed, the time constant (τ_p) was equal to one; after it was installed, τ_p was equal to six. Therefore, the controller markedly slows down the system response.

Problem 2.1.10

Consider a second-order process represented by the transfer function shown below:

$$G_p(s) = \frac{1}{4s^2 + 6s + 1}$$

A proportional controller, $G_c(s) = K_c$, is added, and $G_m(s) = G_f(s) = 1$.

Assuming that the disturbance remains constant;

a) Determine the response type once the proportional controller has been installed.

b) Calculate the parameters of the new response and determine the effect produced when K_c increases.

c) Justify whether K_c must increase or decrease to obtain a faster response.

d) Calculate the offset produced when the set point is a unit step change. Propose a method for reducing this offset.

Solution

a) Substituting the values into the overall response gives:

$$\bar{y}(s) = \frac{G_c(s)G_p(s)G_f(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\bar{y}_{sp}(s) = \bar{y}(s) = \frac{K_c\dfrac{1}{\left(4s^2+6s+1\right)}}{1+K_c\dfrac{1}{\left(4s^2+6s+1\right)}}\bar{y}_{sp}(s)$$

Rearranging and simplifying the above equation gives:

$$\bar{y}(s) = \frac{\dfrac{K_c}{4s^2+6s+1}}{\dfrac{4s^2+6s+1+K_c}{4s^2+6s+1}}\bar{y}_{sp}(s) \rightarrow \bar{y}(s) = \frac{K_c}{4s^2+6s+(1+K_c)}\bar{y}_{sp}(s)$$

Dividing by $1 + K_c$ gives:

$$\bar{y}(s) = \frac{\dfrac{K_c}{1+K_c}}{\left(\dfrac{4}{1+K_c}\right)s^2+\left(\dfrac{6}{1+K_c}\right)s+1}\bar{y}_{sp}(s)$$

The above equation corresponds with a second-order response.

c) Based on the result from part *a* the following can be inferred:

$$(\tau')^2 = \frac{4}{1+K_c} \Rightarrow \tau' = \frac{2}{\sqrt{1+K_c}}$$

$$2\zeta'\tau' = \frac{6}{1+K_c} \Rightarrow \zeta' = \frac{6}{2(1+K_c)}\frac{\sqrt{1+K_c}}{2} \Rightarrow \zeta' = \frac{1.5}{\sqrt{1+K_c}}$$

$$K_p' = \frac{K_c}{1+K_c}$$

When K_c increases, both the natural period and the damping factor decrease. Therefore, an increase in K_c can cause the response to change from overdamped to underdamped, depending on the size of the increase.

c) When K_c increases, ζ' decreases, but the response becomes faster and more oscillatory; contrariwise, when K_c decreases, ζ' increases, but the response becomes slower and less oscillatory.

d) Applying a unit step input $\overline{y}(s)=\frac{1}{s}$ to the above response gives:

$$\overline{y}(s)=\frac{\frac{K_c}{1+K_c}}{\left(\frac{4}{1+K_c}\right)s^2+\left(\frac{6}{1+K_c}\right)s+1}\left(\frac{1}{s}\right)$$

The ultimate value of the response is:

$$y(t\to\infty)=\lim_{s\to 0}\left[s\overline{y}(s)\right]=\lim_{s\to 0}\left[s\left(\frac{1}{s}\right)\frac{\frac{K_c}{1+K_c}}{\frac{4}{1+K_c}s^2+\frac{6}{1+K_c}s+1}\right]=\frac{K_c}{1+K_c}$$

Applying the offset definition gives:

$$\text{Offset}=1-\frac{K_c}{1+K_c}=\frac{1}{1+K_c}$$

Based on the above result, it can be inferred that when K_c increases, the offset decreases.

Problem 2.1.11

Consider the tank illustrated below. A feedback control system regulates its level, varying the input flow rate (F_1) as long as the output flow rate (F_3) remains constant and F_2 must be considered the disturbance.

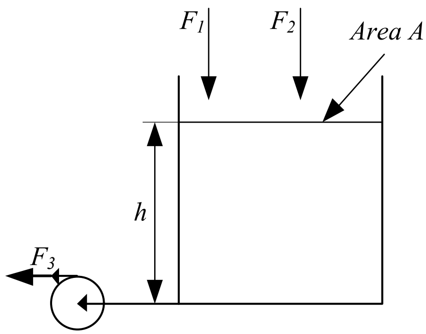
a) Obtain the transfer function that relates the process variables, in which each one of them is expressed as a deviation variable.

Consider that a proportional controller has been installed $G_c(s)=K_c$, also $G_f(s)=G_m(s)=1$.

b) Obtain the overall transfer function of the controlled system.

c) Calculate the offset when the disturbance is a unit step change.

d) Calculate the offset when the set point is a two unit step change.



Solution

a) To obtain the transfer function, an overall mass balance is applied to the entire process:

$$A\frac{dh}{dt} = F_1 + F_2 - F_3$$

The corresponding steady state is:

$$0 = F_{1,s} + F_{2,s} - F_{3,s}$$

Subtracting the above equation from the equation previous to it gives:

$$A\frac{dh}{dt} = (F_1 - F_{1,s}) + (F_2 - F_{2,s}) - (F_3 - F_{3,s})$$

where $F_3 - F_{3,s} = 0$. Furthermore, taking into account that $\frac{dh'}{dt} = \frac{dh}{dt}$ and introducing the deviation variables, gives:

$$A\frac{dh'}{dt} = F_1^{'} + F_2^{'}$$

Applying the Laplace transform to the above equation gives:

$$As\overline{h}'(s) = \overline{F_1^{'}}(s) + \overline{F_2^{'}}(s)$$

Thus, the transfer function that links the process variables is:

$$\overline{h}'(s) = \frac{1}{As}\overline{F_1^{'}}(s) + \frac{1}{As}\overline{F_2^{'}}(s)$$

b) Based on the above equation, the following conclusion can be reached:

$$G_p(s) = \frac{1}{As} \text{ and } G_d(s) = \frac{1}{As}$$

Substituting into the overall equation of a feedback control system gives:

$$\bar{h}'(s) = \frac{K_c\left(\frac{1}{As}\right)}{1+\left(\frac{K_c}{As}\right)}\overline{h_{sp}^{'}}(s) + \frac{\frac{1}{As}}{1+\left(\frac{K_c}{As}\right)}\overline{F_2^{'}}(s) = \frac{1}{\left(\frac{A}{K_c}\right)s+1}\overline{h_{sp}^{'}}(s) + \frac{\frac{1}{K_c}}{\left(\frac{A}{K_c}\right)s+1}\overline{F_2^{'}}(s)$$

c) The following equations are obtained when the disturbance is a unit step change:

$$\overline{F_2^{'}}(s) = \frac{1}{s} \text{ and } \overline{h_{sp}^{'}}(s) = 0$$

Substituting these equations into the overall equation obtained in part *b)* gives:

$$\bar{h}'(s) = \frac{\frac{1}{K_c}}{\left(\frac{A}{K_c}\right)s+1}\left(\frac{1}{s}\right)$$

Calculating the ultimate response value and applying the Final-Value theorem gives:

$$h'(t \to \infty) = \lim_{s\to 0}\left[s\bar{h}'(s)\right] = \lim_{s\to 0}\left[s\left(\frac{1}{s}\right)\frac{\frac{1}{K_c}}{\frac{A}{K_c}s+1}\right] = \frac{1}{K_c}$$

Substituting this equation into the offset definition gives:

$$\text{Offset} = 0 - \frac{1}{K_c} = -\frac{1}{K_c}$$

d) In this case, the values are:

$$\overline{h_{sp}^{'}}(s) = \frac{2}{s} \text{ and } \overline{F_2^{'}}(s) = 0$$

Substituting these values into the overall equation gives:

$$\bar{h}'(s) = \frac{1}{\left(\frac{A}{K_c}\right)s+1}\left(\frac{2}{s}\right)$$

Calculating the ultimate response value and applying the Final-Value Theorem gives:

$$h'(t \to \infty) = \lim_{s \to 0}\left[s\bar{h}'(s)\right] = \lim_{s \to 0}\left[s\left(\frac{2}{s}\right)\frac{1}{\left(\frac{A}{K_c}\right)s+1}\right] = 2 \to \text{offset} = 2-2=0$$

Interestingly, when the set point varies (part *d*) a pure integrator term eliminates the offset, despite the fact a proportional (*P*) controller is used. However, the same is not true when the disturbance varies (part *c*).

Problem 2.1.12

Consider a feedback system having the following transfer functions:

$$G_p(s) = \frac{1}{(s+1)(2s+1)}$$
$$G_c(s) = K_c$$
$$G_m(s) = 1$$
$$G_f(s) = 1$$

a) Plot its root locus (Evans method).

b) Calculate the values of K_c at which the system would be overdamped, critically damped and underdamped.

Solution

a) The feedback system is expressed using the following equation:

$$1 + G_c(s)G_f(s)G_pG_m(s) = 0$$

$$1 + \frac{K_c}{(s+1)(2s+1)} = 0$$

Rearranging the above equation gives:

$$1 + \frac{K_c}{2(s+1)\left(s+\frac{1}{2}\right)} = 0 \to 1 + \frac{K'}{(s+1)\left(s+\frac{1}{2}\right)} = 0 \text{ where } K' = K_c/2$$

At this point, the different steps for plotting the root locus can be taken:

Number of poles and zeros:

There are no zeros (0)

Poles: -1, -1/2 (2)

Number of branches:

$\max(n_p, n_z) = \max(2, 0) = 2$

Segments of the real axis that belong to the root locus:

$$\left[-\infty,-1\right] \notin \text{Root Locus}$$
$$\left[-1,-\frac{1}{2}\right] \in \text{Root Locus}$$
$$\left[-\frac{1}{2},\infty\right] \notin \text{Root Locus}$$

Number of asymptotes:

n_p-n_z = 2 - 0 = 2 asymptotes

Starting point of the asymptotes:

$$\sigma_A = \frac{\Sigma\text{poles} - \sum\text{zeros}}{n_p - n_z} = \frac{-1-1/2}{2} = -0.75$$

Angles of the asymptotes:

$$\theta = \frac{(2k+1)180}{2}$$

Calculating the value of θ at $k = 0$ and at $k = 1$ gives:

$$\theta = 90^{\circ}$$
$$\theta = 270^{\circ}$$

Departure or arrival points on the real axis:

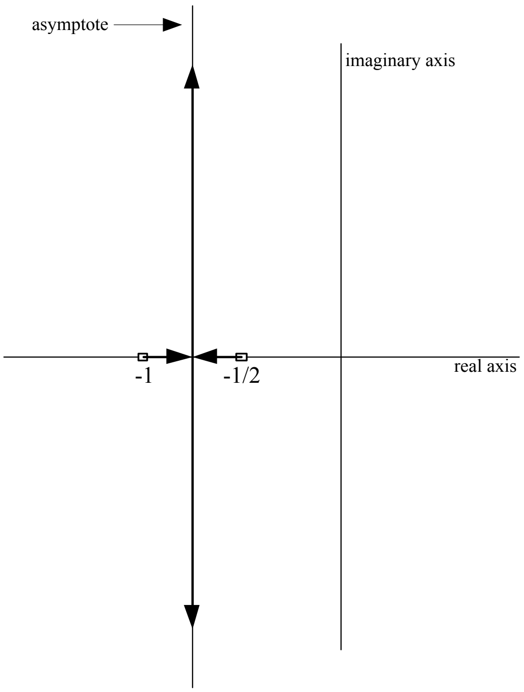
Isolating *K'* from the initial equation gives:

$$K' = -\frac{(s+1)(s+1/2)}{1} = -s^2 - \frac{3}{2}s - \frac{1}{2}$$

Calculating $\dfrac{dK'}{ds} = 0$ gives:

$$\frac{dK'}{ds} = -2s - \frac{3}{2} = 0 \rightarrow s = -\frac{3}{4} = -0.75 \in \text{Root Locus}$$

Thus, the definitive plot will be:



Note: In order to avoid having to perform any unnecessary steps, it is advisable to plot the root locus after each step.

b) First, the following equation must be found:

$$G(s) = \frac{K_c \dfrac{1}{(s+1)(2s+1)}}{1 + K_c \dfrac{1}{(s+1)(2s+1)}} = \frac{K_c}{(s+1)(2s+1) + K_c} = \frac{K_c}{2s^2 + 3s + 1 + K_c}$$

The above equation is a second-order transfer function that can be grouped as follows:

$$G(s) = \frac{K_c}{\left(\dfrac{2}{1+K_c}\right)s^2 + \left(\dfrac{3}{1+K_c}\right)s + 1}$$

At this point, the parameters values that characterize a second-order function transfer can easily be calculated:

$$\tau = \sqrt{\frac{2}{1+K_c}}$$

$$2\zeta\tau = \frac{3}{1+K_c}$$

Substituting the value of τ into the latter above equation gives:

$$2\zeta\sqrt{\frac{2}{1+K_c}} = \frac{3}{1+K_c}$$

$$\zeta = \frac{\dfrac{3}{1+K_c}}{2\sqrt{\dfrac{2}{1+K_c}}} = \frac{1.06}{\sqrt{1+K_c}}$$

When the system is critically damped $\Rightarrow \zeta = 1$:

$$1 = \frac{1.06}{\sqrt{1+K_c}} \Rightarrow K_c = 0.125$$

Based on the above equation the following can be inferred:

When $K_c > 0.125 \rightarrow \zeta < 1 \rightarrow$ Underdamped system
When $K_c < 0.125 \rightarrow \zeta > 1 \rightarrow$ Overdamped system

Problem 2.1.13

Consider a system that has the following transfer function:

$$G_{OL}(s) = \frac{50K}{(s+1)(s+2)(s+10)}$$

a) Plot the root locus for positive values of K from 0 to ∞.

b) Calculate the values of K at the points $\pm j5.65$ and discuss the results obtained.

Solution

a) The equation $1+G_{OL}(s)$ is grouped as follows:

$$1+G_{OL}(s) = 1+\frac{50K}{(s+1)(s+2)(s+10)} = 1+\frac{K'}{(s+1)(s+2)(s+10)}$$

where $K' = 50K$.

At this point, the calculations for plotting the root locus can be done step by step:

Number of poles and zeros:

There are no zeros (0)

Poles: -1, -2, -10 (3)

Number of branches:

$\max(n_p, n_z) = \max(3, 0) = 3$

Segments of the real axis that belong to the root locus:

$[-\infty, -10] \in$ Root Locus

$[-10, -2] \notin$ Root Locus

$[-2, -1] \in$ Root Locus

$[-1, \infty] \notin$ Root Locus

Number of asymptotes:

$n_p - n_z = 3 - 0 = 3$ asymptotes

Starting point of the asymptotes:

$$\sigma_A = \frac{\Sigma pols - \sum zeros}{n_p - n_z} = \frac{-1-2-10}{3-0} = -4.33 \in$$

Angles of the asymptotes:

$$\theta = \frac{(2k+1)180}{3}$$

Calculating the values of θ, at $k = 0$, $k = 1$ and $k = 2$, gives:

$\theta = 60^{\circ}$
$\theta = 180^{\circ}$
$\theta = 300^{\circ}$

Departure or arrival points on the real axis:

Isolating K' from the initial equation gives:

$$K' = -(s+1)(s+2)(s+10) = -s^3 - 13s^2 - 32s - 20$$

Calculating $\frac{dK'}{ds} = 0$ gives:

$$\frac{dK'}{ds} = -3s^2 - 26s - 32 = 0 \rightarrow 3s^2 + 26s + 32 = 0$$

Solving the above equation gives:

$$s = -7.1 \notin \text{Root Locus and } s = -1.5 \in \text{Root Locus}$$

Intersection points with the imaginary axis:

$$1 + \frac{K'}{(s+1)(s+2)(s+10)} = 0 \rightarrow \frac{(s+1)(s+2)(s+10) + K'}{(s+1)(s+2)(s+10)} = 0$$

Rearranging the above equation gives:

$$s^3 + 13s^2 + 32s + 20K' = 0$$

Applying the Routh array gives:

s^3	1	32
s^2	13	20 + K'
s^1	$32 - \frac{20+K'}{13}$	0
s^0	$20 + K'$	

The above array gives:

$$\frac{416 - 20 - K'}{13} = 0 \rightarrow K' = 396$$

This *K'* value must be substituted into the previous row in the Routh array:

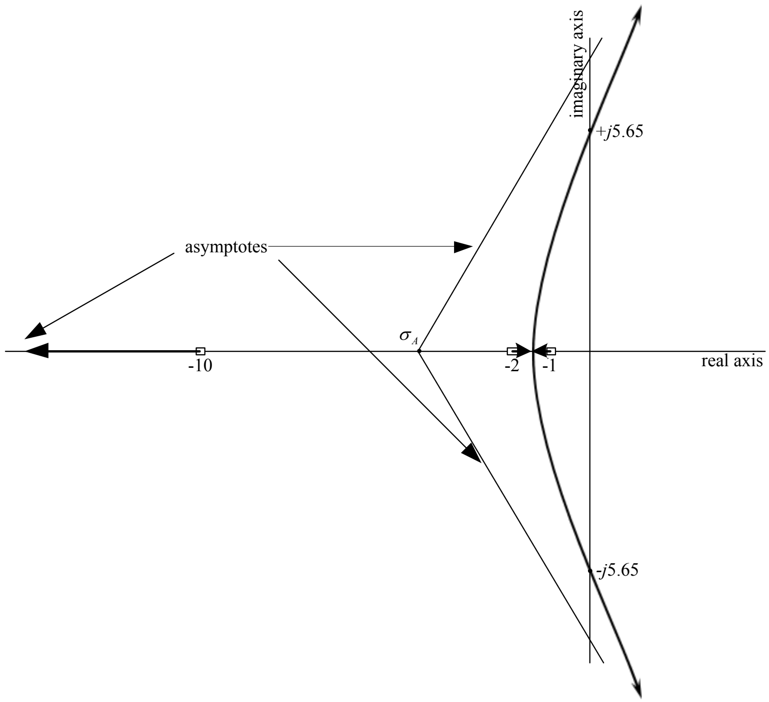
$$13s^2 + 416 = 0 \rightarrow s = \pm j5.65$$

Some of the calculations done to solve this problem are different or have been obviated when solving other problems; this is because, as it has been stated before, the step by

step tracking avoid doing some unnecessary calculations

Calculation of the intersection points with the imaginary axis is highly recommended for solving this problem, as it enables confirmation of the changes in the branches and determination of their stability.

Root locus can be plotted after doing the above steps:



b) To determine the *K* value at any point, the following equation must be applied:

$$K = \frac{\prod \text{distances to poles}}{\prod \text{distances to zeros}}$$

The distances from the statement points to poles must be calculated:

$$d_{(-10)} = \sqrt{(5.65)^2 + (10)^2} = 11.48$$

$$d_{(-2)} = \sqrt{(5.65)^2 + (2)^2} = 6$$

$$d_{(-1)} = \sqrt{(5.65)^2 + (1)^2} = 5.74$$

Substituting the above distance values into the equation to determine *K* gives:

$$K = \frac{(11.48)(6)(5.74)}{1} = 396$$

This value was obtained when the Routh criterion was applied because the *K'* values at the studied points indicate the ones from whose values the system becomes unstable.

The *K* value will be:

$$K = \frac{K'}{50} = \frac{396}{50} = 7.92$$

Problem 2.1.14

Plot the root locus of the below open loop transfer function for positive values of *K* from 0 to ∞.

$$G_{OL}(s) = \frac{K(s+2)}{(s+1)(s^2+6s+10)}$$

Solution

Substituting the above equation into $1+G_{OL}(s)$ gives:

$$1+G_{OL}(s) = 1+\frac{K(s+2)}{(s+1)(s^2+6s+10)}$$

At this point, the different steps that allow plotting the root locus can be applied:

Number of poles and zeros:

Zeros: -2 (1)

Poles: -1, -3+j, -3-j (3)

Number of trajectories:

$\max(n_p, n_z) = \max(3, 1) = 3$

Segments of the real axis that belong to the root locus:

$[-\infty,-3] \notin$ Root Locus

$[-3,-2] \notin$ Root Locus

$[-2,-1] \in$ Root Locus

$[-1,\infty] \notin$ Root Locus

Number of asymptotes:

n_p-n_z = 3-1 = 2 asymptotes

Starting point of the asymptotes:

$$\sigma_A = \frac{\Sigma poles - \sum zeros}{n_p - n_z} = \frac{-1 + (-3 + j) + (-3 - j) - (-2)}{3\text{-}1} = -2.5$$

Angles of the asymptotes:

$$\theta = \frac{(2k+1)\ 180}{2}$$

Calculating the values of θ at $k = 0$ and at $k = 1$ gives:

$\theta = 90\ ^{\mathrm{o}}$

$\theta = 270\ ^{\mathrm{o}}$

Departure or arrival points on the real axis:

Isolating *K'* from the initial equation gives:

$$K = -\frac{(s+1)(s^2+6s+10)}{(s+2)}$$

Calculating $\frac{dK}{ds} = 0$ reveals that the solution does not belong to the real axis.

Intersection points with the imaginary axis:

As observed in the plot the trajectories do not intersect with the imaginary axis.

Angles of departure from the imaginary poles:

$$\sum \varphi_{zeros} - \sum \varphi_{poles} = -180\ ^{\mathrm{o}}$$

To apply the above formula, the angles must be calculated between the positive direction of the real axis and the point studied:

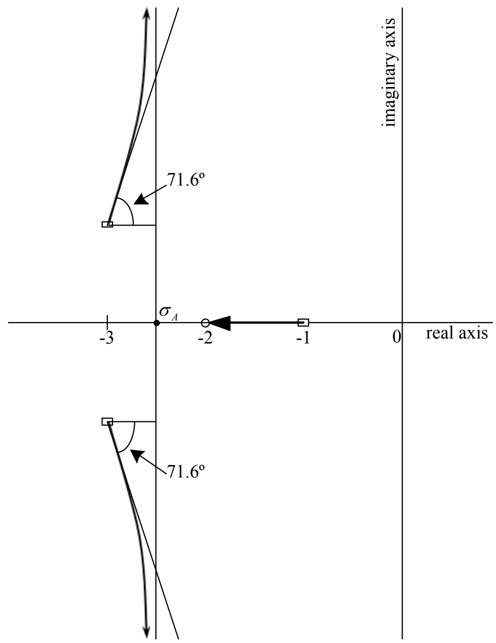
$$135^{\mathrm{o}} - 90^{\mathrm{o}} - 153.4^{\mathrm{o}} - \alpha = -180^{\mathrm{o}} \rightarrow \alpha = 71.6^{\mathrm{o}}$$

For instance, the angle between the pole (-1) and the positive direction of the real axis is calculated as follows:

$$tg^{-1}\left(\frac{1}{2}\right) = 26.56^{\mathrm{o}} \rightarrow 180^{\mathrm{o}} - 26.56^{\mathrm{o}} = 153.4^{\mathrm{o}}$$

Considering that the root locus is symmetric along the real axis and observing the plot, one concludes that no more calculations are required.

Finally, the plot of the root locus will be:



2.2 Study of feedback control systems applying the frequency response

Problem 2.2.1

Consider the transfer function shown below:

$$G(s)=\frac{2}{s^2+7s+1}$$

Calculate its amplitude ratio (AR) and the phase lag (PL).

Solution

Substituting $s \rightarrow j\omega$ into the above transfer function gives:

$$G(j\omega)=\frac{2}{(j\omega)^2+7j\omega+1}=\frac{2}{(1-\omega^2)+7j\omega}$$

Operating the equation in order to eliminate the imaginary number (*j*) from the denominator gives:

$$G(j\omega)=\frac{2}{\left(1-\omega^2\right)+7j\omega}\left(\frac{\left(1-\omega^2\right)-7j\omega}{\left(1-\omega^2\right)-7j\omega}\right)=\frac{2\left[\left(1-\omega^2\right)-7j\omega\right]}{\left(1-\omega^2\right)^2+49\omega^2}=2\left[\frac{\left[\left(1-\omega^2\right)-7j\omega\right]}{\left(1-\omega^2\right)^2+49\omega^2}\right]$$

Finally, separating and rearranging the equation in real and imaginary parts gives:

$$G(j\omega)=\frac{2\left(1-\omega^2\right)}{\left(1-\omega^2\right)^2+49\omega^2}-j\frac{14\omega}{\left(1-\omega^2\right)^2+49\omega^2}$$

At this point, the equation of *AR* can be obtained:

$$AR=\sqrt{\left[\frac{2\left(1-\omega^2\right)}{\left(1-\omega^2\right)^2+49\omega^2}\right]^2+\left[\frac{-14\omega}{\left(1-\omega^2\right)^2+49\omega^2}\right]^2}=\sqrt{\frac{4\left(1-\omega^2\right)^2+196\omega^2}{\left[\left(1-\omega^2\right)^2+49\omega^2\right]^2}}$$

Rearranging the real part gives:

$$AR=2\sqrt{\frac{1}{\left(1-\omega^2\right)^2+49\omega^2}}$$

The *PL* equation is obtained from $G(j\omega)$ by rearranging the imaginary part:

$$PL=tg^{-1}\left\{\frac{-\dfrac{14\omega}{\left(1-\omega^2\right)^2+49\omega^2}}{\dfrac{2\left(1-\omega^2\right)}{\left(1-\omega^2\right)^2+49\omega^2}}\right\}=tg^{-1}\left[-\frac{7\omega}{\left(1-\omega^2\right)}\right]$$

Problem 2.2.2

Consider a feedback control system, whose open loop transfer function is shown below:

$$G(s)=\frac{1}{0.1s^2+0.7s+1}$$

a) Determine the equation in its complex number form (real and imaginary parts).

b) Draw the polar plot and apply the Nyquist stability criterion.

Solution

a) To obtain the complex number form of the transfer function, the $s \to j\omega$ substitution must be performed:

$$G(j\omega) = \frac{1}{0.1(j\omega)^2 + 0.7j\omega + 1}$$

Rearranging gives:

$$G(j\omega) = \frac{1}{\left(1 - 0.1\omega^2 + 0.7j\omega\right)} \frac{\left(1 - 0.1\omega^2 - 0.7j\omega\right)}{\left(1 - 0.1\omega^2 - 0.7j\omega\right)} =$$

$$= \frac{1 - 0.1\omega^2}{0.01\omega^4 + 0.29\omega^2 + 1} - j\frac{0.7\omega}{0.01\omega^4 + 0.29\omega^2 + 1}$$

From the above equation, the values of AR and PL can be inferred:

$$AR = \sqrt{\frac{1 - 0,2\omega^2 + 0,01\omega^4 + 0,49\omega^2}{\left(0,01\omega^4 + 0,29\omega^2 + 1\right)^2}} = \sqrt{\frac{1}{0,01\omega^4 + 0,29\omega^2 + 1}}$$

$$PL = tg^{-1}\left(\frac{-0,7\omega}{1 - 0,1\omega^2}\right) = -tg^{-1}\left(\frac{0,7\omega}{1 - 0,1\omega^2}\right)$$

b) To draw the polar plot, it is enough to calculate some different points assigning different values to ω from 0 to ∞:

ω	AR	PL
0	1	0
0.2	0.994	-8
0.5	0.96	-19.7
1	0.877	-37.87
5	0.263	-113.2
10	0.0877	-142.2
100	≈0	-176
∞	0	-180

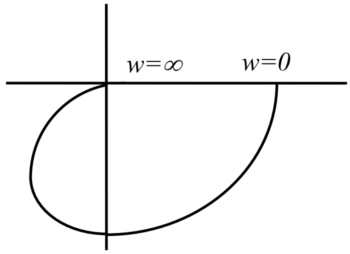
To show how the values have been obtained, the case of $\omega = 5$ can be seen below:

Thus, substituting $\omega = 5$ into the PL equation gives:

$$PL = -tg^{-1}\left(\frac{(0.7)(5)}{1 - (0.1)(25)}\right) = -tg^{-1}\left(\frac{3.5}{-1.5}\right) = -\left[180 - tg^{-1}\left(\frac{3.5}{1.5}\right)\right] = -113.2^{\circ}$$

Proceeding similarly, the other values have been obtained.

To study the stability of the system, the values of the table are plotted in a polar diagram:



According to the plot, the system will always be stable because it never reaches the point (–1, 0).

Problem 2.2.3

Plot the asymptotic Bode diagram of a PI controller.

Solution

Considering that the transfer function of a PI controller is:

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

where K_c and τ_I are the parameters which define the PI controller.

Substituting $s \rightarrow j\omega$, the equation gives:

$$G_c(j\omega) = K_c\left(1+\frac{1}{\tau_I j\omega}\right)$$

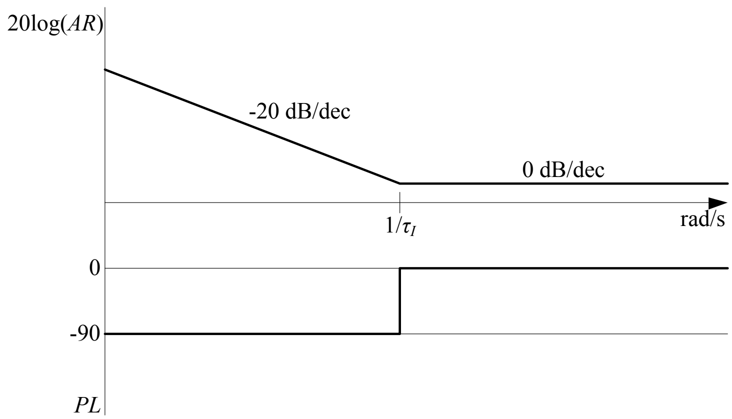
This equation does not allow plotting the asymptotic Bode diagram, because no elementary functions are available. In order to obtain a product of elementary functions whose asymptotic Bode diagrams are known, the above equation must be rearranged:

$$G_c(j\omega) = K_c\left(\frac{\tau_I j\omega+1}{\tau_I j\omega}\right) = \frac{K_c}{\tau_I}\left(\frac{\tau_I j\omega+1}{j\omega}\right)$$

Forming the table to find the asymptotic values of 20log(AR) and PL from the known expressions of $(\tau_I j\omega+1)$ and $\left(\frac{1}{j\omega}\right)$; the values of $\frac{K_c}{\tau_I}$ do not affect the shape of the Bode diagram.

	0^+ – $1/\tau_I$	$1/\tau_I$ – ∞
$(\tau_I j\omega+1)$	0	+1
$\frac{1}{j\omega}$	-1	-1
20log(AR); PL	-1	0

Due to the fact that the values of 20log(AR) and PL are the same in both cases only a table is necessary. Plotting the values of the above table gives:



Problem 2.2.4

Consider the following transfer function corresponding with a PI controller:

$$G_c(s) = 2\left(1+\frac{1}{0.2s}\right)$$

Plot its asymptotic Bode diagram.

Solution

Substituting $s \to j\omega$ gives:

$$G_c\left(j\omega\right) = 2\left(1+\frac{1}{0.2j\omega}\right)$$

Operating the above equation gives:

$$G_c\left(j\omega\right) = 2\left(\frac{0.2j\omega+1}{0.2j\omega}\right) = 10(0.2j\omega+1)\frac{1}{j\omega}$$

The above equation corresponds to the block structure that allows plotting its asymptotic Bode diagram. Thus, it is possible to form the table in order to calculate the asymptotic values of 20log*(AR)* and *PL*:

	0^+ – 5	5 – ∞
$0.20j\omega + 1$	0	+1
$\dfrac{1}{j\omega}$	-1	-1
20log(*AR*); *PL*	-1	0

Also, some values of 20log*(AR)* and *PL* can be calculated from the equations below:

$$20\log(AR) = 20\log 10 + 20\log\sqrt{(0.2)^2\omega^2 + 1} - 20\log\omega$$

$$PL = tg^{-1}\left(\frac{0}{10}\right) + tg^{-1}\left(\frac{0.2\omega}{1}\right) - tg^{-1}\left(\frac{\omega}{0}\right)$$

Some values will be calculated to plot the asymptotic Bode diagram:

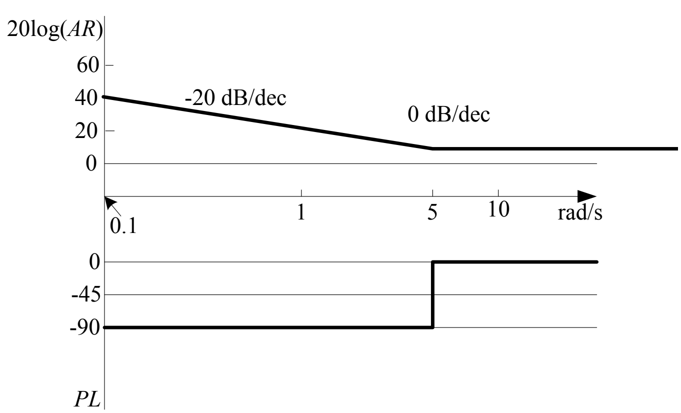
At $\omega = 0.1$:

$$20\log(RA) \approx 20 + 0 + 20 = 40\ dB$$

At $\omega = 5$:

$$PL = 0 + 45 - 90 = -45°$$

At this point, the plot can be drawn:



Problem 2.2.5

Shown below is the transfer function of an open loop, obtained as a product of the elements transfer functions which make up a feedback system:

$$G_{OL}(s)=\frac{5e^{-s}}{(2s+1)(s+1)}$$

Determine the system stability by applying the Bode criterion and, if the system is stable, calculate the amplitude ratio and the phase lag.

Solution

Substituting $s \rightarrow j\omega$ in the above equation gives:

$$G_{OL}(j\omega)=5e^{-j\omega}\frac{1}{(2j\omega+1)}\frac{1}{(j\omega+1)}$$

From $G_{OL}(j\omega)$, the table may be formed to find the asymptotic expression of the Bode diagram, although, in this example it is not necessary for reaching the solution.

The value of 20log(AR) is:

$$20\log(AR)=20\log 5-20\log\sqrt{(2\omega)^2+1}-20\log\sqrt{\omega^2+1}$$

By giving several values to ω for obtaining the corresponding values of 20log(AR), and knowing the value of ω, for which the value of 20log(AR) is zero, we have:

for ω = 0.1 $\Rightarrow 20\log(AR)=13.98-0.17-0.043=13.767$

for ω = 1.5 $\Rightarrow 20\log(AR)=13.98-10-5.19=-1.21$

The previous results show that the value of ω may be found between 0.1 and 1.5; at this point the target value should be optimized:

for ω = 1.4 $\Rightarrow$ $20\log(AR)=13.98-9.46-4.72=-0.2$

for ω = 1.38 $\Rightarrow 20\log(AR)=13.98-9.35-4.63=0$

Calculating PL gives:

$$PL=tg^{-1}\left(\frac{0}{5}\right)-tg^{-1}\left(\frac{2\omega}{1}\right)-tg^{-1}\left(\frac{\omega}{1}\right)-(57.3)\omega=-tg^{-1}(2\omega)-tg^{-1}(\omega)-(57.3)\omega$$

Calculating the value of PL when ω = 1.38 rad/s gives:

$$PL=-70.08-54.07-79.07=-203.22^{\circ}$$

By observing the results, it can be seen that PL intersects -180° before AR intersects 0 dB. Thus, it can be stated that the control system is unstable, and it is not necessary to calculate either the phase margin or the gain margin.

Problem 2.2.6

Calculate the value of K to obtain a phase lag of 45º in a feedback control system whose transfer function in open loop is shown below:

$$G_{OL}(s)=\frac{K}{(0.5s+1)(0.05s+1)(0.005s+1)}$$

Consider that the system must be stable.

Solution

To obtain a phase lag of 45º, given that it is a stable system, the PL must be:

$$PL = -180° + 45° = -135°$$

Substituting $s \rightarrow j\omega$ gives:

$$G_{OL}(j\omega)=\frac{K}{(0.5j\omega+1)(0.05j\omega+1)(0.005j\omega+1)}$$

Thus, the overall expression of PL will be:

$$PL=-tg^{-1}(0.5\omega)-tg^{-1}(0.05\omega)-tg^{-1}(0.005\omega)$$

At this point, several values of ω are assigned in order to find the corresponding value to PL = -135º:

for $\omega = 10 \Rightarrow PL = -78.69-25.56-2.86=-108.11^{o}$

for $\omega = 20 \Rightarrow PL = -84.29-45-5.71=-135^{o}$

and then the overall expression 20log(RA) must be found:

$$20\log(AR)=20\log K-20\log\sqrt{(0.5\omega)^2+1}-20\log\sqrt{(0.05\omega)^2+1}-$$
$$20\log\sqrt{(0.005\omega)^2+1}$$

Substitute ω = 20 in the above equation, assuming that it is the value that makes 20log(RA) = 0:

$$20\log K-20\log\sqrt{(0.5x20)^2+1}-20\log\sqrt{(0.05x20)^2+1}-20\log\sqrt{(0.005x20)^2+1}=0$$

Operating and seeking the value gives:

$$20\log K-20.04-3.01-0.04=0\rightarrow 20\log K=23.09\rightarrow K=14.27$$

Problem 2.2.7

The transfer function in the open loop of a feedback control system corresponds to the equation shown below:

$$G_{OL}(s)=\frac{1.000(s+3)}{s(s+12)(s+50)}$$

a) Plot the asymptotic Bode diagram (gain and phase).

b) Discuss the system stability according to the Bode criterion. Calculate the maximum value of *AR* which allows the system to be stable.

c) If an element with the transfer function 1/s is added to the control system, plot the new asymptotic Bode diagram (gain and phase).

d) Study the system stability including the new element according to the Bode criterion, justify the study in a numerical way. If the result gives a stable system, calculate the phase margin and the gain margin.

Solution

a) To plot the Bode diagram, the transfer function must be found in open loop as a product of functions whose Bode diagrams are previously known:

$$\mathrm{G_{OL}(s)}=\frac{1000(s+3)}{s(s+12)(s+50)}=\frac{1000(3)\left(\frac{s}{3}+1\right)}{12(50)s\left(\frac{s}{12}+1\right)\left(\frac{s}{50}+1\right)}=\frac{5\left(\frac{s}{3}+1\right)}{s\left(\frac{s}{12}+1\right)\left(\frac{s}{50}+1\right)}$$

Substituting $s \rightarrow j\omega$ gives:

$$\mathrm{G_{OL}}(j\omega)=\frac{5\left(\frac{j\omega}{3}+1\right)}{j\omega\left(\frac{j\omega}{12}+1\right)\left(\frac{j\omega}{50}+1\right)}$$

At this point, it is possible to form the table to plot the asymptotic Bode diagram:

	0^+ – 3	3 – 12	12 – 50	50 – ∞
$\dfrac{1}{j\omega}$	-1	-1	-1	-1
$\left(\dfrac{j\omega}{3}+1\right)$	0	+1	+1	+1
$\left(\dfrac{j\omega}{12}+1\right)^{-1}$	0	0	-1	-1
$\left(\dfrac{j\omega}{50}+1\right)^{-1}$	0	0	0	-1
20log(*RA*); *PL*	-1	0	-1	-2

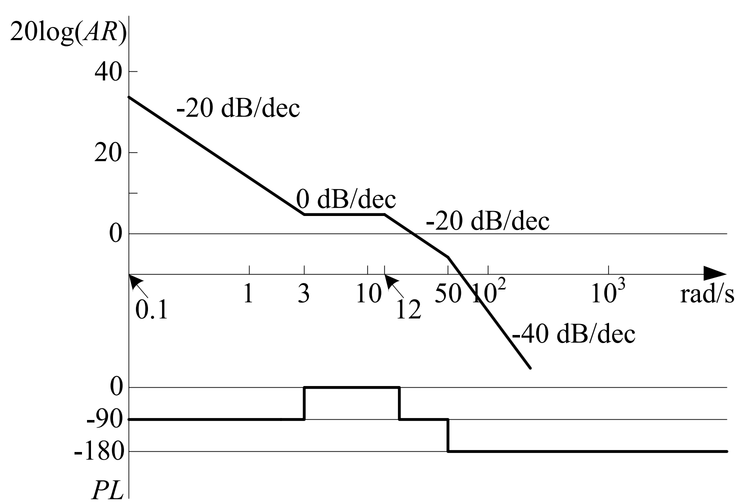
Also, the expression of 20log(*RA*) will be:

$$20\log(AR) =$$

$$= 20\log 5 + 20\log\sqrt{\left(\frac{\omega}{3}\right)^2+1} - 20\log\omega - 20\log\sqrt{\left(\frac{\omega}{12}\right)^2+1} - 20\log\sqrt{\left(\frac{\omega}{50}\right)^2+1}$$

for $\omega = 0.1 \;\Rightarrow\; 20\log(RA) \approx 20 + 14 = 34$

Plotting the results gives:



b) It can be seen from the above graphic that the PL never intersects the straight line corresponding to -180°; therefore, the system will always be stable.

c) Adding a new element $(\frac{1}{j\omega})$ to the above system and forming the table in order to plot the new Bode asymptotic diagram:

	0^+	3	12	50 ∞
$\frac{1}{j\omega}$	-1	-1	-1	-1
$\left(\frac{j\omega}{3}+1\right)$	0	+1	+1	+1
$\left(\frac{j\omega}{12}+1\right)^{-1}$	0	0	-1	-1
$\left(\frac{j\omega}{50}+1\right)^{-1}$	0	0	0	-1
$\frac{1}{j\omega}$	-1	-1	-1	-1
20log(*RA*); *PL*	-2	-1	-2	-3

It can be seen that the effect of adding a term like 1/s makes all the values increase by one unit, both for plotting the 20log(*AR*) as well as the PL.

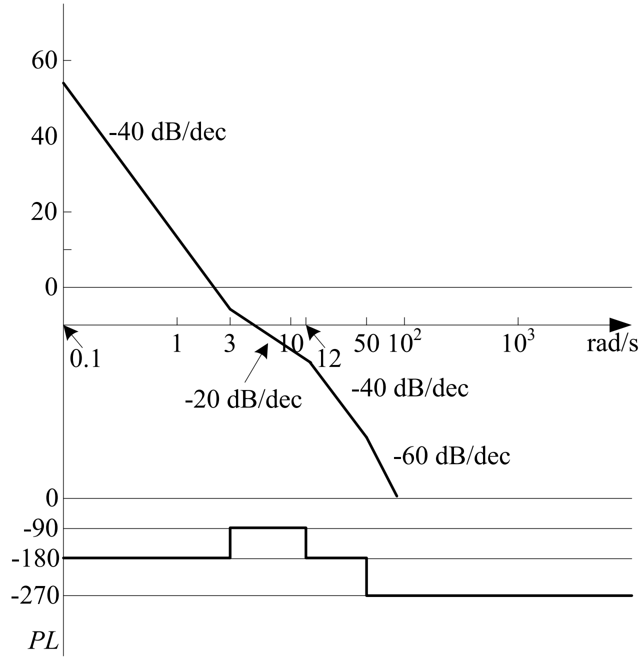
The expression of 20log(*AR*) will be:

$$20\log AR =$$
$$= 20\log 5 + 20\log\sqrt{\left(\frac{\omega}{3}\right)^2+1} - (2)(20)\log\omega - 20\log\sqrt{\left(\frac{\omega}{12}\right)^2+1} - 20\log\sqrt{\left(\frac{\omega}{50}\right)^2+1}$$

To plot the asymptotic Bode diagram, the intersection point with the y-axis must be calculated:

for $\omega = 0.1 \;\Rightarrow\; 20\log AR \approx 14 + 40 = 54$

Plotting the diagram gives:



d) To calculate the phase margin from the Bode diagram it is necessary to find the value of ω that makes 20log(*RA*) = 0:

$$20\log(AR) =$$

$$= 20\log 5 + 20\log\sqrt{\left(\frac{\omega}{3}\right)^2 + 1} - 2x20\log\omega - 20\log\sqrt{\left(\frac{\omega}{12}\right)^2 + 1} - 20\log\sqrt{\left(\frac{\omega}{50}\right)^2 + 1}$$

for ω = 2 $\Rightarrow 20\log(AR) = 3.48$

for ω = 2.5 $\Rightarrow 20\log(AR) = 0.21$

for ω = 2.6 $\Rightarrow 20\log(AR) = -0.37$

for ω = 2.55 $\Rightarrow 20\log(AR) = -0.09$

for ω=2.54 $\Rightarrow 20\log(AR) = -0.03$

Calculating the value of PL when $\omega = 2.54$ gives:

$$PL = tg^{-1}\left(\frac{\omega}{3}\right) - 2tg^{-1}\left(\frac{\omega}{0}\right) - tg^{-1}\left(\frac{\omega}{12}\right) - tg^{-1}\left(\frac{\omega}{50}\right) =$$

$$tg^{-1}\left(\frac{\omega}{3}\right) - 180 - tg^{-1}\left(\frac{\omega}{12}\right) - tg^{-1}\left(\frac{\omega}{50}\right)$$

for $\omega = 2.54 \Rightarrow PL = 40.25 - 180 - 11.95 - 2.91 = -154.61^o$

The value of the phase margin is 25.39º above -180º; thus, the system is stable.

It is necessary to find the value of ω that makes PL = -180º in order to calculate the margin gain:

for ω= 15 $\Rightarrow PL$ = -169.34º

for ω= 25 $\Rightarrow PL$ = -187.76º

for ω= 20 $\Rightarrow PL$ = -179.34º

for ω= 20.5 $\Rightarrow PL$ = -180.23º

for ω= 20.4 $\Rightarrow PL$ = -180.07º

The value of PL for ω=20.4 is close to -180º; thus, the gain margin will be:

If ω= 20.4 $\Rightarrow 20\log(AR) = 14 + 16.74 - 52.38 - 5.9 - 0.67 = -28.21$

Problem 2.2.8

Consider the transfer function in open loop:

$$G_{OL}(s) = \frac{K}{s(0.01s+1)(0.1s+1)}$$

a) Calculate the K value that makes the phase margin 45º.

b) Calculate the K value that makes the gain margin 5 dB.

Remark: The system must be considered as stable in both sections.

Solution

Substituting $s \to j\omega \Rightarrow G_{OL}(j\omega) = \dfrac{K}{j\omega(0.01j\omega+1)(0.1j\omega+1)}$

The expression of PL is:

$$PL = tg^{-1}\left(\frac{0}{K}\right) - tg^{-1}\left(\frac{\omega}{0}\right) - tg^{-1}\left(\frac{\omega}{100}\right) - tg^{-1}\left(\frac{\omega}{10}\right) = -90 - tg^{-1}\left(\frac{\omega}{100}\right) - tg^{-1}\left(\frac{\omega}{10}\right)$$

a) If the phase margin is 45º $\Rightarrow PL = -180 + 45 = -135^o$ (45º above -180^o). The value of $20\log(RA)$ = 0 when seeking the phase margin. At this point the value of ω that makes PL = -135º must be found by iteration:

for $\omega = 10 \Rightarrow PL$ = -140.71º

for $\omega = 5 \Rightarrow PL$ = -119.42º

for $\omega = 8 \Rightarrow PL$ = -133.23º

for $\omega = 8.5 \Rightarrow PL$ = -135.22º

for $\omega = 8.45 \Rightarrow DF$ = -135.02º

Substituting $\omega = 8.45$ in the equation 20log(RA) = 0, and calculating the K value gives:

$$20\log K - 18.54 - 0.030 - 2.34 = 0 \Rightarrow 20\log K = 20.91 \Rightarrow K = 11.1$$

b) When the value of the phase margin is -5 dB (the system is stable), the PL value is -180º; for this reason, (and by taking advantage of the previous results), the value of ω which makes PL =-180 must be found by iteration.

for $\omega = 20 \Rightarrow PL$ = -164.7º

for $\omega = 30 \Rightarrow PL$ = -178.26º

for $\omega = 31 \Rightarrow PL$ = -179.34º

for $\omega = 31.7 \Rightarrow PL$ = -180.07º

Substituting $\omega = 31.7$ in the expression 20log(AR) = -5 gives:

$$20\log K = 30.02 + 0.415 + 10.42 - 5 \Rightarrow K = 62.05$$

Problem 2.2.9

The following transfer function in open loop corresponds to a feedback control system:

$$G_{OL}(s) = \frac{0.6}{s(s+1)(4s+1)}$$

a) Plot the asymptotic diagram of the transfer function in open loop.

b) Study the stability of the system in closed loop. Calculate the phase margin and the gain margin if the system is stable.

Solution

a) Firstly, substitute $s \to j\omega \Rightarrow G_{OL}(j\omega) = \dfrac{0.6}{j\omega(j\omega+1)(4j\omega+1)}$

From the above equation, the expressions of 20log(AR) and PL can be drawn below:

$$20\log(AR) = 20\log(0.6) - 20\log\omega - 20\log\sqrt{\omega^2+1} - 20\log\sqrt{16\omega^2+1}$$

$$PL = tg^{-1}\left(\frac{0}{0,6}\right) - tg^{-1}\left(\frac{\omega}{0}\right) - tg^{-1}(\omega) - tg^{-1}(4\omega) = -90 - tg^{-1}(\omega) - tg^{-1}(4\omega)$$

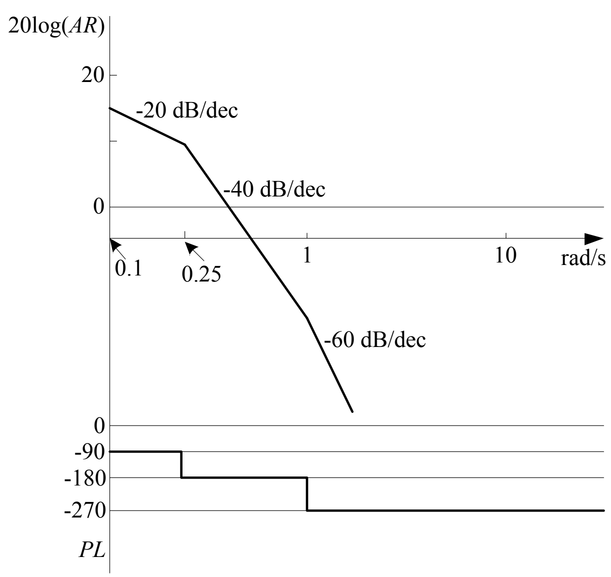
From the above expression of $G_{OL}(j\omega)$, the following table is formed:

	0^+ – 0.25	0.25 – 1	1 – ∞
$(j\omega)^{-1}$	-1	-1	-1
$(j\omega+1)^{-1}$	0	0	-1
$(4j\omega+1)^{-1}$	0	-1	-1
20log(RA); PL	-1	-2	-3

The value of 20log(AR) for $\omega = 0.1$ to plot the Bode diagram:

for $\omega = 0.1 \Rightarrow 20\log(AR) = -4.44 + 20 - 0.043 - 0.64 = 14.877$

The asymptotic diagram is shown below:



b) By observing the above diagram, we can see that it is not possible to extract reliable results regarding the stability of the system, because it is just an approach to the real values. The first values for making an iteration can be approximately extracted from the Bode diagram.

For $\omega = 1 \Rightarrow PL = -210.96^{\circ}$; this points out that lesser values must be tried.

Testing $\omega = 0.5 \Rightarrow PL = -180^{\circ}$, which matches with the sought value.

Substituting $\omega = 0.5$ in the expression 20log(RA) gives:

$$\omega = 0.5 \Rightarrow 20\log(AR) = -6.4$$

The sign of 20log(AR) points out that the system is stable.

Also, it is possible to find the value of ω that makes 20log(RA) = 0, and the phase margin to be calculated:

for $\omega = 0.4 \Rightarrow 20\log(AR) = -2.63$

for $\omega = 0.3 \Rightarrow 20\log(AR) = 1.77$

for $\omega = 0.33 \Rightarrow 20\log(AR) = 0.36$

for $\omega = 0.34 \Rightarrow 20\log(AR) = -0.085$

Now, the value of PL is calculated:

for $\omega = 0.34 \Rightarrow DF = -90 - 18.78 - 53.67 = -162.45^{\circ}$

The value of phase margin is 17,55º above –180º; so, the system is stable.

Problem 2.2.10

A feedback control system is formed by the following transfer functions:

$$G_c(s) = 4\left(1 + \frac{1}{0.25s}\right)$$

$$G_f(s) = \frac{10}{(0.1s+1)}$$

$$G_p(s) = \frac{5e^{-t_d s}}{(2s+1)(s+1)}$$

$$G_m = \frac{2}{(0.5s+1)}$$

Considering $t_d = 0$:

a) Plot the Bode diagram in open loop.

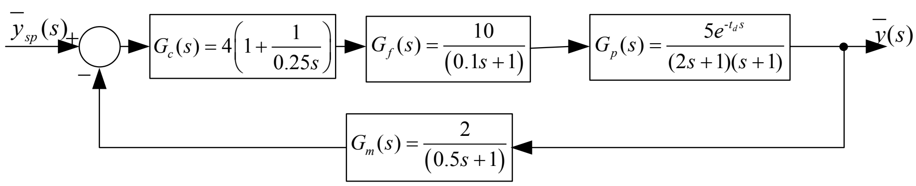
b) Study the stability of the system and calculate the phase margin and the gain margin if the system is stable.

Considering $t_d \neq 0$:

c) Briefly discuss its influence on the system.

Solution

According to the statement (there are no data about disturbances) forming the block diagram of the system gives:



a) Assuming $t_d = 0$, the transfer function in open loop of the system will be:

$$G_{OL}(s) = 4\left(1 + \frac{1}{0.25s}\right)\frac{10}{(0.1s+1)}\frac{5}{(2s+1)(s+1)}\frac{2}{(0.5s+1)}$$

Operating and simplifying gives:

$$G_{OL}(s) = 400\left(\frac{1+0.25s}{0.25s}\right)\frac{1}{(0.1s+1)}\frac{1}{(2s+1)}\frac{1}{(s+1)}\frac{1}{(0.5s+1)}$$

Substituting $s \rightarrow j\omega$ gives:

$$G_{OL}(j\omega) = 1600\frac{1}{(j\omega)}(1+0.25j)\frac{1}{(0.1j\omega+1)}\frac{1}{(2j\omega+1)}\frac{1}{(j\omega+1)}\frac{1}{(0.5j\omega+1)}$$

The expression of 20log(AR) will be:

$$20\log(AR) = 20\log(1600) - 20\log(\omega) + 20\log\sqrt{1+(0.25\omega)^2} - 20\log\sqrt{(0.1\omega)^2+1} -$$
$$20\log\sqrt{(2\omega)^2+1} - 20\log\sqrt{\omega^2+1} - 20\log\sqrt{(0.5\omega)^2+1}$$

The expression of PL will be:

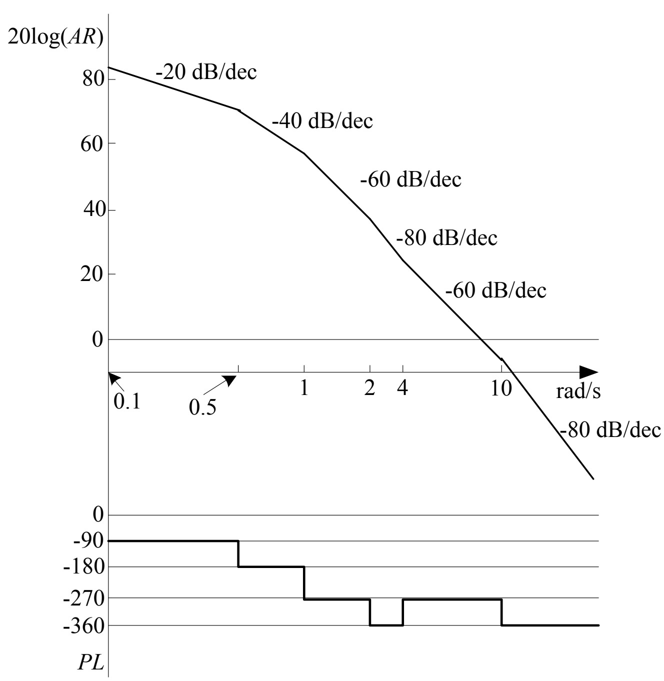
$$PL = tg^{-1}(0) + tg^{-1}(0.25\omega) - tg^{-1}\left(\frac{\omega}{0}\right) - tg^{-1}(0.1\omega) - tg^{-1}(2\omega) - tg^{-1}(\omega) - tg^{-1}(0.5\omega) =$$
$$= tg^{-1}(0.25\omega) - 90^{\circ} - tg^{-1}(0.1\omega) - tg^{-1}(2\omega) - tg^{-1}(\omega) - tg^{-1}(0.5\omega)$$

From the above expression of $G_{OL}(j\omega)$, the below table of AR and PL is formed:

	0^+	½	1	2	4	10	∞
$(j\omega)^{-1}$	-1	-1	-1	-1	-1	-1	
$(1+0.25j\omega)$	0	0	0	0	+1	+1	
$(1+0.1j\omega)^{-1}$	0	0	0	0	0	-1	
$(1+2j\omega)^{-1}$	0	-1	-1	-1	-1	-1	
$(1+j\omega)^{-1}$	0	0	-1	-1	-1	-1	
$(1+0.5j\omega)^{-1}$	0	0	0	-1	-1	-1	
20log(RA); PL	-1	-2	-3	-4	-3	-4	

For $\omega = 0.01 \Rightarrow 20\log(RA) \approx 64 + 20 = 84\ dB$

Considering the above results, the asymptotic Bode diagram is plotted:



b) From the above Bode diagram in open loop it is possible to calculate the value of ω which makes $20\log(AR) = 0$:

for $\omega = 3 \;\Rightarrow\; 20\log(AR) = 25.22$
for $\omega = 5 \;\Rightarrow\; 20\log(AR) = 10.28$
for $\omega = 7 \;\Rightarrow\; 20\log(AR) = 0.31$
for $\omega = 7{,}1 \;\Rightarrow\; 20\log(AR) = -0.03$

for $\omega = 7.1 \Rightarrow\rightarrow\; PL = -307.06^o \rightarrow$ This result points out that the system is unstable.

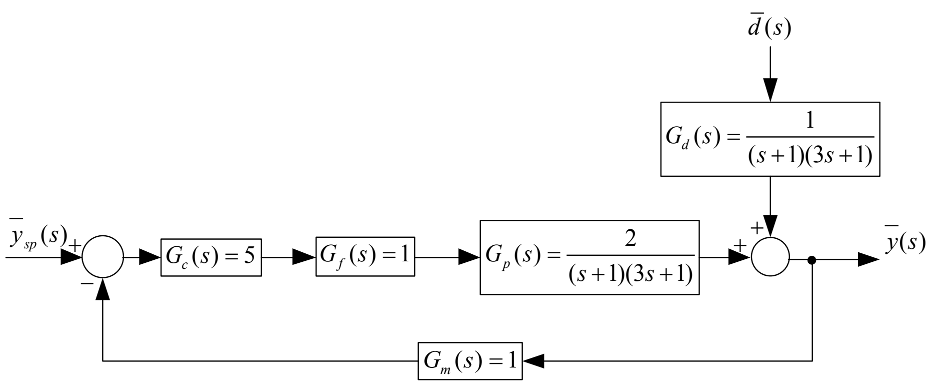
c) If the value of $t_d \neq 0$, $20\log(AR)$ will not be affected; however, the value of PL will be more negative because the term $-\omega t_d\,(57.3)^{\circ}$ has been added.

If the system was stable, the effect of $t_d \neq 0$ would make the system unstable. Regardless, due to the system being unstable, the value of ω which makes PL = -180º decreases and the system is more unstable.

2.3 Applying several techniques to the study of feedback systems

Problem 2.3.1

The figure below shows the block diagram of a feedback control system:



a) Determine the overall expression for the closed loop response.

b) Considering only the block $G_p(s)$, calculate its time response when a unit step input is applied.

c) Calculate the *offset* of the system if the *set point* increases suddenly by two units, assuming that the disturbance does not vary.

d) Determine the stability of the system by applying the Routh-Hurwitz criterion.

e) Calculate the range of values that makes the system stable when $G_c(s) = K_c$.

f) Point out two different ways to avoid *offset* due to the proportional controller, $G_c(s)$, when the *set point* changes.

g) Draw the polar plot of the control system in open loop.

h) Determine the stability of the system from the above plot by applying the Nyquist stability criterion.

i) Calculate the transfer function of the Process Reaction Curve (PRC) in accordance with the Cohen and Coon method.

Solution

a) The overall expression of the closed loop response will be:

$$\bar{y}(s) = \frac{G_c(s)G_p(s)G_f(s)}{1+G_c(s)G_p(s)G_f(s)G_m}\bar{y}_{sp}(s) + \frac{G_d(s)}{1+G_c(s)G_p(s)G_f(s)G_m(s)}\bar{d}(s)$$

Substituting the values from the problem statement gives:

$$\bar{y}(s) = \frac{5(1)\dfrac{2}{(s+1)(3s+1)}}{1+5(1)\dfrac{2}{(s+1)(3s+1)}(1)}\bar{y}_{sp}(s) + \frac{\dfrac{1}{(s+1)(3s+1)}}{1+5(1)\dfrac{2}{(s+1)(3s+1)}(1)}\bar{d}(s)$$

Operating and simplifying gives:

$$\bar{y}(s) = \frac{10}{(s+1)(3s+1)+10}\bar{y}_{sp}(s) + \frac{1}{(s+1)(3s+1)+10}\bar{d}(s)$$

b) Applying a unit step input to $G_f(s)$ gives:

$$\bar{f}(s) = \frac{1}{s} \longrightarrow \boxed{G_p(s) = \frac{2}{(s+1)(3s+1)}} \longrightarrow \bar{y}(s)$$

The output expression will be:

$$\bar{y}(s) = G_p(s)\bar{f}(s) = \frac{2}{(s+1)(3s+1)}\left(\frac{1}{s}\right)$$

In order to find the inverse Laplace transform, a partial-fraction expansion must be performed:

$$\frac{2}{s(s+1)(3+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{3s+1}$$

$$2 = A(s+1)(3s+1) + Bs(3s+1) + Cs(s+1)$$

$$2 = A(3s^2 + 4s + 1) + B(3s^2 + s) + C(s^2 + s)$$

Matching coefficients gives:

$$\begin{cases} 0 = 3A + 3B + C \\ 0 = 4A + B + C \\ 2 = A \end{cases}$$

Solving the system gives:

$$A = 2$$

$$B = 1$$

$$C = -9$$

So, the simplified expression for calculating the inverse Laplace transform will be:

$$\overline{y}(s) = \frac{2}{s} + \frac{1}{s+1} - \frac{9}{3s+1}$$

The above expression enables calculation of the time response almost immediately:

$$y(t) = \mathcal{L}^{-1}[\overline{y}(s)] = \mathcal{L}^{-1}\left[\frac{2}{s} + \frac{1}{s+1} + \frac{-9}{3s+1}\right] = 2\,\mathcal{L}^{-1}\left[\frac{1}{s}\right] + \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] - \frac{9}{3}\,\mathcal{L}^{-1}\left[\frac{1}{s+\frac{1}{3}}\right]$$

The time response will be:

$$y(t) = 2 + e^{-t} - 3e^{-\frac{t}{3}}$$

c) According to the statement, the value of the set point increases by two units; this implies $\overline{y}_{sp}(s) = \frac{2}{s}$, and applying the overall response formula gives:

$$\overline{y}(s) = \frac{10}{(s+1)(3s+1)+10}\overline{y}_{sp}(s) + \frac{1}{(s+1)(3s+1)+10}\overline{d}(s)$$

Substituting gives:

$$\overline{y}(s) = \frac{10}{(s+1)(3s+1)+10}\left(\frac{2}{s}\right)$$

applying the Final-Value Theorem gives:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0}\left[s\left(\frac{10}{(s+1)(3s+1)+10}\right)\left(\frac{2}{s}\right)\right] = \frac{20}{11}$$

By applying the definition of offset, the difference between the new set point value and the final response value gives:

$$Offset = 2 - \frac{20}{11} = \frac{2}{11}$$

d) The characteristic equation of the system is:

$$1 + G_p(s)G_c(s)G_f(s)G_m(s) = 0$$

Substituting the values of the block diagram gives:

$$1 + 5(1)\frac{2}{(s+1)(3s+1)}(1) = 0$$

Operating gives:

$$(s+1)(3s+1) + 10 = 0$$

Rearranging the above equation gives:

$$3s^2 + 4s + 11 = 0$$

Applying the Routh criterion gives:

s^2	3	11
s^1	4	0
s^0	11	

According to the Routh array, the system is always stable because there are no sign changes in the first column and its stability does not depend on any of its parameters.

e) Considering $G_c(s) = K_c$ gives:

$$1 + K_c\frac{2}{(s+1)(3s+1)} = 0$$

$$(s+1)(3s+1) + 2K_c = 0$$

Operating the characteristic equation is obtained:

$$3s^2 + 4s + 1 + 2K_c = 0$$

Forming the Routh array gives:

s^2	3	$1 + 2K_c$
s^1	4	0
s^0	$1 + 2K_c$	

The system will be stable if:

$$1 + 2K_c > 0$$

$$K_c > -\frac{1}{2}$$

f) To avoid offset due to the proportional controller, two solutions can be chosen:

f.1 Adding to the system a pure capacity element which has a transfer function like 1/s.

f.2 Increasing the values of K_c to achieve offset values close to zero, although the offset will not be avoided.

g) From the transfer function in open loop and substituting the values shown in the block diagram gives:

$$G_{OL}(s) = 5(1)\frac{2}{(s+1)(3s+1)}(1) = \frac{10}{(s+1)(3s+1)}$$

Substituting $s \to j\omega$ gives:

$$G_{OL}(s) = \frac{10}{(j\omega+1)(3j\omega+1)}$$

Calculating the *AR* and *PL* expressions gives:

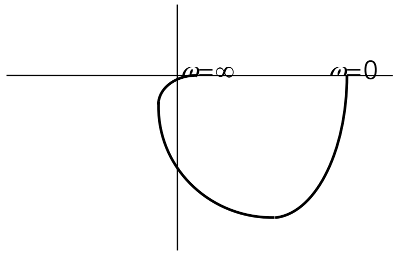
$$RA = \frac{10}{\sqrt{1+\omega^2}\sqrt{1+(3\omega)^2}}$$

$$DF = tg^{-1}(0) - tg^{-1}(\omega) - tg^{-1}(3\omega)$$

To plot the Nyquist diagram, several values of ω from 0 to ∞ are assigned and shown on the table below:

ω	AR	PL
0	10	0
0.25	7.76	-50.86º
0.5	5	-82.9º
1	2.24	-116.56°
10	0.03	-172.37°
∞	0	-180°

Plotting the above table gives:



h) In the above graphic, it can be seen that the plot does not encircle the point (-1, 0) when its values vary from -∞ to +∞; this implies that the system is stable.

i) The transfer function will be:

$$G_{PRC}(s) = \frac{\overline{y}_m(s)}{\overline{c}(s)} = G_f(s)G_p(s)G_m(s) = (1)\frac{2}{(s+1)(3s+1)}(1)$$

Operating the above equation gives:

$$G_{PRC}(s) = \frac{2}{(s+1)(3s+1)}$$

Problem 2.3.2

A feedback system control is used to maintain the temperature in a tank. The transfer functions of the elements are shown below:

$$G_p(s) = \frac{10}{(s+1)}$$

Measuring device:

$$G_m(s) = \frac{0.95}{(0.01s+1)}$$

Proportional controller:

$$G_c(s) = 10$$

Final control element:

$$G_f(s) = 1$$

a) Calculate the expression of the liquid temperature in function of the desired temperature while considering one disturbance.

b) Plot the asymptotic Bode diagram of the transfer function in open loop.

c) Determine the system's stability by using the simplified Bode criterion and calculate the phase and the gain margins.

d) Calculate the values of K_c (the constant characterizing the proportional controller), which stabilize the system.

e) Using the Bode diagram, determine how the system's stability would be affected when the time delay $e^{-0,1s}$ is included.

f) Discuss whether the proportional controller is the most appropriate kind of controller for the given process. If not, justify which would be the most suitable controller.

g) Calculate and plot the time output if the controller transfer function is $G_c(s) = K_c\left(1+\dfrac{1}{\tau_I s}\right)$ and the error is 1/s.

Solution

a) The equation below corresponds to the closed-loop response:

$$\bar{y}(s) = \frac{G_c(s)G_f(s)G_p}{1+G_c(s)G_f(s)G_p(s)G_m(s)}\bar{y}_{sp}(s) + \frac{G_d(s)}{1+G_c(s)G_f(s)G_p(s)G_m(s)}\bar{d}(s)$$

Substituting the values of the transfer functions gives:

$$\bar{y}(s) = \frac{10\left(\dfrac{0.95}{0.01s+1}\right)\left(\dfrac{10}{s+1}\right)}{1+10\left(\dfrac{0.95}{0.01s+1}\right)\left(\dfrac{10}{s+1}\right)(1)}\bar{y}_{sp}(s) + \frac{G_d(s)}{1+10\left(\dfrac{0.95}{0.01s+1}\right)\left(\dfrac{10}{s+1}\right)(1)}\bar{d}(s)$$

Simplifying the above expression gives:

$$\bar{y}(s) = \frac{95}{(0.01s+1)(s+1)+95}\bar{y}_{sp}(s) + \frac{(0.01s+1)(s+1)G_d(s)}{(0.01s+1)(s+1)+95}\bar{d}(s)$$

b) Substituting to obtain the expression of $G_{OL}(s)$ gives:

$$G_{OL}(s) = G_c(s)G_f(s)G_p(s)G_m(s) = 10(1)\frac{10}{(s+1)}\frac{0.95}{(0.01s+1)} = \frac{95}{(s+1)(0.01s+1)}$$

Substituting $s \to j\omega$ gives:

$$G_{OL}(j\omega) = \frac{95}{(j\omega+1)(0.01j\omega+1)}$$

The equations for obtaining 20log(AR) and PL are:

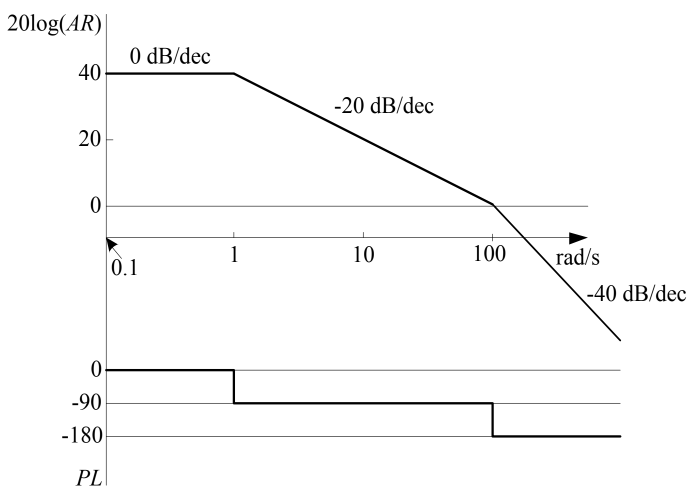
$$20\log(AR) = 20\log 95 - 20\log\sqrt{(j\omega)^2+1} - 20\log\sqrt{(0.01\omega)^2+1}$$

$$DF = tg^{-1}\left(\frac{0}{95}\right) - tg^{-1}\left(\frac{\omega}{1}\right) - tg^{-1}\left(\frac{0.01}{1}\right) = -tg^{-1}(\omega) - tg^{-1}(0.01\omega)$$

Forming the table to plot the asymptotic Bode diagram (from elementary functions whose Bode diagram is known) gives:

	0^+ – 1	1 – 100	100 – ∞
95	0	0	0
$(j\omega+1)^{-1}$	0	-1	-1
$\left(\frac{j\omega}{100}+1\right)^{-1}$	0	0	-1
20log(*AR*); *PL*	0	-1	-2

Plotting the values of the above table gives:



In addition, the value of 20log(*AR*) for $\omega = 0.1$ has been calculated to help plot the Bode diagram:

for $\omega = 0.1 \Rightarrow 20\log(AR) = 39.507$

c) As can be seen in the Bode diagram, the gain margin is infinite because *PL* never crosses the straight line corresponding to -180º.

The value of the phase margin will be the value of *PL* when 20log(*AR*) intersects 0 dB. First, it must be found the value of ω which makes 20log(AR) = 0:

for $\omega = 80 \Rightarrow 20\log(RA) = -0.65$

for ω =76 $\Rightarrow 20\log(RA) = -0.11$

for ω =75.5 $\Rightarrow 20\log(AR) \approx 0$

Calculating the value of *PL* for ω=75.5 gives:

for ω= 75.5 $\Rightarrow PL = -126.29^{o} \Rightarrow$ The value of the phase margin is 53.71º above –180º.

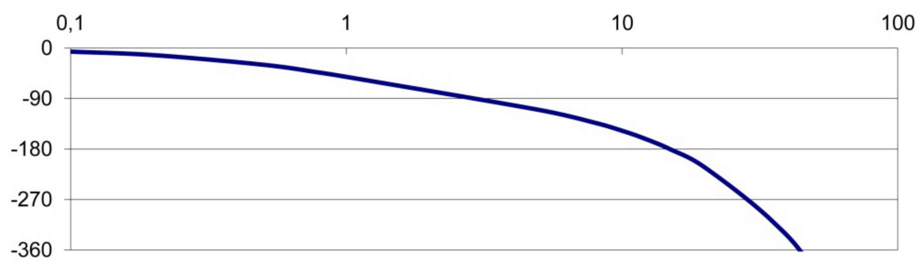
d) Any value of K_c can be taken, as the closed loop system is always stable. This fact can be extracted from the Bode diagram in open loop.

e) The value of 20log(AR) is not affected when the term $e^{-0.1s}$ is added. However, the *AR* value becomes more negative and it could intersect with *PL* = -180º. This suggests that the system could be unstable.

The new expression of $G_{OL}(j\omega)$ will be:

$$G_{OL}(j\omega) = \frac{95}{(j\omega+1)(0.01j\omega+1)} e^{-0.1j\omega}$$

To study the evolution of PL, various other values are plotted by calculating several values of ω. In the figure below it can be seen that the system is unstable as *PL* intersects with -180º before 20log(*AR*) can intersect with 0 dB; since for ω= 15 $\Rightarrow$ *DF* = -180º.



f) The process under study is characterized by a heat transfer whose response is very slow; to improve the response speed it is advisable to install a PID controller. This controller has the additional advantage of eliminating the offset.

g) According to the statement, the value $\overline{m}(s)$ must be calculated if the transfer function is $Kc\left(1+\frac{1}{\tau_I s}\right)$ and the input is $\bar{e}(s) = \frac{1}{s}$, as shown in the block diagram below:

$$\bar{e}(s) \longrightarrow \boxed{G_c(s) = Kc\left(1+\frac{1}{\tau_I s}\right)} \longrightarrow \overline{m}(s)$$

The output will be:

$$\overline{m}(s) = G_c(s)\overline{e}(s) = K_c\left(1+\frac{1}{\tau_I}\right)\left(\frac{1}{s}\right)$$

To obtain the time response, a partial-fractions expansion must be performed:

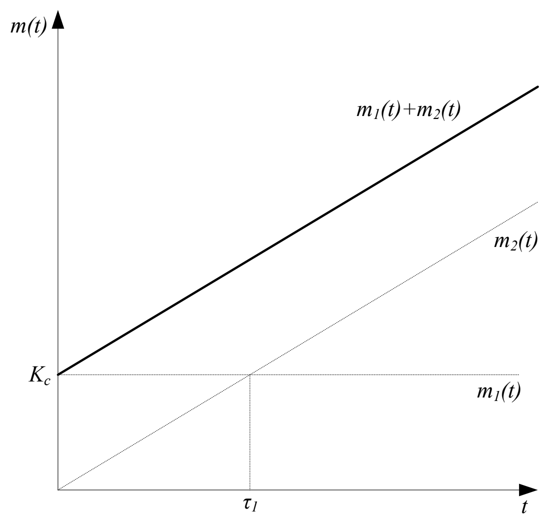
$$\overline{m}(s) = \frac{K_c}{s} + \frac{K_c}{\tau_I s^2}$$

The operation for obtaining the time response gives:

$$m(t) = \mathcal{L}^{-1}\left[\frac{K_c}{s} + \frac{K_c}{\tau_I s^2}\right] = \mathcal{L}^{-1}\left[\frac{K_c}{s}\right] + \mathcal{L}^{-1}\left[\frac{K_c}{\tau_I s^2}\right] \rightarrow m(t) = \underset{(m_1(t))}{K_c} + \underset{(m_2(t))}{\frac{K_c}{\tau_I}t} = K_c\left[1+\frac{t}{\tau_I}\right]$$

To plot the output, the above expression is split into two simpler functions, $m_1(t)$ and $m_2(t)$, and the overall function will be the sum of the two functions:

for $t = 0 \Rightarrow m_2(t) = 0$; for $t = \tau_I \Rightarrow m_2(t) = K_c$



Problem 2.3.3

The transfer functions of a feedback control system are shown below:

$$G_c(s) = K_c \qquad G_f(s) = \frac{2}{(s+1)} \qquad G_p(s) = \frac{3}{(4s+1)} \qquad G_d(s) = \frac{8}{(s+2)} \qquad G_m(s) = \frac{5}{(3s+1)}$$

a) Calculate the largest value of K_c before the system becomes unstable.

b) Calculate the value of the offset in function of K_c, assuming that

$$\bar{d}(s) = 0 \text{ and } \bar{y}_{sp}(s) = \frac{1}{s}$$

c) Calculate K_c if the phase margin is 20º; assuming that the system is stable.

d) Calculate the gain margin if $K_c = 0.2$.

e) Use the Bode criterion to discuss the stability of the feedback system if $G_m(s) = 5$.

f) Determine the stability of the feedback system if

$$G_m(s) = \frac{5e^{-0.1s}}{(3s+1)} \text{ and } K_c = 0.2 .$$

Solution

a) The characteristic equation for applying the Bode criterion is:

$$1 + G_c(s)G_f(s)G_p(s)G_m(s) = 0$$

Substituting gives:

$$1 + K_c \frac{2}{(s+1)} \frac{3}{(4s+1)} \frac{5}{(3s+1)} = 0$$

Rearranging to apply the Routh criterion gives:

$$12s^3 + 19s^2 + 8s + 1 + 30K_c = 0 \Rightarrow 12s^3 + 19s^2 + 8s + (1 + 30K_c)\ s^0 = 0$$

Forming the Routh array gives:

s^3	12	8
s^2	19	$(1+30K_c)$
s^1	$\frac{140 - 360K_c}{19}$	0
s^0	$(1+30K_c)$	

The conditions obtained from the Routh array are:

$$140 - 360K_c > 0 \Rightarrow K_c < 0.389$$

and also:

$$1 + 30K_c > 0 \Rightarrow K_c > -\frac{1}{30} \Rightarrow K_c > -0.033$$

The largest value of K_c is 0.389.

b) Remembering that offset is defined as:

offset = (new set point) - (ultimate value of the response)

and calculating the ultimate value of the response gives:

$$y(t)(t\to\infty)=\lim_{s\to 0} s\left(\frac{G_c(s)G_f(s)G_p(s)}{1+G_c(s)G_f(s)G_p(s)G_m(s)}\right)y_{sp}(s)=$$

$$=\lim_{s\to 0} s\left(\frac{K_c\dfrac{2}{(s+1)}\dfrac{3}{(4s+1)}}{1+K_c\dfrac{2}{(s+1)}\dfrac{3}{(4s+1)}\dfrac{5}{(3s+1)}}\right)\left(\frac{1}{s}\right)$$

Rearranging the above equation gives:

$$y(t)=\frac{6K_c}{1+30K_c}$$

The offset expression will be:

$$\text{offset}=1-\frac{6K_c}{1+30K_c}=\frac{1+24K_c}{1+30K_c}$$

c) Since the system is stable, if the phase margin is 20º, the *PL* value is -160º.

First, the value of ω which makes PL = -160º is calculated:

for $\omega = 0.6 \Rightarrow DF = -159.28^{\circ}$

for $\omega = 0.61 \Rightarrow DF = -160.44^{\circ}$

The sought value of K_c is one which makes 20log(AR) = 0 for $\omega = 0.61$.

The expression of $G_{OL}(j\omega)$ is:

$$G_{OL}(j\omega)=\frac{30K_c}{(j\omega+1)(4j\omega+1)(3j\omega+1)}$$

The expression of 20log(AR) will be:

$$20\log(AR)=20\log(30)+20\log(K_c)-20\log\sqrt{(\omega^2+1)}-$$
$$20\log\sqrt{(16\omega^2+1)}-20\log\sqrt{(9\omega^2+1)}$$

Substituting $\omega = 0.61$ and making 20log(AR) = 0 gives:

$$20\log(AR)=20\log(K_c)+29.54-1.37-8.42-6.38=0$$

The value of K_c will be:

$$20\log(K_c) = -13.37 \Rightarrow K_c = 0.214$$

d) Substituting K_c to obtain the new expression of $G_{OL}(j\omega)$ gives:

$$G_{OL}(j\omega) = 0.2\frac{2}{(j\omega+1)}\frac{3}{(4j\omega+1)}\frac{5}{(3j\omega+1)} = \frac{6}{(j\omega+1)(4j\omega+1)(3j\omega+1)}$$

The value of ω which makes PL = -180º is calculated:

for $\omega = 10 \Rightarrow DF$ = -260.9º

for $\omega = 1 \Rightarrow DF$ = -192.5º

for $\omega = 0.5 \Rightarrow DF$ = -146.3º

for $\omega = 0.8 \Rightarrow DF$ = -178.68º

for $\omega = 0.82 \Rightarrow DF$ = -180.27º

The expression of 20log(AR) will be:

$$20\log(AR) = 20\log(6) - 20\log\sqrt{(\omega^2+1)} - 20\log\sqrt{(16\omega^2+1)} - 20\log\sqrt{(9\omega^2+1)}$$

Adding substitutions in the above equation to obtain the value of 20log(AR) for ω = 0.82 gives:

20log(AR) = -5.85 dB (This value is negative since the system is stable.)

e) The new expression of $G_{OL}(j\omega)$ will be:

$$G_{OL}(j\omega) = \frac{30K_c}{(j\omega+1)(4j\omega+1)}$$

From the above expression, it can be concluded that the system will always be stable, since the value of PL never intersects –180º.

f) The new expression of $G_{OL}(j\omega)$ will be:

$$G_{OL}(j\omega) = 0.2\frac{2}{(j\omega+1)}\frac{3}{(4j\omega+1)}\frac{5e^{-0.1j\omega}}{(3j\omega+1)}$$

The value of PL will be:

$$PL = -\tan^{-1}(\omega) - \tan^{-1}(4\omega) - \tan^{-1}(3\omega) - (57.3)(0.1)\omega$$

Calculating the value of ω which makes PL = -180º gives:

for $\omega = 0.1 \Rightarrow PL = -44.78^\circ$

for $\omega = 0.6 \Rightarrow PL = -162.72^\circ$

for $\omega = 0.75 \Rightarrow DF = -178.7^\circ$

for $\omega = 0.77 \Rightarrow DF = -180.61^\circ$

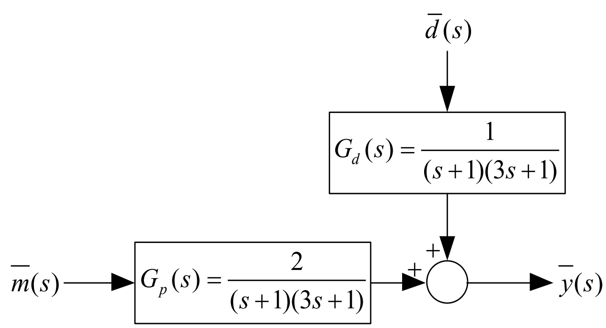
Calculating the value of $20\log(AR)$ for $\omega = 0.77$ gives:

$20\log(AR) = -4.685$; the system remains stable, although its stability is less than before.

Note: The asymptotic Bode diagram can be plotted to facilitate the calculation of the ω values, although it is not difficult to find these values if some very simple trials are performed, as seen before.

Problem 2.3.4

The transfer function of a process is shown in the block diagram below:



The following elements are added to form a feedback loop:

$$G_c(s) = K_c$$

$$G_f(s) = \frac{2}{(2s+1)}$$

$$G_m(s) = 2$$

a) Draw the block diagram corresponding to the feedback control system and calculate the output expression in closed loop.

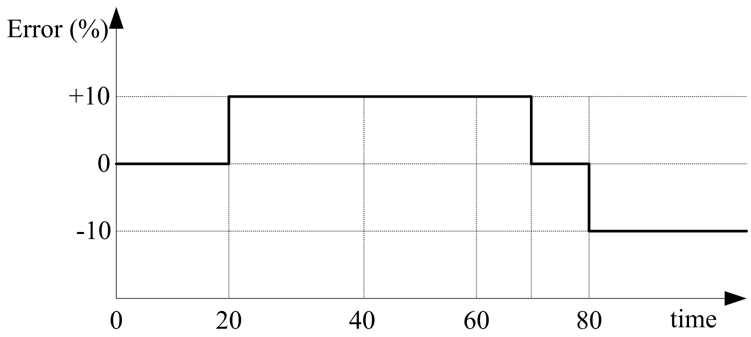
b) Plot its root locus for values of K_c from 0 to ∞.

c) Calculate the Kc values which can be used in this control system without becoming unstable.

d) Obtain the value (time expression) that enters the process if the disturbance is a unit step.

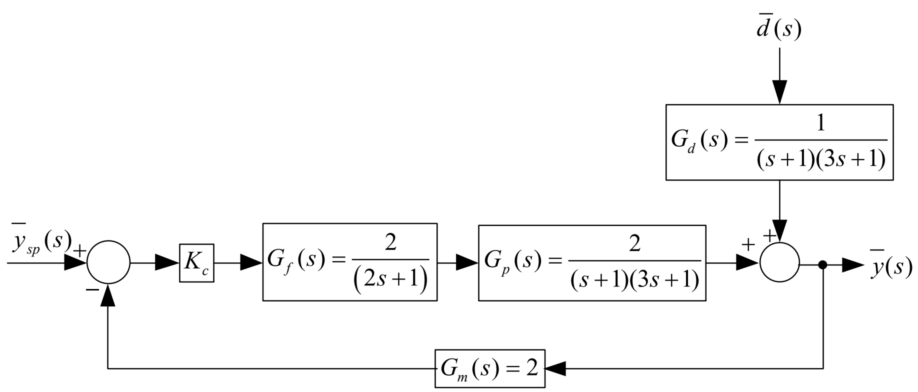
e) Plot the asymptotic Bode diagram if the proportional controller is substituted with the PI controller $G_c(s) = \left(1 + \frac{1}{0.5s}\right)$.

f) Calculate and plot the controller response value if the error signal follows the plot below.



Solution

a) The elements given in the statement must be arranged as components of a feedback control system:



To calculate the output expression, it is possible to apply the formula which relates the output to the blocks and the system inputs:

$$\bar{y}(s)=\frac{K_c\dfrac{2}{(2s+1)}\dfrac{2}{(s+1)(3s+1)}}{1+K_c\dfrac{2}{(2s+1)}\dfrac{2}{(s+1)(3s+1)}(2)}\bar{y}_{sp}(s)+\frac{\dfrac{1}{(s+1)(3s+1)}}{1+K_c\dfrac{2}{(2s+1)}\dfrac{2}{(s+1)(3s+1)}(2)}\bar{d}(s)$$

Operating the above formula gives:

$$\bar{y}(s)=\frac{4K_c}{(2s+1)(s+1)(3s+1)+8K_c}\bar{y}_{sp}(s)+\frac{(2s+1)}{(2s+1)(s+1)(3s+1)+8K_c}\bar{d}(s)$$

b) The expression below is needed for plotting the root locus:

$$1+K_c\frac{2}{(2s+1)}\frac{2}{(s+1)(3s+1)}(2)=0$$

Operating gives:

$$1+\frac{8K_c}{(2s+1)(s+1)(3s+1)}=1+\frac{8K_c}{(2)(3)\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)}=$$

$$=1+\frac{\frac{8}{6}K_c}{\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)}=0$$

Simplifying gives:

$$1+\frac{K'}{\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)}=0$$

where:

$$K'=\frac{8}{6}K_c=1.33K_c$$

Number of poles and zeros:

There are no zeros (0)

Poles: -1, -1/2, -1/3 (3)

Number of branches:

$\max(n_p, n_z) = \max(3, 0) = 3$

Zones of the real axis that belong to the root locus:

$$[-\infty,-1]\in \text{Root Locus}$$

$$\left[-1,-\frac{1}{2}\right]\notin \text{Root Locus}$$

$$\left[-\frac{1}{2},-\frac{1}{3}\right]\in \text{Root Locus}$$

$$\left[-\frac{1}{3},\infty\right]\notin \text{Root Locus}$$

Number of asymptotes:

n_p-n_z = 3-0 = 3 asymptotes

Starting point of the asymptotes:

$$\sigma_A=\frac{\Sigma poles-\sum zeros}{n_p - n_z}=\frac{-1-\frac{1}{2}-\frac{1}{3}}{3-0}=-0.61$$

Angles of the asymptotes:

$$\theta=\frac{(2k+1)\ 180}{3}$$

Calculating the value of $k = 0$, 1 and 2 gives:

$$\theta=60^o$$

$$\theta=180^o$$

$$\theta=300^o$$

Departure or arrival points on the real axis:

Isolating *K'* in the above equation gives:

$$K'=-s^3-\frac{11}{6}s^2-s-\frac{1}{6}$$

Calculating $\frac{dK'}{ds}=0$ gives:

$$\frac{dK'}{ds}=-3s^2-\frac{22}{6}s-1=0\rightarrow 3s^2+\frac{22}{6}s+1=0$$

Solving the above equation gives:

$$s = -0.41 \in \text{Root Locus}$$

The other value does not belong to the root locus.

Intersection points with the imaginary axis:

$$1+\frac{K'}{\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)}=0 \rightarrow \frac{\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)+K'}{\left(s+\frac{1}{2}\right)(s+1)\left(s+\frac{1}{3}\right)}=0$$

Rearranging the above equation gives:

$$s^3+\frac{11}{6}s^2+s+\left(\frac{1}{6}+K'\right)s^0=0$$

Applying the Routh array gives:

s^3	1	1
s^2	$\frac{11}{6}$	$\frac{1}{6}+K'$
s^1	$\frac{10-6K'}{11}$	0
s^0	$\frac{1}{6}+K'$	

The above array gives:

$$\frac{10-6K'}{11}=0 \rightarrow K'=\frac{10}{6}$$

This *K'* value must be substituted into the previous row in the Routh array:

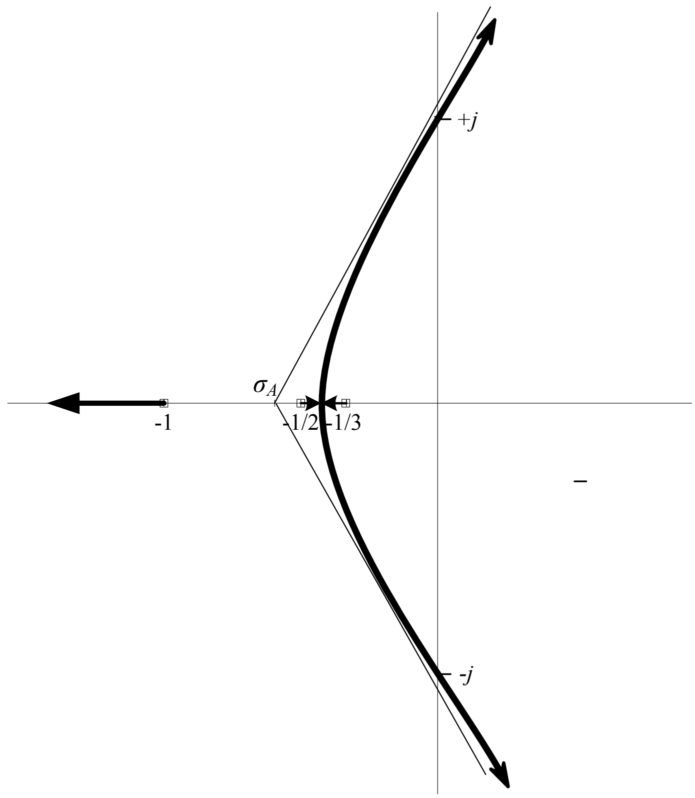
$$\frac{11}{6}s^2+\frac{1}{6}+\frac{10}{6}=0 \rightarrow s^2+1=0$$

Solving the above equation gives:

$$s=\pm j$$

In this case, finding the points which intersect the imaginary axis is strongly advised, because they allow the branch evolution to be confirmed, and the stability to be studied, if necessary.

The plot depicting the root locus is shown below.



c) Substituting to obtain the characteristic equation gives:

$$1+G_{OL}(s)=0 \rightarrow 1+\frac{K_c(2)(2)(2)}{(2s+1)(s+1)(3s+1)}=0 \rightarrow 1+\frac{8K_c}{(2s+1)(s+1)(3s+1)}=0$$

Rearranging to find the expression needed to apply the Routh criterion gives:

$$(2s+1)(s+1)(3s+1)+8K_c=0 \rightarrow 6s^3+11s^2+6s+1+8K_c=0$$

Applying the Routh array gives:

s^3	6	6
s^2	11	$1+8K_c$
s^1	$\frac{60-48K_c}{11}$	0
s^0	$1+8K_c$	

From the above table, it can be extracted the K_c values which allow the system to be stable:

Applying the first condition gives:

$$\frac{60-48K_c}{11}>0 \rightarrow 60-48K_c>0 \rightarrow K_c<1.25$$

Applying the second condition gives:

$$1+8K_c>0 \rightarrow K_c>-0.125$$

Finally, considering the above conditions, the Kc values which allow the system to be stable will be:

$$-0{,}125<K_c<1{,}25$$

d) The disturbance transfer function is:

$$G_d(s)=\frac{1}{(s+1)(3s+1)}$$

If a unit step input is applied, the output is:

$$\bar{r}(s)=\frac{1}{(s+1)(3s+1)}\left(\frac{1}{s}\right)$$

A partial-fractions expansion must be made to calculate the time expression and obtain:

$$\bar{r}(s)=\frac{1}{(s+1)(3s+1)}\left(\frac{1}{s}\right)=\frac{1}{s(s+1)(3s+1)}=\frac{\frac{1}{3}}{s(s+1)\left(s+\frac{1}{3}\right)}=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{\left(s+\frac{1}{3}\right)}$$

Rearranging the above expression:

$$\frac{1}{3}=A(s+1)\left(s+\frac{1}{3}\right)+Bs\left(s+\frac{1}{3}\right)+Cs(s+1)\rightarrow$$

$$\frac{1}{3}=\left(s^2+\frac{4}{3}s+\frac{1}{3}\right)A+\left(s^2+\frac{1}{3}s\right)B+\left(s^2+s\right)C$$

Computing the constant values gives:

$$\begin{cases} 0 = A + B + C & A = 1 \\ 0 = \dfrac{4}{3}A + \dfrac{1}{3}B + C \rightarrow & B = \dfrac{1}{2} \\ \dfrac{1}{3} = \dfrac{1}{3}A & C = -\dfrac{3}{2} \end{cases}$$

Substituting in the above equation gives:

$$\bar{r}(s) = \frac{1}{s(s+1)(3s+1)} = \frac{1}{s} + \frac{\frac{1}{2}}{(s+1)} - \frac{\frac{3}{2}}{\left(s+\frac{1}{3}\right)}$$

Applying the inverse Laplace transform to calculate the time expression gives:

$$r(t) = \mathcal{L}^{-1}\left[\bar{r}(s)\right] = \mathcal{L}^{-1}\left[\frac{1}{s}\right] + \mathcal{L}^{-1}\left[\frac{\frac{1}{2}}{(s+1)}\right] - \mathcal{L}^{-1}\left[\frac{\frac{3}{2}}{\left(s+\frac{1}{3}\right)}\right] \rightarrow r(t) = 1 + \frac{1}{2}e^{-t} - \frac{3}{2}e^{-t/3}$$

e) Now, the transfer function of the controller is:

$$G_c(s) = \left(1 + \frac{1}{0.5s}\right)$$

Substituting $s \rightarrow j\omega$ gives:

$$G_c(j\omega) = \left(1 + \frac{1}{0.5\,j\omega}\right)$$

To facilitate the construction of the asymptotic Bode diagram, the above expression must be arranged as:

$$G_c(j\omega) = \left(\frac{0.5\,j\omega + 1}{0.5\,j\omega}\right) = (0.5\,j\omega + 1)(0.5\,j\omega)^{-1}$$

Forming the table to plot the asymptotic Bode diagram (from elementary functions whose Bode diagram is known) gives:

	0^+ – 2	2 – ∞
$(0.5\,j\omega + 1)$	0	+1
$(0.5j\omega)^{-1}$	-1	-1
20log(AR); PL	-1	0

As in previous cases, only a table is needed because the values of 20log(AR) and PL are the same.

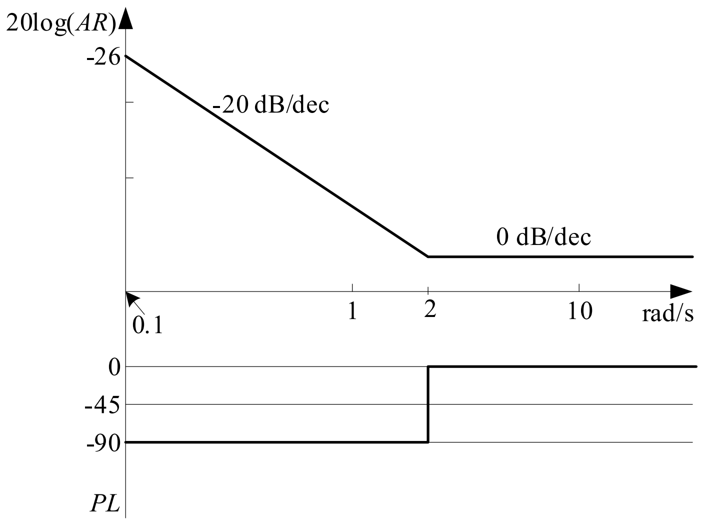
Several values must be calculated to complete the Bode diagram:

$$\text{for } \omega = 0{,}1 \Rightarrow 20\log(AR) = 20\log 1 + 20\log\sqrt{(0.5)^2\omega^2+1} - 20\log\left(0.5\omega\right) = -26\ dB$$

$$DF = \tan^{-1}\left(0.5j\omega\right) - \tan^{-1}\left(\frac{0.5\omega}{0}\right) = \tan^{-1}\left(0.5j\omega\right) - 90$$

$$\omega = 2 \rightarrow DF = 45^{o} - 90^{o} = -45^{o}$$

Plotting the Bode diagram gives:



g) According to the graph indicating the values of the error, the controller response in function of time can be found by applying the following formula:

$$c(t) = (1)e(t) + \frac{1}{0.5}\int_0^t e(t)dt$$

Calculating the controller output values for different times gives:

$\text{for } t = 0 \rightarrow e(t) = 0 \Rightarrow c(t) = 0$

$\text{for } t = 20- \rightarrow e(t) = 0 \Rightarrow c(t) = 0$

$\text{for } t = 20+ \rightarrow e(t) = 10 \Rightarrow c(t) = 10 + (2)(0) = 10$

$$\text{for } t = 70- \rightarrow e(t) = 10 \Rightarrow c(t) = (10)(1) + (2)(50)(10) = 1010$$

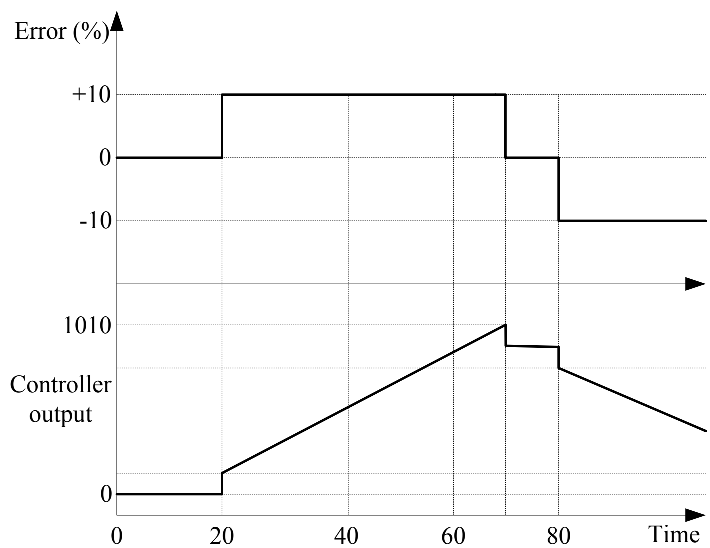
$$\text{for } t = 70+ \rightarrow e(t) = 0 \Rightarrow c(t) = 1000$$

$$\text{for } t = 80- \rightarrow e(t) = 0 \Rightarrow c(t) = (0)(1) + (2)(50)(10) = 1000$$

$$\text{for } t = 80+ \rightarrow e(t) = -10 \Rightarrow c(t) = -10 + (2)(50)(10) = 990$$

$$\text{for } t = 100+ \rightarrow e(t) = -10 \Rightarrow c(t) = -10 + (2)\left[(50)(10) - (20)(10)\right] = 590$$

Considering the specific values obtained so far, the controller response can be plotted as a function of the input of the error.



It is advisable to join the two graphics in order to see the evolution of the controller output.

Note: the last graphic has not been scaled and only the evolution is shown.

Problem 2.3.5

The components of a control loop are shown below:

$$G_f(s) = \frac{0.1}{4s+1} \quad G_p(s) = \frac{10}{20s+1} \quad G_m(s) = \frac{2}{5s+1}$$

If a PI controller is added to close the feedback control system, calculate its parameters (K_c and τ_I) by applying the Ziegler-Nichols method.

Solution

The expressions used in this problem are:

$$G_{OL}(j\omega) = \frac{0.1}{(4j\omega+1)}\frac{10}{(20j\omega+1)}\frac{2}{(5j\omega+1)}K_c^{'} = \frac{2K_c^{'}}{(4j\omega+1)(20j\omega+1)(5j\omega+1)}$$

$$20\log(AR) = 20\log 2 + 20\log K_c^{'} 20\log\sqrt{(4\omega)^2+1} -$$
$$20\log\sqrt{(20\omega)^2+1} - 20\log\sqrt{(5\omega)^2+1}$$

$$PL = -\tan^{-1}(4\omega) - \tan^{-1}(20\omega) - \tan^{-1}(5\omega)$$

Calculating the value of ω (the value that makes PL = -180º) gives:

for $\omega = 0.1 \to PL = -111.79$

for $\omega = 0.2 \to PL = -152.62$

for $\omega = 0.3 \to PL = -187.03$

for $\omega = 0.35 \to PL = -195.98$

for $\omega = 0.25 \to PL = -175.03$

for $\omega = 0.27 \to PL = -180.17$

The value of $K_c^{'}$ must be obtained when ω = 0.27 and 20log(PL) = 0; this involves solving the equation below:

$$6.02 + 20\log K_c^{'} - 3.357 - 14.79 - 4.51 = 0 \to K_c^{'} = 6.792$$

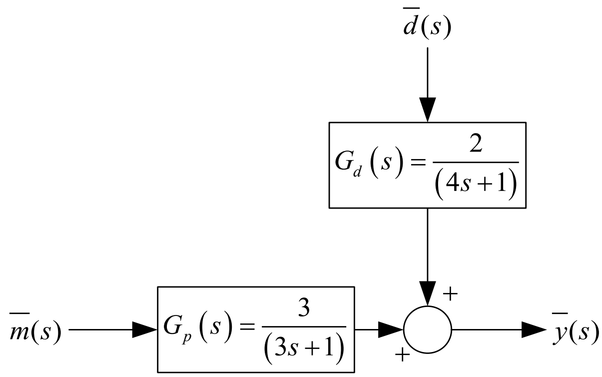
The value of P_n is: $P_n = \frac{2\pi}{\omega} = \frac{2\pi}{0.27} = 23.27$

Applying the Ziegler-Nichols method for a PI controller gives:

$$K_c = \frac{6.792}{2.2} = 3.087 \quad \text{and} \quad \tau_I = \frac{23.27}{1.2} = 19.39$$

Problem 2.3.6

The figure below shows the block diagram of a process:



a) Calculate the value of *y(t)* after much time has passed, assuming $\overline{d}(s)=0$ and $\overline{m}(s)=\frac{2}{s}$.

After adding the elements below to the system:

$G_f(S)=\frac{1}{2s+1}$ $G_m(s)=\frac{2}{s+1}$ and a proportional controller $G_c(s)=K_c$

b) Calculate the values that stabilize the control system.

c) Calculate the offset value (in function of K_c) when the disturbance is $\overline{d}(s)=\frac{1}{s}$ and the set point does not vary. In addition, calculate the maximum and minimum values of the offset that stabilize the system.

d) If the P controller is substituted with a PI controller, tune the latter by using the Ziegler-Nichols method to calculate the parameters K_c and τ_I.

e) Calculate the gain margin of the system with the PI controller and discuss the result obtained.

f) The PI controller is now substituted with a P controller whose K_c is 0.1, calculate the gain margin and discuss the result.

Solution

a) The value of *y(t)* can be obtained by applying the Final-Value Theorem:

$$\bar{y}(s)=\frac{3}{3s+1}\frac{2}{s}=\frac{6}{s\left(3s+1\right)};\ \text{if t}\rightarrow\infty\rightarrow y(t)=\lim_{s\to 0}s\,\bar{y}(s)=\lim_{s\to 0}s\frac{6}{s(3s+1)}=6$$

b) Substituting this into the equation below to obtain the characteristic equation gives:

$$1+G_c(s)G_f(s)G_p(s)G_m(s)=0\rightarrow 1+K_c\frac{1}{\left(2s+1\right)}\frac{3}{\left(3s+1\right)}\frac{2}{\left(s+1\right)}=0$$

Rearranging the above equation gives:

$$\left(2s+1\right)\left(3s+1\right)\left(s+1\right)+6K_c=0\rightarrow 6s^3+11s^2+6s+\left(1+6K_c\right)s^0=0$$

Applying the Routh array gives:

s^3	6	6
s^2	11	$1+6K_c$
s^1	$\frac{60\text{-}36K_c}{11}$	0
s^0	$1+6K_c$	

From the above table, it can be extracted the K_c values which stabilize the system

Applying the first condition gives:

$$\frac{60-36K_c}{11}>0\rightarrow 60-36K_c>0\rightarrow K_c<1.67$$

Applying the second condition gives:

$$1+6K_c>0\rightarrow 6K_c>-1\rightarrow K_c>-0.16$$

Finally, considering the above conditions, the K_c values which stabilize the system will be:

$$-0.167<K_c<1.67$$

c) Substituting the transfer functions from the statement gives:

$$\bar{y}(t\rightarrow\infty)=\lim_{s\to 0}s\frac{\frac{2}{\left(4s+1\right)}}{1+K_c\frac{1}{\left(2s+1\right)}\frac{3}{\left(3s+1\right)}\frac{2}{\left(s+1\right)}}\frac{1}{s}=\frac{2}{\left(1+6K_c\right)}$$

$$offset=0-\frac{2}{1+6K_c}=-\frac{2}{1+6K_c}$$

$$\text{if } K_c = -0.167 \rightarrow \textit{offset} = -\frac{2}{1-1} = -\infty \text{ (minimum value)}$$

$$\text{if } K_c = 1.67 \rightarrow \textit{offset} = -\frac{2}{1+(6)(1.67)} = -0.181 \text{ (maximum value)}$$

d) Calculating the $G_{OL}(s)$ expression gives:

$$G_{OL}(s) = K_c^{'} \frac{1}{(2s+1)} \frac{3}{(3s+1)} \frac{2}{(s+1)}$$

Substituting $s \rightarrow j\omega$ gives:

$$G_{OL}(j\omega) = K_c^{'} \frac{1}{(2j\omega+1)} \frac{3}{(3j\omega+1)} \frac{2}{(j\omega+1)} = \frac{6K_c^{'}}{(2j\omega+1)(3j\omega+1)(j\omega+1)}$$

Calculating the expressions of 20log(AR) and PL gives:

$$20\log(AR) = 20\log(6) + 20\log K_c^{'} - 20\log\sqrt{(4\omega^2+1)} -$$
$$20\log\sqrt{(9\omega^2+1)} - 20\log\sqrt{(\omega^2+1)}$$

$$PL = -\tan^{-1}(2\omega) - \tan^{-1}(3\omega) - \tan^{-1}(\omega)$$

$$\text{if } \omega = 1 \rightarrow PL = -179.99^{o}$$

Substituting $\omega = 1$ into the expression 20log(AR) = 0 gives:

$$0 = 15.56 + 20\log K_c^{'} - 6.99 - 10 - 3.01 \rightarrow \log K_c^{'} = 0.222 \rightarrow K_c^{'} = 1.67$$

Tuning the PI controller by applying the Ziegler-Nichols method gives:

$$Pu = \frac{2\pi}{1} = 2\pi \rightarrow K_c = \frac{K_u}{2.2} = \frac{1.67}{2.2} = 0.759 \text{ and } \tau_I = \frac{P_u}{1.2} = 5.236$$

e) Obtaining the new value of $G_{OL}(j\omega)$ gives:

$$G_{OL}(s) = 0.76\left(1 + \frac{1}{1+5.236s}\right) \frac{1}{(2s+1)} \frac{3}{(3s+1)} \frac{2}{(s+1)} =$$

$$= 0.76\left(\frac{5.236s+1}{5,236s}\right) \frac{1}{(2s+1)} \frac{3}{(3s+1)} \frac{2}{(s+1)}$$

The expression of *PL* will be:

$$PL = \tan^{-1}(5.236\omega) - \tan^{-1}(\infty) - \tan^{-1}(2\omega) - \tan^{-1}(3\omega) - \tan^{-1}(\omega) =$$

$$= \tan^{-1}(5.236\omega) - 90 - \tan^{-1}(2\omega) - \tan^{-1}(3\omega) - \tan^{-1}(\omega)$$

Calculating the ω value that makes PL = -180º gives:

for $\omega = 1 \rightarrow PL = -190.78$

for $\omega = 0.9 \rightarrow PL = -184.56$

for $\omega = 0.8 \rightarrow PL = -177.47$

for $\omega = 0.83 \rightarrow PL = -179.71$

$$\text{If } \omega = 0.83 \rightarrow 20\log(AR) = 20\log(0.870) + 20\log\sqrt{(5.236\omega)^2+1} \text{ -}$$

$$-20\log(\omega) - 20\log\sqrt{(4\omega^2+1)} - 20\log\sqrt{(9\omega^2+1)} - 20\log\sqrt{(\omega^2+1)} = -3.21$$

As expected, the system is stable because the acquired parameters are the same as those used in the Ziegler-Nichols method.

f) The values of $G_{OL}(s)$ and *PL* are:

$$G_{OL}(s) = 0.1\frac{1}{(2s+1)}\frac{3}{(3s+1)}\frac{2}{(s+1)} = 0.6\frac{1}{(2s+1)}\frac{1}{(3s+1)}\frac{1}{(s+1)}$$

$$PL = -\tan^{-1}(2\omega) - \tan^{-1}(3\omega) - \tan^{-1}(\omega)$$

$$\text{if } \omega = 1 \rightarrow 20\log(AR) = -20\log(0.6) - 20\log\sqrt{(4+1)} -$$

$$20\log\sqrt{(9+1)} - 20\log\sqrt{(1+1)} = -24.437\text{dB}$$

As expected, the feedback control system is stable because the value of K_c is within the values -0.167 and 1.67.

→ 3

Feedforward and other Control Systems

Introduction

Although feedback control is the most widely used control system, it is necessary to take into account other control systems that are based on different techniques which give excellent results in Chemical Engineering; sometimes they complement feedback control and sometimes they substitute it. In this chapter some techniques are shown, which are applied to chemical processes.

Sometimes these control systems are based on known principles of feedback control (cascade control, inferential control, etc.). In addition, they can present concepts that cannot be predicted or known intuitively from the feedback systems. For this reason, it has been considered useful to include some comment in order to clarify their basic operation mode.

3.1 Systems based on feedforward control

Problem 3.1.1

Consider a process whose transfer functions are:

$$\frac{\bar{y}(s)}{\bar{m}(s)} = G_p(s) = \frac{10}{2s+1}$$

$$\frac{\bar{y}(s)}{\bar{d}(s)} = G_d(s) = \frac{5}{2s+1}$$

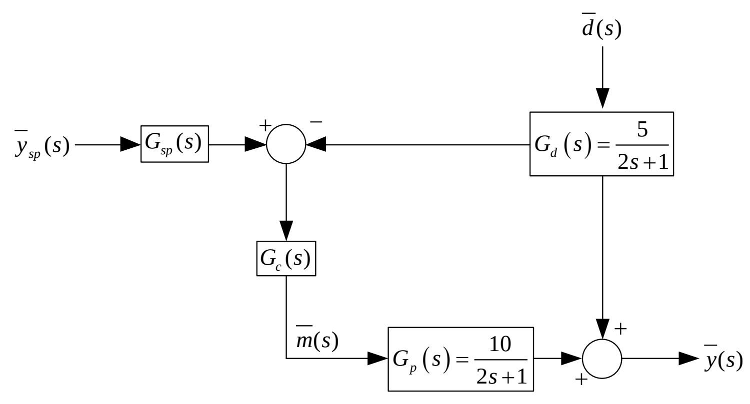
a) Design a feedforward control system to oppose the disturbance and to keep the set point.

b) Using the feedforward control obtained in the previous section, calculate and plot graphically, as a function of time, the controlled output obtained when the disturbance is a unit step change.

c) Assuming that $G_p(s) = \dfrac{15}{2s+1}$, and the value of $G_d(s)$ does not change, use the feedforward system from the first section to calculate and plot graphically, as a function of time, the controlled output obtained when the disturbance is a unit step change, but with the new values indicated.

Solution

a) To design a feedforward control system, the values of the transfer functions must be calculated, corresponding to the blocks $G_{sp}(s)$ and $G_c(s)$, which characterize the control system. They are a function of the other blocks that make up the process.



From the above block diagram, the following equation can be obtained:

$$\bar{y}(s) = \left(G_{sp}(s)\bar{y}(s) - \bar{d}(s)\right)G_c(s)G_p(s) + G_d(s)\bar{d}(s)$$

Rearranging gives:

$$\bar{y}(s) = G_{sp}(s)G_c(s)G_p(s)\bar{y}(s) + \left(G_d(s) - G_c(s)G_p(s)\right)\bar{d}(s)$$

The two conditions of a feedforward system must be achieved: disturbance rejection and set point tracking. Applying the first one gives:

$$G_d(s) - G_c(s)G_p(s) = 0$$

The above equation allows calculation of the value of $G_c(s)$:

$$G_c(s) = \frac{G_d(s)}{G_p(s)}$$

Substituting the values of the statement gives:

$$G_c(s) = \frac{\frac{5}{2s+1}}{\frac{10}{2s+1}} = \frac{1}{2}$$

Applying the second condition gives:

$$G_{sp}(s)G_c(s)G_p(s) = 1$$

From the above expression the value of $G_{sp}(s)$ can be obtained:

$$G_{sp}(s) = \frac{1}{G_c(s)G_p(s)} = \frac{1}{G_d(s)}$$

Substituting the values of the statement gives:

$$G_{sp}(s) = \frac{2s+1}{5}$$

With the values of $G_c(s)$ and $G_{sp}(s)$ the feedforward system can be constructed.

b) To know the response of the process, the following equation must be applied:

$$\bar{y}(s) = G_d(s)\bar{d}(s) + G_c(s)G_p(s)\left(-\bar{d}(s)\right)$$

Substituting the numerical values of the statement, $\bar{y}_{sp}(s) = 0$ and $\bar{d}(s) = \frac{1}{s}$ gives:

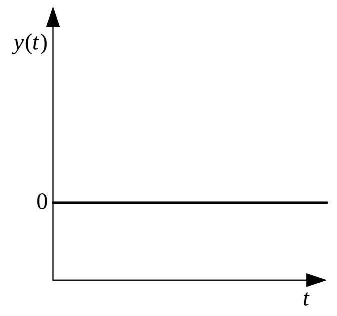
$$\bar{y}(s) = \frac{5}{(2s+1)}\frac{1}{s} - \frac{1}{2}\frac{10}{\left(2s+1\right)}\frac{1}{s}$$

Rearranging and simplifying gives:

$$\bar{y}(s) = 0$$

This result was expected because an ideal feedforward control system has been designed. This means that the disturbance effect will not be felt.

Plotting the response as a function of time gives:



It should not be forgotten that the values are changes in the response; they are not absolute values of the response. This justifies the graphic plotted above.

c) From now on, the values obtained in the first section will not be used, and thus the response will not correspond to an ideal feedforward control system:

$$\bar{y}(s) = G_d(s)\bar{d}(s) - G_c(s)G_p(s)\bar{d}(s)$$

Substituting the numerical values gives:

$$\bar{y}(s) = \frac{5}{(2s+1)}\frac{1}{s} - \frac{1}{2}\frac{15}{(2s+1)}\frac{1}{s}$$

Rearranging gives:

$$\bar{y}(s) = \frac{1}{s}5\left[\frac{1}{2s+1} - \frac{3}{2(2s+1)}\right] = -\frac{5}{s}\left[\frac{1}{2(2s+1)}\right] = -\frac{1.25}{s}\left(\frac{1}{s+0.5}\right)$$

Calculating the roots of the above function gives two real and distinct roots (0 and 0.5):

$$\frac{1}{s}\left(\frac{1}{s+0.5}\right) = \frac{A}{s} + \frac{B}{s+0.5}$$

Rearranging gives:

$$1 = A(s+0.5) + Bs = As + 0.5A + Bs$$

Identifying the coefficients gives:

$A = 2$

$B = -2$

The time response will be:

$$y(t) = \mathcal{L}^{-1}\left[-\frac{1.25}{s} + \frac{1.25}{s+0.5}\right] = (-1.25)\,\mathcal{L}^{-1}\left[\frac{2}{s}\right] + (1.25)\,\mathcal{L}^{-1}\left[\frac{2}{s+0.5}\right]$$

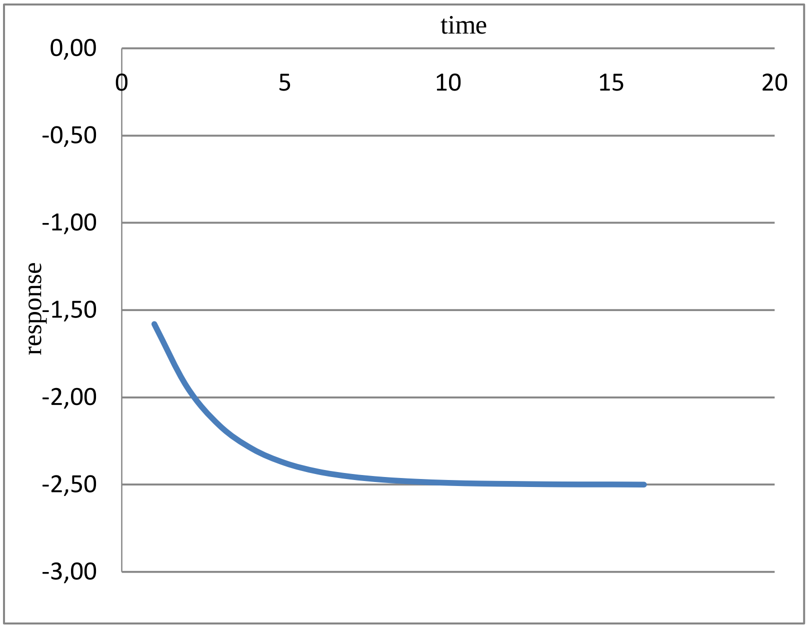
Rearranging and simplifying the above equation, the overall expression of the time response will be:

$$y(t) = -2.5 + (2.5)e^{-\frac{1}{2}t} = (-2.5)\left(1 - e^{-\frac{1}{2}t}\right)$$

To study the time response evolution, several time values are substituted (see the table below):

t	**y(t)**	**t**	**y(t)**
0	0.0	9	-2.472
1	-0.984	10	-2.483
2	-1.580	11	-2.490
3	-1.942	12	-2.494
4	-2.162	13	-2.496
5	-2.295	14	-2.498
6	-2.376	15	-2.499
7	-2.425	16	-2.499
8	-2.454	17	-2.499

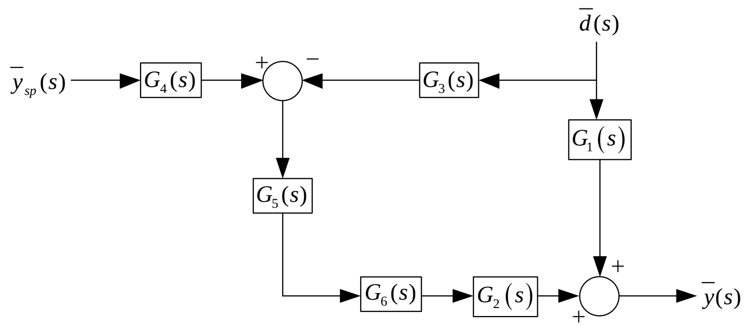
The plot obtained is shown below:



It can be seen in the above plot that the output is evolving towards a value that is not exactly the set point, since the ideally obtained transfer functions have been replaced by other values.

Problem 3.1.2

A feedforward system applied to the process described by $G_1(s)$ and $G_2(s)$ is shown in the figure below:



Where G3 and G6 are, respectively, the measuring device and the final control element. In addition, G_4 and G_5 are the elements of the feedforward control system.

a) Calculate the values of G4 and G5 as a function of the other system transfer functions. All the steps followed must be justified.

b) Discuss whether the feedforward control system, obtained in the previous section, may ensure that the output will always be the desired one.

c) Suggest one way to improve the indicated feedforward control system.

Solution

a) Applying the properties of block diagrams gives:

$$\bar{y}(s) = G_1(s)\bar{d}(s) + G_2(s)G_6(s)G_5(s)\left(G_4(s)\bar{y}_{sp}(s) - G_3(s)\bar{d}(s)\right)$$

Rearranging the above equation gives:

$$\bar{y}(s) = G_2(s)G_4(s)G_5(s)G_6(s)\bar{y}_{sp}(s) + \left(G_1(s) - G_2(s)G_6(s)G_5(s)G_3(s)\right)\bar{d}(s)$$

Assuming that the disturbance has no effect on the system (disturbance rejection), the above output equation allows the values of G_5 to be obtained:

$$G_1(s) - G_2(s)G_6(s)G_5(s)G_3(s) = 0 \Rightarrow G_5(s) = \frac{G_1(s)}{G_2(s)G_3(s)G_6(s)}$$

In addition, the set point tracking gives the value of $G_4(s)$:

$$G_2(s)G_4(s)G_5(s)G_6(s) = 0 \Rightarrow G_4(s) = \frac{1}{G_2(s)G_5(s)G_6(s)} = \frac{G_3(s)}{G_1(s)}$$

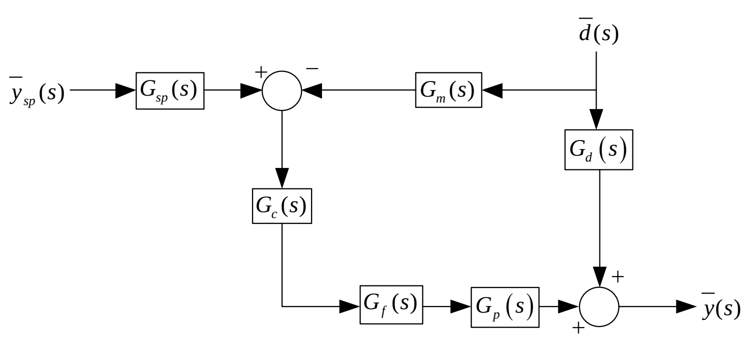
b) If the working conditions of the process do not vary and the transfer functions obtained follow the input-output ratios, it is feasible that the output will be the desired one. However, this is a situation that does not tend to occur in any real process related to chemical engineering.

c) There is the possibility of adding a feedback control system that operates when some error is detected (i.e., a difference between the desired value and the obtained value). This system will correct the error, which the feedforward control cannot correct due to its way of operating.

Problem 3.1.3

The figure below shows the block diagram of a feedforward control system. The known values of the transfer functions are:

$$G_m(s) = K_m; \quad G_d(s) = \frac{2}{2s+1}; \quad G_f(s) = \frac{1}{s+1}; \quad G_p(s) = \frac{7}{3s+1}$$



a) Determine the output expression $\bar{y}(s)$ a function of $\overline{y_{sp}}(s)$ and $\bar{d}(s)$.

Calculate the value of Gc(s).

Calculate the value of Gsp(s).

Solution

a) Applying the principles of algebra for block diagrams, the following expression can be obtained:

$$\overline{y}(s) = \left(G_{sp}(s)\overline{y}_{sp}(s) - G_m(s)\overline{d}(s)\right)G_c(s)G_f(s)G_p(s) + G_d(s)\overline{d}(s) =$$

$$= G_{sp}(s)G_c(s)G_f(s)G_p(s)\overline{y}_{sp}(s) + \left(G_d(s) - G_m(s)G_c(s)G_f(s)G_p(s)\right)\overline{d}(s)$$

From the above expression, the behaviour of the feedforward system can be described.

c) If the controller is able to completely eliminate the disturbance, it can be written:

$$G_d(s) - G_m(s)G_c(s)G_f(s)G_p(s) = 0$$

From the above expression the value of $G_c(s)$ can be obtained:

$$G_c(s) = \frac{G_d(s)}{G_m(s)G_f(s)G_p(s)} = \frac{\frac{2}{2s+1}}{K_m \frac{1}{(s+1)}\frac{7}{(3s+1)}} = \frac{2(s+1)(3s+1)}{7K_m(2s+1)}$$

c) From the expression obtained in section *a)*, and by taking into account the set point tracking, imposing $\overline{y}(s) = \overline{y}_{sp}(s)$ gives:

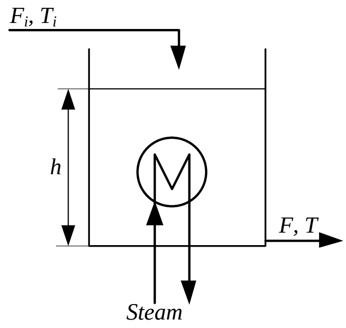
$$G_{sp}(s)G_c(s)G_f(s)G_p(s) = 1$$

and maintaining $G_d(s) - G_m(s)G_c(s)G_f(s)G_p(s) = 0$ gives:

$$G_{sp}(s) = \frac{1}{G_c(s)G_f(s)G_p(s)} = \frac{G_m(s)G_f(s)G_p(s)}{G_f(s)G_p(s)G_d(s)} = \frac{G_m(s)}{G_d(s)} = \frac{K_m}{\frac{2}{2s+1}} = \frac{K_m}{2}(2s+1)$$

Problem 3.1.4

The figure below shows a tank that contains a heat exchanger. The aim is to maintain the temperature of a liquid whose density is ρ(Rg/m^3) at a given value T_{sp}. Assume that T_i (input temperature) is the disturbance and q (amount of heat exchanged per unit time *Watts*) is the manipulated variable. The value of the tank section is A (constant).



a) Determine the expression of the overall mass balance.

b) Determine the expression of the overall heat balance.

c) Design the feedforward controller in steady state for this process and plot the appropriate block diagram.

d) Design the feedforward controller by taking into account the process dynamic response.

e) Explain when the two controllers previously designed are the same.

Solution

a) Applying an overall mass balance gives:

$$\rho_i F_i - \rho F = \frac{d(\rho A h)}{dt}$$

Assuming that A and ρ are constant, arranging the above expression gives:

$$A\frac{dh}{dt} = F_i - F$$

b) Applying an overall heat balance gives:

$$\frac{d\left(\rho A h c_p (T - T_0)\right)}{dt} = \rho F_i c_p (T_i - T_0) - \rho F c_p (T - T_0) + q$$

To obtain the above equation, T_0 is the assumed reference temperature and the heat capacity is assumed to be the same in both streams.

A more simplified expression can be obtained if $T_0 = 0$:

$$A\frac{d(hT)}{dt} = F_iT_i - FT + \frac{q}{\rho c_p}$$

Operating the expression $A\frac{d(hT)}{dt}$ gives:

$$A\frac{d(hT)}{dt} = Ah\frac{dT}{dt} + AT\frac{dh}{dt} = Ah\frac{dT}{dt} + T(F_i - F) = Ah\frac{dT}{dt} + (F_i - F)T = F_iT_i - FT + \frac{q}{\rho c_p}$$

Rearranging the above equation gives:

$$Ah\frac{dT}{dt} + F_iT - FT = F_iT_i - FT + \frac{q}{\rho c_p}$$

Finally, it is:

$$Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{q}{\rho c_p}$$

c) Assuming that there is no accumulation in steady state, $\left(\frac{dT}{dt} = 0\right)$ gives:

$$0 = F_i(T_i - T) + \frac{q}{\rho c_p} \rightarrow T = T_i + \frac{q}{F_i\rho c_p}$$

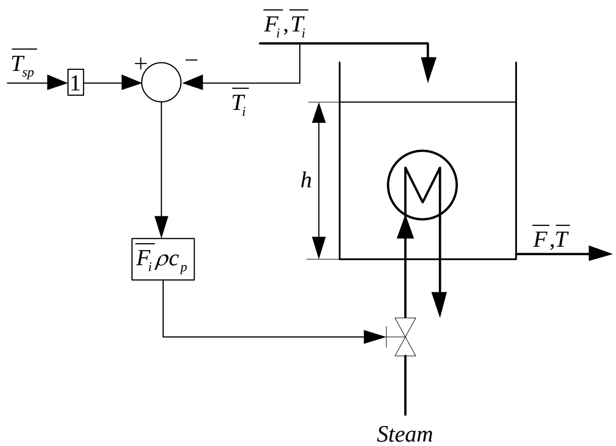
Assuming that $T = T_{sp}$, the process design equation will be obtained:

$$q = F_i\rho c_p(T_{sp} - T_i)$$

The above equation can be stated using Laplace transforms:

$$\overline{q} = F_i\rho c_p(\overline{T_{sp}^{'}} - \overline{T_i^{'}})$$

To plot the feedforward system diagram, the process variables must be arranged to follow an identical structure as that of the design equation:



d) From now on $\frac{dT}{dt} \neq 0$, the same steps of the previous section must be repeated:

$$Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{q}{\rho c_p}$$

Substituting *Ah* for *V* in the above equation gives:

$$\frac{V}{F_i}\frac{dT}{dt} + T = T_i + \frac{q}{\rho c_p F_i}$$

Introducing deviation variables gives:

$$\frac{V}{F_i}\frac{dT'}{dt} + T' = T_i' + \frac{q}{\rho c_p F_i}$$

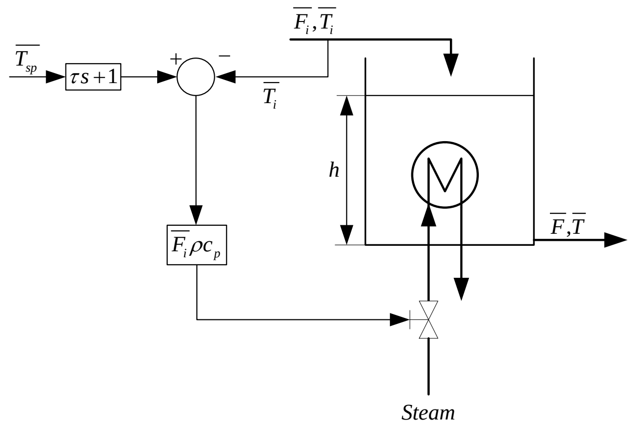
Substituting (residence time) τ for $\frac{V}{F_i}$ and applying the Laplace transform gives:

$$\overline{T}'(s) = \frac{\overline{T_i'}(s)}{(\tau + 1)} + \frac{1}{F_i c_p \rho}\frac{1}{(\tau + 1)}\overline{q}(s)$$

If $\overline{T}'(s) = \overline{T_{sp}'}(s)$, the design equation is obtained:

$$\overline{q}(s) = F_i \rho c_p \left((\tau + 1)\overline{T_{sp}'}(s) - \overline{T_i'}(s)\right)$$

As in section c, to build the feedforward system scheme, the values of the process variables must be arranged to follow an identical structure as that of the design equation.



e) If only the disturbance undergoes a change, the input values are:

$$\overline{T}_{sp}(s) = 0 \text{ and } \overline{T}_i(s) \neq 0$$

Substituting in the above equation gives:

$$\overline{q} = -F_i \rho c_p \overline{T_i'}$$

It can be seen that the two controllers have the same behaviour.

Problem 3.1.5

The behaviour of a process can be modelled by means of the following transfer functions:

$$G_p(s) = \frac{\overline{y}(s)}{\overline{m}(s)} = \frac{s+1}{(s+2)(2s+3)}$$

$$G_d(s) = \frac{\overline{y}(s)}{\overline{d}(d)} = \frac{5}{s+2}$$

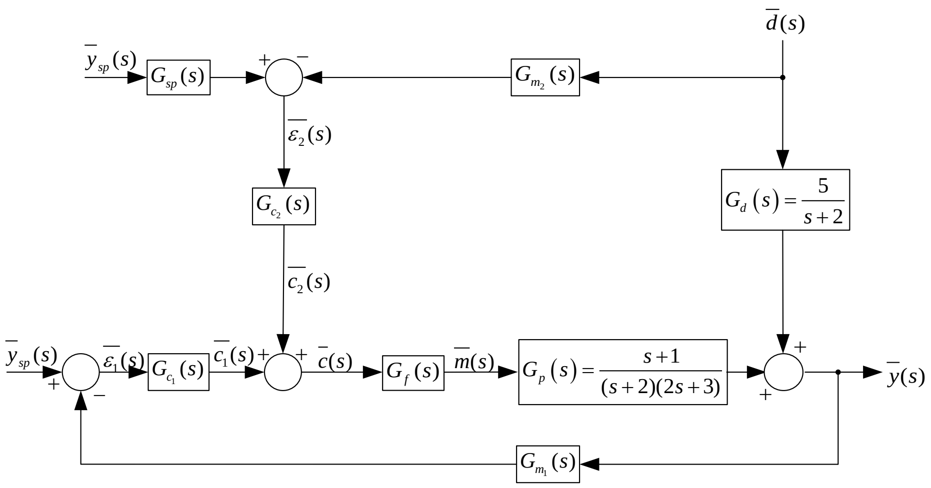
a) Design a feedforward-feedback control system that must have the following specifications: a PI controller in the feedback loop and the feedforward loop must be able to oppose disturbances and set point changes. To simplify, assume that the other transfer functions are unity.

b) Discuss the stability problems in the feedback loop caused by the presence of a feedforward loop.

c) Discuss whether the presence of the feedback loop is essential for optimal performance of the system.

Solution

a) First, following the statement conditions, a schema of the feedforward-feedback control is plotted. All the blocks and the values of their transfer functions are indicated. It should not be forgotten that there is only one final control element and that the values which must be calculated are $G_{sp}(s)$ and $G_{c2}(s)$.



According to the above scheme, and by applying the rules of block algebra, the following equations can be inferred:

$$\overline{y}(s)=\frac{s+1}{(s+2)(2s+3)}\overline{m}(s)+\frac{5}{s+2}\overline{d}(s)$$

$$\overline{m}(s)=G_f(s)\overline{c}(s)=G_f(s)\left(\overline{c}_1(s)+\overline{c}_2(s)\right)=G_f(s)\left(G_{c_1}(s)\overline{\varepsilon}_1(s)+G_{c_2}(s)\overline{\varepsilon}_2(s)\right)=$$

$$=G_f(s)G_{c_1}(s)\overline{\varepsilon}_1(s)+G_f(s)G_{c_2}(s)\overline{\varepsilon}_2(s)$$

Rearranging gives:

$$\overline{m}(s)=G_f(s)G_{c_1}(s)\left(\overline{y}_{sp}(s)-G_{m_1}(s)\overline{y}(s)\right)+G_f(s)G_{c_2}(s)\left(G_{sp}(s)\overline{y}_{sp}(s)-G_{m_2}(s)\overline{d}(s)\right)$$

Substituting the above equation and the statement values of $G_p(s)$ and $G_d(s)$ in the expression of $\overline{y}(s)$ gives:

$$\bar{y}(s)=\frac{\frac{s+1}{(s+2)(2s+3)}G_f(s)\left[G_{c_1}(s)+G_{c_2}(s)G_{sp}(s)\right]}{1+\frac{s+1}{(s+2)(2s+3)}G_f(s)G_{c_1}(s)G_{m_1}(s)}\bar{y}(s)+$$

$$+\frac{\frac{5}{(s+2)}-\frac{s+1}{(s+2)(2s+3)}G_f(s)G_{c_2}(s)G_{m_2}(s)}{1+\frac{s+1}{(s+2)(2s+3)}G_f(s)G_{c_1}(s)G_{m_1}(s)}\bar{d}(s)$$

Applying the conditions of a feedforward system gives:

$$\frac{5}{s+2}-\frac{s+1}{(s+2)(2s+3)}G_{c_2}(s)=0 \qquad (*)$$

$$\frac{\frac{s+1}{(s+2)(2s+3)}\left[G_{c_1}(s)+G_{c_2}(s)G_{sp}(s)\right]}{1+\frac{s+1}{(s+2)(2s+3)}G_{c_1}(s)}=1 \qquad (**)$$

Rearranging the equation (*) gives:

$$G_{c_2}(s)=\frac{5(2s+3)}{s+1}$$

Rearranging the equation (**) gives:

$$\frac{(s+1)\left(G_{c_1}(s)+G_{c_2}(s)G_{sp}(s)\right)}{(s+2)(2s+3)+(s+1)G_{c_1}(s)}=1$$

Rearranging the above equation gives:

$$(s+1)G_{c_1}(s)+(s+1)G_{c_2}(s)G_{sp}(s)=(s+2)(2s+3)+(s+1)G_{c_1}(s)$$

Finally, the value of $G_{sp}(s)$ is obtained:

$$G_{sp}(s)=\frac{(s+2)(2s+3)}{(s+1)G_{c_2}(s)}=\frac{s+2}{5}$$

b) The equation that gives the response has the expression $1+G_p(s)G_f(s)G_{c_1}(s)G_{m_1}(s)$ in its denominator. This expression is exactly the one corresponding to the characteristic equation of a feedback system; so the feedforward loop has no influence on the stability.

c) If the values of $G_p(s)$ and $G_d(s)$ correspond exactly to the true expression of transfer functions, the feedback loop will not be necessary. In addition, any process characteristic should not change with time.

Problem 3.1.6

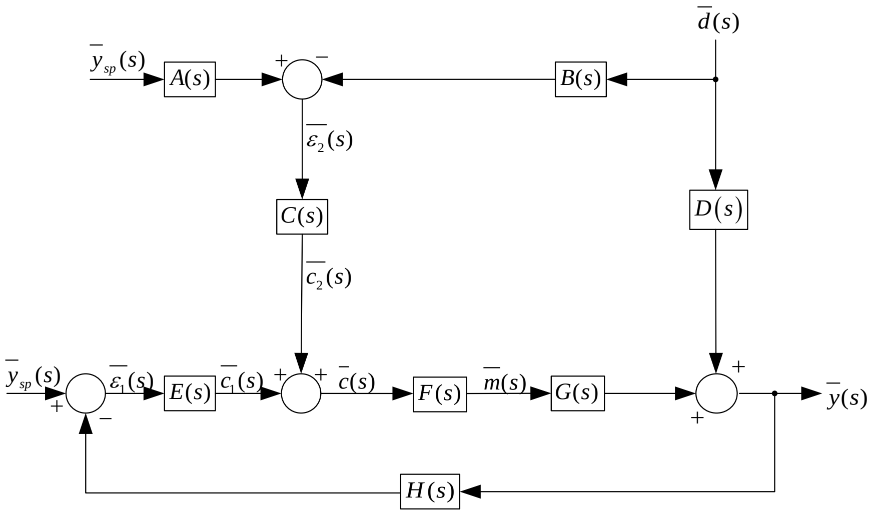
Consider the feedforward-feedback control system illustrated below.

a) Explain whether the disturbance must be measured.

b) Explain whether the controlled variable must be measured.

c) Specify when the feedback system will act.

d) Study the stability of this control system



Solution

a) The very nature of the operation of a feedforward control system involves the measurement of the disturbance, which is a basic measure when the aim is to achieve the advantages of the feedforward systems.

b) The feedforward system control presents several problems that are not detectable, since it does not measure the controlled variable. However, the feedback system does, and it allows checking whether the controlled variable is not maintained within the desired value, correcting the error if necessary.

c) As stated in the previous section, the feedback system will perform when the feedforward system is unable to correct the errors introduced by the disturbance.

d) Calculating the value of the controlled variable gives:

$$\overline{y}(s) = G(s)\overline{m}(s) + D(s)\overline{d}(s)$$

Calculating the value of the manipulated variable gives:

$$\overline{m}(s) = F(s)\overline{c}(s) = F(s)\left(\overline{c}_1(s) + \overline{c}_2(s)\right) = F(s)E(s)\overline{\varepsilon}_1(s) + F(s)C(s)\overline{\varepsilon}_2(s)$$

Calculating the value of $\overline{y}(s)$ from the previous expressions and arranging gives:

$$\overline{y}(s) = \frac{G(s)F(s)\left(E(s) + \overline{c}(s)A(s)\right)}{1 + E(s)F(s)G(s)H(s)}\overline{y}_{sp}(s) + \frac{D(s) - G(s)F(s)C(s)B(s)}{1 + E(s)F(s)G(s)H(s)}\overline{d}(s)$$

Looking at the overall response of the system, it can be seen that the denominator is the same and has the characteristic equation expression of the feedback control. In this way, studying the feedback stability will reveal the stability of the overall loop.

Problem 3.1.7

The following transfer functions have been identified for a distillation column:

$$\frac{\overline{x_D}}{\overline{x_F}} = \frac{25e^{-5s}}{5s+1}$$

$$\frac{\overline{x_D}}{\overline{R}} = \frac{5e^{-0.5s}}{s+1}$$

where: x_D is the composition of the distillate product (controlled variable).

x_F is the feed composition (disturbance).

R is the reflux flow (manipulated variable).

a) Design a feedforward control system that rejects any changes in the disturbance or the set point. In addition, determine its transfer functions by assuming that the transfer functions not indicated are unity.

b) Calculate the value of x_D when x_F increases suddenly by 0.1 units.

c) Assuming that $\frac{\overline{x_D}}{\overline{x_F}} = \frac{20e^{-4s}}{35s+1}$ and, considering the control system designed before, calculate the value of x_D when x_F increases suddenly by 0.1 units. Compare the results with that obtained in section b).

Solution

a) Considering the statement values gives:

$$G_p(s) = \frac{\overline{x}_D}{\overline{R}} = \frac{5e^{-0.5s}}{s+1}$$

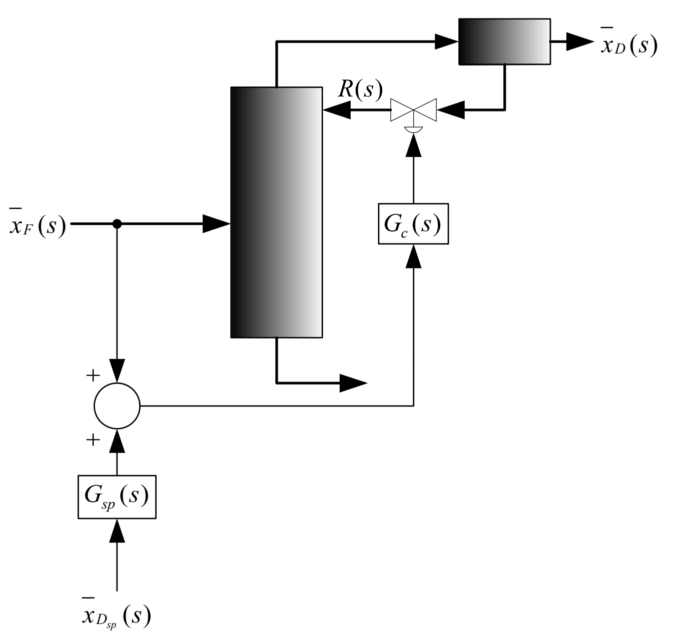
$$G_d(s) = \frac{\overline{x}_D}{\overline{x}_F} = \frac{25e^{-5s}}{5s+1}$$

The values of $G_{sp}(s)$ and $G_c(s)$ can be calculated using the above expressions, taking into account that the other transfer functions are unity:

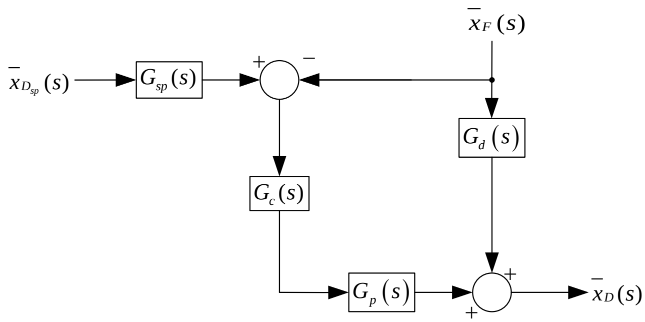
$$G_{sp}(s) = \frac{1}{\overline{G}_d(s)} = \frac{5s+1}{25e^{-5s}}$$

$$G_c(s) = \frac{G_d(s)}{G_p(s)} = \frac{25e^{-5s}}{5s+1} : \frac{5e^{-0.5s}}{s+1} = \frac{s+1}{5s+1}(5)e^{-4.5s}$$

The process is depicted by the following schema:



b) The block diagram is shown below to make the calculation of $\overline{x}_D(s)$ easier:



Considering the above block diagram and assuming that the set point does not vary:

$$\bar{x}_D(s) = G_d(d)\bar{x}_F(s) - G_p(s)G_c(s)\bar{x}_F = \frac{25e^{-5s}}{5s+1}\frac{0,1}{s} - \frac{5e^{-0.5s}}{s+1}\frac{s+1}{5s+1}5e^{-4.5s}\frac{0,1}{s} = 0$$

c) Considering the new value of the transfer function gives:

$$\bar{x}_D(s) = G_d(d)\bar{x}_F(s) - G_p(s)G_c(s)\bar{x}_F(s) = \frac{20e^{-4s}}{3s+1}\frac{0.1}{s} - \frac{5e^{-0.5s}}{s+1}\frac{s+1}{5s+1}5e^{-4.5s}\frac{0.1}{s} =$$

$$= \frac{0.1}{s}\left[\frac{20e^{-4s}}{3s+1} - \frac{25e^{-5s}}{5s+1}\right]$$

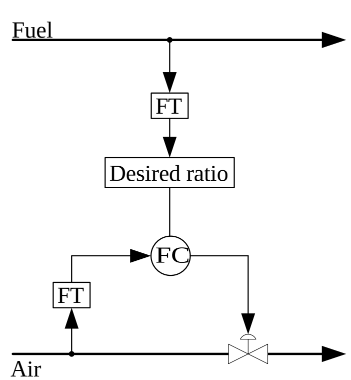
Applying the final-value theorem when t→∞ gives:

$$x_D(t \to \infty) = \lim_{s\to 0} s\frac{(0.1)}{s}\left[\frac{20e^{-4s}}{3s+1} - \frac{25e^{-5s}}{5s+1}\right] = -0.5$$

3.2 Other control systems

Problem 3.2.1

The figure below shows a ratio control system for keeping the ratio of fuel air constant:



a) Justify whether the fuel steam must be the wild stream.

b) Explain briefly how this control system works.

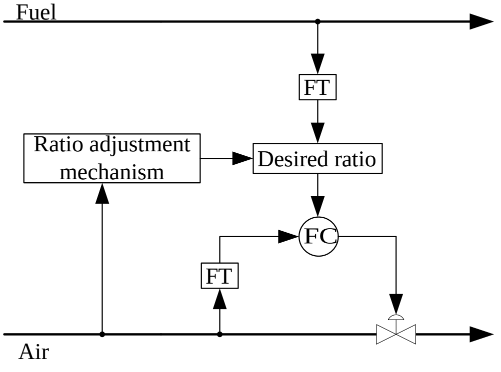
c) Propose a system to improve the performance of this ratio control system and include a graphic representation.

Solution

a) In any chemical industrial plant, there is a demand for fuel that depends on the specific needs of production. For this reason, the fuel stream will have sudden oscillations that are linked to punctual needs of production. The air stream must be regulated according to the fuel flow rate in order to maintain the best ratio between them.

b) The requirement for fuel flow is detected by a flow measurement system and sent to a converter that multiplies this value by the ratio fuel/air, which has been defined by the control system user. This value is that of the air flow rate, and it is regulated by the control system shown in the figure above.

c) The optimal fuel/air ratio is not a constant value throughout the development of an industrial process because temperature has a profound influence. For this reason, varying the fuel/air ratio according to the air temperature would be suitable in order to improve the performance of combustion. One way to optimize the combustion process is by introducing a programmed adaptive control system in which measuring the air temperature allows a choice of the most suitable fuel/air ratio. The diagram of this system is shown in the figure below.



Problem 3.2.2

Two gaseous streams (nitrogen and hydrogen) are fed into a reactor to synthetize ammonia and obtain a determined production. Justify a ratio control system for maintaining the desired ratio between the streams entering the reactor. Indicate the wild stream, the controlled stream and the ratio between them.

Solution

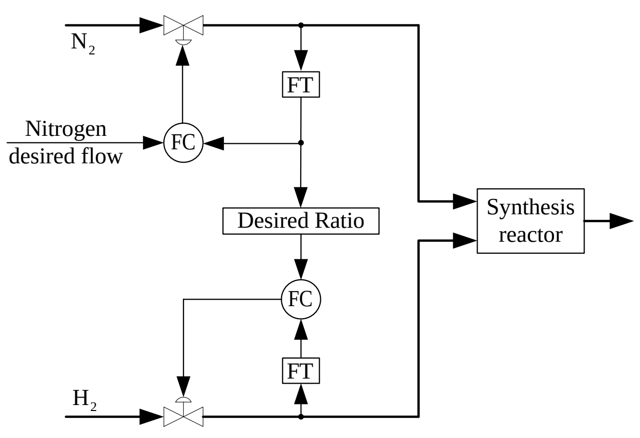
This is a typical example of where a ratio control system should be applied, since two reactives are introduced into a reactor and a stequiometric ratio between them must be maintained.

The reaction of ammonia synthesis is:

$$3H_2 + N_2 \rightleftarrows 2NH_3$$

It is known that the molar ratio matches the volume ratio; so the regulation of the volumetric flow by means of a ratio control system will allow the necessary molar ratio to be achieved. The stequiometric ratio is: $\frac{N_2}{H_2} = \frac{1}{3}$.

From a chemical point of view, either of them can be considered the wild flow; thus, the other one will be related to the value of the free stream. The figure below shows a ratio control system where it can be seen that the nitrogen is controlled in order to obtain a previously established flow. The hydrogen flow is regulated by the ratio control system. In addition, this system has a feedback control system.

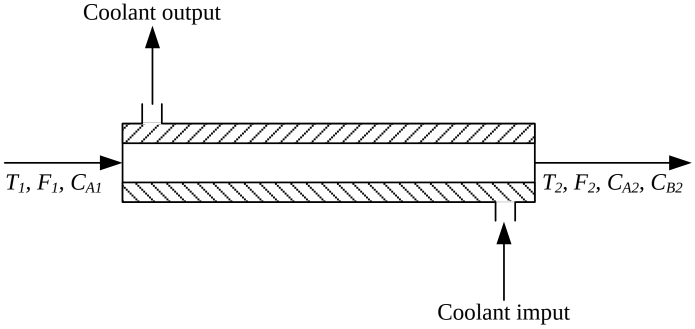


Problem 3.2.3

To control the product composition that leaves the catalytic tubular reactor shown below, some temperature measurements are taken throughout the whole reactor. Consider that the reaction is exothermic and heat is removed by a coolant. The reaction that takes place in the reactor is: $A \rightarrow B$.

a) Plot the corresponding inferential control system. Indicate each variable and determine the output expression. Consider the input concentration as the disturbance.

b) Due to continued operation, a decrease in the activity of the catalyst is detected. Plot the scheme for an adaptive control system which adjusts the controller parameters using the information from the output composition, which is periodically supplied by a gas chromatograph.



Solution

a) The corresponding variables are:

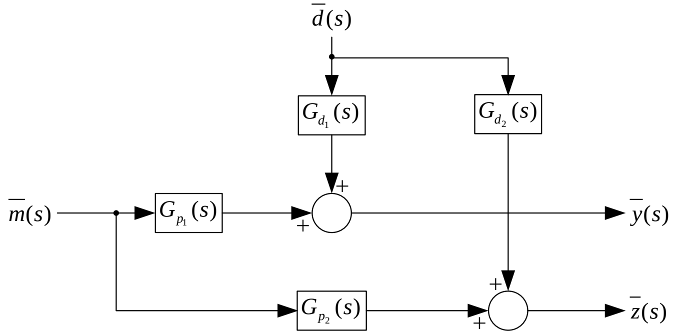
Controlled variable: output product composition (unmeasured value), $\bar{y}$

Secondary measurement (auxiliary): reaction temperature, $\bar{z}$

Manipulated variable: coolant flow, $\bar{m}$

Disturbance: input concentration, $\bar{d}$

The scheme corresponding to the inferential control system is shown below:



From the above block diagram the following equations can be inferred:

$$\bar{y}(s)=G_{p1}(s)\bar{m}(s)+G_{d1}(s)\bar{d}(s)$$

$$\bar{z}(s)=G_{p2}(s)\bar{m}(s)+G_{d2}(s)\bar{d}(s)$$

Calculating the value of $\bar{d}(s)$ from the second equation above gives:

$$\bar{d}(s)=\frac{1}{G_{d_2}(s)}\bar{z}(s)-\frac{G_{p_2}(s)}{G_{d_2}(s)}\bar{m}(s)$$

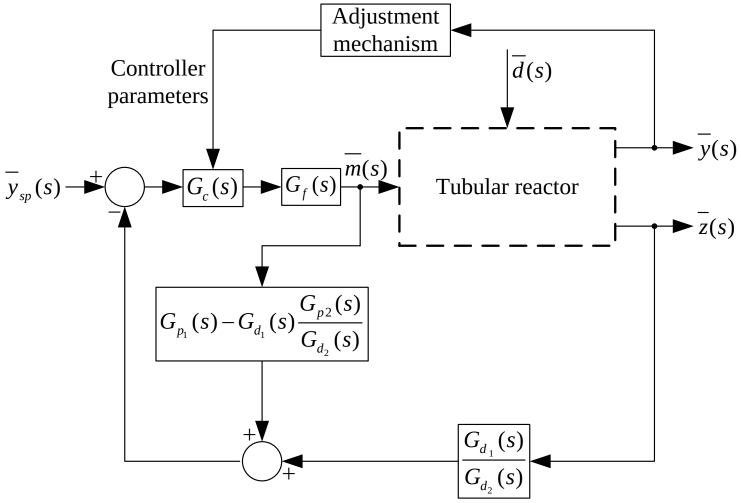
Substituting the value of $\bar{d}(s)$ in the first equation gives:

$$\bar{y}(s)=G_{p_1}(s)\bar{m}(s)+G_{d_1}(s)\left[\frac{1}{G_{d_2}(s)}\bar{z}(s)-\frac{G_{p_2}(s)}{G_{d_2}(s)}\bar{m}(s)\right]$$

Rearranging the above equation gives:

$$\bar{y}(s)=\left[G_{p_1}(s)-G_{d_1}(s)\frac{G_{p_2}(s)}{G_{d_2}(s)}\right]\bar{m}(s)+\frac{G_{d_1}(s)}{G_{d_2}(s)}\bar{z}(s)$$

b) The adaptive control applied to the inferential control system is shown in the diagram below:



Problem 3.2.4

Studying the variables of a distillation column gives the following equations:

$$\bar{y}(s) = \frac{2e^{-s}}{50s+1}\bar{d}(s) + \frac{e^{-s}}{40s+1}\bar{m}(s)$$

$$\bar{z}(s) = \frac{e^{-s}}{80s+1}\bar{d}(s) + \frac{2}{10s+1}\bar{m}(s)$$

Where: $\bar{y}$ is the distillate product composition (not measurable).

$\bar{z}$ is the top tray temperature (secondary measurement).

$\bar{d}$ is the feed composition (disturbance).

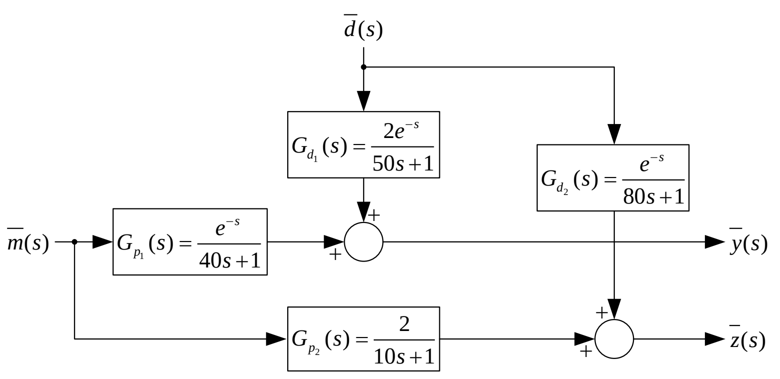
$\bar{m}$ is the reflux ratio (manipulated variable)

a) Plot the block diagram of the distillation column, including the variables indicated before.

b) Plot the corresponding inferential control system.

Solution

a) Forming a block diagram according to the statement data gives:



The equations shown below can be obtained from the above block diagram:

$$\bar{y}(s) = G_{p1}(s)\bar{m}(s) + G_{d1}(s)\bar{d}(s)$$

$$\bar{z}(s) = G_{p2}(s)\bar{m}(s) + G_{d2}(s)\bar{d}(s)$$

b) To plot the inferential system, the value of $\bar{y}(s)$ must be obtained in function of the system variables.

Isolating $\bar{d}(s)$ from the second equation gives:

$$\bar{d}(s)=\frac{1}{G_{d2}(s)}\bar{z}(s)-\frac{G_{p2}(s)}{G_{d2}(s)}\bar{m}(s)$$

Substituting in the first equation gives:

$$\bar{y}(s)=G_{p1}(s)\bar{m}(s)+G_{d1}(s)\left[\frac{1}{G_{d2}(s)}\bar{z}(s)-\frac{G_{p2}(s)}{G_{d2}(s)}\bar{m}(s)\right]=$$
$$=\left[G_{p1}(s)-\frac{G_{d1}(s)}{G_{d2}(s)}G_{p2}(s)\right]\bar{m}(s)+\frac{G_{d1}(s)}{G_{d2}(s)}\bar{z}(s)$$

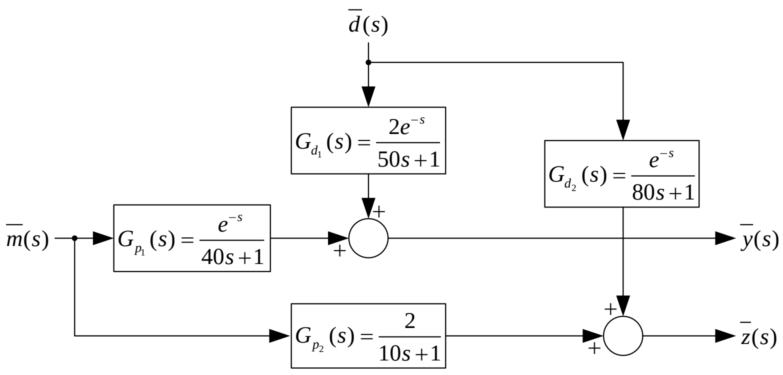
Substituting the statement values in the equation of $\bar{y}(s)$ gives:

$$\bar{y}(s)=\left[\frac{e^{-s}}{40s+1}-\frac{2e^{-s}}{50s+1}\frac{80s+1}{e^{-s}}\frac{2}{10s+1}\right]\bar{m}(s)+\frac{2e^{-s}}{50s+1}\frac{80s+1}{e^{-s}}\bar{z}(s)$$

Rearranging gives:

$$\bar{y}(s)=\left[\frac{e^{-s}}{40s+1}-\frac{4(80s+1)}{(50s+1)(10s+1)}\right]\bar{m}+\frac{2(80s+1)}{50s+1}\bar{z}(s)$$

From the above equations, the schema corresponding to the inferential control system can be plotted:



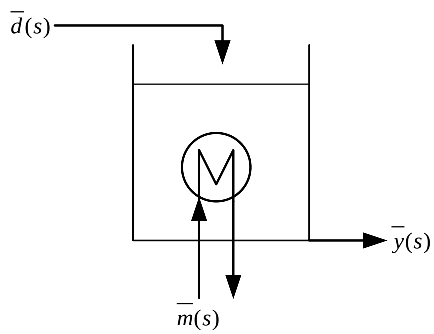
Problem 3.2.5

a) Develop an inferential control system for a CSTR (Continuous Stirred Tank Reactor), using temperature measurements to keep the output concentration within a desired value. Assume that the reaction is exothermic and that the reacting mass is cooled by water that goes through a heat exchanger around the reactor. Also, suppose that the disturbance is the feed concentration (not measurable).

b) The overall coefficient of heat transfer between water and reacting mass decreases because the exchanger walls get dirty. Draw an adaptive control system that uses measurements of output composition at intervals of time in order to adjust the controller parameters.

Solution

a) According to the problem statement, the CSTR schema is plotted below:



It is essential to define the variables in each case and to know the ones which can be measured or not. The system variables are:

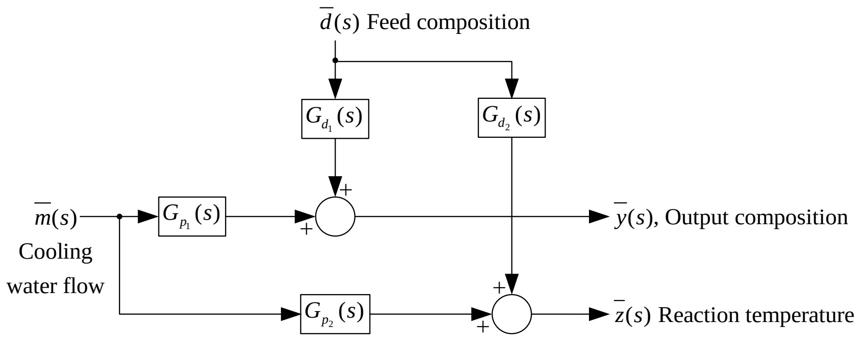
Controlled variable, $\bar{y}(s)$: output concentration (not measured).

Secondary variable: $\bar{z}(s)$: temperature of the reacting mass (measured variable).

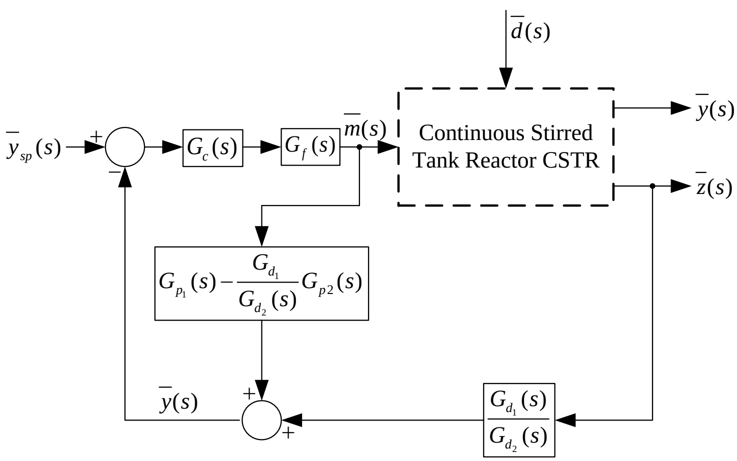
Manipulated variable, $\bar{m}(s)$: cooling water.

Disturbance, $\bar{d}(s)$: feed composition.

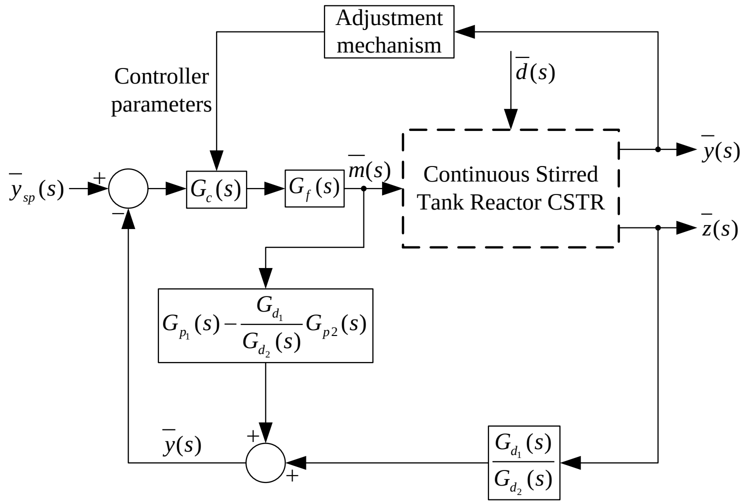
Considering the above system and variables, the block diagram that represents the CSTR inferential control is:



Considering the overall process, the concepts of block algebra which were used in previous problems are applied, and a new scheme is drawn by taking into account the above inferential system.



b) Taking the above scheme into account, an adaptive control mechanism is introduced which allows the generation of controller parameter values from the values of the output composition.


Adjustment mechanism
Controller parameters
$\bar{d}(s)$
$\bar{y}_{sp}(s)$
+
−
$G_c(s)$
$G_f(s)$
$\bar{m}(s)$
Continuous Stirred Tank Reactor CSTR
$\bar{y}(s)$
$\bar{z}(s)$
$G_{p_1}(s) - \frac{G_{d_1}}{G_{d_2}(s)} G_{p2}(s)$
$\bar{y}(s)$
+
+
$\frac{G_{d_1}(s)}{G_{d_2}(s)}$

→4

Automation of Discrete Processes

Introduction

Technological elements that enable practical development are important factors for controlling and automating processes. For this reason, this chapter focusses on several practical elements and methods that allow automation and control projects to be carried out in a real environment.

The Programmable Logic Controllers (PLC's) have become part of the most relevant industrial equipment for developing projects in chemical engineering and other branches of the industries. A good knowledge of PLC programming requires previous study of Boolean algebra for combinational and sequential systems.

For this initial study, the problems have been designed with increasing difficulty. They include combinational and sequential exercises of pneumatic systems that introduce the philosophy of technology without requiring extensive knowledge for their comprehension. Care has been taken to propose some problems that are intuitive, allowing other more sophisticated and complex techniques to be extrapolated.

The second section of this chapter introduces the Ladder as the elementary programming of PLC's. This is the most extended programming language based on the association of contacts and coils whose combination with timers and counters allows a great number of combinational and sequential automation processes to be designed and applied in chemical industries.

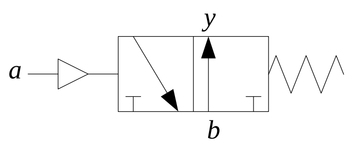
The ease of decomposing a process into different steps with actions associated to each one allows the use of a method named SFC (Sequential Function Chart), also known as GRAFCET. This simplifies the study of processes and their programming by graphically representing their sequential steps. Transitions between steps allow the process' behaviour to be defined, as well as the subsequent Ladder program.

It should also be kept in mind that in many cases a recommended solution is not always the only solution; other viable approaches may be possible.

4.1 Combinational and sequential pneumatic systems

Problem 4.1.1

Obtain the truth table corresponding to the pneumatic valve of the schema, where *a* and *b* are the inputs and *y* is the output.



Solution

The above figure correspond to a 3/2 pneumatically piloted valve with spring return. The first step consists of using their behaviour to determine the truth table. This is done by applying the logic levels of the inputs (*a* and *b*) and inferring the value of the output (*y*).

The truth table obtained is shown below:

a	*b*	*y*
0	0	0
0	1	1
1	0	0
1	1	0

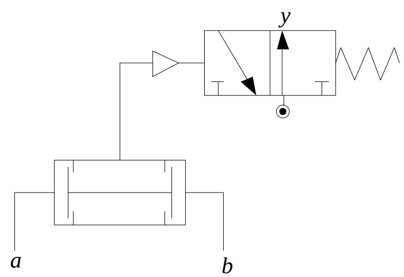
From the above table the equation of the output for any combination of inputs can be obtained:

$$y = \overline{a} \cdot b$$

A valve with the behaviour indicated by the table is named *inhibition* valve.

Problem 4.1.2

Obtain the truth table and the simplified equation that satisfies the valves system from the schema below.

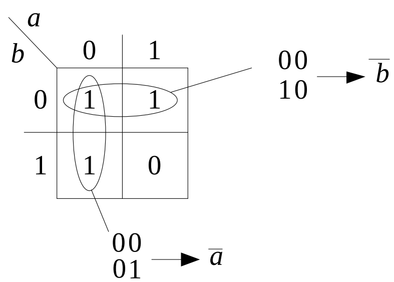


Solution

By using *a* and *b* as inputs and *y* as output, as in the above two-valve schema, the truth table corresponds with the one below:

a	***b***	***y***
0	0	1
0	1	1
1	0	1
1	1	0

Simplifying the Karnaugh method gives:



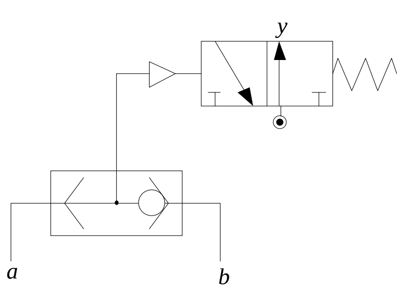
The groups of values in the above table make the output true. They are used to simplify the function, which leads to the result:

$$y = \overline{a} + \overline{b} = \overline{a \cdot b}$$

The final expression is calculated by applying the De Morgan theorem.

Problem 4.1.3

Obtain the truth table of the two-valve system below. Calculate the most simplified expression.



Solution

There are two valves connected in series: a 3/2 pneumatic piloted valve with a spring return, and a specific "OR" function valve. The system behaviour is obtained by applying signals to the two inputs (*a* and *b*) and inferring the value of its output (*y*). The truth table corresponds to the one below:

a	*b*	*y*
0	0	1
0	1	0
1	0	0
1	1	0

From the above table, the equation of the output for any combination of inputs can be obtained:

$$y = \overline{a} \cdot \overline{b} = \overline{a + b}$$

The final expression is calculated by applying the De Morgan theorem.

Problem 4.1.4

A combinational system with four inputs is used to add two reactives to a reactor. The inputs are: temperature (*a*), pressure (*b*), level (*c*) and composition (*d*). The addition of the reagent *A* or the reagent *B* accomplishes the truth table shown below.

a	*b*	*c*	*d*	*A*	*B*
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	1	0	0

Design a pneumatic system that conforms to the above truth table. Obtain the most simplified expression of the function.

Solution

The system has two outputs, *A* and *B* that are not related. This allows studying each one separately.

Addition of reagent *A*:

The first step is to form the grid below, which is extracted from the truth table for reagent *A*:

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	0	0	0	0
10	0	0	0	0

The Karnaugh simplification must be applied by grouping the elements in the grid as well as those indicated in the above truth above table. Also, the largest number of elements must be grouped in a power of two, when possible:

$$\left.\begin{matrix} 0100 \\ 1100 \\ 0101 \\ 1101 \end{matrix}\right\} \rightarrow b \cdot \overline{c}$$

$$\left.\begin{matrix} 1101 \\ 1001 \end{matrix}\right\} \rightarrow a \cdot \overline{c} \cdot d$$

In order to add the reagent A , the simplified expression corresponds to:

$$A = b \cdot \overline{c} + a \cdot \overline{c} \cdot d = \overline{c} \cdot (b + a \cdot d)$$

Adding reagent *B*:

Proceeding as in the previous case and extracting values for reagent *B* gives:

cd \ ab	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	1	1	0

The function is simplified from the above groups:

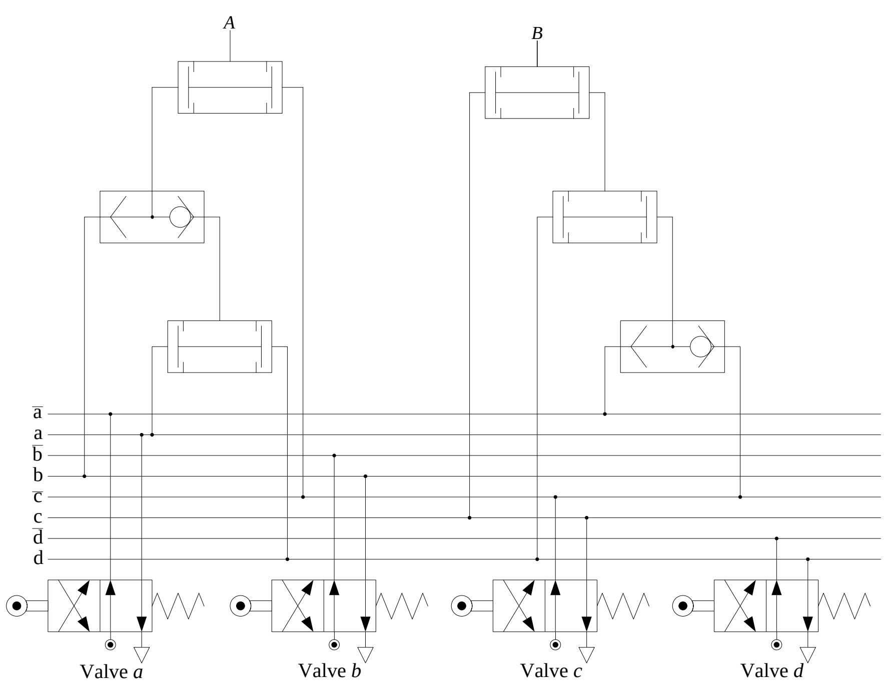
$$\left.\begin{matrix}0110\\1110\end{matrix}\right\} \rightarrow b{\cdot}c{\cdot}\overline{d}$$

$$\left.\begin{matrix}0010\\0110\end{matrix}\right\} \rightarrow \overline{a}{\cdot}c{\cdot}\overline{d}$$

In order to add reagent *B*, the simplified expression corresponds to:

$$B = b{\cdot}c{\cdot}\overline{d} + \overline{a}{\cdot}c{\cdot}\overline{d} = c{\cdot}\overline{d}\,(\overline{a} + b)$$

Taking the above simplified expressions of *A* and *B*, the combinational pneumatic circuit can be designed:



In order to generate binary input signals, 4/2 valves have been used. The two outputs of each 4/2 valve are used to generate the ON and the OFF states. It's possible to design an equivalent system using two valves to generate each signal, although it is a more expensive solution. The activation of the valves may be of a distinct nature.

Problem 4.1.5

The heating system of a reactor is handled by the signal generated by three temperature sensors, their combination lets the steam flow by opening or closing the valve. These actions are performed according to the combination indicated in the truth table (O: open; C: Closed).

a	***b***	***c***	***s***
0	0	0	O
0	0	1	O
0	1	0	C
0	1	1	C
1	0	0	O
1	0	1	C
1	1	0	O
1	1	1	C

Design a combinational simplified pneumatic system that corresponds with the above truth table and generates two signals (valve open or closed). Note: Use valves 4/2.

Solution

Before solving the problem, it is necessary to separate the two situations considered in the table: when the valve is open and when it is closed. All the data are extracted from the truth table.

To open the valve:

c \ *ab*	00	01	11	10
0	O		O	O
1	O			

The Karnaugh simplification must be applied as in the previous problem:

$$\left.\begin{matrix}000\\001\end{matrix}\right\} \rightarrow \bar{a}\cdot\bar{b}$$

$$\left.\begin{matrix}110\\100\end{matrix}\right\} \rightarrow a\cdot\bar{c}$$

From the above expressions:

$$O = \bar{a}\cdot\bar{b} + a\cdot\bar{c}$$

To close the valve:

$c \backslash ab$	00	01	11	10
0		C		
1		C	C	C

From the elements selected, the simplified groups are described below:

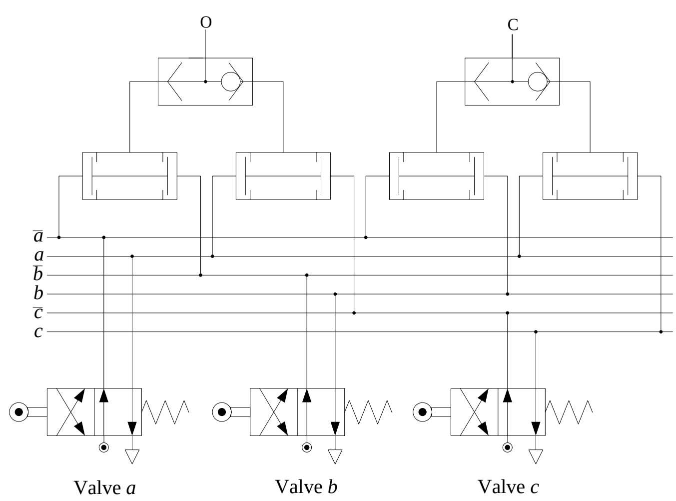
$$\left.\begin{matrix} 010 \\ 011 \end{matrix}\right\} \rightarrow \bar{a}\cdot b$$

$$\left.\begin{matrix} 111 \\ 101 \end{matrix}\right\} \rightarrow a\cdot c$$

Finally, the second expression corresponds to:

$$T = \bar{a}\cdot b + a\cdot c$$

At this point, the combinational pneumatic circuit can be designed:



As can be seen, a solution which is similar to the previous problem has been chosen. Valves 4/2, which allow generating two binary signals, are used as inputs for the combinational system. The two outputs must operate an element that allows the steam valve to open or close. These actions could be accomplished by using a double acting cylinder.

Problem 4.1.6

A valve that lets cooling water flow in a reactor where there is an exothermic reaction is handled according to the signal generated by three temperature sensors. The signals combination keeps the valve either open or closed. The valve opens (0) if there are at least two sensors that send signals; otherwise, the valve remains closed (1). The activation is made by a double acting cylinder commanded by a 4/2 valve. Design a combinational pneumatic circuit that complies with these requirements.

Solution

The logic state of sensors T_1, T_2 and T_3 are "0" when the temperature has not reached the programmed value, and "1" when the temperature is equal to or greater than the programmed value.

The truth table below has been derived from the data of the statement. In this case the table has three inputs and one output:

T1	***T2***	***T3***	***V***
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

The output is used to perform two signals: one to open and the other one to close. Two Karnaugh grids must be solved: one to close and the other to open.

To close the valve, the combinations that make the output be true must be taken. From the truth table:

T_3 \ T_1T_2	00	01	11	10
0	1	1		1
1	1			

As shown in the map, an element located at the end of the table has been grouped with another element at the opposite end. This kind of grouping is in accordance with the simplification rules of the Karnaugh method, so as to generate more simplified functions.

The simplifications are below:

$$\left.\begin{matrix}000\\010\end{matrix}\right\}\rightarrow \overline{T_1}\cdot\overline{T_3}$$

$$\left.\begin{matrix}000\\001\end{matrix}\right\}\rightarrow \overline{T_1}\cdot\overline{T_2}$$

$$\left.\begin{matrix}000\\010\end{matrix}\right\}\rightarrow \overline{T_2}\cdot\overline{T_3}$$

The simplified expression to close the valve corresponds to:

$$V=\overline{T_1}\cdot\overline{T_3}+\overline{T_1}\cdot\overline{T_2}+\ \overline{T_2}\cdot\overline{T_3}$$

This expression could be simplified by applying Boolean algebra rules; but that is not necessary for solving the problem.

To open the valve, the grid is filled with the combinations that make the output false. From the truth table:

T_3 \ T_1T_2	00	01	11	10
0			0	
1		0	0	0

Grouping the values in the above table gives:

$$\left.\begin{matrix}110\\111\end{matrix}\right\}\rightarrow T_1\cdot T_2$$

$$\left.\begin{matrix}011\\111\end{matrix}\right\}\rightarrow T_2\cdot T_3$$

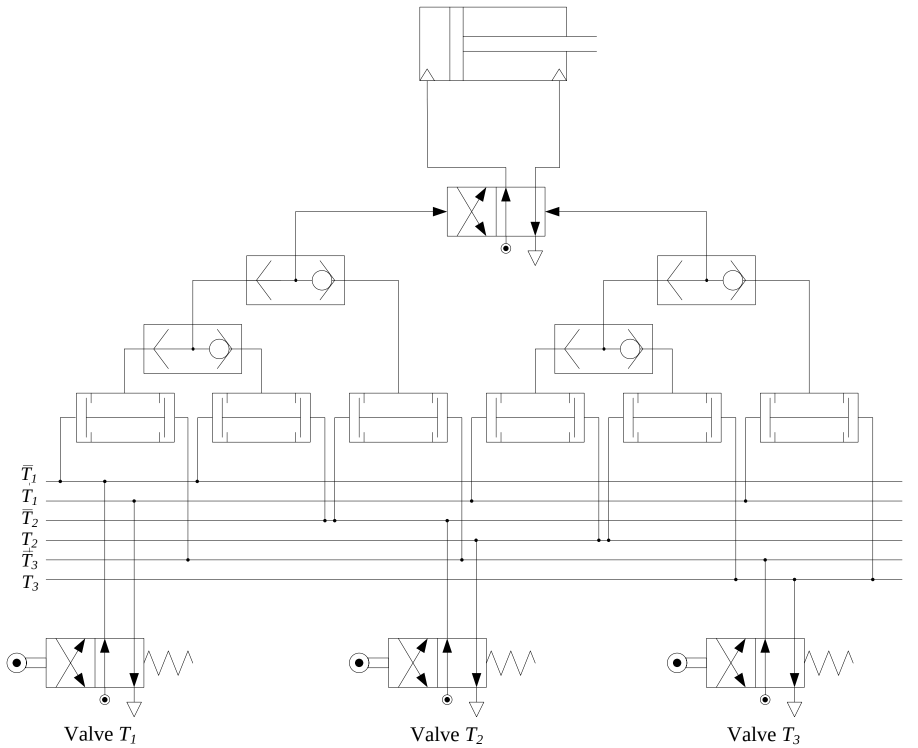
$$\left.\begin{matrix}111\\101\end{matrix}\right\}\rightarrow T_1\cdot T_3$$

The simplified expression for opening the valve corresponds to:

$$\overline{V}=T_1\cdot T_2+T_2\cdot T_3+T_1\cdot T_3$$

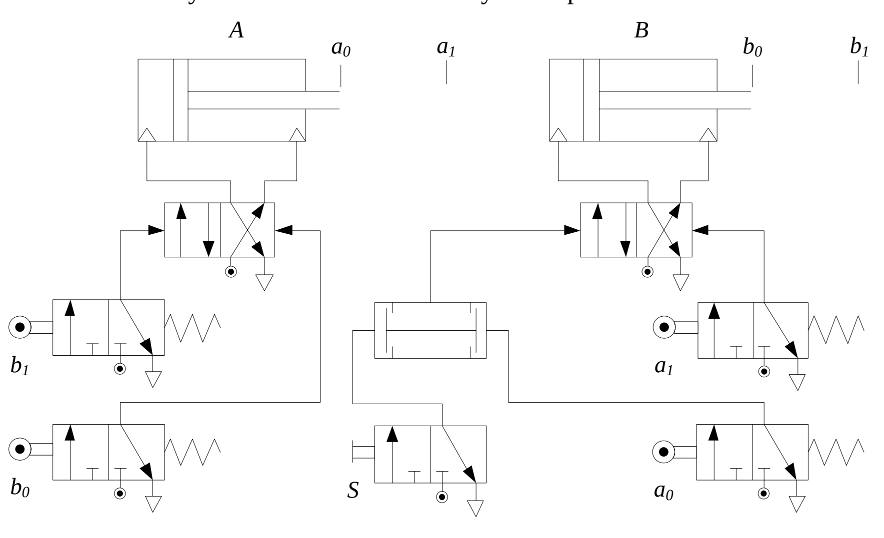
As above, this expression could be more simplified.

In this case, a double acting cylinder has been incorporated in the schema, as indicated by the statement. The cylinder will be commanded by a 4/2 valve, which will receive the orders from the two combinational circuits described above.



Problem 4.1.7

Describe the sequence performed by the pneumatic circuit when switch S is pressed in the schema below. Cylinders A and B are initially in the position shown.



Solution

To solve this example, a signal is applied to the *S* button. The sequence of the cylinders will be the one below:

$$B+A+B-A-$$

Problem 4.1.8

Design the logical functions to develop the sequence *A+ A- B+ B-* for a pneumatic circuit using double acting cylinders, and draw its schema.

Solution

The first step in solving the problem is to assemble the table with the movements indicated by the sequence of the statement. The table presents the columns, where the combination of sensor signals (*a*,*b*) generate each movement of the sequence. They are indicated by “0” or “1”, depending on the position (e.g., a_o or a_1, respectively). In each row, a circle indicates the variable that causes each phase of the sequence. The last column shows the decimal value of each phase. This value is obtained from the binary combination assigned to its sensors. This value allows to identify repeated phases by seeking repeated numerical values. In this case, there are several repeated phases.

	a	***b***	***n***	
A+	0	0	0	x+
A-	1	0	2	
B+	0	0	0	x-
B-	0	1	1	

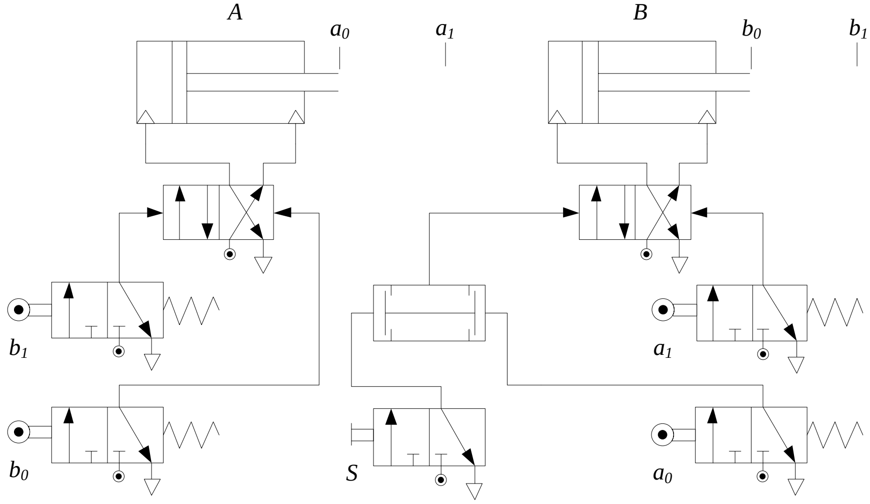
As shown in the table, there are some repeated values; so, a new sequence should be created to avoid repeating them. First of all, new phases must include (*x*). In the repeated phase, it is necessary to go back a row and half to insert a new sequence. Even phases must be inserted always; first by placing the positive values, then the negative ones.

	a	***b***	***x***	***n***
A+	0	0	0	0
X+	1	0	0	4
A-	1	0	1	5
B+	0	0	1	1
X-	0	1	1	3
B-	0	1	0	2

From now on, the simplified expression of each movement can be obtained. The combination that defines the phase must not be repeated in the opposite movement, as seen below:

$$a+ = b_0 x_0$$
$$x+ = a_1$$
$$a- = x_1$$
$$b+ = a_0 x_1$$
$$x- = b_1$$
$$b- = x_0$$

With the above results, the pneumatic sequential circuit can be designed. In order to develop the pneumatic circuit from the statement, two double-acting cylinders must be used together with their commanding 4/2 valves. In addition, another valve without any cylinder must be placed in the circuit to solve the new sequence. To start the sequence, one 3/2 valve commanded by a push button must be installed. The schema below corresponds to the solution.



Problem 4.1.9

A pneumatic circuit follows the sequence below:

A+B+C+C-B-A-

Determine the simplified functions that accomplish this sequence.

Solution

Proceeding as in the previous problem, the phase table is mounted by adding the new rows that avoid repeating the phases.

	a	*b*	*c*	*n*	
***A*+**	0	0	0	0	
***B*+**	1	0	0	4	
***C*+**	1	1	0	6	*x*+
***C*-**	1	1	1	7	
***B*-**	1	1	0	6	
***A*-**	1	0	0	4	
					x-

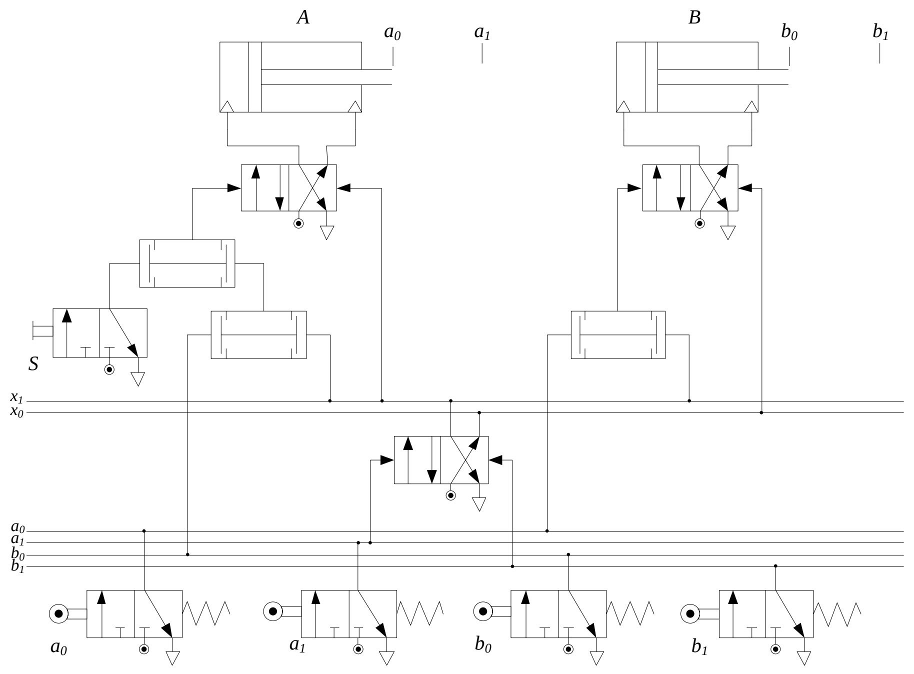
The arrow corresponding to x- could be placed at the beginning, because it deals with a cyclical process. The table containing the phases which correspond to the sequence and its auxiliary rows is shown below.

	a	*b*	*c*	*x*	*n*
***A*+**	0	0	0	0	0
***B*+**	1	0	0	0	8
***C*+**	1	1	0	0	12
***X*+**	1	1	1	0	14
***C*-**	1	1	1	1	15
***B*-**	1	1	0	1	13
***A*-**	1	0	0	1	9
***X*-**	0	0	0	1	1

The simplified functions for each movement are:

$$a+ = x_0$$
$$b+ = a_1 x_0$$
$$c+ = b_1 x_0$$
$$x+ = c_1$$
$$c- = x_1$$
$$b- = c_0 x_1$$
$$a- = b_0 x_1$$
$$x- = a_0$$

Once the simplification has been completed, the following step is to draw the schema:



Problem 4.1.10

Simplify and design a system with double acting cylinders corresponding to the sequence shown below:

$$A+A-B+C+C-B-A+A-B+B-$$

Solution

Firstly, the phase table is mounted:

	a	*b*	*c*	*n*
A+	0	0	0	0
A-	1	0	0	4
B+	0	0	0	0
C+	0	1	0	2
C-	0	1	1	3
B-	0	1	0	2
A+	0	0	0	0
A-	1	0	0	4
B+	0	0	0	0
B-	0	1	0	2

The movements that eliminate the repetitions are added to the table.

	a	*b*	*c*	*n*	
A+	0	0	0	0	*x*+
A-	1	0	0	4	
B+	0	0	0	0	
C+	0	1	0	2	*y*+
C-	0	1	1	3	
B-	0	1	0	2	
A+	0	0	0	0	*x*-
A-	1	0	0	4	
B+	0	0	0	0	*y*-
B-	0	1	0	2	

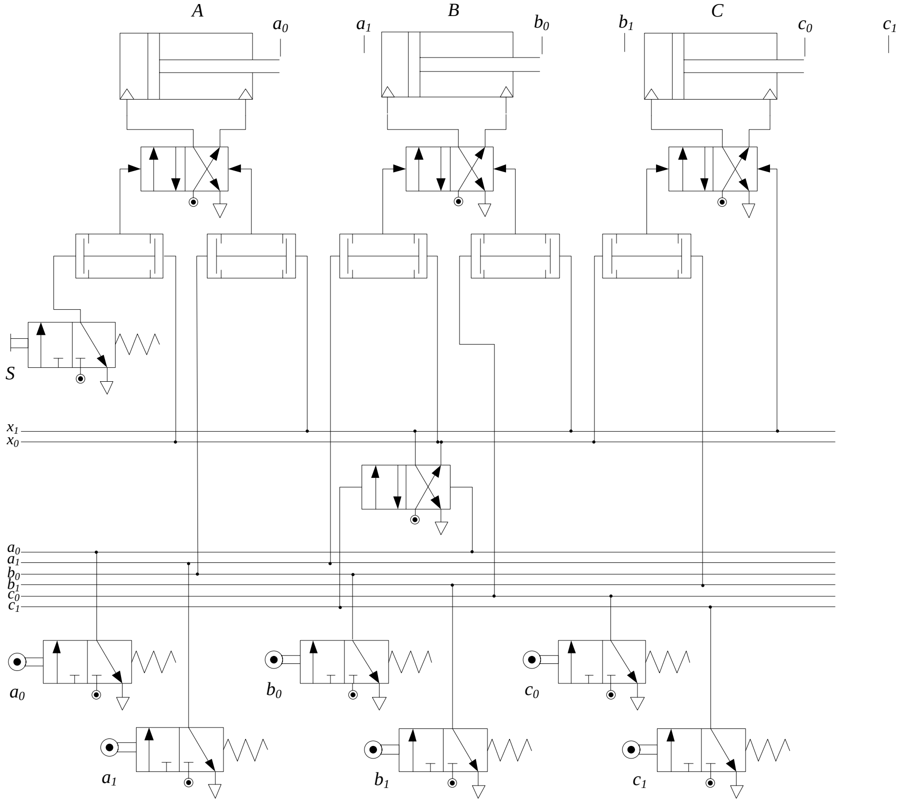
The rows corresponding to the added phases have been inserted into the table.

	a	*b*	*c*	*x*	*y*	*n*
A+	0	0	0	0	0	0
X+	1	0	0	0	0	16
A-	1	0	0	1	0	18
B+	0	0	0	1	0	2
C+	0	1	0	1	0	10
Y+	0	1	1	1	0	14
C-	0	1	1	1	1	15
B-	0	1	0	1	1	11
A+	0	0	0	1	1	3
X-	1	0	0	1	1	19
A-	1	0	0	0	1	17
B+	0	0	0	0	1	1
Y	0	1	0	0	1	9
B-	0	1	0	0	0	8

The simplified expressions of each movement are shown below:

$$a+ = b_0 x_0 y_0 + b_0 x_1 y_1 = b_0 \ (x_0 y_0 + x_1 y_1)$$
$$x+ = a_1 y_0$$
$$a- = x_1 y_0 + x_0 y_1$$
$$b+ = a_0 x_1 y_0 + a_0 x_0 y_1 = a_0 (x_1 y_0 + x_0 y_1)$$
$$c+ = b_1 x_1 y_0$$
$$y+ = c_1$$
$$c- = y_1$$
$$b- = c_0 x_1 y_1 + x_0 y_0$$
$$x- = a_1 y_1$$
$$y- = b_1 x_0$$

With the above expressions, it is possible to design the pneumatic circuit that accomplishes the sequence of the statement.



4.2 Programming a basic discrete process in Ladder and SFC

The Ladder language is the most used PLC programming system; this language tries to transfer the symbols used in the wired schemas to a programmable system. The implementation of this language differs slightly among different manufacturers. The conventions used for representing the program variables as inputs, outputs and auxiliary variables are described below, as well as the set of basic programming instructions that will be used in solving the problems. They should be reviewed before starting the exercises.

The standard Boolean variables used to solve the exercises are:

– Inputs: They refer to the Boolean value of a PLC input that indicates the state of an element in the process. They will be represented by Ix, where x indicates the input channel number.

– Outputs: They refer to the Boolean value of a PLC output that enables or disables an element connected to a channel in the process. They will be represented by Qx , where x indicates the output channel number.

– Auxiliary variables: They refer to internal Boolean variables of a PLC. They are not connected to any physical elements, but they are used to maintain the internal state values of the process. They will be represented by Mx, where x indicates the variable number.

The standard contacts used to solve the exercises are defined below:

—| |—	Open contact: If the associated value of the variable is "1" the contact is closed, and it could activate the coil. Conversely, if the value is "0", the contact is open and it prevents the activation of the next element.
—|/|—	Closed contact: If the associated value of the variable is "0" the contact is closed, and it could activate the coil. Conversely, if the value is "1", the contact is open and it prevents the activation of the next element.
—|R|—	Rising edge contact: This contact acts as a one-short rising input. When the signal transition from a low state to a high state occurs, the contact closes in the same SCAN cycle in which the edge is detected; otherwise, it remains open.
—|F|—	Falling edge contact: This contact acts as a one-short falling input. When the signal transition from a high state to a low state occurs, the contact closes in the same SCAN cycle in which the edge is detected; otherwise, it remains open.

The standard coils used to solve the exercises are defined below:

–( )–	Normal coil: When the coil is energized by the previous contacts of the rung, the output turns ON (“1”).
–(/)–	Inverse coil: When the coil is energized by the previous contacts of the rung, the output turns OFF (“0”).
–(S)–	SET coil: When the coil is energized by the previous contacts of the rung, the output turns ON (“1”); this output is remains activated, even when the coil is not energized.
–(R)–	RESET coil: When the coil is energized by the previous contacts of the rung, the output turns OFF (“0”). This coil has been previously activated by a SET coil.

Problem 4.2.1

Plot the Boolean functions described below in a Ladder diagram, and mount their corresponding truth table:

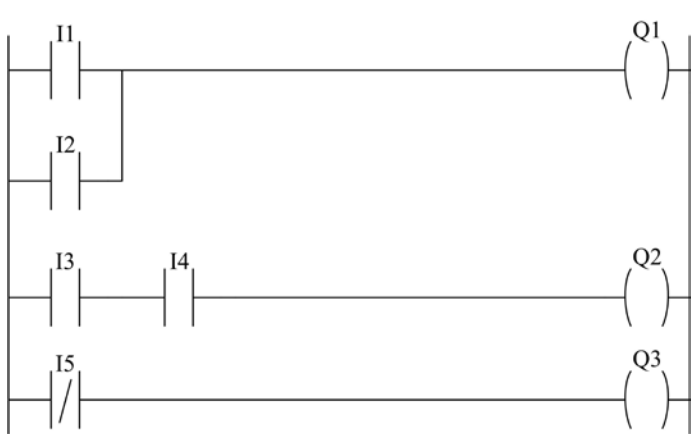
Q1=I1+I2

Q2=I3*I4

Q3=/I5

Solution

As is shown in the figure, the basic Boolean functions of the contact connections are represented as serial (AND function) or parallel (OR function).



The true tables for each output are shown below:

Q1=I1+I2

I1	I2	Q1
0	0	0
0	1	1
1	0	1
1	1	1

Q2=I3*I4

I3	I4	Q2
0	0	0
0	1	0
1	0	0
1	1	1

Q3=/I5

I5	Q3
0	1
1	0

Problem 4.2.2

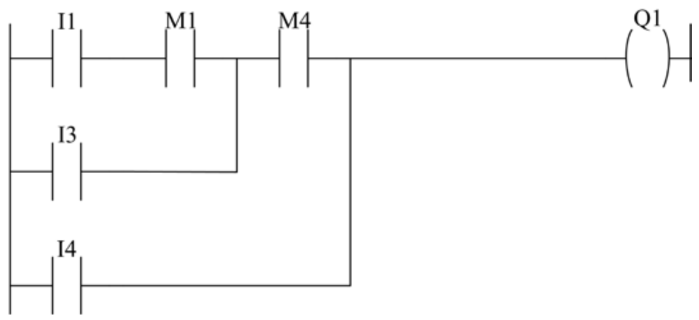
The control of an element in a chemical plan is described by the combinational function below:

Q1= ((I1*M1)+I3)*M4+I4

Plot its Ladder diagram.

Solution

After applying the basic rules to represent the functions AND and OR in Ladder language, as seen before, the result is:



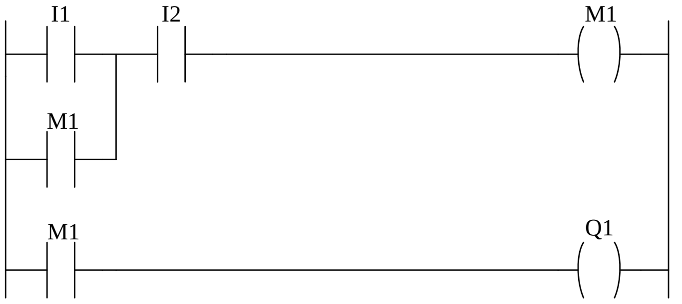
Problem 4.2.3

Design the Ladder diagram to control a valve connected to the PLC output Q1. The control is carried out by means of two buttons: *S1* (Normally Open, NO), connected to the input I1, which opens the valve; and the button *S2* (Normally Closed, NC), connected to the input I2, which closes the valve.

Solution

The system is designed using two buttons which operate in two different ways. *S1* is normally open, implying that the PLC has a true state logic in its input when the user pushes the button. Alternatively, when the user pushes *S2*, the PLC has a false state logic in its input. This situation entails using a NO contact for I2 in order to break the continuity of the rung when *S2* is pushed.

Moreover, due to the buttons returning to their initial state when they are no longer pushed, it is necessary to memorize the state of the valve by means of M1, which maintains its state. After applying the above considerations, the diagram is shown below:

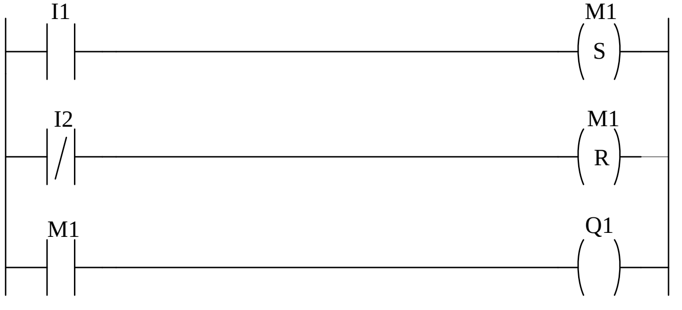


Problem 4.2.4

Repeat the above exercise, but this time by applying the SET and RESET instructions to design the Ladder program:

Solution

The SET and RESET instructions memorize and disable, respectively, the state of a variable. By using these instructions, it is not necessary to program a feedback contact, as it has already been done by placing M1 in the first rung of the above problem.



From the solution of the above schema, NC is also the contact associated to I2 (input from the NC button to close the valve). This means that when the button is pushed, the logic value of the I2 input is equal to 0; thus, the NC contact enables the rung to reset the M1 Boolean state.

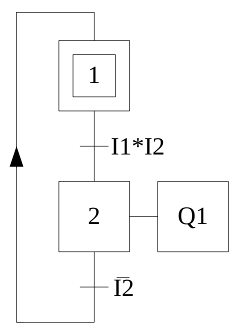
Problem 4.2.5

Plot the corresponding SFC Level 3 of the above problem.

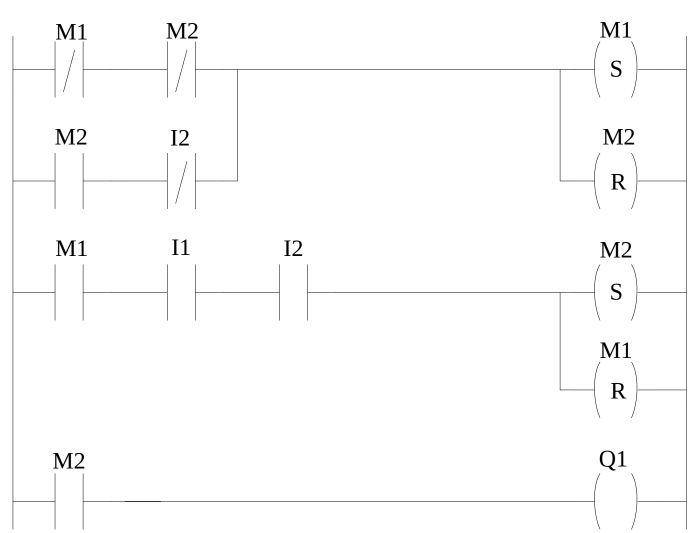
Solution

SFC Level 3 represents the sequential evolution of the process and includes the variables of the system that will be implemented. In this case, the variables correspond to I1, I2 and Q1.

The SFC is plotted below:



From the SFC, each step is assigned to each PLC variable Mx , with the same numbers indicated by the SFC. By solving the sequential evolution with SET and RESET instructions, and by assigning to each step the associated action, the following schema is plotted:



The first rung of the program initializes the SFC. This rung activates the initial step when all the steps are deactivated. In addition, to initialize the SFC, most PLCs have a system bit that indicates the first cycle of SCAN.

Problem 4.2.6

Design the program to control a valve connected to the PLC output Q1. The control is carried out through a single *S1* button (NO) connected to input I1. A rising edge in the input changes the state of the output by behaving in the following way: if the valve is open when *S1* is pressed, the PLC closes it; and if the valve is closed when *S1* is pressed, the PLC opens it.

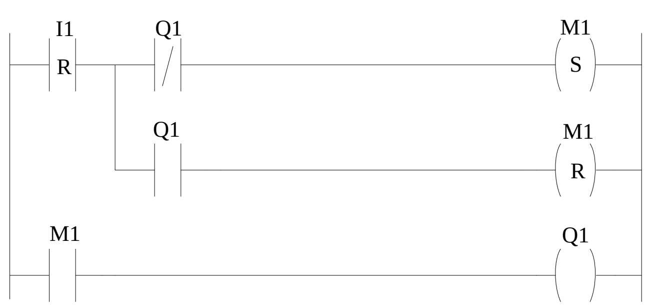
Design the process using two methods:

a) Directly with a Ladder program.

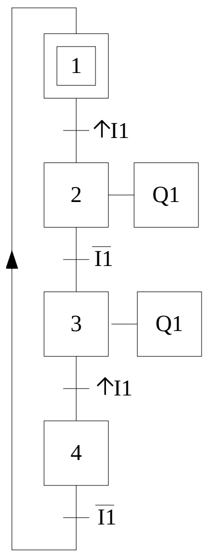
b) Using the SFC method to obtain its equivalent Ladder.

Solution

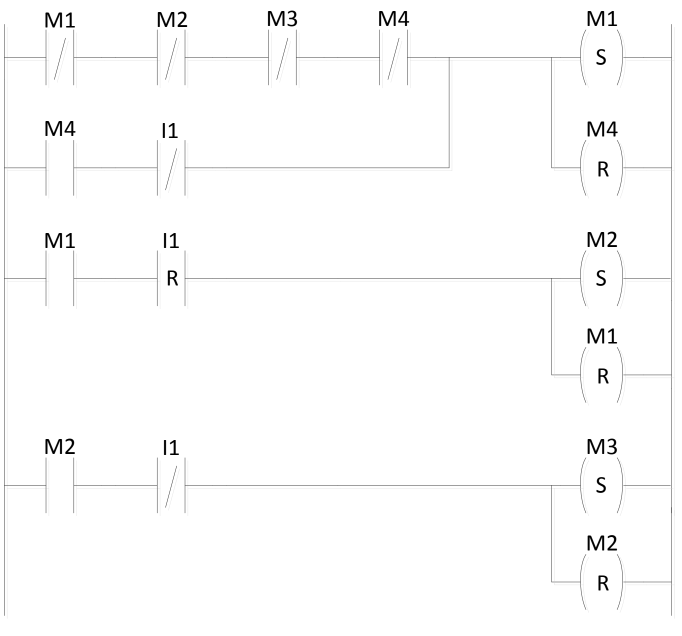
a) The rising edge contact must be used to solve the control of the valve. When an edge is detected in input I1, the SCAN cycle evaluates the Boolean state of output Q1 and changes its state on the last rung through the variable %M1.

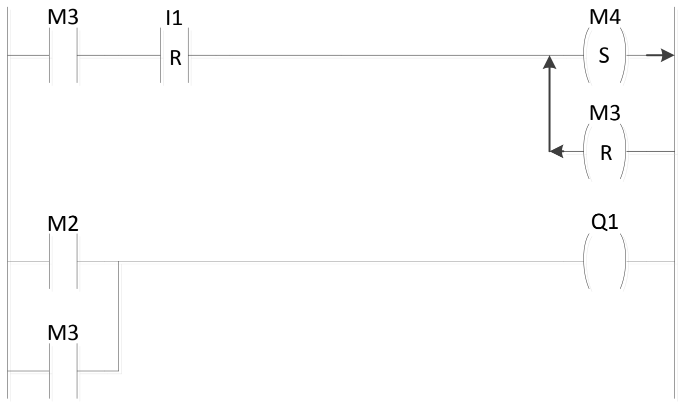


b) Solving the problem using the SFC method gives:



The SFC shows the states of the process and the steps necessary for describing it. To obtain the Ladder program from its SFC, each step must have assigned to a PLC variable Mx, whose number must match that in the SFC. In this way, it is possible to solve the sequential evolution of the SFC using SET and RESET instructions and finally assigning the associated action to each step. Following these instructions gives:



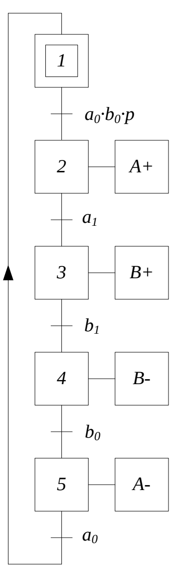


Problem 4.2.7

Design the SFC for the pneumatic sequence of two pneumatic double-acting cylinders: *A*+ *B*+ *B*- *A*-. The sequence starts when the user pushes the *p* button (NO). Each cylinder has a pair of sensors that indicate the position of the corresponding cylinder: sensors a_0, b_0, when the cylinders are in their retracted position; and sensors a_1,b_1, when the cylinders are extended. Consider both cylinders to be double acting and that they are activated by one 5/2 valve with a coil in the two actions, *A*+, *A*- and *B*+, *B*-.

Solve the SFC by using the rules for designing the ladder program.

Solution

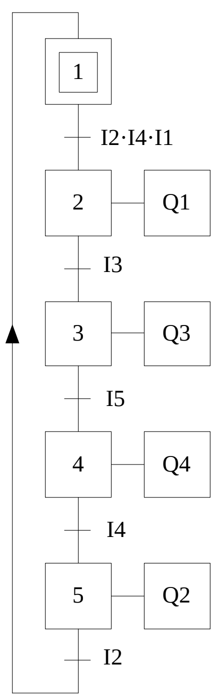


Initially, SFC level 2 is plotted by assigning the names of the elements to actions and transitions.

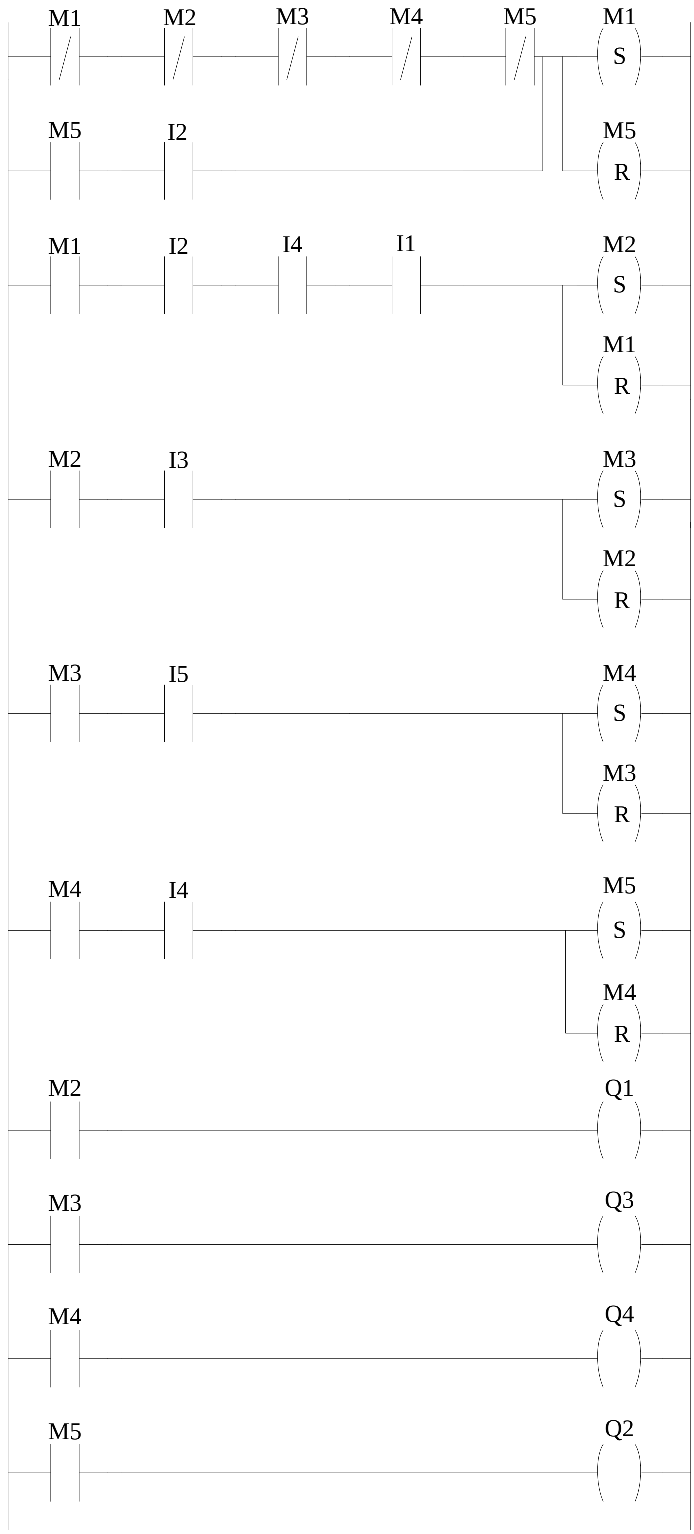
When SFC Level 2 has been plotted; the next step is to assign an input or output of the PLC to each system's physical elements and assigning an internal variable (Mx) to each step of the SFC:

Inputs		**Outputs**		**Variables**	
Element	Input	Element		Step	Variable
p	I1	A+	Q1	1	M1
ao	I2	A-	Q2	2	M2
a1	I3	B+	Q3	3	M3
b0	I4	B-	Q4	4	M4
b1	I5			5	M5

With these assignments, the corresponding SFC is plotted as:

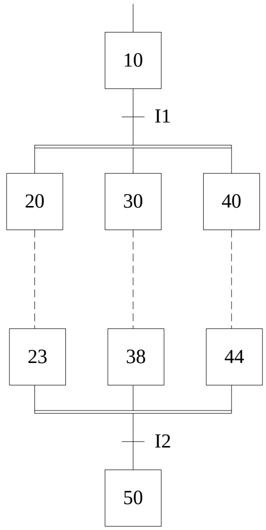


Finally, developing the SFC from the Ladder instructions gives:


M1
M2
M3
M4
M5
M1
S
M5
I2
M5
R
M1
I2
I4
I1
M2
S
M1
R
M2
I3
M3
S
M2
R
M3
I5
M4
S
M3
R
M4
I4
M5
S
M4
R
M2
Q1
M3
Q3
M4
Q4
M5
Q2

Problem 4.2.8

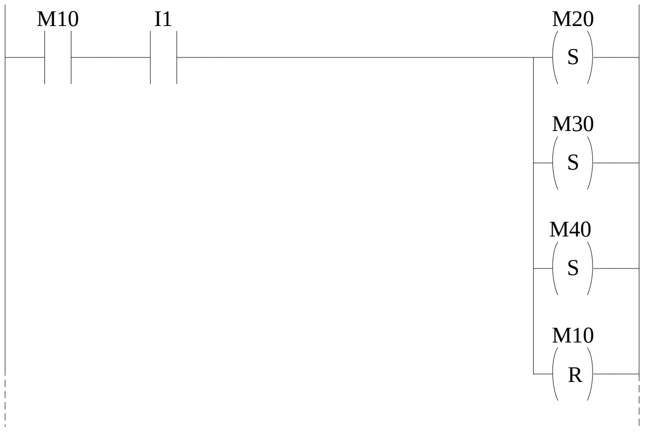
Solve the Ladder program that corresponds to the partial SFC shown below, and which also describes the "AND" divergence and the "AND" convergence. In the Ladder program, use the same notation as that which is shown in the schema.

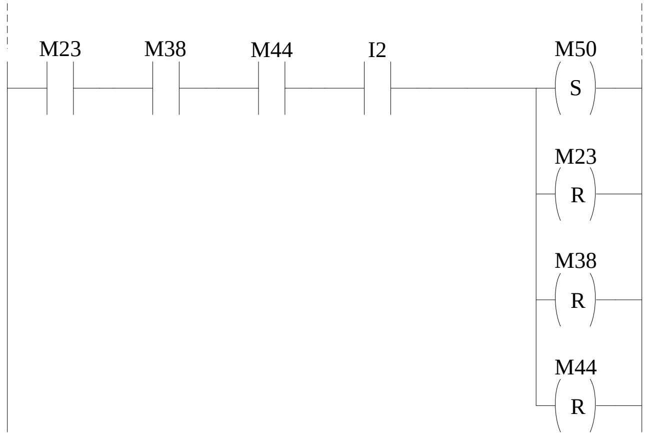


Solution

By applying the evolution rules of an "AND" divergence, where both, step 10 and transition I1 are active, steps 20, 30 and 40 are turned on simultaneously and the previous step is disabled.

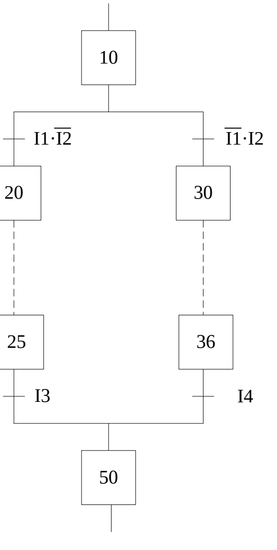
On the other hand, an "AND" divergence must end with its convergence. So, to activate step 50, all the sequences must comply (steps 23, 38 and 44 are active) and one single condition of the transition must be activated (I2). In this case, step 50 is turned on and all the precedent steps are disabled.





Problem 4.2.9

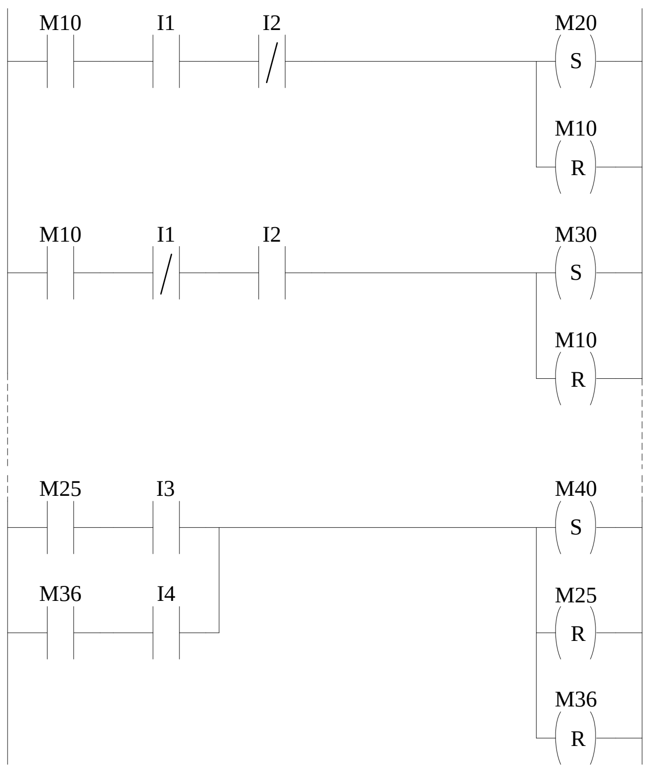
Solve the Ladder program corresponding to the partial SFC shown below, which describes the “OR” divergence and “OR” convergence. Use the same notation in the Ladder program that is shown in the schema.



Solution

When applying the evolution rules of an “OR” divergence, only one of the subsequent transition conditions can be activated if step 10 is active. Therefore, there must be a transition condition that is mutually exclusive to each sequence.

Convergence can be established from any of the active sequences. This means that, to activate step 50, the “OR” function must be included in the program, with the product of the last step and its corresponding transition condition for each sequence.



Problem 4.2.10

The rotation of a stirrer is controlled by a PLC. The system has three buttons: Counterclockwise motion (CCW), Clockwise motion (CW) and Stop (NC). Starting from the idle state, the user must press the rotation movement (CW or CCW) to activate the motor. To change the direction of rotation, the user must press the stop button and then press the new direction desired.

a) Plot SFC Level 1 of the process

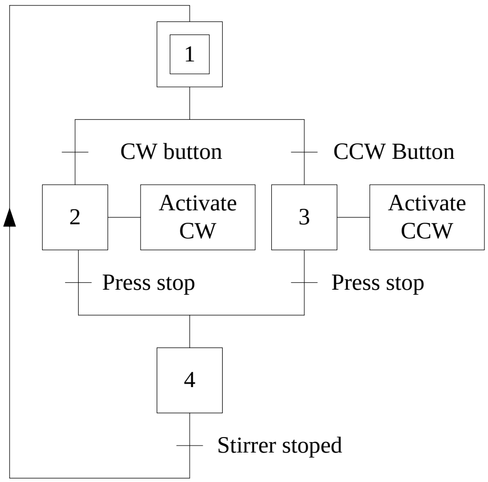
b) Assign to each element an input or output of the PLC

c) Plot the SFC Level 3 inputs and outputs above.

d) Perform the Ladder program.

Solution

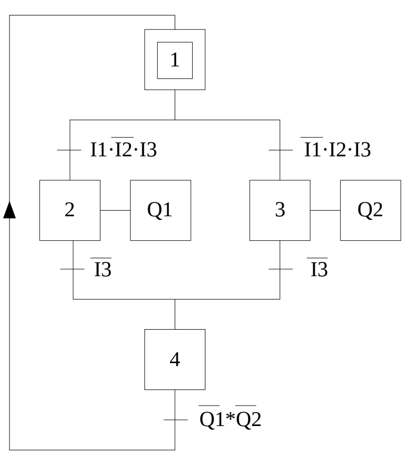
a) SFC Level 1 is solved from the work cycle defined in the statement of the problem. After the first step, the OR divergence permits choosing the direction of rotation.



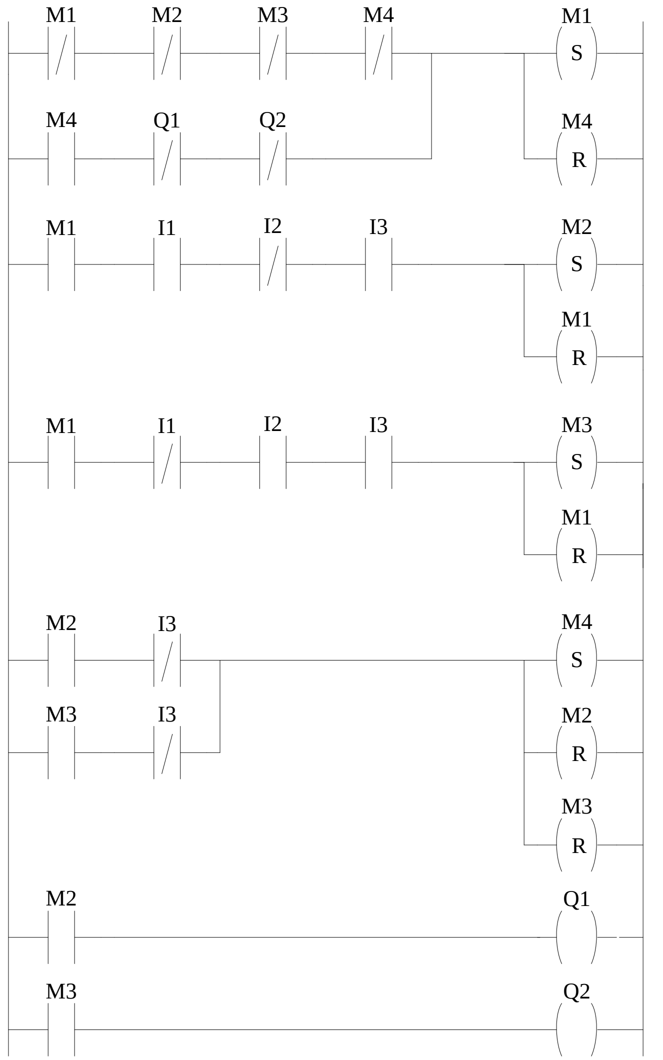
b) Assigning the PLC inputs and outputs to each element gives:

Inputs		Outputs		Variables	
Element	Input	Element		Step	Variable
CW button (NO)	I1	CW	Q1	1	M1
CCW button (NO)	I2	CCW	Q2	2	M2
Stop button (NC)	I3			3	M3
				4	M4

c) After assigning the system's physical elements, it is possible to translate SFC level 1 to SFC level 3.



d) Finally, based on the translation rules of SFC to Ladder, the program can be plotted:

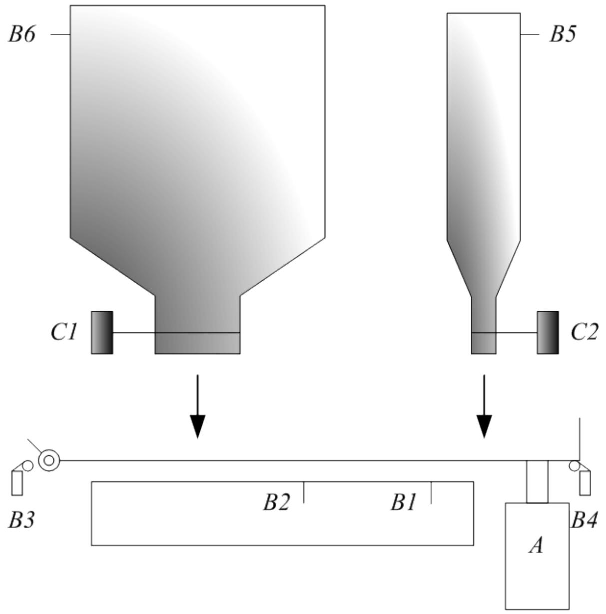


Problem 4.2.11

Before being added to a reactor, the weight of a reactive is controlled by the system shown in the picture. Two gates, driven by two single-acting cylinders, control the reactive coarse and fine adjustment. Each cylinder is actuated by a 5/2 valve with a spring return. The work cycle corresponds to the description below:

The desired weight is adjusted by sensor *B1*. To limit the filling coarse value, sensor *B2* must be adjusted. When the system is at initial conditions and the start button (*S1*) is pressed, the two gates open. When sensor *B2* is activated, cylinder *C1* closes. And when sensor *B1* is activated, cylinder *C2* closes. In addition, both cylinders are equipped with sensors to detect their position.

After reaching the desired weight, the platform tilts from being activated by the double-acting cylinder *A*, which is commanded by a 5/2 valve that has a coil for both movements of the cylinder. The position of the platform is detected by two sensors, *B3* and *B4*. Sensors *B5* and *B6* detect whether the amount of reactive is sufficient.



The correspondence between the system elements and the Inputs/Outputs of the PLC is shown in the table below:

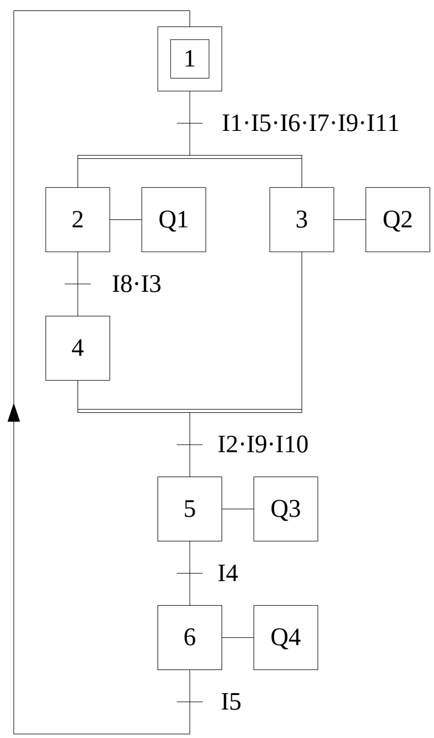
Inputs		**Outputs**	
Element	Input	Element	Output
Start button (NO)	I1	*C1+*	Q1
Sensor *B1*	I2	*C2+*	Q2
Sensor *B2*	I3	*A+*	Q3
Sensor *B3*	I4	*A-*	Q4
Sensor *B4*	I5		
Sensor *B5*	I6		
Sensor *B6*	I7		
Position *c1+*	I8		
Position *c1-*	I9		
Position *c2+*	I10		
Position *c2-*	I11		

a) Plot SFC Level 3.

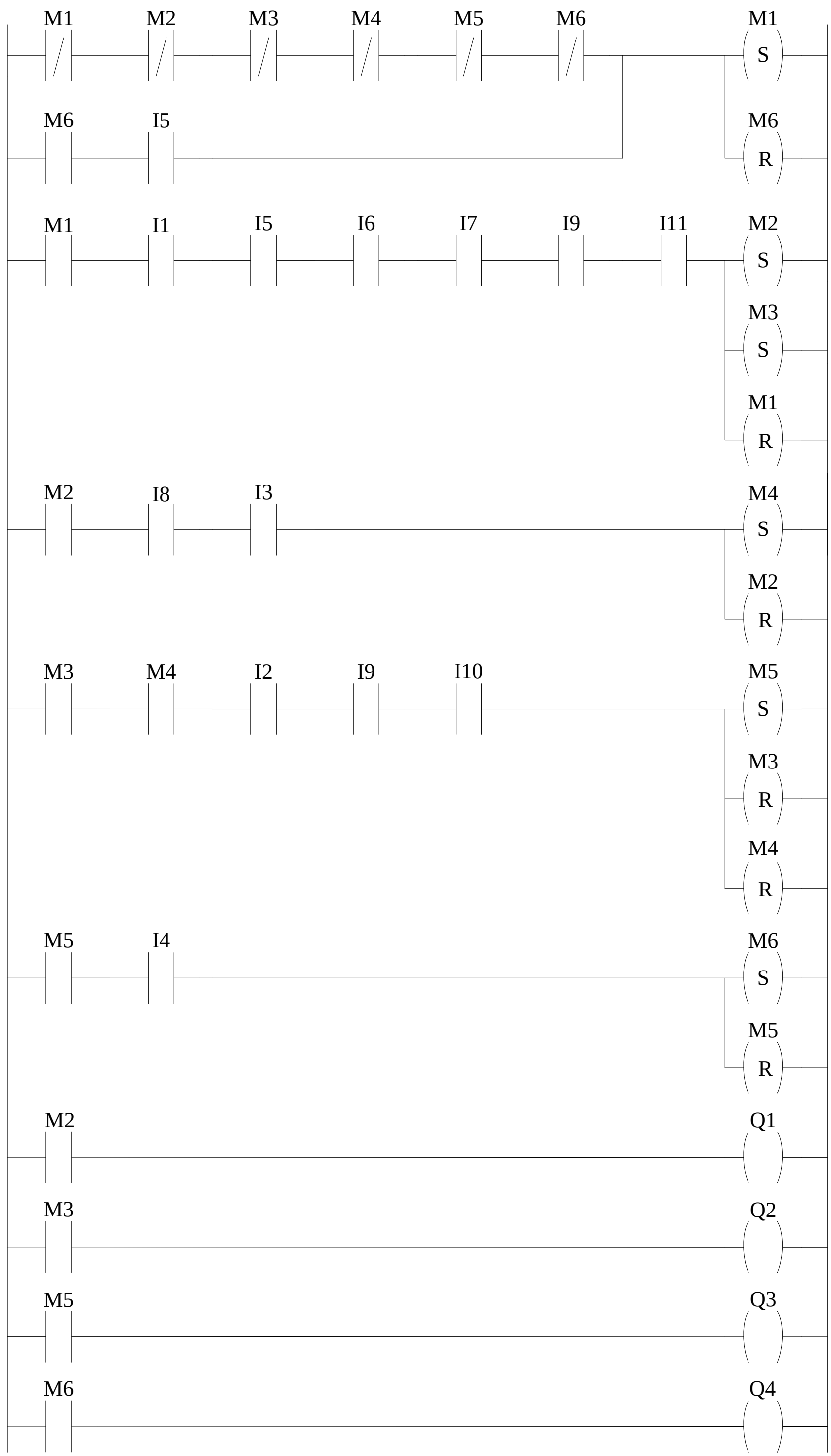
b) From SFC Level 3, design the Ladder program.

Solution

a) The solution of the problem arises from the point of view of an AND divergence (it could be solved by a single sequence), which starts the process when the user pushes *Start* while the system is at initial conditions.



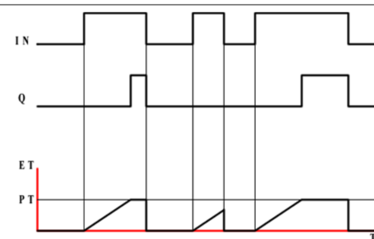
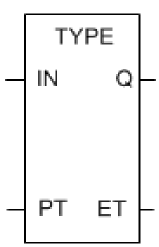
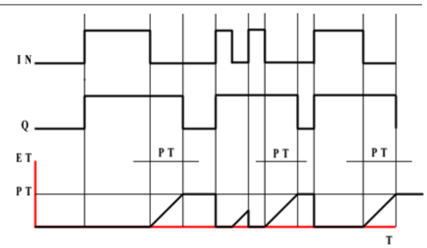
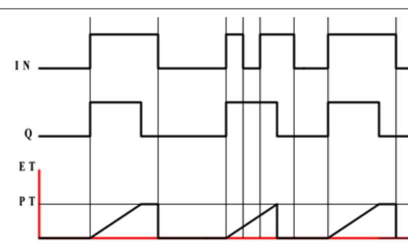
b)


M1
M2
M3
M4
M5
M6
M1
S
M6
I5
M6
R
M1
I1
I5
I6
I7
I9
I11
M2
S
M3
S
M1
R
M2
I8
I3
M4
S
M2
R
M3
M4
I2
I9
I10
M5
S
M3
R
M4
R
M5
I4
M6
S
M5
R
M2
Q1
M3
Q2
M5
Q3
M6
Q4

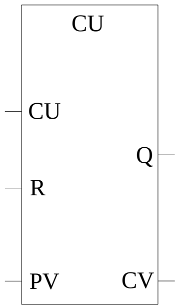
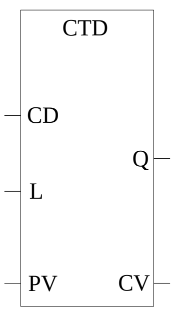
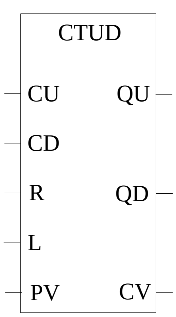
4.3 Programming TIMERS and COUNTERS

Most chemical processes require the programming of some events that influence on their evolution related with time or they count the number of times an event has been repeated. PLCs have timing and counting instructions that control these processes.

The timer block has two inputs on the left side: IN, which is a Boolean signal that enables and controls the timing of the function; and PT, a numeric value corresponding to the time being controlled. On the right side of the block the functions have two outputs: Q, a Boolean value which is active when the timing has finished; and ET, a numeric value that indicates the value of the elapsed time. In the table below the typical block ins

Block	**TYPE**	**name**	
	TON	On-Delay timer	
	TOFF	Off-delay Timer	
	TP	Pulse Timer	

The counter block enables the PLC to control the events in a process. Most PLCs have three types of counters, shown in the table below:

Block	**Name**	**Input / Output**	**Description**
	UP COUNTER	CU	A rising edge in the input increases the register CV by one unit.
		R	A true value in the input, sets the CV value to 0.
		PV	Numeric value that indicates the number of events to be counted.
		Q	Q=1 when CV>= PV
		CV	Counter current value.
	DOWN COUNTER	CD	A rising edge in the input decreases the register CV by one unit.
		L	A true value in the input. Sets the CV value to the PV value.
		PV	Numeric value that indicates the number of events to be counted.
		Q	Q=1 when CV=0
		CV	Counter current value
	UP/DOWN COUNTER		This counter corresponds to the combination of the above two counters, and its inputs and outputs have the same function as described above.

Problem 4.3.1

The figure below shows a tank used to heat a liquid from room temperature to a higher temperature (T1) and maintain it for 30 minutes. The sequence of the automatism must be the following:

1. When button S1 is pushed and the tank is empty (L2 = 0), a continuous cycle starts by opening V1 to fill the tank.

2. When the tank is full (L1 = 1) and V1 is closed, the system heats up (V2 = 1) and stirs (K1 = 1) the liquid until reaching the desired temperature T1 (the sensor is active when T ≥ T1) and maintains this action for 30 minutes.

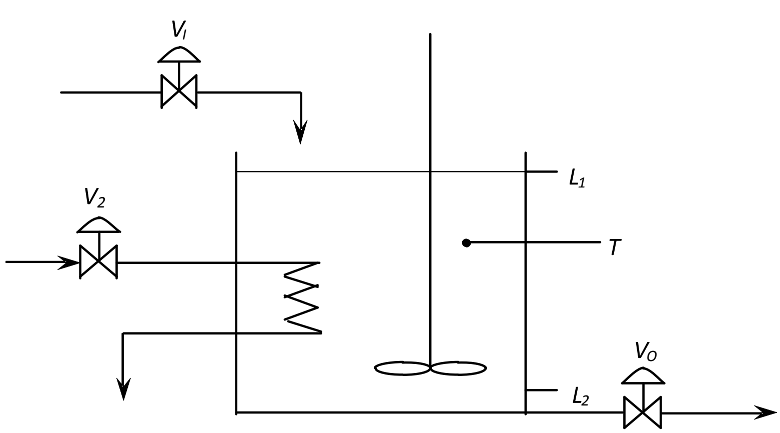
3. After maintaining the same temperature for thirty minutes, the stirrer and the heating systems are switched off. Finally, valve V0 opens until the liquid level reaches L2.

4. When the tank is empty, a new cycle starts to fill it again.

5. When button S2 is pushed, the current cycle ends and the process stops.

6. The correspondence between the process elements and the PLC inputs/outputs is shown in the table below:

Inputs		**Outputs**	
Element	Input	Element	
S1 (NO)	I1	V0	Q1
S2 (NC)	I2	V1	Q2
L1	I3	V2	Q3
L2	I4	K1	Q4
T1	I5		



a) Plot the SFC Level 3.

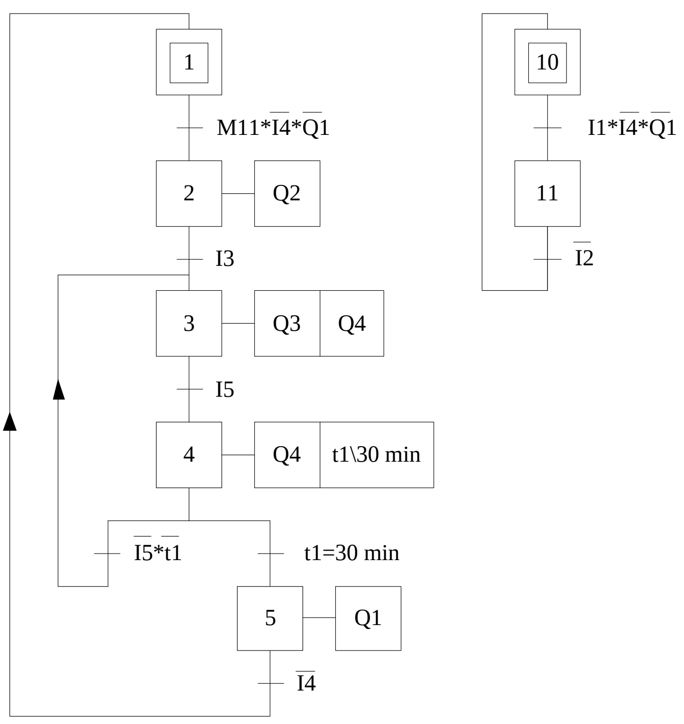
b) From the above SFC, design the Ladder program

Solution

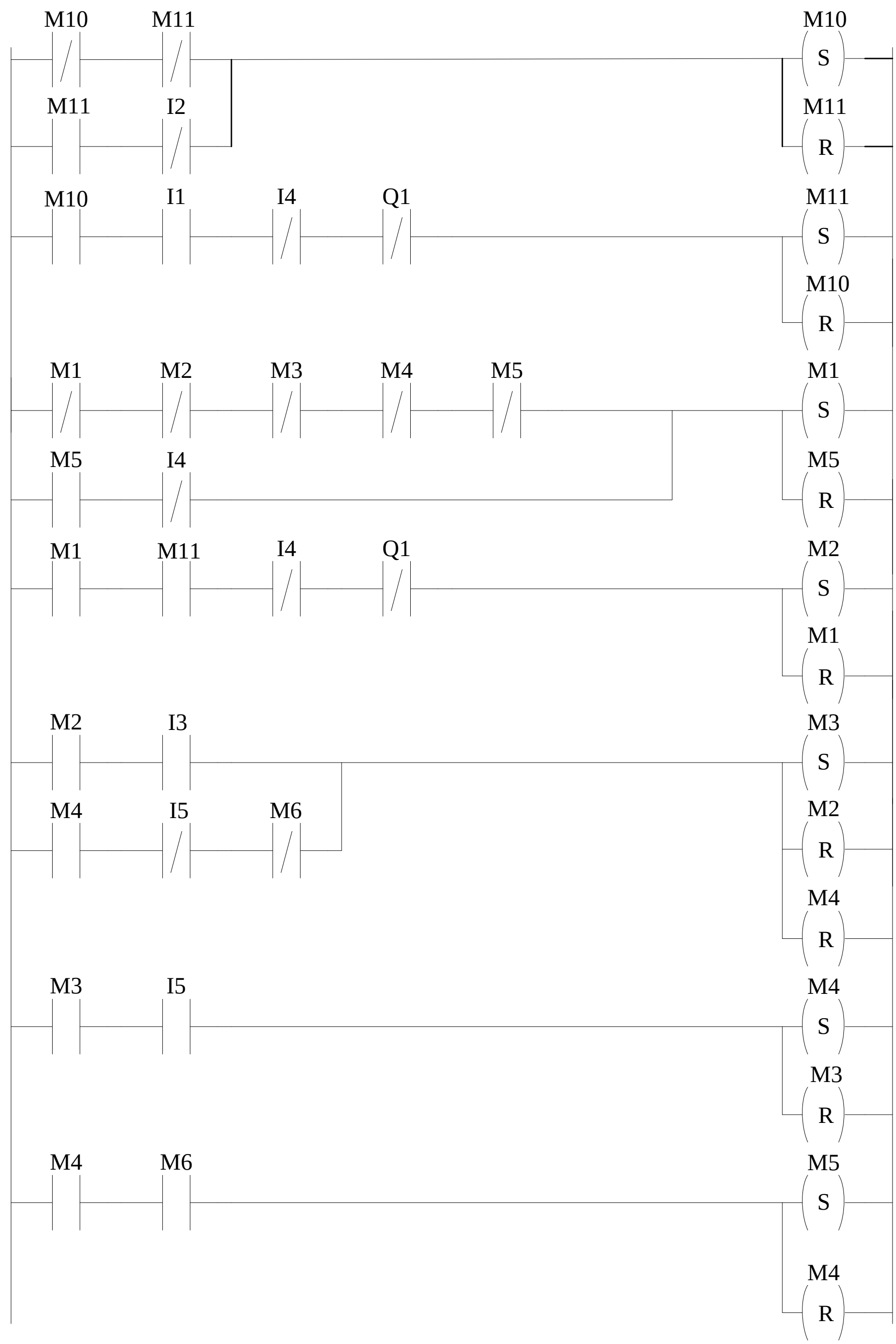
a) The system is solved with the two SFCs shown in the figure below. The first one (left) describes the automated sequence and the second one (right) shows the control of the continuous cycle.

In order to build the Ladder program, the two SFCs must be solved independently; the continuous cycle is controlled in accordance with the state of internal memory M11: when S1 is pushed in the initial conditions, M11 is activated; and when S2 is pushed at any moment of the cycle, M11 is disabled.

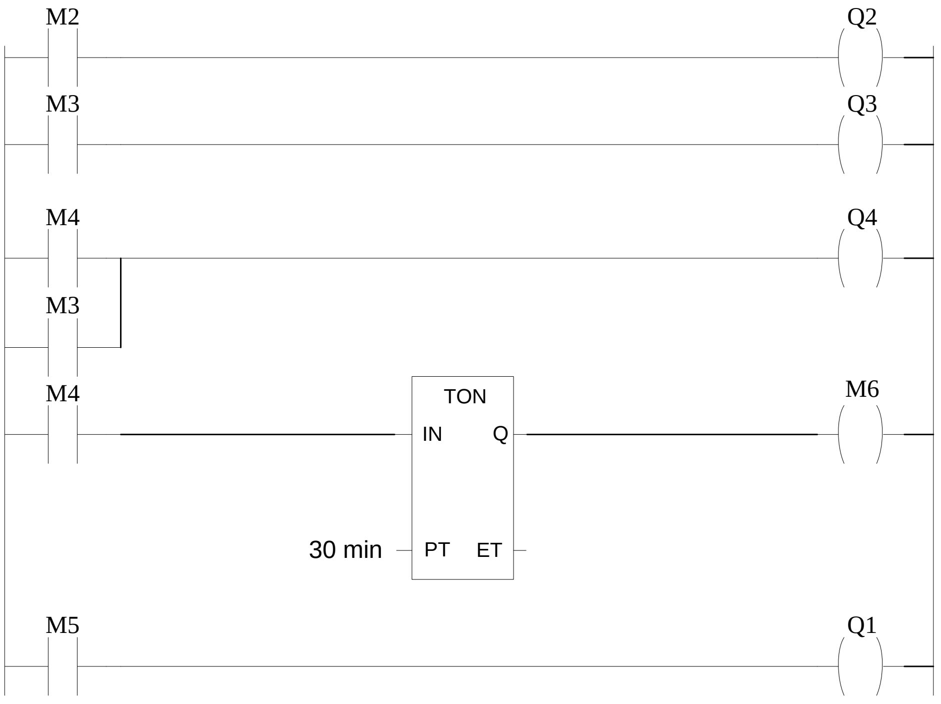
To start a new cycle when the previous one has finished, M11 must be active; otherwise, the system remains in an idle state.



b) Once the SFC is drawn, the Ladder program can be plotted using the translation rules from SFC to Ladder. First, the sequence of the two SFCs is developed:

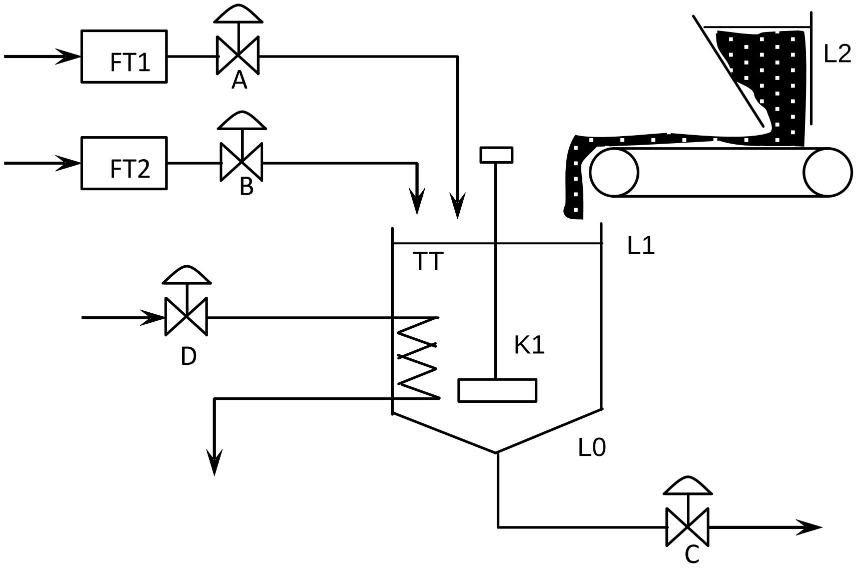


When the sequence has been programmed, each action is associated with its step:



Problem 4.3.2

The figure below shows a system for mixing two liquids and one solid by taking advantage of the solubility improvement when increasing the temperature.



The process working cycle has the following actions sequence:

1. Before pressing the start button (S1, NO), valves A, B, C and D must be closed, the hopper level must be above L2 and the tank must be empty. The continuous cycle begins opening A and B to pass the set flow rate of each liquid until it detects that the tank is full (L1).

2. Once L1 is activated, valves A and B are closed and the tank stirrer is activated (K1). At the same time, valve D opens and a heating liquid passes into the mixture in order to increase its temperature.

3. When the mixture reaches the required temperature (TT), the conveyor belt is activated for 30 seconds to add the solid to the mixture.

4. Once the conveyor belt is stopped, the stirrer continues for 15 minutes. After the indicated time, valve D is closed and the stirrer is stopped. The mixture is then discharged through opening of the valve C.

5. When the tank is empty (L0=0), the cycle ends and a new one starts.

6. If there is not enough solid (L2=0) at the start of a new cycle, a flashing pilot is triggered at one-second intervals. The operator then fills the hopper manually and validates the action by pressing S3 (NO), which starts a new cycle.

7. The system has a button, S2 (NC), which ends the current cycle when pressed and stops the process.

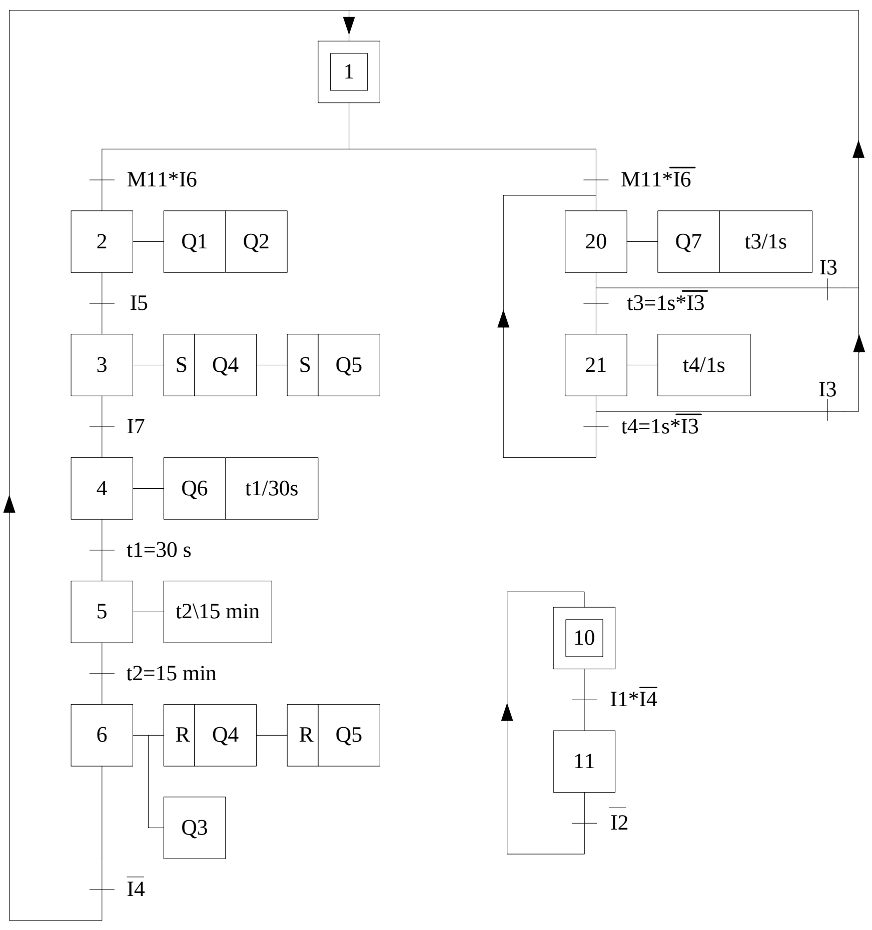
a) Plot Level 3 SFC.

Solution

To solve Level 3 SFC, the first step is to assign each physical element of the process to the inputs/outputs of the PLC. This allocation corresponds to the table below:

Inputs		**Outputs**	
Element	Input	Element	Output
S1 (NO)	I1	VA	Q1
S2 (NC)	I2	VB	Q2
S3 (NA)	I3	VC	Q3
L0	I4	VD	Q4
L1	I5	K1	Q5
L2	I6	Conveyor	Q6
TT	I7	Pilot	Q7

From the above table, Level 3 SFC corresponds to:



Problem 4.3.3

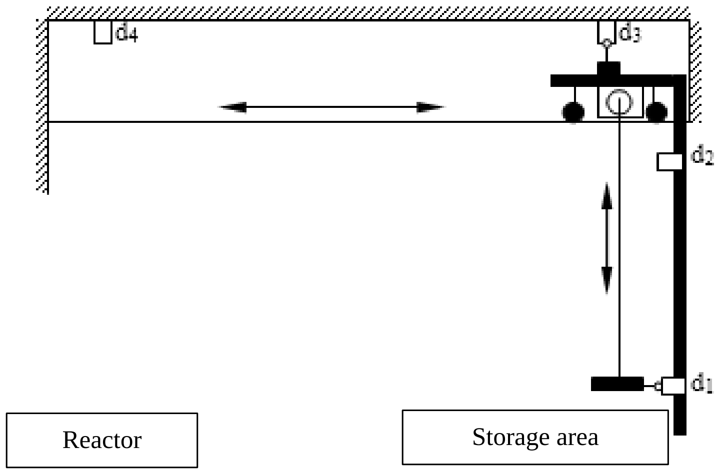
The figure below shows a system for transporting the reagents, using a crane which has two engines: M1, which is actuated by K1 (system goes up) and K2 (system goes down); and M2, which is actuated by K3 (system goes left) and K4 (system goes right). Both are commanded by the following cycle:

1. When the system detects d1 and d3, an operator places the reagents on the platform and press button S1 (NO) to start the process, actuating K1 to start the vertical movement.

2. When the crane is in the upper position (d2), K1 is stopped and K3 starts the left movement until d4 is activated.

3. When the platform has reached d4, K2 lowers the platform to d1.

4. In the last position, the system pauses for 15 seconds. After that, it returns to the home position, which activates K1 until d2 is detected. Once it is detected, K4 moves the platform until d3 is activated. Finally, K2 is activated until the rest position (d1 and d3) is reached.



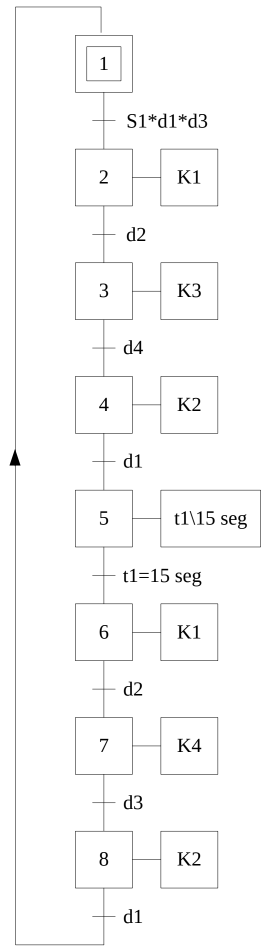
a) Plot Level 2 SFC, with the name of the variables described in the exercise

b) Draw a scheme which indicates the inputs and outputs of the control system and assign the corresponding input or output of the PC:

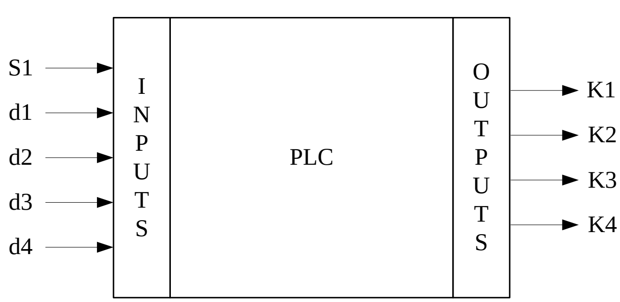
c) From the Level 2 SFC and the allocation of inputs and outputs, design the Ladder program

Solution

a) Level 2 SFC, with the name of the variables described in the exercise, corresponds to the figure shown below:



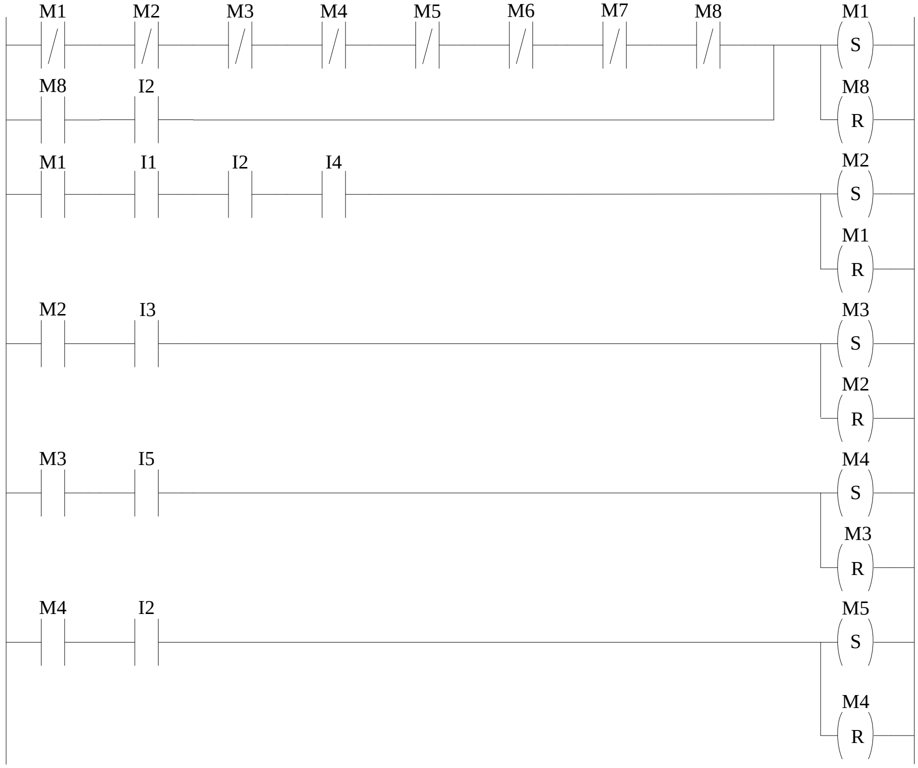
b) In accordance with the process described above, the connection of the inputs and outputs to the PLC correspond to the scheme shown below

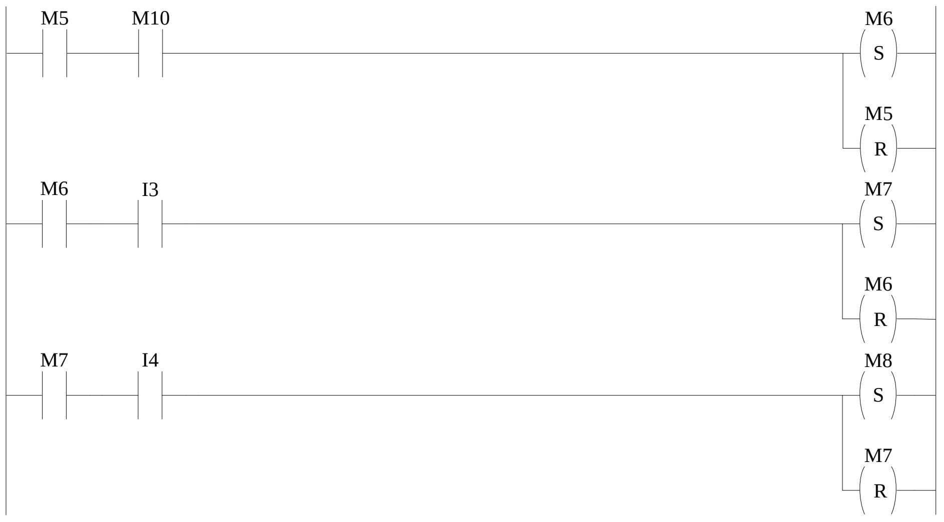


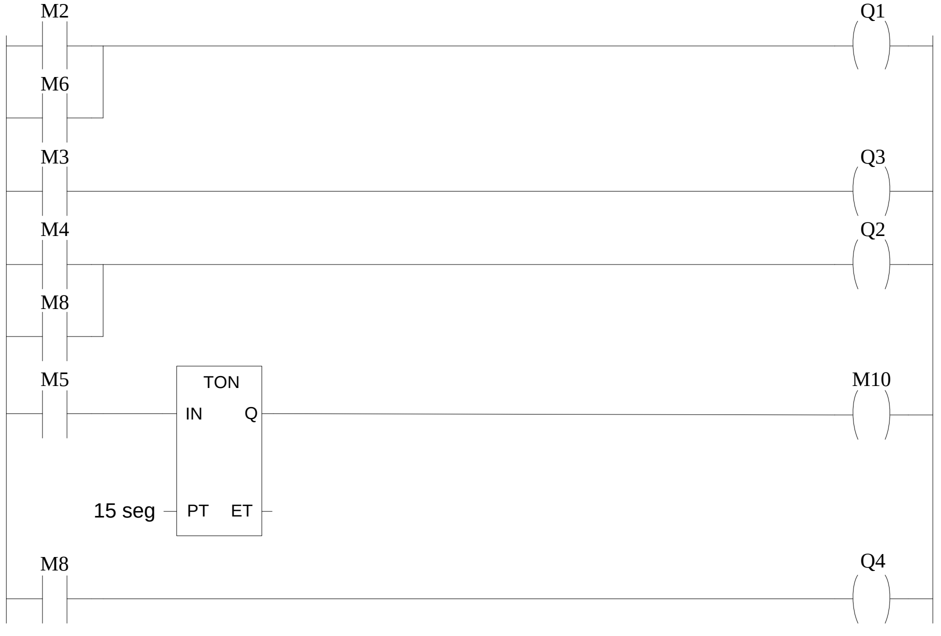
From the above scheme, it is possible to assign the corresponding input or output to each element.

Inputs		**Outputs**	
Element	Input	Element	Output
S1	I1	K1	Q1
d1	I2	K2	Q2
d2	I3	K3	Q3
d3	I4	K4	Q4
d4	I5		

Once the Level 2 SFC is solved and the inputs and outputs are assigned to the physical elements, the program can be plotted using the translation rules from SFC to Ladder:




M5
M10
M6
S
M5
R
M6
I3
M7
S
M6
R
M7
I4
M8
S
M7
R


M2
Q1
M6
M3
Q3
M4
Q2
M8
M5
TON
IN
Q
M10
15 seg
PT
ET
M8
Q4

Problem 4.3.4

1. The system shown in the figure below consists of weighting a product by opening and closing two single-acting cylinders (B and C) with sensors in the two end positions (b_0 and c_0 when they are closed and b_1 and c_1 when they are open). The work cycle of the process consists of the following steps:

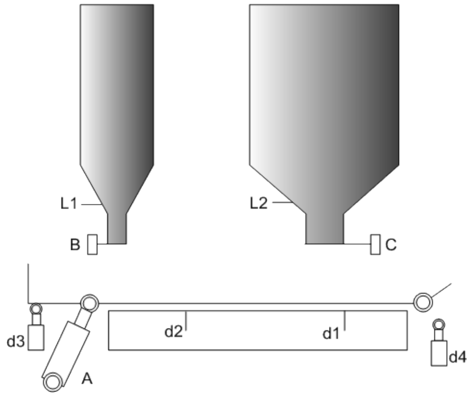
2. The user must establish the desired weight (d1), and then push the start button (S1).

3. If there is enough product in the two tanks (L1 an L2 activated), cylinders A and B open the hopper outlet simultaneously.

4. When the weight reaches sensor d2, which is less than d1, hopper outlet C is closed.

5. When the weight reaches sensor d1, hopper outlet B is closed.

6. After reaching the desired weight, a double acting cylinder (A) tilts the platform from sensor d3 to sensor d4, and pauses for 15 seconds in this position while pouring the product onto a conveyor belt. After this time elapses, cylinder A is deactivated and the system remains in the rest position until receiving a new order from the user.



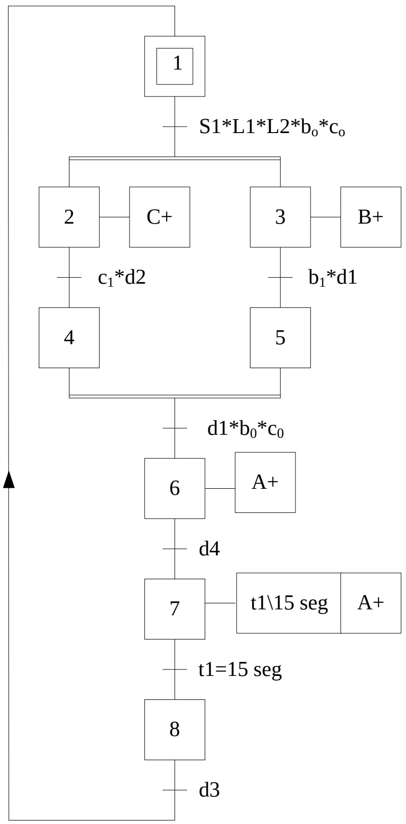
a) Plot Level 2 SFC, with the name of the variables described in the statement of the problem.

b) Draw a scheme which indicates the inputs and outputs of the control system and assign the corresponding input or output of the PLC.

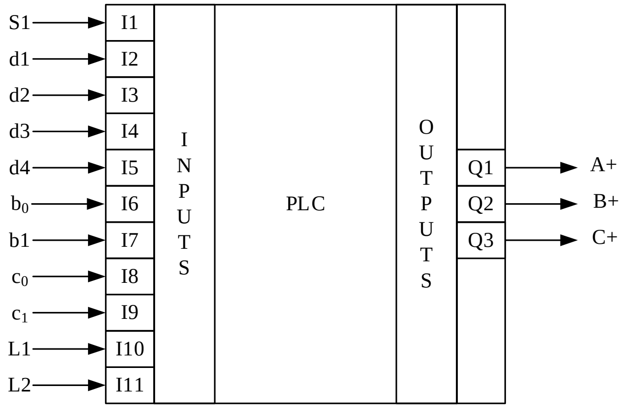
c) From the Level 2 SFC and the allocation of inputs and outputs, design the Ladder program

Solution

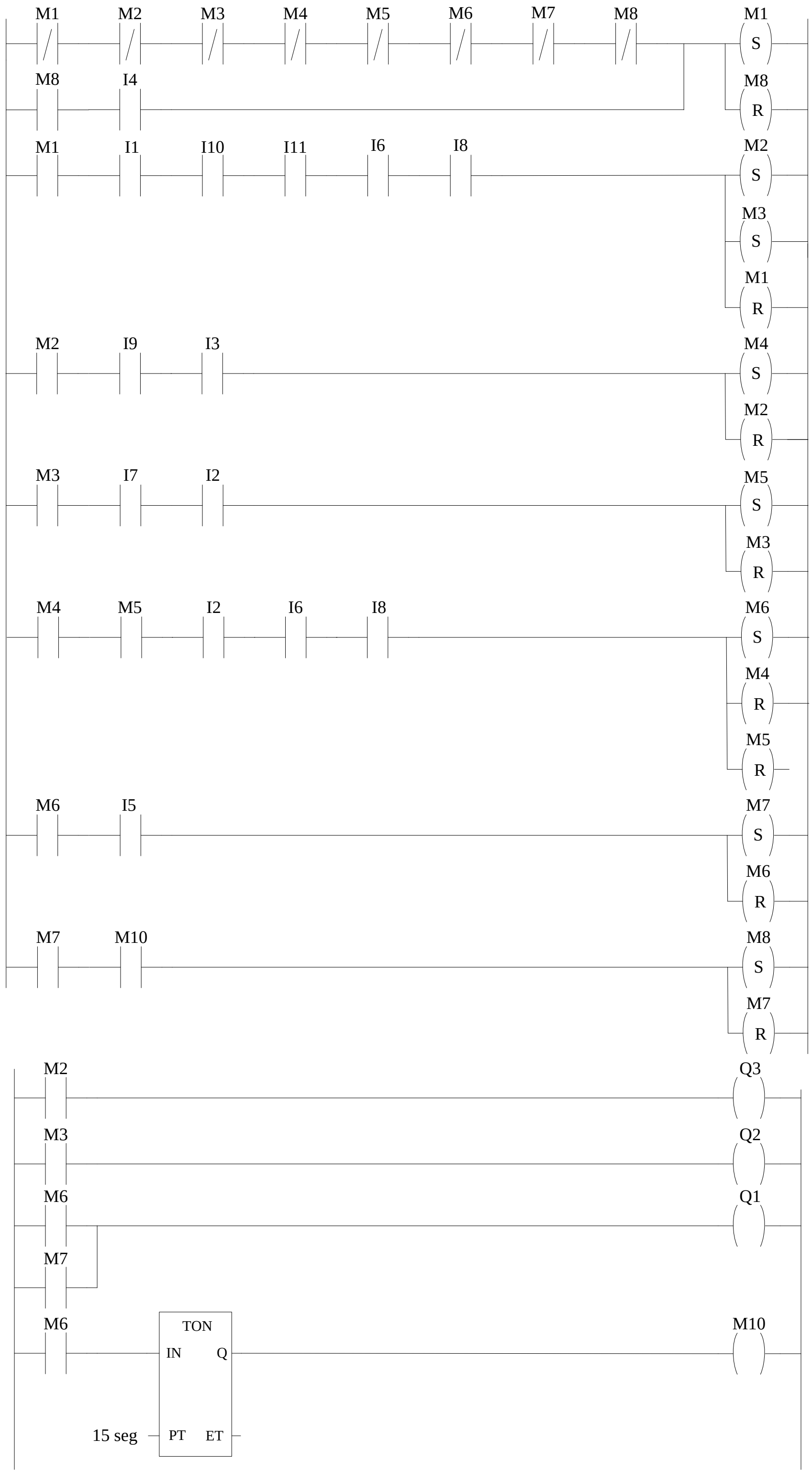
a) Level 2 SFC, with the names of the variables described in the exercise, corresponds to:



b) The scheme connecting the process elements with the PLC inputs and outputs is shown below:



c) From the Level 2 SFC and the allocation of inputs and outputs, the Ladder program corresponds to the figure shown below:


M1 M2 M3 M4 M5 M6 M7 M8 M1 S
M8 I4 M8 R
M1 I1 I10 I11 I6 I8 M2 S
M3 S
M1 R
M2 I9 I3 M4 S
M2 R
M3 I7 I2 M5 S
M3 R
M4 M5 I2 I6 I8 M6 S
M4 R
M5 R
M6 I5 M7 S
M6 R
M7 M10 M8 S
M7 R
M2 Q3
M3 Q2
M6 Q1
M7
M6 TON IN Q M10
15 seg PT ET

Problem 4.3.5

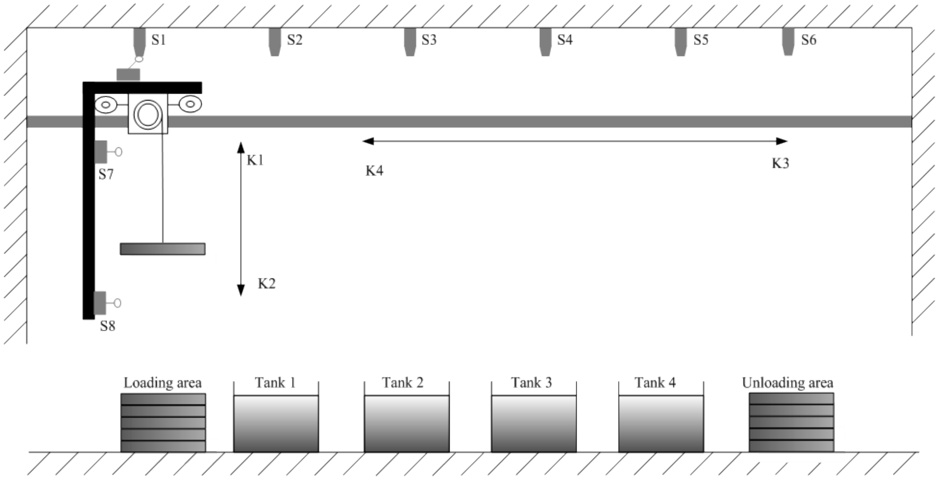
The scheme below represents a surface treatment system that involves sequentially introducing metallic pieces into tanks which contain chemical products. The operating cycle consists of the following steps:

1. An operator places the piece to be treated in the initial position (S1 and S8 are activated) and presses S10 to start the cycle.

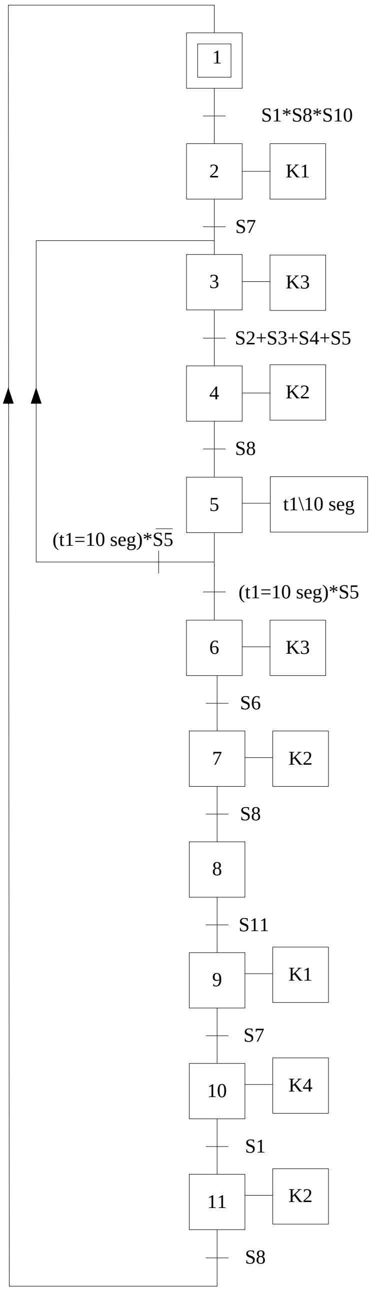
2. Once the cycle has started, the motor K1 raises the piece until S7 is detected. Then K1 stops and K3 begins its rightward movement until reaching S2. K2 lowers the piece to S8 and remains in this position for 10 seconds.

3. After this time, the action will be repeated when reaching positions S3, S4 and S5.

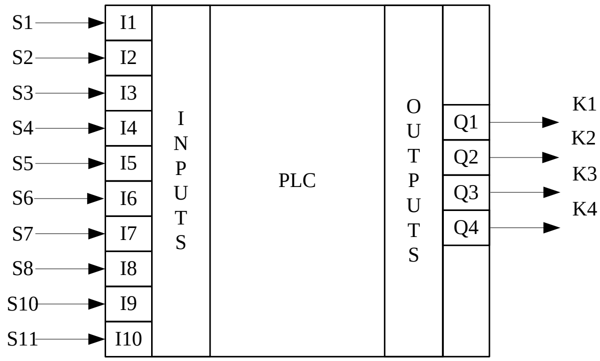
4. When the piece arrives at S6, it is unloaded by another operator, who then presses S11 to return the crane to the initial position. This occurs by sequentially activating the single movements of motors K1, K4 and K2.



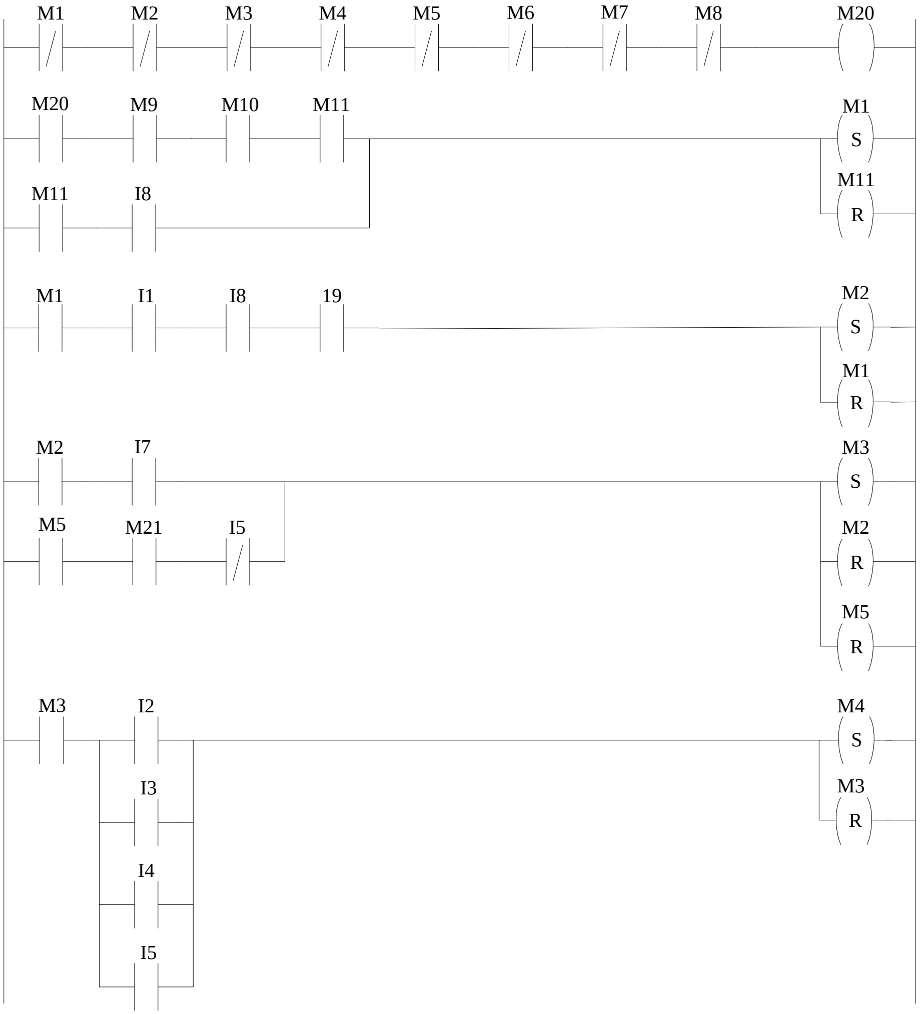
a) Level 2 SFC, using the names of the variables described in the problem, corresponds to:

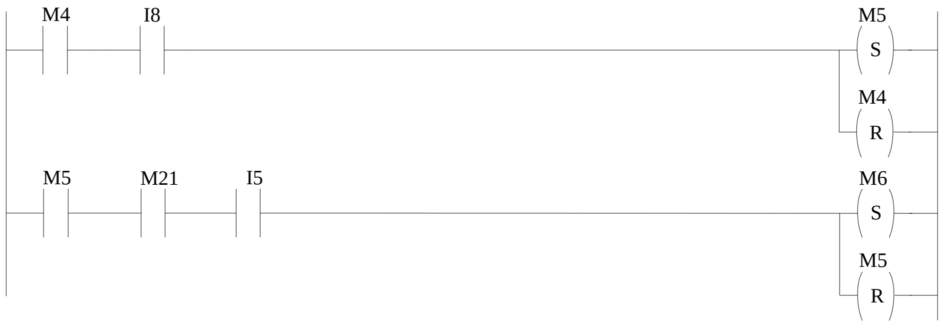


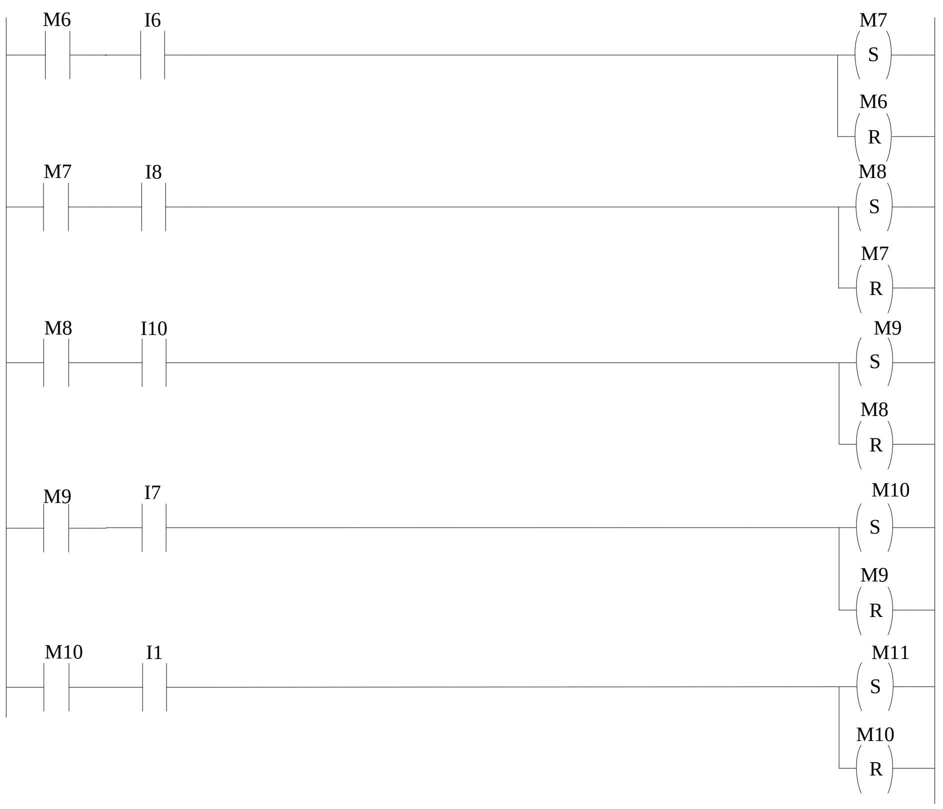
b) The scheme connecting the process elements with the PLC inputs and outputs is shown below:

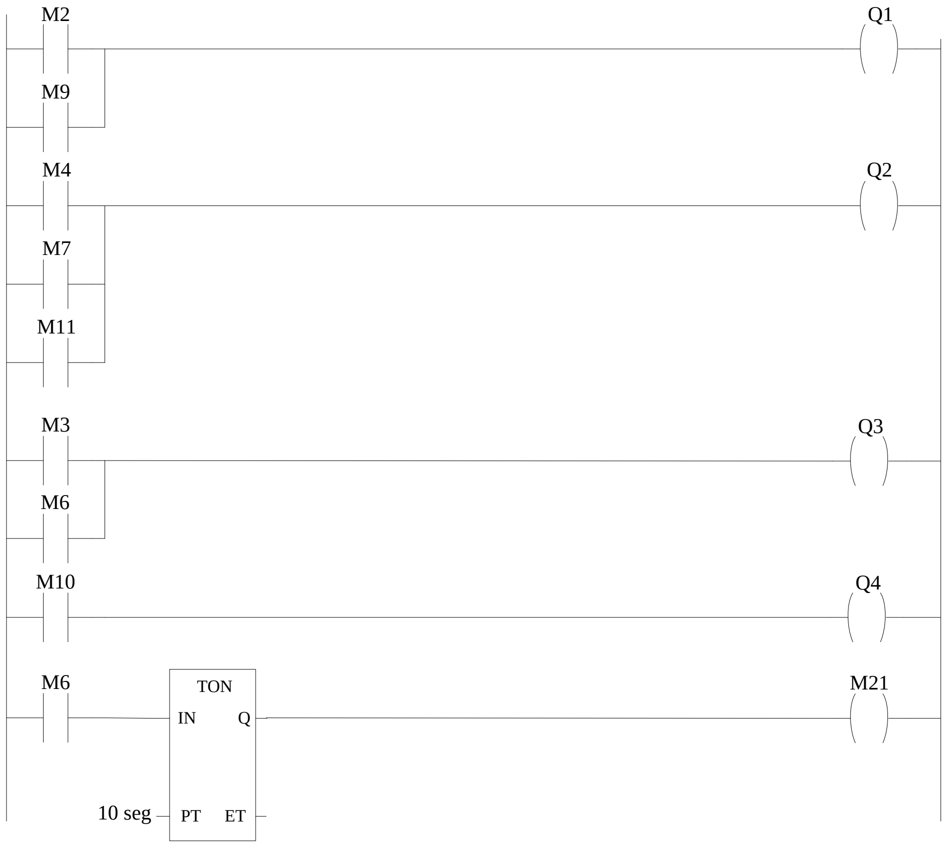


c) From the Level 2 SFC and the allocation of inputs and outputs, the Ladder program corresponds to the following figures:




M4
I8
M5
S
M4
R
M5
M21
I5
M6
S
M5
R


M6
I6
M7
S
M6
R
M7
I8
M8
S
M7
R
M8
I10
M9
S
M8
R
M9
I7
M10
S
M9
R
M10
I1
M11
S
M10
R



Problem 4.3.6

A packaging system is controlled by two conveyor belts: K1 for conveyor belt 1, and K2 for conveyor belt 2. The process starts when pushing S1 (NO), and instantly stops when pushing S2 (NC). The system work cycle consists of the following steps:

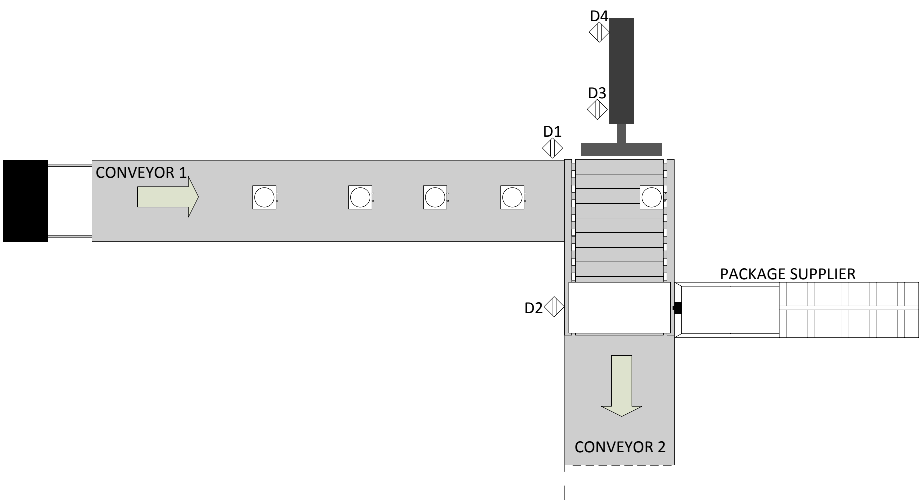
1. When the process starts by pressing S1, conveyor belts 1 and 2 are activated simultaneously.

2. Detector D1 is at the end of conveyor belt 1. It counts the number of bottles that arrive at the packaging area until reaching four bottles. At this point, conveyor belt 1 stops and a single-acting cylinder is activated (K3). It is controlled by a 5/2 valve with coil and spring.

3. After 1.5 seconds the bottles are placed in the box and they arrive at conveyor belt 2, which moves them to the palletizing area.

4. Once the cylinder is in its rest position, K4 is activated and it places a new box into position. When D2 is activated, conveyor belt 1 reinitiates for packaging another four bottles.

5. Pressing stop at any time will put the system into rest, and all the actuators will stop until the system is restarted.



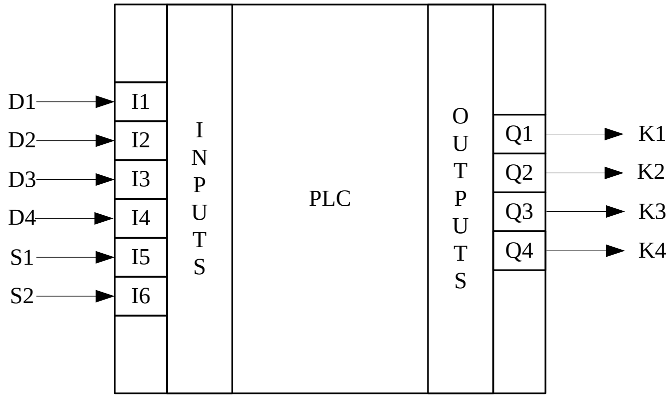
a) Draw a scheme which indicates the inputs and outputs of the control system and assigns the corresponding PLC input or output:

b) Plot Level 3 SFC using the allocated variables.

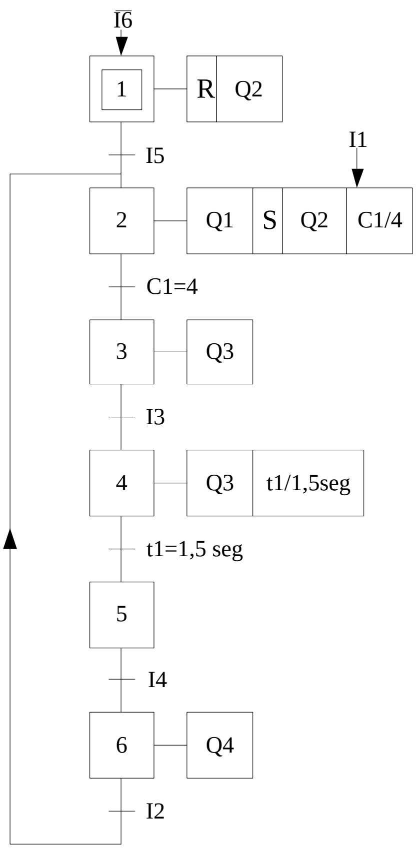
c) Using the Level 3 SFC, design the Ladder program

Solution

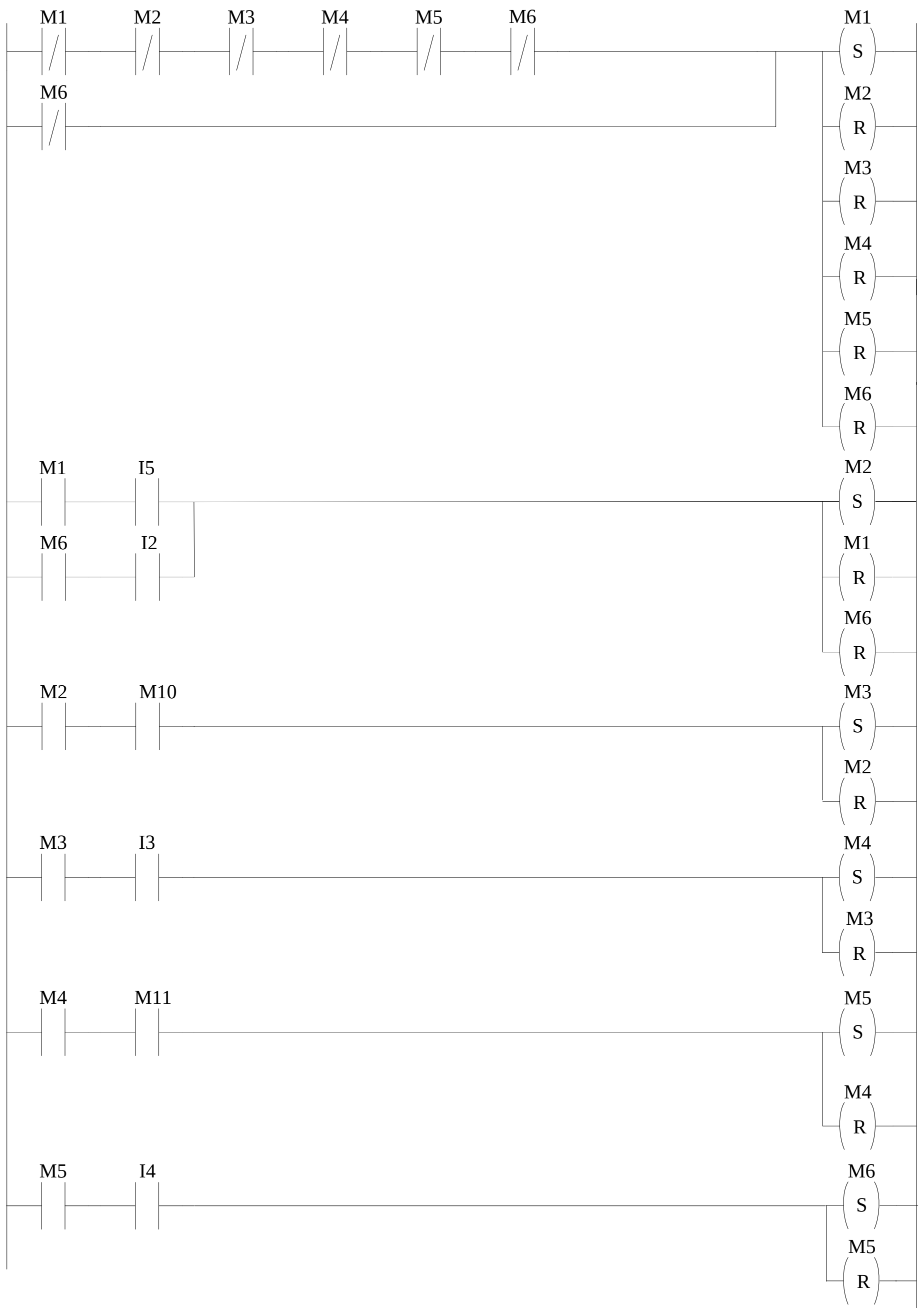
a) First, it is necessary to assign a PLC input or output to each system element in order to create the Level 3 SFC.

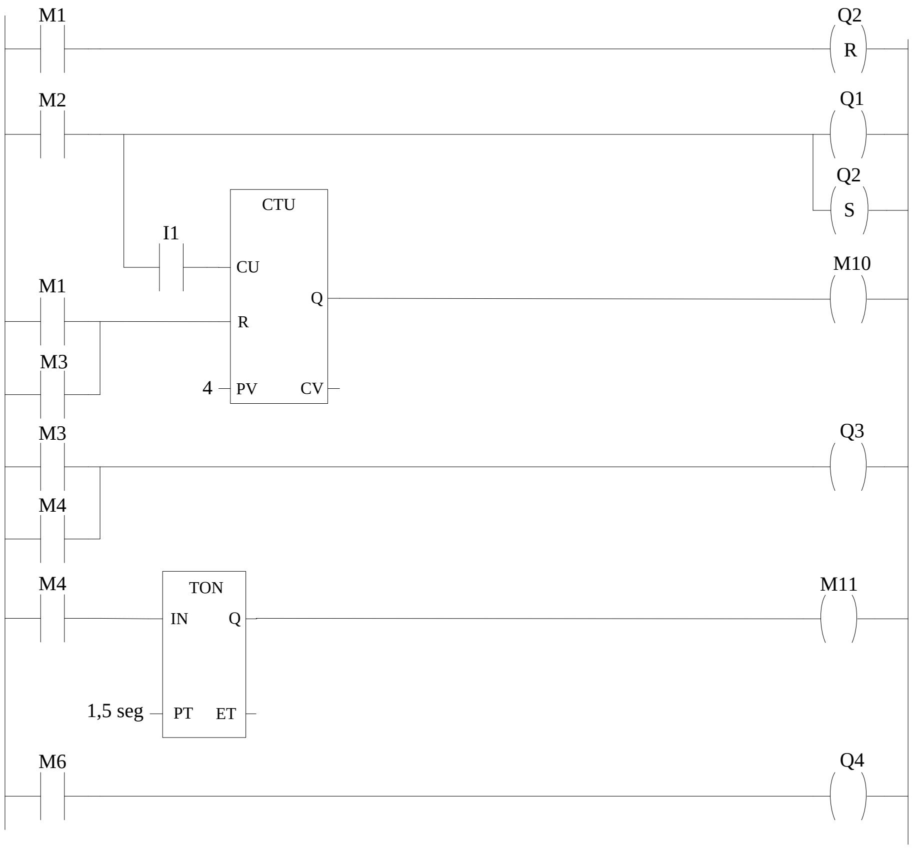


b) When the inputs and outputs are allocated, the Level 3 SFC can be designed as shown in the figure below:



c) Using the Level 3 SFC, the Ladder program corresponds to the figure shown below:




M1
Q2
R
M2
Q1
Q2
S
CTU
I1
CU
M1
Q
M10
R
M3
4
PV
CV
M3
Q3
M4
M4
TON
M11
IN
Q
1,5 seg
PT
ET
M6
Q4

→5

Automation and control of continuous processes with PLC

Introduction

The exercises solved in the previous chapter have enabled the automation of discrete processes using the set of basic instructions in Ladder Language (contacts and coils), and the use of timers and counters. Furthermore, chemical production involves a set of physical variables (temperature, pH, level, etc.) that correspond to numerical variables changing continuously over time. The PLC, as an automation and control element, provides both physical and programming tools that allow this kind of control. Below will explain a short description of these tools for solving the problems of this chapter, but it is recommended reviewing the literature on hardware, analogical inputs and outputs for a better understanding of these tools.

PLCs have analog input modules that capture the physical variables involved in processes. These variables can be pressure, flow, temperature, level or composition, among others. Processes of this kind provide an analogic value, which varies continuously over time within a specific range. The physical variables involved in the control processes are converted to an electrical signal by transducers, which are connected to the analogic input module that provides a range of values. Usually, proportional voltages or currents are associated to the physical variables.

Furthermore, PLCs have analogic output modules, commonly used to proportionally drive the final control elements to the levels of voltage or current supplied by the PLC. The number of elements adapted to analogic outputs is large and varied, some of which are:

- *Analog valves:* The opening of these valves is adjustable and the aperture is directly proportional to the analog signal applied, which can adjust the flow rate through the valve.

- *Set points for electric motors:* Speed regulation of the electric motors by an analog set point allows regulating the behaviour of the devices coupled to them (agitators, pumps, and so on).

- *Analog Meters*: These can be coupled to both of the PLC outputs digital and analog meters, which provide information to whoever uses the analog variables.

- *Set points*. Analog set points can usually be applied to regulate and stabilize processes by controllers that are programmed into the designed Ladder.

In controlling continuous processes, the programmed variables are not Boolean. To perform the automation and control of these kinds of processes, the PLC has numeric variables in which the analog values are stored. To solve the exercises, four types of numeric variables will be used. The first three (*IW*, *QW* and *MW*) correspond to numerical variables of 16 bits, which store a signed integer; so values between -32768 and +32767 can be stored. The last one (*MF*) corresponds to floating point values:

- *IWx*: Variable that refers to the analog input *x*, which stores an integer value proportional to the analog signal in the input of the PLC; therefore, it is proportional to the physical parameter that is being measured.

- *QWy*: A variable that refers to the analog output *y*. The integer value is stored in this variable and it is converted to a proportional voltage or current which will modify the final control element.

- *MWx*: Internal integer variable used to store the results of operations needed for controlling continuous processes.

- *MFx*: Internal floating point variable used to store the results of operations needed for controlling continuous processes.

When programming with numeric variables and performing operations for cotrolling processes, standard numerical functions are implied for use in the PL programming language.

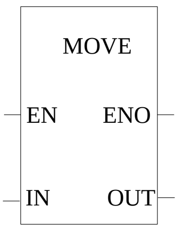
In order to classify the different functions, the standard functions have been divided into different groups, depending on the utility and functionality within the program.

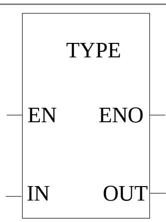
- Type conversion functions
- Arithmetic functions
- Bit shift functions
- Selection functions
- Bitwise Boolean functions
- Comparison functions
- String functions
- Time functions
- Enumerated data functions

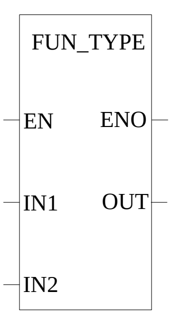
In the next section a brief description of the syntax and functionality of these instructions is developed. Only the most commonly used instructions are explained, in order to solve the exercises developed in this chapter.

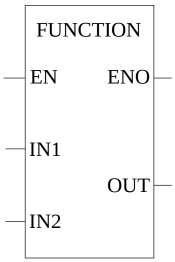
5.1 Syntax for the numeric instructions

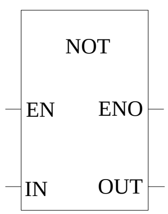
The most typical block functions are shown in the next tables. These functions are used for programming a great number of applications in chemical industries, both for discrete and continuous systems. They will be used to solve the exercises in this chapter.

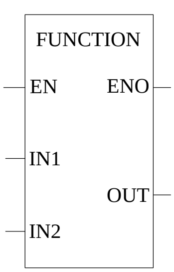
Block	Function	Description
	MOVE	Transfer and copy data from the source memory (IN) to the destination memory location (OUT).

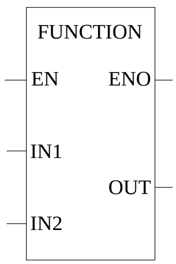
Block	Function TYPE	Description
	INC DEC	The function increases (INC) or decreases (DEC) the input variable (IN). The result is stored in the variable output (OUT) in each SCAN cycle of the PLC.

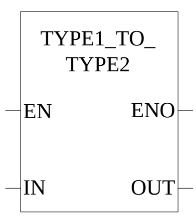
Block	Function	Description
	ADD_*TYPE*	The function adds the variables located in the inputs IN1 and IN2. The result is stored in the variable located in the output OUT.
	SUB_*TYPE*	The function subtracts the variables located in the inputs IN1 and IN2. The result is stored in the variable located in the output OUT.
	MUL_*TYPE*	The function multiplies the variables located in the inputs IN1 and IN2. The result is stored in the variable located in the output OUT
	DIV_*TYPE*	The function divides the variables located in the inputs IN1 and IN2. The result is stored in the variable located in the output OUT.

Block	Function	Description
	AND	The instruction implements the function AND between the bits located at the same positions of the variable IN1 and IN2. The result is stored in the variable connected to the output OUT.
	OR	The instruction implements the function OR between the bits located at the same positions of the variable IN1 and IN2. The result is stored in the variable connected to the output OUT.
	XOR	The instruction implements the function XOR between the bits located at the same positions of the variable IN1 and IN2. The result is stored in the variable connected to the output OUT.

Block	Function	Description
	NOT	The instruction implements the NOT function of all the bits in variable IN. The result is stored in the variable connected to the output OUT.

Block	Function	Description
	SHL	The function shifts N bits of the input (IN) to the left and fills the shifted bits with the value 0. The result is stored in the variable located at the output OUT.
	SHR	The function shifts N bits of the input (IN) to the right and fills the bits shifted with the value 0. The result is stored in the variable located at the output OUT.
	ROL	The function rotates N bits of the input (IN) to the left and reintroduces the displaced bits to the right. The result is stored in the variable located at the output OUT.
	ROR	The function rotates N bits of the input (IN) to the right and reintroduces the displaced bits to the left. The result is stored in the variable located at the output OUT.

Block	Function	Description
	GT	The block output (OUT) is enabled when the value of IN1 is greater than IN2.
	GE	The block output (OUT) is enabled when the value of IN1 is greater than or equal to IN2.
	LT	The block output (OUT) is enabled when the value of IN1 is less than IN2.
	LE	The block output (OUT) is enabled when the value of IN1 is less than or equal to IN2.
	EQ	The block output (OUT) is enabled when the value of IN1 is equal to IN2.
	NE	The block output (OUT) is enabled when the value of IN1 is different than IN2.

Block	Function	Description
	Conversion from variable TYPE1 to TYPE2	The function takes the value of the variable IN in the TYPE1 format, and converts and stores the result in OUT in the TYPE2 format. TYPE1 and TYPE2 can be, among others, ASCII, STRING, BCD, BYTE, WORD, INT, REAL, TIME and its derivatives.

5.2 Basic control of continuous processes

Problem 5.2.1

An analog input module of a PLC controls the process temperature through a sensor connected to the input IW0 with the following relationships:

	Signal input	Temperature measured	IW0
Min	4 mA	10 ºC	0
Max	20 mA	30 ºC	10000

The user defines the set point of the process by a potentiometer-based system connected to the input IW1 with the following relationships:

	Signal input	**Set point temperature**	**IW1**
Min	0 V	15 ºC	0
Max	10 V	25 ºC	10000

Design a program that starts the process by pushing the NO button (I0) and stops it by pushing the NC button (I1).

With the heating system connected to the PLC digital output Q0 and the process running (I0 has been activated), Q0 will be activated when the temperature is below the set point, and it will turn off either when it exceeds 10% of the set point or the user pushes the stop button.

Solution

The solution of a chemical control system can have multiple analog values provided by sensors, set points or other elements. The different measurement ranges of these values must be adjusted to the PLC register range. In this problem, for the same range of values of the PLC register IW (0 to 10,000), the sensor attached to IW0 captures different ranges of temperature (10 º C to 30 º C) whose set point is attached to IW1 (15 º C to 25 º C). This condition makes it inadvisable to work directly with the value of the analogue register IW when solving these cases, because the same value of the PLC provides different information about the physical value measured.

Thus, when dealing with these kind of problems, it is advisable to use the application variables, temperature in this case. The solution of the problem implies that the equations for converting the register value to the application value should be included in the Ladder program by using the following ratio rule:

$$\frac{XWn - Offset}{(Max - Min)_{XWn}} = \frac{V_{aplic} - Offset}{(Max - Min)_{aplic}}$$

In the above formula *XWn* corresponds to the analog register (input or output); V_{aplic} is the value that corresponds to the application variable; *offset* is the deviation of the signal with respect to 0; and, finally, *Max* and *Min* are the maximum and minimum values that the PLC register and the application can have.

From the above equation, it can be inferred the relationship between the sensor temperature and the register IW0:

$$\frac{IW0 - 0}{(10000 - 0)} = \frac{T_{measured} - 10}{(30 - 10)}$$

Solving and simplifying gives:

$$T = \frac{IW0}{500} + 10$$

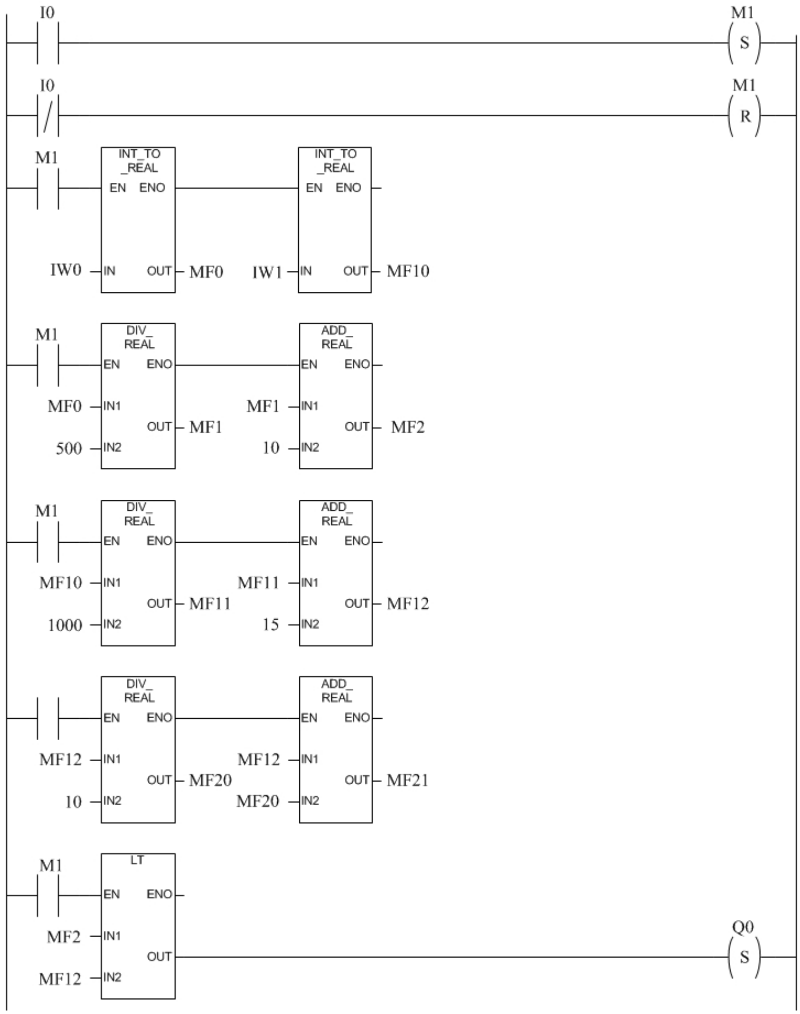
In the same way, the relationship between the set point and IW1 correspond to:

$$\frac{IW1 - 0}{(10000 - 0)} = \frac{T_{sp} - 15}{(25 - 15)}$$

Solving and simplifying gives:

$$T = \frac{IW1}{1000} + 15$$

With the above two equations, the Ladder program can be solved as shown below:



As can be seen in the solution, M1 controls the process. The values captured from IW0 and IW1 are converted to a real type (MF0 and MF10). Once these values are converted by means of the equation found above, they become a range of temperatures (MF2 and MF12). Finally, the output temperature value (MF2) is compared with the set point (MF12) in order to activate the control output (Q0). In the last step MF2 is compared with MF21 (ten per cent above the set point) to reset de control output (Q0).

Problem 5.2.2

The PLC internal bit M1 activates a pressure process control. When this bit is active, a pressure gauge connected to the IW0 channel provides the following information:

	Signal input	Pressure measured	IW0
Min	4 mA	5 bar	0
Max	20 mA	30 bar	10000

Thus, each of the following digital outputs will be activated, depending on the system pressures indicated:

Q0 if $14 \text{ bar} > p \geq 12 \text{ bar}$

Q1 if $18 \text{ bar} > p \geq 14 \text{ bar}$

Q2 if $p \geq 18 \text{ bar}$

a) Solve the exercise using the register values of the analog inputs.

b) Convert the register values of the analog inputs to pressure values using the PLC instructions, and solve the exercise using the results of the equations that convert register values to pressure values.

Solution

a) Solving the exercise with the values obtained directly from the analogue input register involves performing the calculation before applying the values in the Ladder program. Thus, the values of the register for each pressure correspond to the equation below:

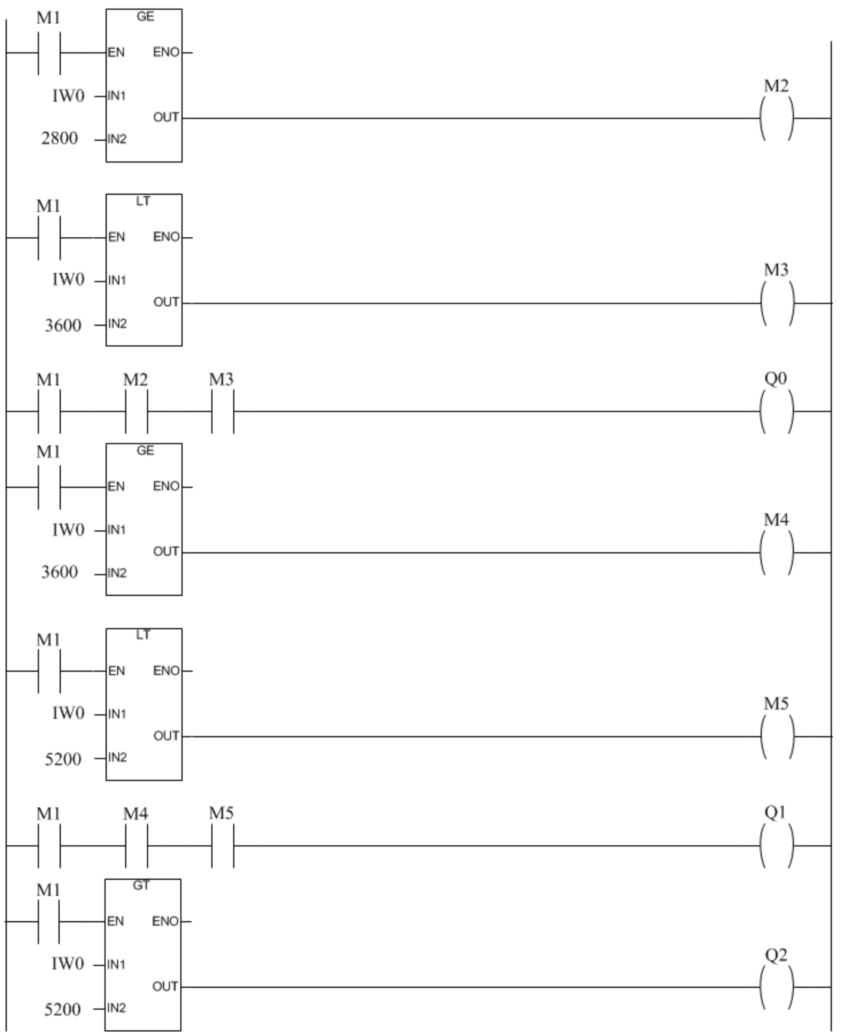
$$\frac{IW0-0}{(10000-0)}=\frac{P_{measured}-5}{(30-5)}$$

From the above equation, the register value for each pressure value can be inferred:

$$IW0=400\cdot P_{measured}-2000$$

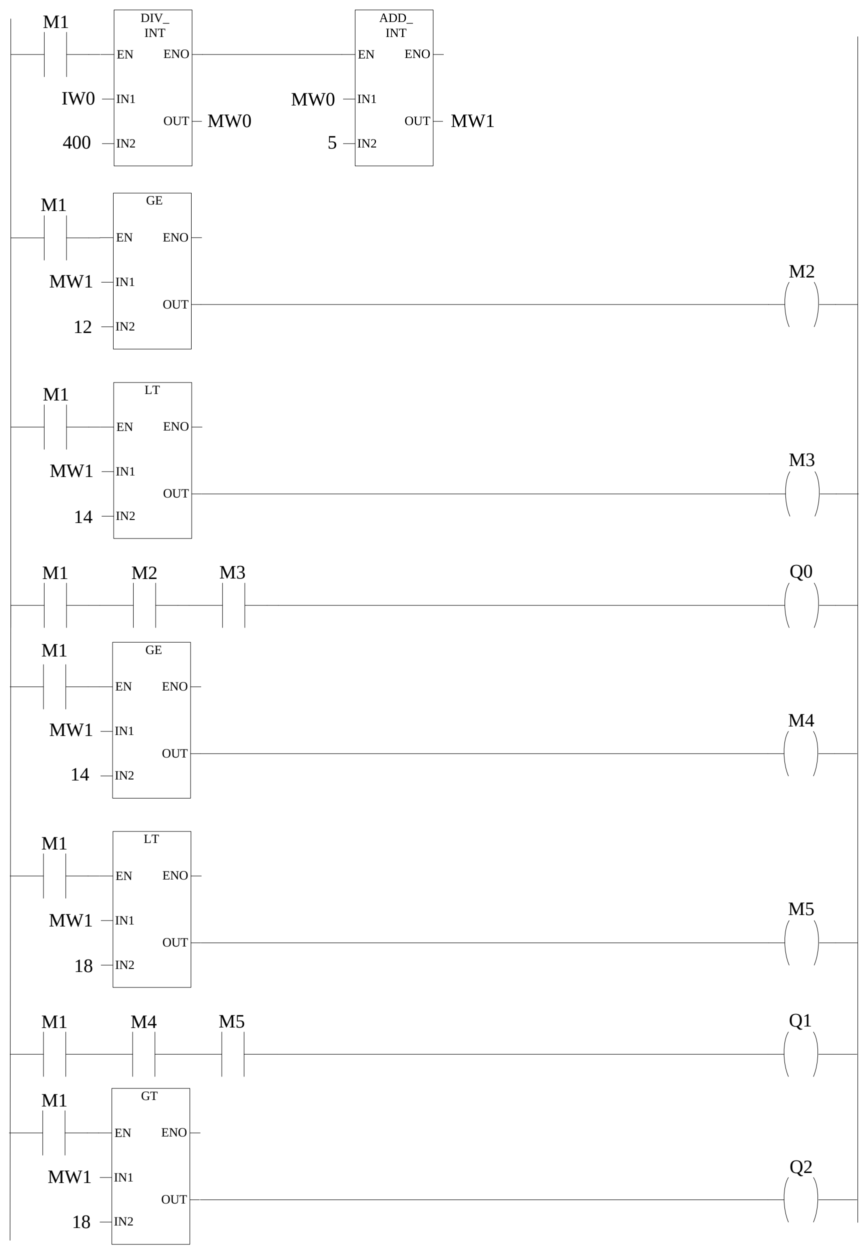
– 12 bar of pressure correspond to IW0 equals 2800.
– 14 bar of pressure correspond to IW0 equals 3600.
– 16 bar of pressure correspond to IW0 equals 5200.

With these values, the Ladder program can be created:



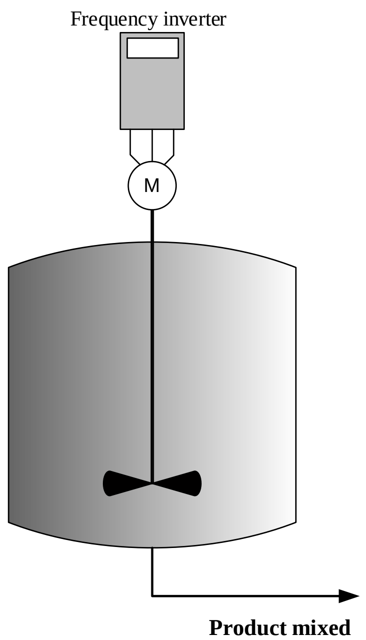
b) Including the formula instead of the calculated values in the program (as has the application without performing the previous calculations.

$$P_{measured} = \frac{IW0}{400} + 5$$



Problem 5.2.3

The main challenge of using an industrial mixer in Chemical Industry is maintaining the speed at the desired value without depending on the properties of treated materials.



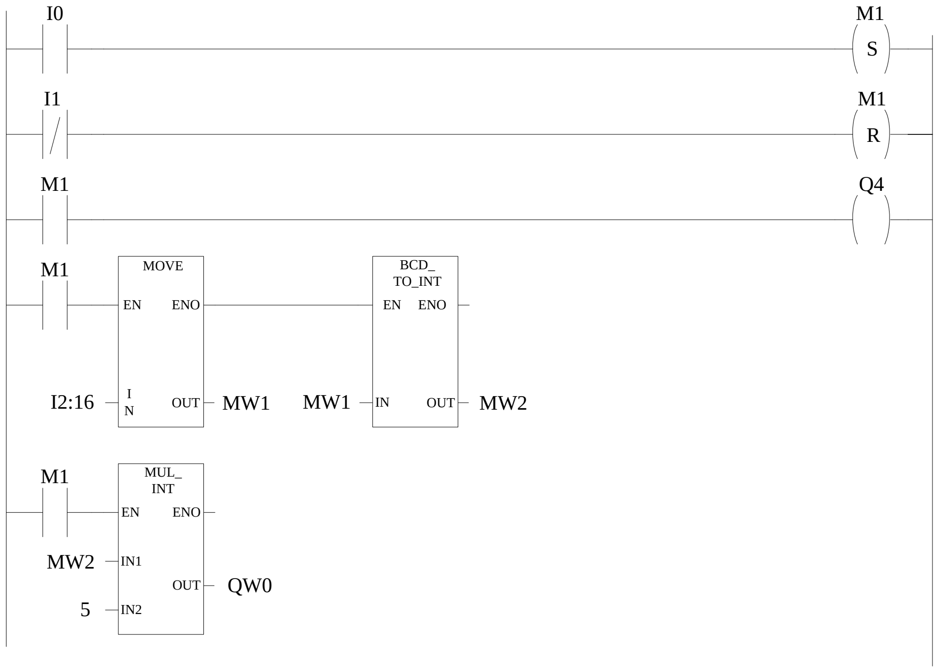
Design the Ladder program of the process so that it keeps the speed of an AC motor constant. In this case the mixer is controlled by a frequency inverter, which in turn is commanded by the analog output QW0 with the following range of values:

	Signal output	**Speed**	**QW0**
Min	0 V	0 rpm	0
Max	10 V	2000 rpm	10000

The user will start the system with a NO button (I0) and will stop it with an NC button (I1). When the process is running, the digital signal Q4 will be activated to start the rotation of the motor.

The user must enter the speed of the motor through a four-digit binary encoder BCD connected to the digital inputs I2:16 (inputs form I2 to I17), indicating the speed in rpm. This set point will be sent in analogue format to the inverter through QW0.

Solution



As can be observed in the program, M1 controls the activation of the engine through the start and stop push buttons. In order to control the motor speed, the user introduces the set point by means of the digital outputs I2:16. The set point is then stored in MW1 in BCD format and converted to INT in MW2.

With the set point in MW2, the relationship between this value and the analog output signal is determined by the following equation:

$$QW0 = 5 \cdot V_{rpm}$$

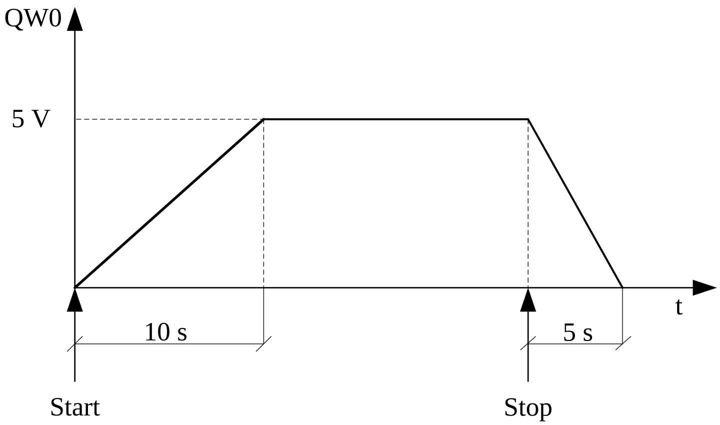
where the V_{rpm} variable corresponds to the value stored in MW2

Problem 5.2.4

Design the program to control the start-up and shutdown of an analog output (QW0) by following a ramp in both cases, as shown in the figure:

The relationship between the analog output and its associated register corresponds to:

	Output Signal	**%QW0**
Min	0 V	0
Max	10 V	10000



The ramp starts by pressing the button (NO) connected to the digital input I0. After the system reaches the steady state it will stop when the button I1 (NC) is pressed.

a) Plot the Level 3 GRAFCET.

b) From the Level 3 GRAFCET, plot the Ladder program of the process.

Solution

a) In order to solve the problem, the first step consists of determining the ramp equations for start and stop, which correspond to the straight-line equation shown below.

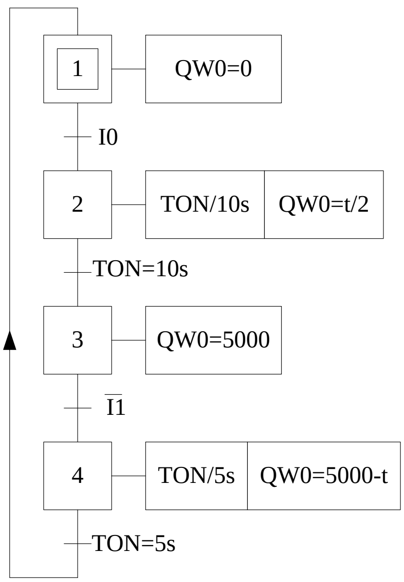
$$y = mx + b,$$

where "*m*" is the slope and "*b*" is the interception with the y axis. In order to include the equation in the Ladder program, the slope is calculated as the relationship between time (ms) and voltage. From these premises, the start and stop equations correspond to:

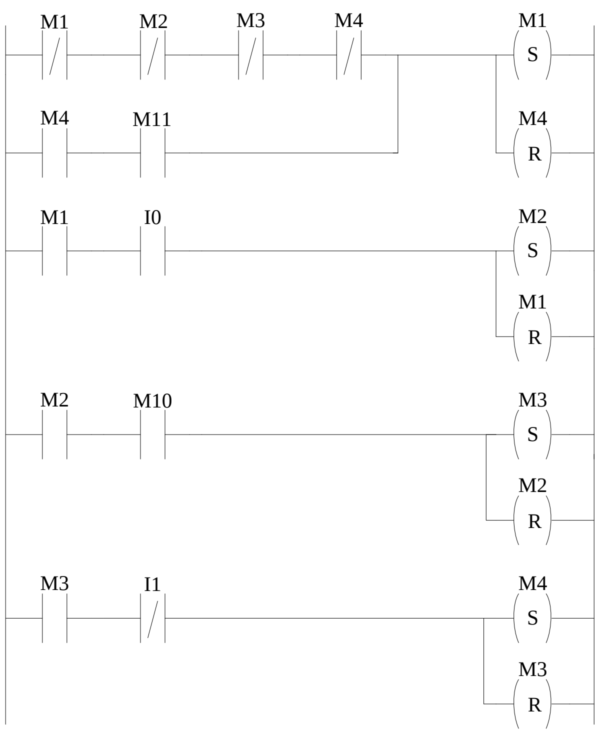
$$Start : QW0 = \frac{t}{2}$$

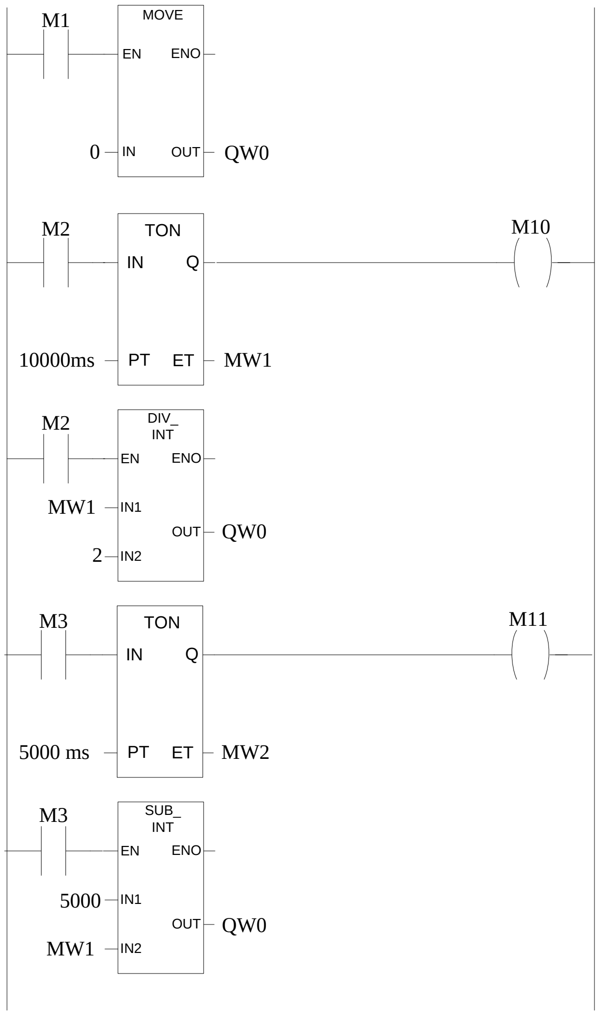
$$Stop : QW0 = 5000 - t$$

From these equations the Level 3 GRAFCET can be plotted:



b) From the above GRAFCET, the Ladder program is plotted below:





Problem 5.2.5

Design the Ladder program to perform an on/off control of an oven with the following elements:

– A single start/stop button (NO) connected to the PLC digital input I0.

– A 16-bit switch binary encoder BCD connected to I1:16.

– A heater resistance activated by the digital output Q0.

– The temperature sensor connected to IW0, with the following relationship:

	Input Signal	Temperature measured	%IW0
Min	0 V	0ºC	0
Max	10 V	1500 ºC	10000

The cycle of the program corresponds to the following steps:

Step 1: The user starts and stops the process by means of the button connected to I0.

Step 2: The user must select the oven temperature between the values 200ºC and 800ºC by means of the 16-bit switch binary encoder (0000 to 9999). If the selected value is less than 200ºC, the system automatically adjusts the temperature up to 200ºC. If the set value is greater than 800ºC, the system automatically adjusts the temperature down to 800ºC.

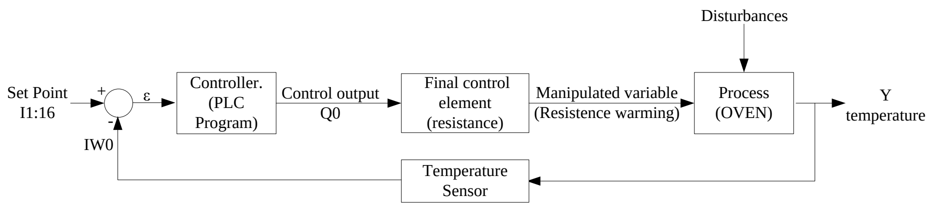
Step 3: When the system is running, the heater is activated when the actual temperature is lower than 90% of the set point, and it will be deactivated when the actual temperature is higher than 110% of the set point.

a) Plot the block diagram of the closed-loop control, identifying each input/output of the PLC with the variables that shape a closed-loop control.

b) Design and plot the Ladder program to perform the control.

Solution

a) The correspondence between a closed-loop control and the physical variables of a system (in this case the PLC) is shown in the next figure:



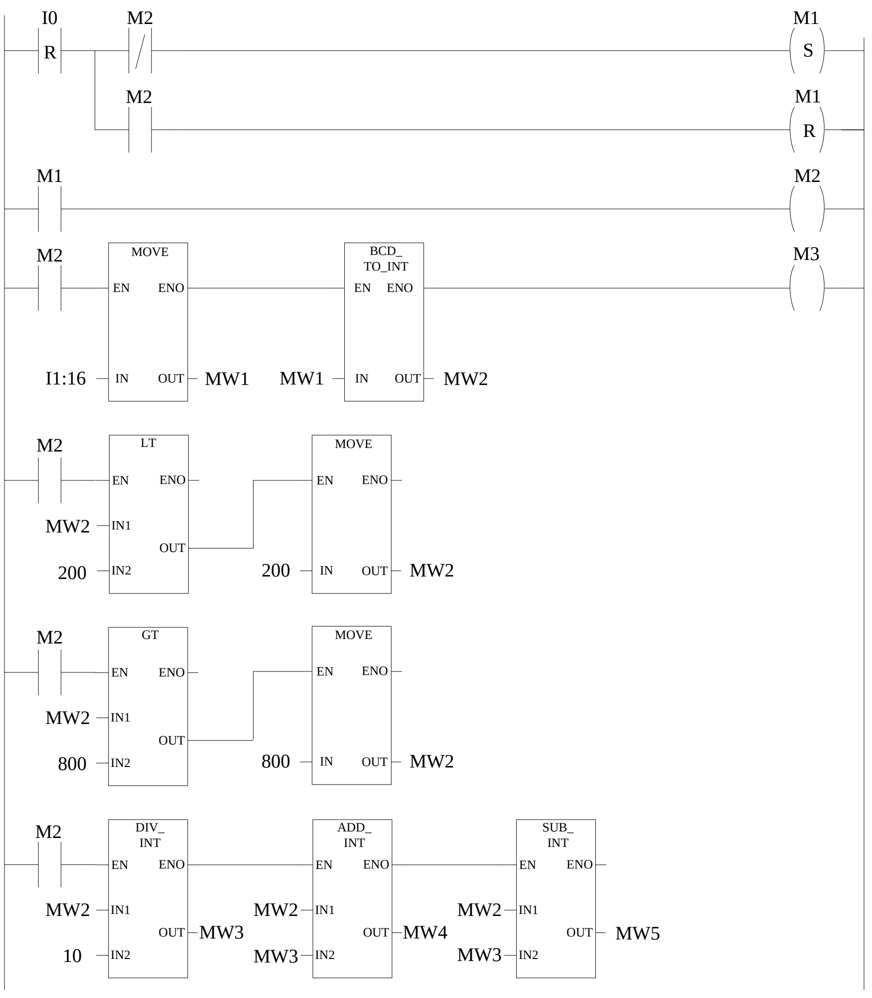
As can be observed, the set point introduced by the operator corresponds to a digital value connected to 16 digital inputs (I1 to I16). This value must be compared with the values of the temperature sensor (IW0). According to its design, the Ladder program generates the control output (Q0) that activates or deactivates the warming resistance, which modifies the oven temperature. Disturbances (like heat losses, oven opening or new materials placed inside) produce decreases in temperature that the control system must maintain constant.

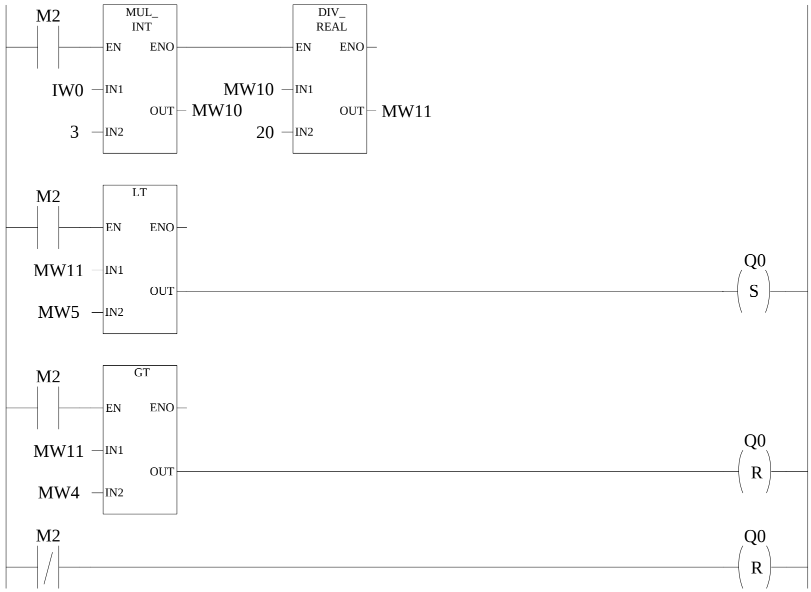
b) Firstly, M1 is programmed as the bit that starts or stops the process. The value of the set point (I1:16) is saved in the register MW2, converting the BCD value to INT format in order to operate with it. This variable is compared with the limits and is modified when the set point exceeds these limits.

Then the values that activate (MW5) or deactivate (MW4) the control output are calculated. The next line reads the value of IW0 and converts it into a range of temperatures (MW11) according to the equation below:

$$T^{a} = \frac{3 \cdot IW0}{2\phi}$$

Finally, the vaue of M11 is compared with those of MW5 and MW4, in order to control the state of Q0.





Problem 5.2.6

Design the ladder program for controlling the volume of a tank, which varies dynamically depending on the process conditions. The system has the following elements:

– A single start/stop push button (NO) connected to the discrete input I0.

– The user defines the set point of the process by means of a potentiometer system connected to the input IW0 with the following relationships:

	Input signal	Volume in the tank	IW0
Min	0 V	1,500 L	0
Max	10 V	2,000 L	10000

– An analog control valve that linearly regulates the input flow rate using the analog output of the PLC with the following relationships.

	Output signal	Analog Valve	QW0
Min	0 V	Closed	0
Max	10 V	Full open	10000

– A load cell which provides information on the tank volume with the following ratio values:

	Input signal	Volume	IW1
Min	0 V	0 L	0
Max	10 V	2,500 L	10000

Program a proportional controller when the process is running, so that the process control signal (signal applied to the analog valve, QW0) is proportional to the error signal.

– The error signal is defined as the difference between the set point minus the real volume of the tank. ε = Set point – Volume.

– If the error is positive (set point of the volume > actual volume), the control signal will be a 5 * Error signal. If the value of the product exceeds 10000, the output value will be 10000, to avoid saturating the valve.

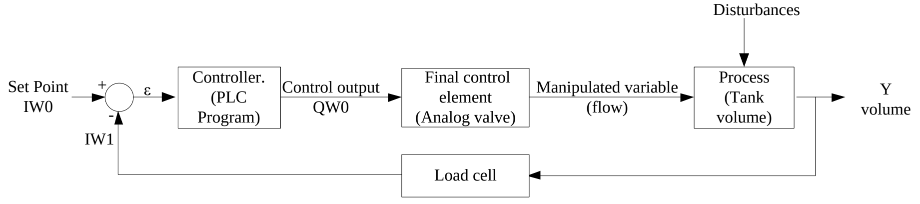
– If the error is negative (set point Volume < actual volume), the valve will be closed (QW0 = 0).

a) Plot the block diagram of the closed-loop control, identifying each input/output of the PLC with the variables that shape a closed-loop control.

b) Design and plot the Ladder program to perform the control.

Solution

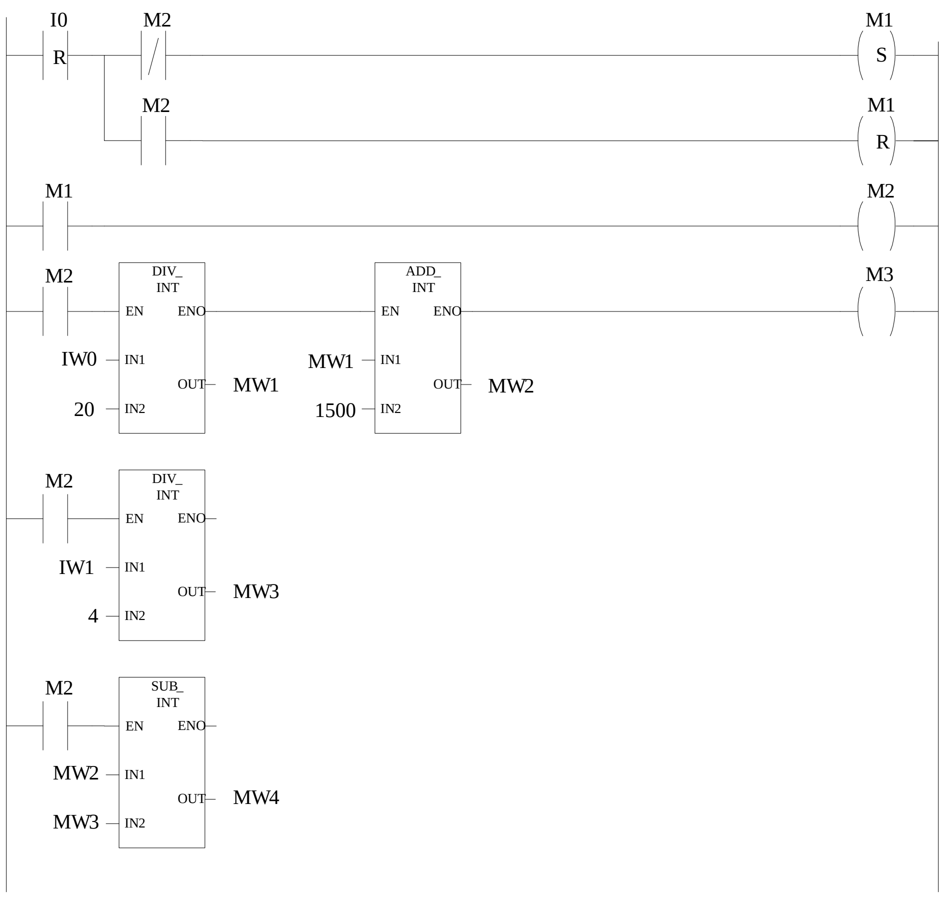
a) The correspondence between a closed-loop control and the physical variables of a system (in this case the PLC) is shown in the figure below:

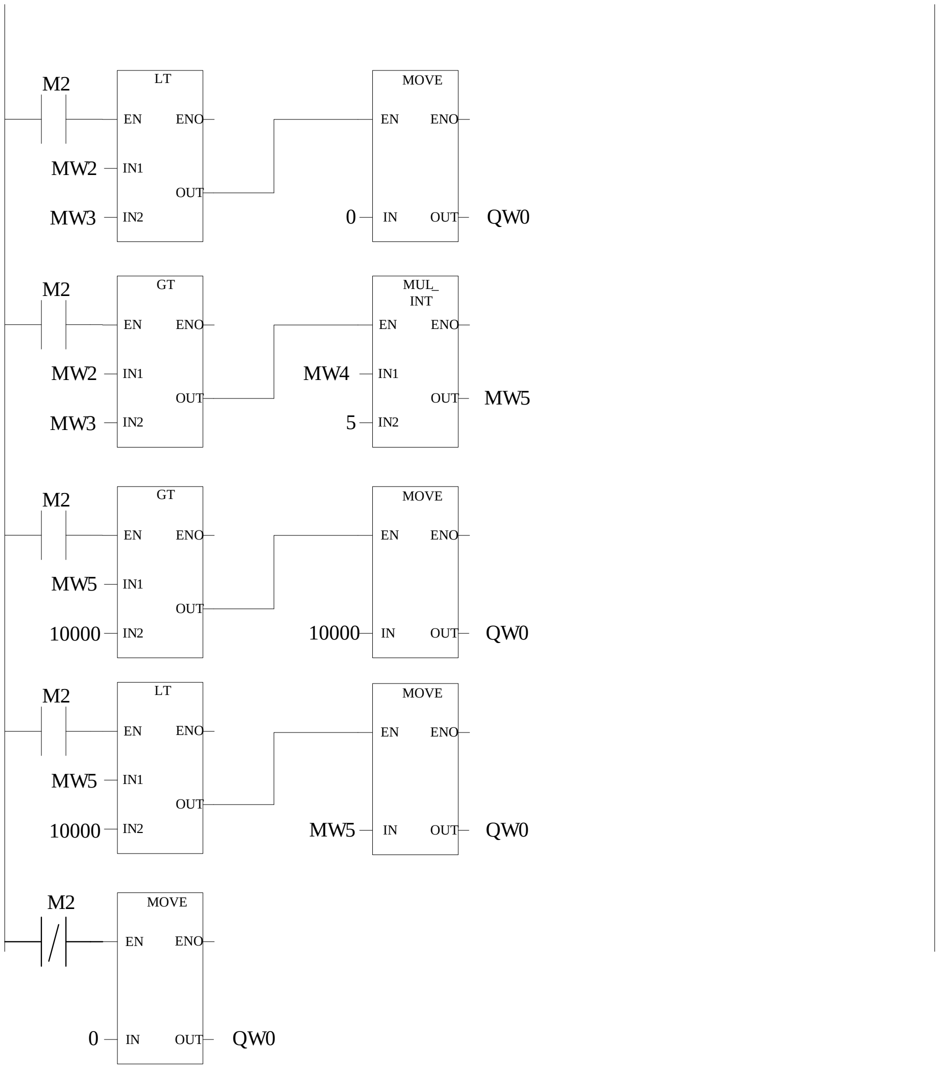


As can be observed, the set point introduced by the operator corresponds to an analog value that is directly proportional to the desired level. This value must be compared with the value supplied by the analog input IW1 from the load cell. These two registers provide different ranges of volume for the same range of the analog input, so the ladder program must change the value reading supplied by the analog inputs, in order to compare an equivalent value in volume.

From the error signal, the program executes the proportional controller by assigning an analog value to the final control element, which regulates the flow rate entering the tank.

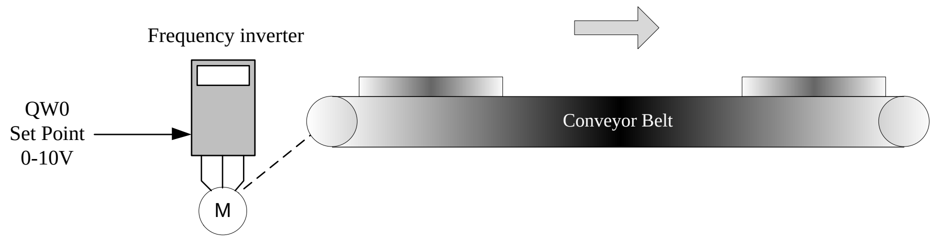
b) The image below shows the solution for controlling the process. The first two lines allow the activation of M2 in order to start the process. The following three lines calculate the set point (MW2), the volume of the tank (MW3) and the error (MW4). Finally, by comparing the error, the control signal applied over the analog output QW0 is calculated.





Problem 5.2.7

A control speed of a conveyor belt has two buttons: Start (I0, N0) and Stop (I1, NC).The set point of the speed control of the conveyor is supplied by the analog input IW0:



	Input Signal V	Set point speed cm/min	IW0
Min	0	0	0
Max	10	400	10000

A 100:1 reducer is placed in the motor shaft, with a tachometric input which supplies information about the rotational speed in rpm to the PLC (IW1).

	Input Signal V	Speed rpm	IW1
Min	0	0	0
Max	10	20	10000

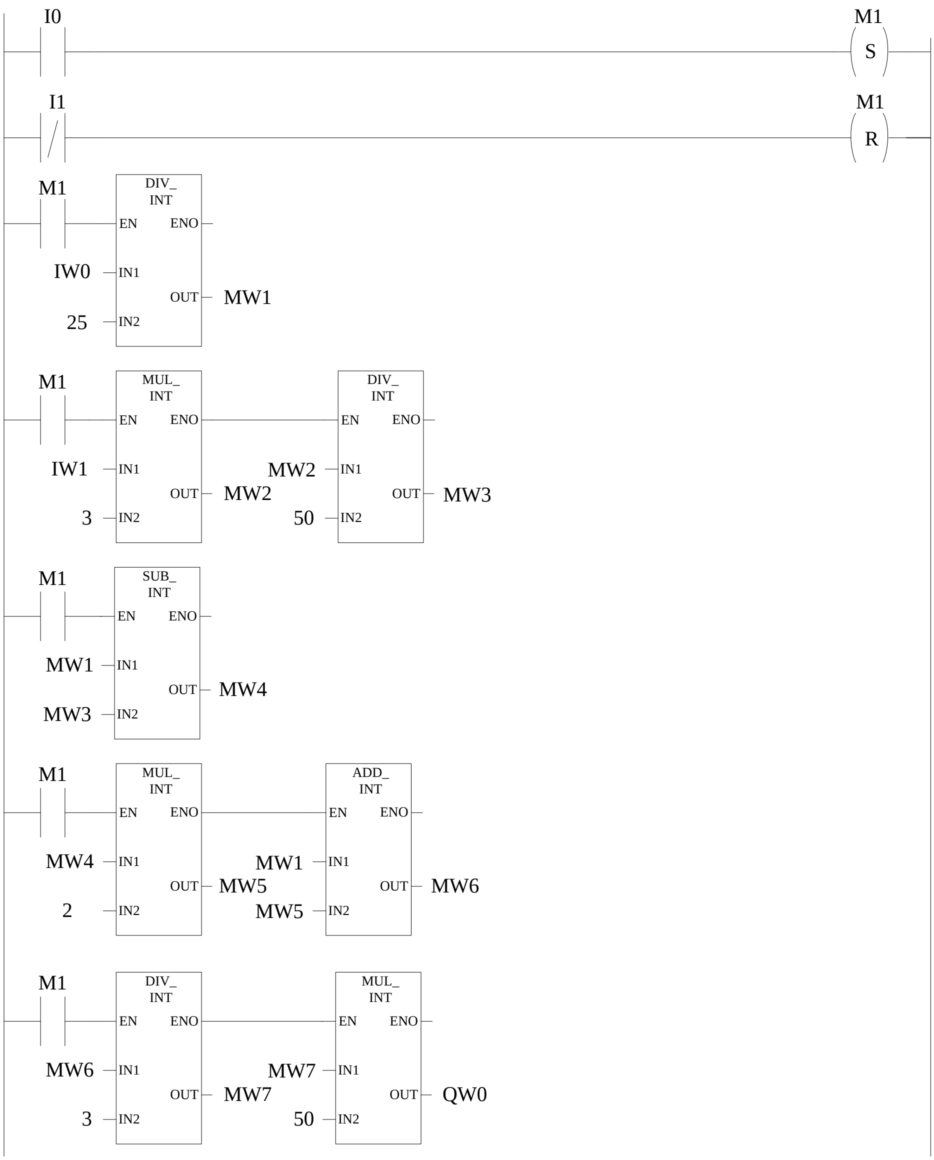
The conveyor advances 30 cm when the motor completes one revolution. Design a Ladder program for calculating the signal assigned to a frequency inverter (QW0) and which follows the value of the set point. The relationship is in the table below:

	Output Signal V	Set point speed rpm	QW0
Min	0	0	0
Max	10	20 rpm	10000

The value of the control signal is equal to:

$$control = SP + 2 \cdot \varepsilon$$

Solution



In the solution, the variable M1 controls the activation of the process. Firstly, set point IW0 is captured, converting the variable to a range in cm/min (MW1). In the same way, the actual motor speed is acquired by IW1, and converted to cm/min (MW3).

In the next line, the value of the error is stored in MW4, and the control signal is calculated in cm/min (MW6). Finally MW6 is converted to the range of QW0 values.

5.3 Control of continuous processes with PID function

As seen in the previous problems, the basic condition in a regulation process is that the output must follow the set point. This aim is achieved by generating a control signal that should command the final control element, which regulates the manipulated variable. Many PLCs include the PID block function that implements the algorithm.

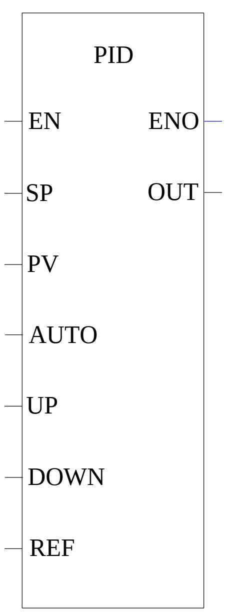
The PID controller includes three actions over the error signal:

- Proportional action P: proportional to the actual error
- Integral action I: proportional to the accumulated error
- Derivative action D: proportional to the speed error

Depending on the type of action, the following combinations are available: P, I, PI, PD and PID.

The representation of the PID function block, with its input and output parameters, is shown below:

PID Function



Parameter	TYPE	Description
EN	BOOL	Input for enabling the block function
SP	INT	Integer variable with the set point of the process.
PV	INT	Integer variable with the value for the control output.
AUTO	BOOL	Input for automatic (1) or manual (0) mode control
UP	BOOL	Increases the value of the OUT variable in one unit for each rising edge at its input.
DOWN	BOOL	Decreases the value of the OUT variable in one unit for each rising edge at its input.
REF	INT	Base register from which the parameters determining the PID algorithm are stored.
ENO	BOOL	Active output whenever the PID algorithm is executed.
OUT	INT	Result of the control signal.

It is necessary to program in the instruction the register (REF), which will be the first of a series of registers with the different parameters needed by the PID algorithm. Each manufacturer uses a number of PLC registers and their allocation within its memory map, so that they can implement the variables needed by the algorithm. The next table shows an allocation with the common parameters that this function uses to solve the following problems.

Register	Parameter	Description
REF	Loop	Integer variable that identifies the control loop within the program.
REF + 1	Algorithm	Integer variable that indicates the type of algorithm which is applied to the function (ISA is defined as 1 and IND is defined as 2)
REF + 2	Execution period	Time elapsed between two executions of the algorithm if the EN input is active. The value zero implies that the algorithm is continuously executed.
REF + 3	Dead Band +	When the PV value is comprised between SP and SP + Dead Band+, the algorithm does not work.
REF + 4	Dead Band -	When the PV value is comprised between SP and SP - Dead Band-, the algorithm does not work.
REF + 5	Proportional Gain	Integer variable that sets the proportional gain of the PID algorithm.
REF + 6	Integral Tine	Integer variable that sets the integral time.
REF + 7	Derivative time	Integer variable that sets the derivative time.
REF + 8	Bias	Integer variable that is added to the control signal which results from fitting the algorithm to the range of the final control element.
REF + 9	Upper value of CV	Integer variable that indicates the maximum value that can reach CV, in order to avoid saturating the final control element.
REF + 10	Lower value of CV	Integer variable that indicates the minimum value that can reach CV, in order to avoid saturating the final control element.
REF + 11	Minimum rise time	Positive integer variable that defines the minimum rise time of the control signal from 0 to 100%, required for the final control element to continue the output variable.
REF + 12	SP	Integer variable where the set point is stored by the user.

Register	Parameter	Description
REF + 13	CV	Integer variable resulting from the application of the control algorithm.
REF + 14	PV	Integer variable with the value of the process output.
REF + 15	OUT	Integer variable with the control signal.

The parameters given in the above table must be pre-programmed before using the PID function. Most variables depend on structural process values, such as the final control element, the PV and the set point values; these values are determined from the characteristic of the physical elements. However, there are some parameters that are characteristic of the process, such as the proportional gain as well as integral and derivative times whose time values must be experimentally determined by tuning the PID controller.

Problem 5.3.1

Describe the PID algorithm and how each term affects the control signal and the process.

Solution

The PID controller includes three actions acting on the error signal: One action (P) is directly proportional to the error; another action is proportional to the integral error (I); and, finally, another action is proportional to the error derivative. Depending on the type of action that forms the control signal, different controllers are available: P, PI or PID. Then, the corresponding individual actions and their effect on the controlled variable are described:

– *Proportional action*. The controller generates an output that is directly proportional (Kp) to the evolution of the error signal over time. This single control has a limited use because it generates a stable process error (offset).

$$u(t) = K_p \cdot \varepsilon(t)$$

– *Integral action.* This action generates an output that is proportional (K_i) to the accumulated error (integral error), which implies a slow mode control that can reach the saturation of the final control element. However, it eliminates the permanent offset.

$$u(t) = K_i \int \varepsilon(t) dt$$

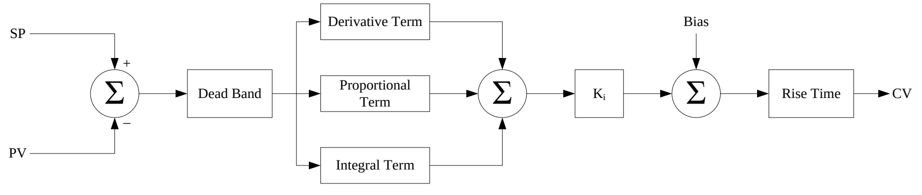
– *Derivative action.* The derivative action generates a signal which is proportional (K_d) to the derivative of the error. The control signal generated is proportional to the slope of the error, so it tends to anticipate and minimize error variations.

$$u(t) = K_d \frac{d\varepsilon(t)}{dt}$$

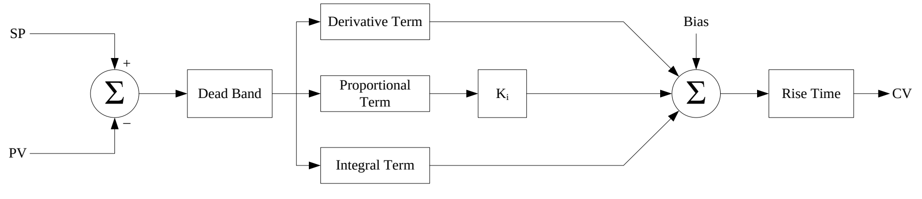
From the combined actions of three independent controls, the equation for the PID controller is determined:

$$u(t) = K_p \cdot \varepsilon(t) + K_i \int \varepsilon(t)dt + K_d \frac{d\varepsilon(t)}{dt}$$

In the figure below, the two typical structures of the operation of a PID function block can be seen: the standard PIDISA (a), where a proportional term is applied to each of the control actions; and the PIDIND (b), where the proportional term is applied only to the proportional action.



(a)



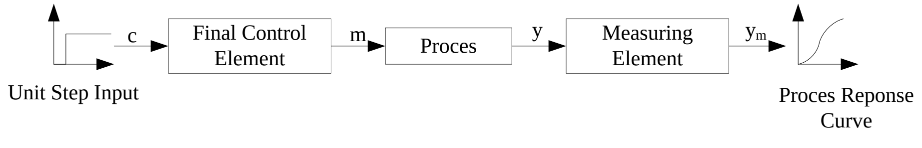
(b)

Problem 5.3.2

Explain the tuning method for a PID controller proposed by Ziegler and Nichols, based on the process reaction curve in open loop.

Solution

The tuning method of Ziegler and Nichols is based on determining the reaction curve of the process by applying a step signal when it is working in open loop:



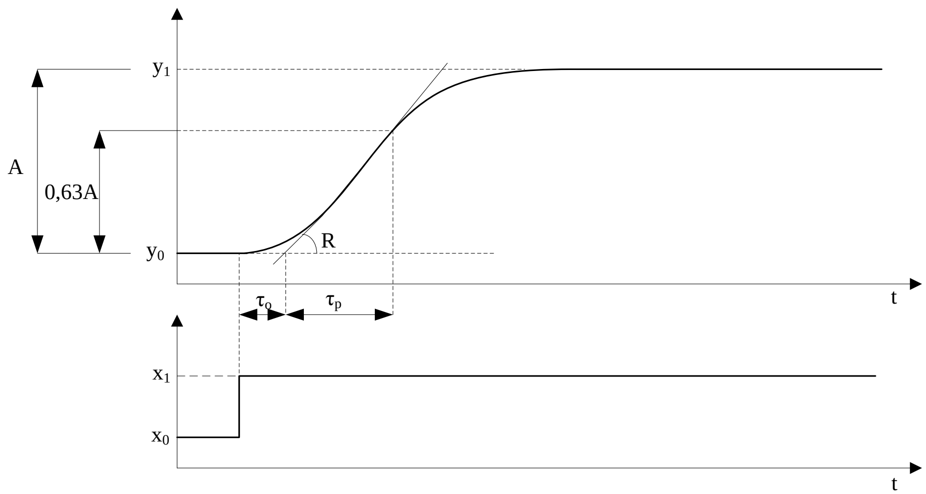
From this point, and by following the next steps, it is possible to determine its characteristic parameters:

1. With the elements in open loop, the process is carried out manually to a stable output y_0 by applying a value of set point x_0.

2. The set point is changed from x_0 to x_1, a value from 10% to 20% of the value of the final control element.

3. The output response of the process continues until a stabilized value y_1 is reached. Such a response, when plotted, is known as *the process response curve.*

The response of most chemical processes to a step input is similar to the figure below, which corresponds with a first order system with dead time. In this figure, it can be seen as the output evolves from y_0 to y_1. The step input amplitude is the difference between x_1 and x_0.



The parameters of the reaction curve are determined as follows:

1. *Process gain constant (K).* This value corresponds to the relationship between the increases of the output signal and the set point signal, defined by the formula.

$$K = \frac{y_1 - y_0}{x_1 - x_0}$$

2. *Slope of the response (R).* This parameter is defined by the slope of the tangent line at the inflection point of the process reaction curve.

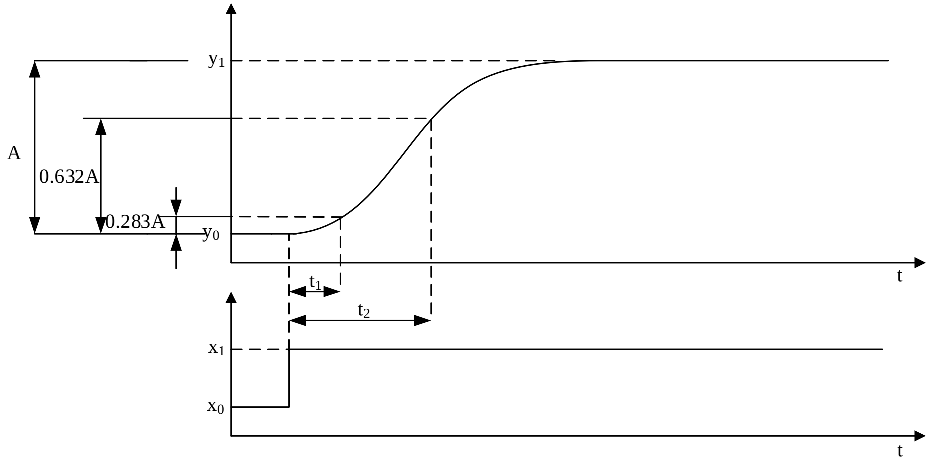
3. Process time constant (τp). This parameter is defined by the time elapsed from when the tangent crosses the initial value of the controlled variable (y0) until the process reaches 63% of the final response value.

4. *Dead Time or delay* (τ_0). This value corresponds to the time elapsed between the raising edge of the set point and the time where the tangent intersects with y_0.

Determining the tangent is the problem with this method, because it may lead to different results. The method of the two points, proposed by C. L. Smith simplifies the calculation of the parameters of the process reaction curve. This method consists of determining the time elapsed when the response reaches 28.3% (t_1) and 63.2% (t_2) of its final value. From these values, τ_p and τ_0 are calculated:

$$\tau_p = 1.5 \cdot (t_2 - t_1)$$

$$\tau_0 = (t_2 - \tau_p)$$



Now, the Ziegler and Nichols method sets the parameters of the PID controller according to the following table:

Controller	K_p	T_i	T_d
P	τ_p / τ_0		
PI	$0.9 \cdot (\tau_p / \tau_0)$	$\tau_0 / 0.3$	
PID	$1.2 \cdot (\tau_p / \tau_0)$	$2 \cdot \tau_p$	$0.5 \cdot \tau_p$

Thus, the complete formula of the PID algorithm described at the beginning: is implemented with the values obtained:

$$u(t) = K_p \cdot \varepsilon(t) + K_i \int \varepsilon(t) dt + K_d \frac{d\varepsilon(t)}{dt}$$

$$u(t)=K_p\cdot\varepsilon(t)+\frac{1}{T_i}\int\varepsilon(t)dt+T_d\frac{d\varepsilon(t)}{dt}$$

Problem 5.3.3

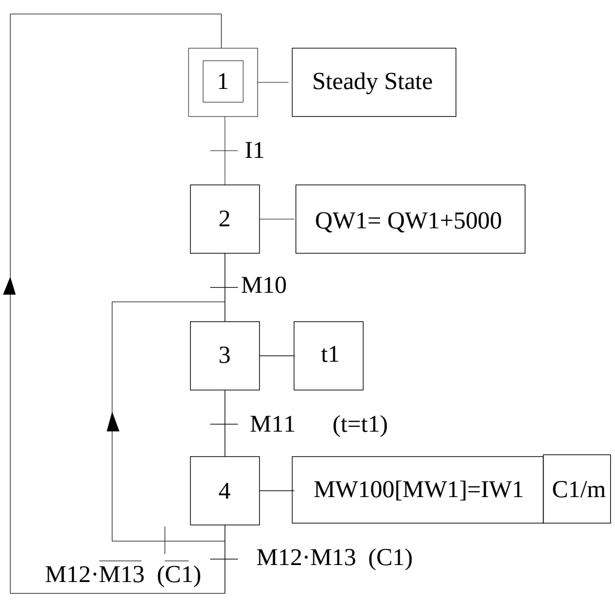
Design the Ladder program to determine the reaction curve of a process controlled by PLC, in which the value of the output variable is read by the analog input IW1, with an input range from 0 to 32,000. The control signal applied to the final control element corresponds to the analog output QW1, with a range of values from 0 to 32,000.

a) Design the Level 3 SFC of the program.

b) From the SFC, determine the Ladder program.

Solution

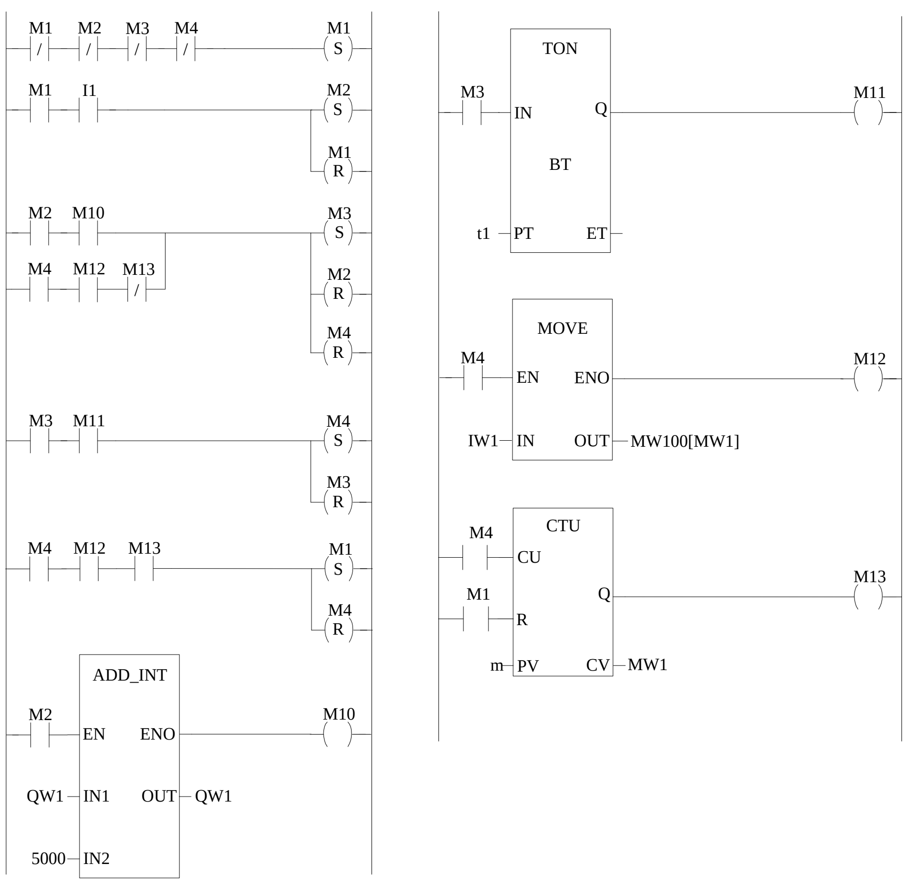
a) First, the user sets an output value, QW1, until the system is stabilized. When stabilized, the program for determining the process response curve corresponds to the figure of the SFC below:



The process for determining the reaction curve starts when pressing I1, generating a step in the set point signal of 5000 in QW1 (QW1 = QW1+5000).

After this operation is executed (M10), a TON timer with a preset value equalto t1 is programmed. This value is set depending on the speed of the system response, and determines the sampling period of the output signal. Then, the process value is captured through the analog input IW1, which is stored as an indexed register that MW100 [MW1] points to. That MW100 will be the first value read and it will be stored in consecutive registers, depending on the value of MW1. The value of %MW depends on a counter with module *m*, which determines the number of samples. So if the current count value is 10 (MW1 = 10), the value for that sample is stored in MW100 [10]. Therefore, the value will be in MW110. Once these two operations have been performed, the system goes to resting state.

b) From the SFC, the program is solved by Ladder instructions, as shown in the figure below:

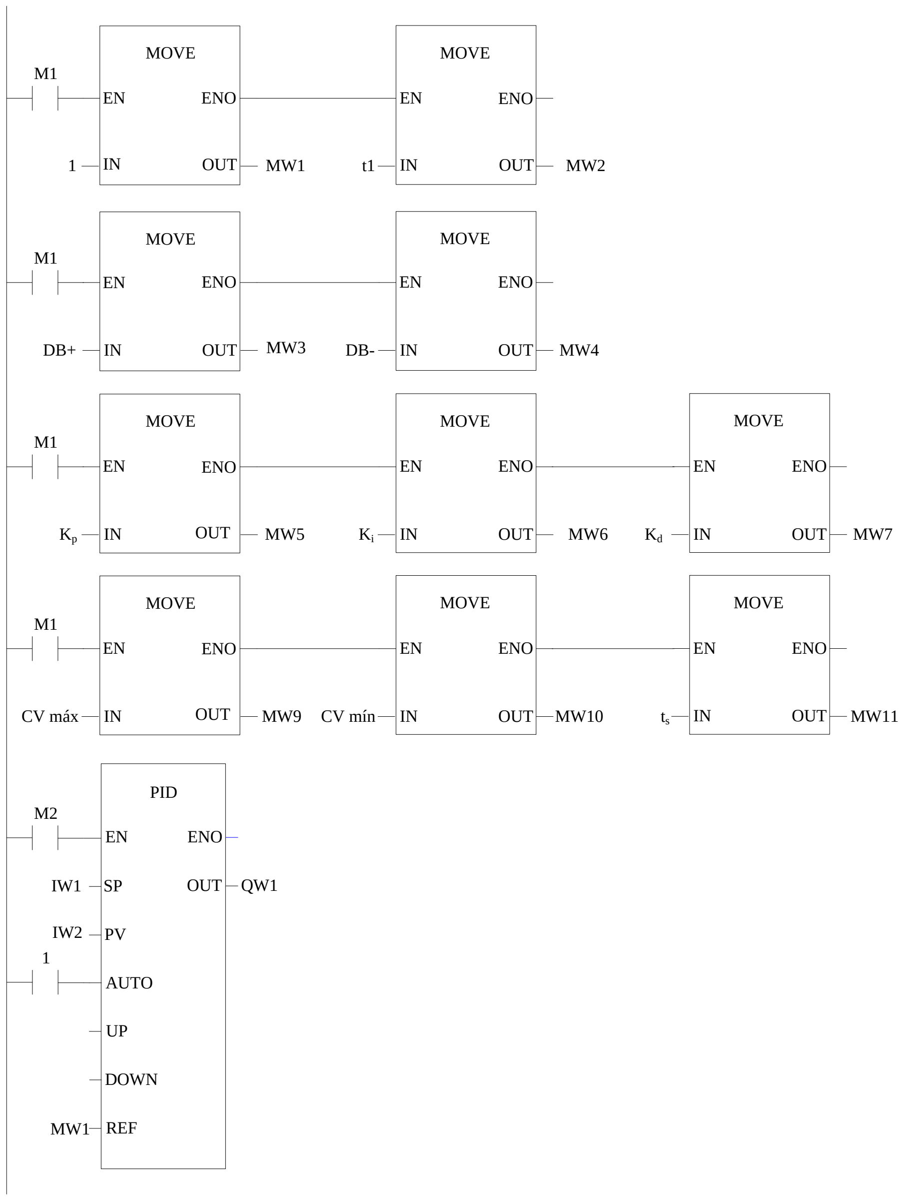


Problem 5.3.4

Design and explain a Ladder program to configure and use the PID instruction.

Solution

From the statement of the exercise, the next Ladder program is designed.



The above figure shows how the PID instruction can be programmed using the values of the process. Before programming the PID instruction, the parameters required by the function must be assigned.

The register MW1 is programmed as the reference base memory in the REF input of the PID instruction. This variable determines where the parameters required by the function must be stored.

The first line of the program, when M1 is activated, assigns the PIDISA algorithm to the function, and the value for the execution period is equal to t1. The next line (2) assigns the dead band values, which are applicable to the execution of the algorithm. In line 3, the characteristic parameters of the PID algorithm (Kp, Ki, Kd) are programmed.

In the fourth line, the maximum and minimum output values are stored, in order not to saturate the final control element and the minimum rise time of the signal (ts). If either of these values reaches the limit, the algorithm stops running and adapts the integral action to match the maximum or minimum value of the output.

Finally, in the last line (5), the PID instruction is programmed with the necessary inputs and outputs of the function: the value IW1, corresponding to the set point; and IW2, corresponding to the output value of the process. The manual input is not implemented in the function block. The output signal (OUT), to be applied to the final control element, is associated to an analog output of the PLC (QW1).

Problem 5.3.5

A large number of industrial continuous processes can be modelled as a first-order system with delay. The study of these processes in discrete devices requires implementing a discrete algorithm that approximates the continuous system.

A first-order system is represented by its transfer function *G (s)*:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$

Determine the discretization of the system to apply the model in a PLC program.

Solution

Starting from the transfer function of the exercise statement, the temporal model of the transfer function can be achieved as below:

$$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = \frac{K}{\tau} u(t)$$

where *y(t)* is the output and *Y(s)* is its Laplace transform, *u(t)* is the input and *U(s)* is the Laplace transform, *K* is the gain and τ is the time constant.

Applying the above formula, the derivative of the output will be:

$$\frac{dy(t)}{dt} = \frac{1}{\tau}(Ku(t) - y(t))$$

From the above equation, the discrete approximation of the derivative output is:

$$\frac{dy}{dt}(n) = \dot{y}(n) = \frac{y(n+1) - y(n)}{dt}$$

where $\dot{y}(n)$ is the derivative of the output sample at time *n*, *y(n)* is the system output sample at time *n*, *y(n +1)* is the system output sample at time *n +1* (subsequent sample), and *dt* corresponds to sampling time *T* of the system.

Consequently the output *y (n +1)*, corresponds to:

$$y(n+1) = \dot{y}(n)T + y(n)$$

Substituting in the above formula the value of $\dot{y}(n)$ found by the previous equation gives:

$$\dot{y}(n) = \frac{1}{\tau}(Ku(n) - y(n))$$

Finally, discretization of the first order continuous system is obtained:

$$y(n+1) = \frac{1}{\tau}(Ku(n) - y(n))T + y(n)$$

Problem 5.3.6

The discretization of a first order process with one second sampling time corresponds to the following equation:

$$y(n+1) = 0.244 \cdot u(n) + 0.951 y(n)$$

From the above discrete function, determine the transfer function of the system.

Solution

According to the previous exercise, the discretization of a first order equation corresponds to:

$$y(n+1) = \frac{1}{\tau}(Ku(n) - y(n))T + y(n)$$

If the sampling period value is one second, the equation will be:

$$y(n+1) = \frac{1}{\tau}(Ku(n) - y(n)) + y(n)$$

Grouping the terms:

$$y(n+1)=\frac{K}{\tau}u(n)+\frac{\tau-1}{\tau}y(n)$$

Then, matching each term in the above equation with the expression of the statement gives:

$$\frac{K}{\tau}=0.244$$

$$\frac{\tau-1}{\tau}=0.951$$

Solving the above equations gives:

$$\tau=20.4$$

$$K=4.97$$

Therefore, the Laplace transform of the process is:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{4.97}{20.4s+1}$$

Problem 5.3.7

A chemical process behaves as a first order system with delay according to the following transfer function.

$$G(s)=(Y(s))/(U(s))=K/(\tau s+1)e^{(-t_0 s)}$$

where the value of t_o corresponds to the response delay of the output when a change in its input is applied. Considering a delay time of 4 seconds, a transfer function and its discretization model like the previous exercise:

$$y(n+1)=0.244\cdot u(n)+0.951y(n)$$

a) Create a spreadsheet with the system response in open loop when an input of 25 units is applied for 120 seconds.

b) Draw the response.

c) Determine the parameters of the process reaction curve.

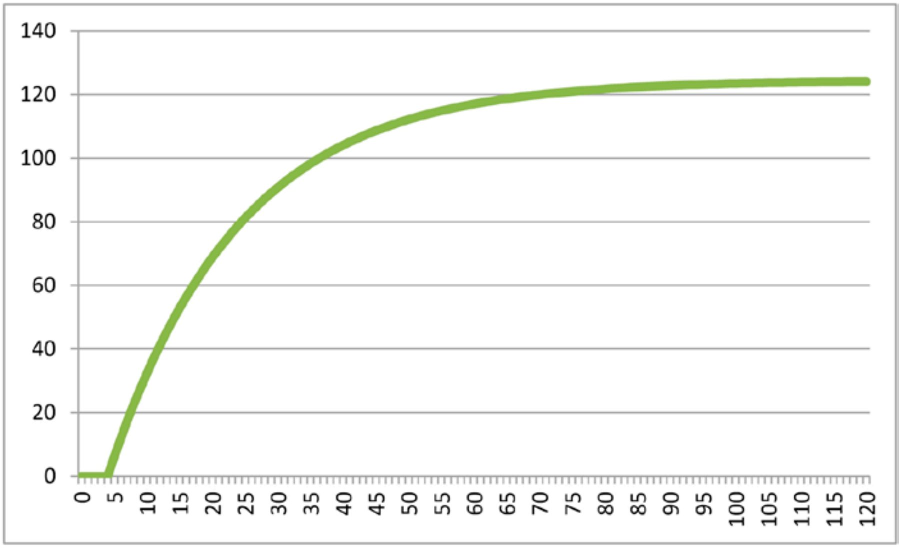
d) Using the Ziegler and Nichols method, determine the parameters of the PID controller (*Kp*, *Ti*, *Td*)

Solution

a) Introducing the discretization of the function in a spreadsheet with the values of the statement gives:

t(s)	output	t(s)	output	t(s)	output	t(s)	output	t(s)	output	t(s)	output
1	0.00	**21**	71.50	**41**	105.09	**61**	117.39	**81**	121.89	**101**	123.54
2	0.00	**22**	74.10	**42**	106.04	**62**	117.74	**82**	122.02	**102**	123.58
3	0.00	**23**	76.56	**43**	106.94	**63**	118.07	**83**	122.14	**103**	123.63
4	0.00	**24**	78.91	**44**	107.80	**64**	118.38	**84**	122.25	**104**	123.67
5	6.10	**25**	81.15	**45**	108.62	**65**	118.68	**85**	122.36	**105**	123.71
6	11.90	**26**	83.27	**46**	109.40	**66**	118.96	**86**	122.47	**106**	123.75
7	17.42	**27**	85.29	**47**	110.14	**67**	119.24	**87**	122.57	**107**	123.79
8	22.66	**28**	87.21	**48**	110.84	**68**	119.49	**88**	122.66	**108**	123.82
9	27.65	**29**	89.04	**49**	111.51	**69**	119.74	**89**	122.75	**109**	123.85
10	32.40	**30**	90.77	**50**	112.15	**70**	119.97	**90**	122.84	**110**	123.88
11	36.91	**31**	92.43	**51**	112.75	**71**	120.19	**91**	122.92	**111**	123.91
12	41.20	**32**	94.00	**52**	113.33	**72**	120.40	**92**	122.99	**112**	123.94
13	45.28	**33**	95.49	**53**	113.87	**73**	120.60	**93**	123.07	**113**	123.97
14	49.16	**34**	96.91	**54**	114.39	**74**	120.79	**94**	123.14	**114**	123.99
15	52.86	**35**	98.26	**55**	114.89	**75**	120.97	**95**	123.20	**115**	124.02
16	56.37	**36**	99.55	**56**	115.36	**76**	121.15	**96**	123.27	**116**	124.04
17	59.70	**37**	100.77	**57**	115.81	**77**	121.31	**97**	123.33	**117**	124.06
18	62.88	**38**	101.93	**58**	116.23	**78**	121.47	**98**	123.38	**118**	124.08
19	65.90	**39**	103.04	**59**	116.64	**79**	121.61	**99**	123.44	**119**	124.10
20	68.77	**40**	104.09	**60**	117.02	**80**	121.76	**100**	123.49	**120**	124.12

b) From the above values, the response of the process can be drawn:



c) Parameters of the process reaction curve:

c1. Process gain constant:

$$K = \frac{\Delta y}{\Delta x} = \frac{124.12}{25} = 4.97$$

c2. Response at 28.3% and 63.2% of the final value:

The final value is equal to 124.12. Then, the values that correspond to 28.3% and 63.2% are, respectively, 35.12 and 78.44. By seeking these output values in the table response, the values of *t1* and *t2* can be found:

t1 = 10.6 s
t2= 23.8 s

c3. τp = 1.5 (t2-t1)= 1.5(23.7-10.6)= 19.65 s
c4. τ0= (t2-tp)= 23.7-19.65= 4.05 s

These values have been obtained from the table based on the discretization of the function. These values are approximately the same as those which were obtained by applying the Laplace transform and obviating the errors stemming from the approximation of the values found in the response curve.

d) Using the table with the formulas proposed by Ziegler and Nichols to set the parameters of the PID controller gives:

– $K_p = 1.2\cdot(\tau_{p/}\,\tau_0)$= 1.2(19.65/4.05)= 5,82

– $T_i = 2\cdot\tau_p$=2·19.65 = 39.3 s

– $T_d = 0.5\cdot\ \tau_p$ = 0.5·19.65 = 9.825 s

Problem 5.3.8

Design a Ladder program to study the behaviour of a first order process with delay using a PLC. The design of the program must fulfil the criteria in the sections below:

1. Main program: Acting on the inputs I0 and I1, the user will select which control to develop for the system according to the next combinations of inputs:

Input I0	Input I1	Control
0	0	Open loop
0	1	Closed Loop without controller
1	0	Closed Loop with PID controller

After selecting the type of control, a rising edge in the I3 input will activate the process by calling a subroutine which implements the control selected. In order to know the output response, an array of 120 values will save the output from the rising edge in one-second intervals.

2. To simulate the first order with delay process, a subroutine must be programmed with its discretization, as shown in the previous exercises. This subroutine will be called from the main program while the control is being developed.

$$y(n+1) = 0.244 \cdot u(n) + 0.951 y(n)$$

Depending on the type of control, the value of *u(n)* will correspond to:

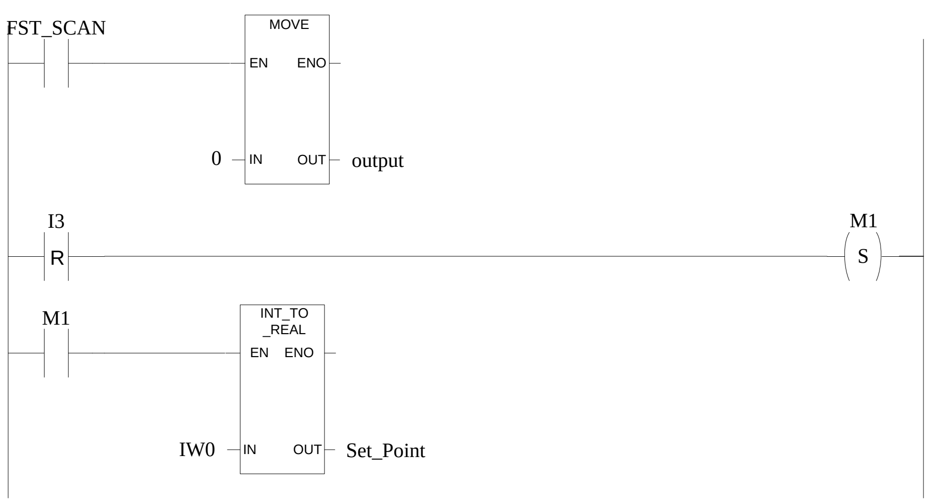
a) Open Loop: *u(n)* will be the value of the set point introduced by the user.

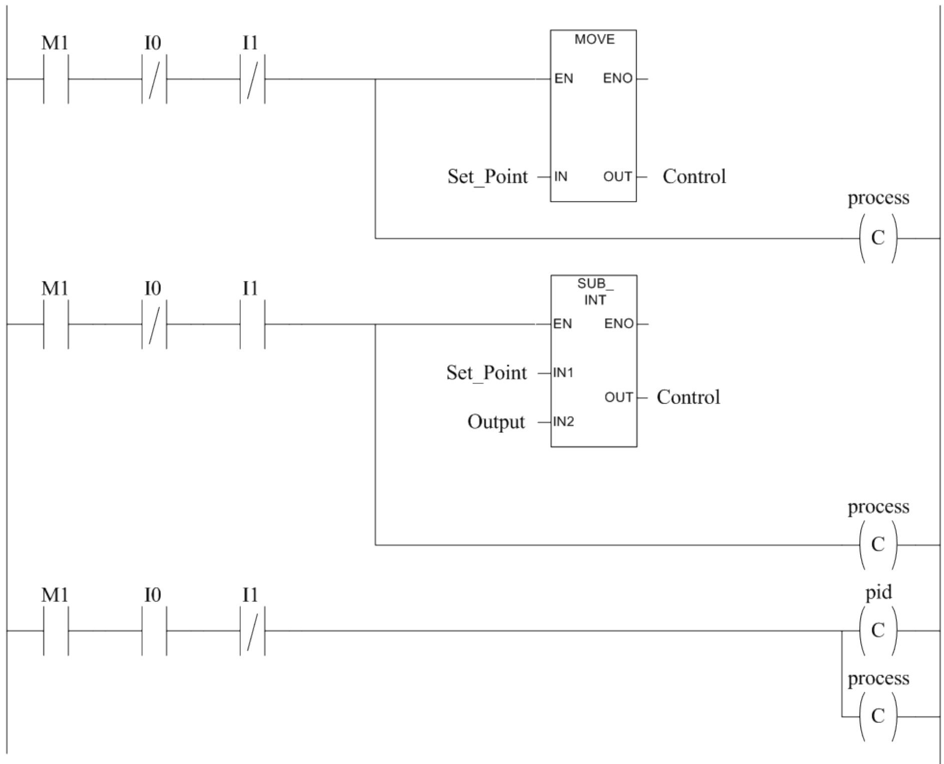
b) Closed Loop: *u(n)* will be the output process subtracted from the set point.

c) Closed Loop with PID controller: *u(n)* will correspond to the output value of the PID instruction

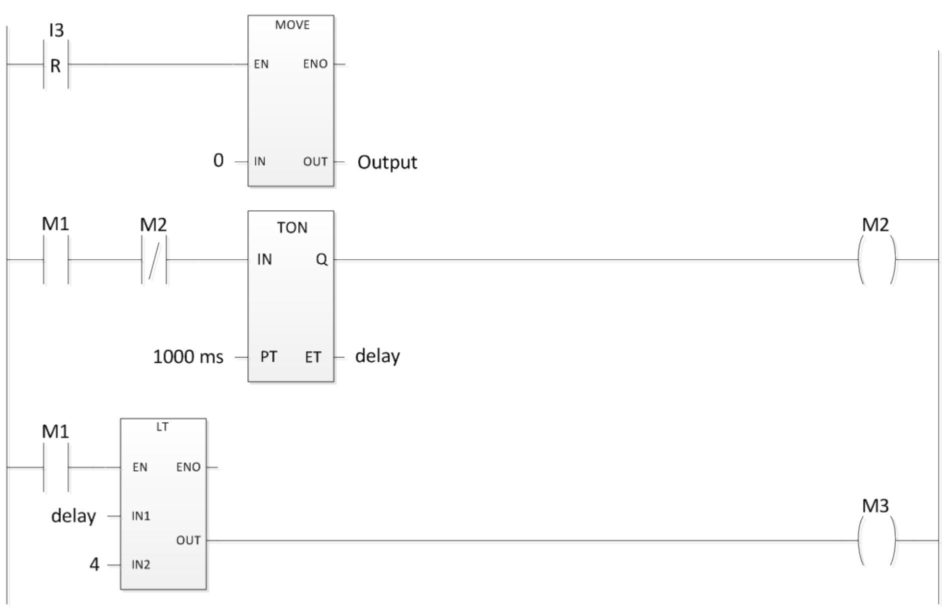
Solution

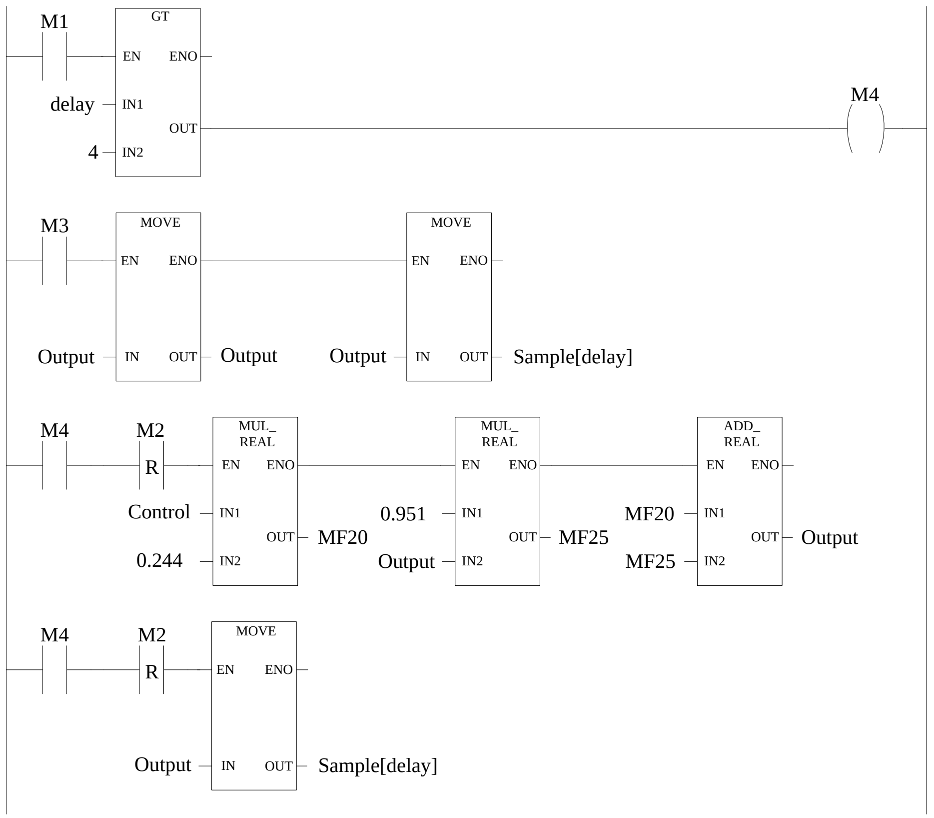
a) The solution of the exercise begins by creating the main routine. In order to manage the values, three real variables are created (*Control*, Set Point and output). Once the process activating I3 is started, and depending on the inputs I0 and I1, the respective control is implemented, calling the subroutines *process* and *PID.*





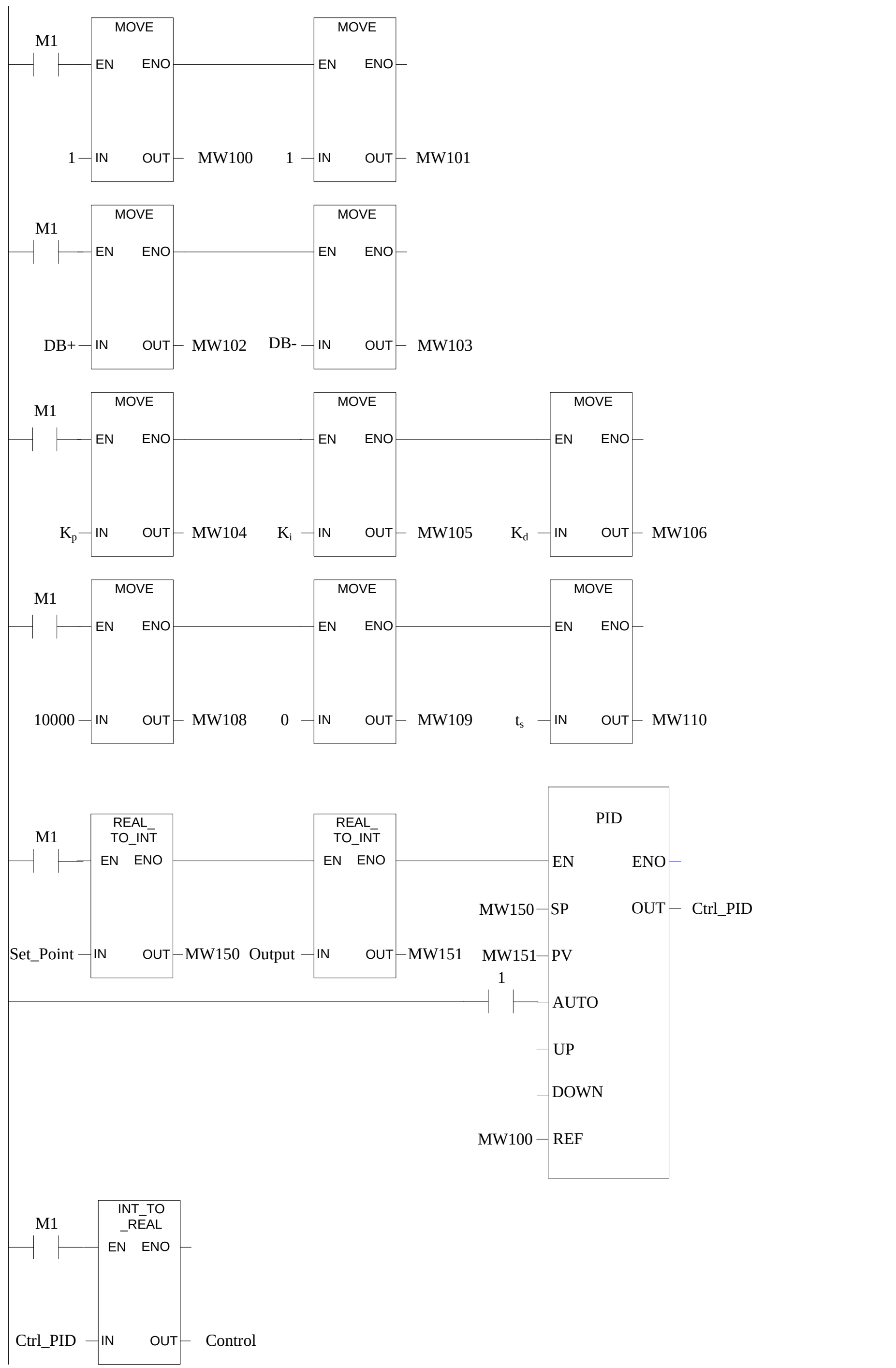
b) Once the main routine is programmed, the next step consists of developing the *process* subroutine in order to discretize the process and obtain the output:





As can be seen, when the rising edge in I3 is detected, the output is set to the initial value 0. Then, a Timer is programmed in order to create the one-second sampling period for saving the value of the output in the array. The Counter programmed in the next line controls the number of the sample with the variable *delay* programmed in its Current Value output (CV). Finally, the two last lines perform the discretization of the model, the first one introducing the delay and the second one obtaining the discretized output. The value is stored in the array named *sample*, whose index corresponds to the current value of the counter (*sample [time]).*

c) Finally the PID subroutine is programmed, with the values of K, T_i and T_d, being used for the control of the process.



M1
MOVE
EN ENO
1 IN OUT MW100
MOVE
EN ENO
1 IN OUT MW101
M1
MOVE
EN ENO
DB+ IN OUT MW102
MOVE
EN ENO
DB- IN OUT MW103
M1
MOVE
EN ENO
K_p IN OUT MW104
MOVE
EN ENO
K_i IN OUT MW105
MOVE
EN ENO
K_d IN OUT MW106
M1
MOVE
EN ENO
10000 IN OUT MW108
MOVE
EN ENO
0 IN OUT MW109
MOVE
EN ENO
t_s IN OUT MW110
M1
REAL_ TO_INT
EN ENO
Set_Point IN OUT MW150
REAL_ TO_INT
EN ENO
Output IN OUT MW151
PID
EN ENO
MW150 SP OUT Ctrl_PID
MW151 PV
1
AUTO
UP
DOWN
MW100 REF
M1
INT_TO _REAL
EN ENO
Ctrl_PID IN OUT Control