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1. INTRODUCTION

The actual demand of Internet services is increasing fast and the contents are becoming heavier because of the higher demand of video and voice services as VoIP and HDTV. This fact is pushing the actual infrastructure to the limit and service providers are lead to invest in new technologies and new infrastructure. The solution seems to be found in the use of cable and fiber optics but while this technologies are capable of offering video and voice services they cannot offer the 3 services together (voice, video and Internet access). Also the use of this technologies means that the whole actual infrastructure need to be changed.

Meanwhile this technologies are not ready, the solution is to improve the performance of the exiting copper pair infrastructure and nowadays the technology that does this better is VDSL which can provide up to 52 Mbps on a single cooper pair loop. Besides, VDSL, as other DSL technologies, is capable of providing Internet and voice services because it is compatible with POTS and ISDN and in addition, VDSL can provide video services thanks to the higher throughput.

To achieve this higher bit rates, VDSL uses the frequency range up to 30 MHz and DMT modulations as OFDM that divides the frequency spectrum into a number of tones as FDM but with the particular characteristic that the tones are orthogonal providing a very limited intercarrier interference (ICI) and a better performance.

Summed to VDSL, in the last years, the use of multiple transmitters and receivers is revolutionizing telecommunications. MIMO systems has been proved to be a very useful tool to improve capacity in wireless communications by using several antennas at the transmitter and the receiver side but it is also possible to use MIMO in wired communications and in particular in DSL transmissions. For example, using 4 bonded lines it is possible to operate at more than 6 times the rate of a single line.

In this paper we will combine the benefits of VDSL and MIMO technologies to implement simulations of the capacity on a system composed of 8 lines transmitting together. The simulations have the purpose of seeing how the capacity of the system is affected by internal and external phenomena as for example different kind of interferences. In the part "Simulations of the capacity" you will find different simulations of a binder of 8 lines using MIMO and VDSL with thermal noise and external noise affecting as well as different power constraints. The external noise here comes from a parallel transmission of another 8-line binder. There is also a simulation of a SISO system with an external noise coming from the parallel lines of the original 8-line binder.

We have considered 2 scenarios for the different simulations: the fist one where we evaluate capacity with loops of different lengths and the second one where we evaluate capacity for binders with different number of lines. The different simulations are:

- Capacity under total power constraints with Additive White Gaussian Noise (AWGN).
- Capacity under total power constraints with AWGN and external noise.
- Capacity under per-modem total power constraints with AWGN and external noise.
- Capacity under per-modem total power constraints and spectral mask constraints with AWGN and external noise.
- Capacity of a SISO system with AWGN and external noise (coming from the parallel lines).

After evaluating each simulation separately there is a comparison of all them together where we are able to see their achievements and their behaviors.

2. <u>VDSL</u>

2.1 Introduction

Both consumers and businesses are using the many benefits of the Internet, while innovative service providers are tapping into the convergent technologies to offer more complete and higher-speed services. As the Internet gains more users, the content becomes heavier, and the delivery of services becomes more difficult.

The ADSL Standard provides up to 6 Mbps downstream and 640 kbps upstream in a transmission loop distance of over 4 kilometers but while it delivers high-speed Internet access over the existing narrow band network, it falls far short of being able to deliver full services that include video.

Nowadays, while the cable technologies are still developing to offer Internet access plus video and voice, the most promising technology capable of this is VDSL (Very-high-bit-rate DSL).

In simple terms, VDSL transmits high speed data over short reaches of twisted-pair copper telephone lines, with range of speeds depending upon the actual line length.

VDSL can be both symmetric and asymmetric and it can provide up to 52Mbps of bandwidth on a single twisted-pair loop in short distances (≈300 meters) as a result of the compromise that always takes part in DSL technologies between capacity and distance.

2.2 Architecture

The model of VDSL architecture exposed in Figure 1 is the most common in which, from the homes with full access to voice, data and video services, the twisted pair reaches the Network Units and from that point the information gets to the Central Office on fiber optic.

As mentioned before, the huge range of frequencies and the high bit-rates allow short length twisted pairs and, as a result, service providers have to improve their infrastructures to approach fiber optic to the homes which is one of the biggest challenges for VDSL.

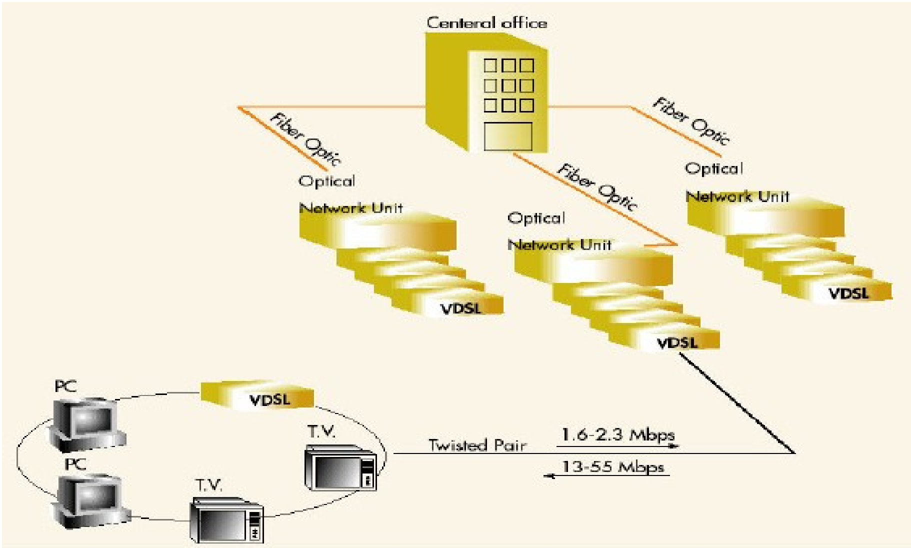


Figure 1. VDSL – General Architecture

2.3 Frequency band

Like other DSL technologies, VDSL uses the higher-frequency spectrum available over standard copper above the frequencies used for POTS and ISDN. This technology enables service providers to use the existing copper infrastructure for the delivery of broadband services over the same physical plant. The VDSL spectrum is specified to range from 138 kHz to 30 MHz.

Shown in Figure 2 are the band plans below 12 MHz defined for ETSI in Europe and for ANSI in U.S, Japan and Europe.

In the frequency range between 12MHz and 30MHz, VDSL2 specifies at least one additional downstream or upstream band. Bands above 12 MHz are specified by additional band separating frequencies. The number of bands separating frequencies depends on the number of bands defined between 12 MHz and 30MHz.

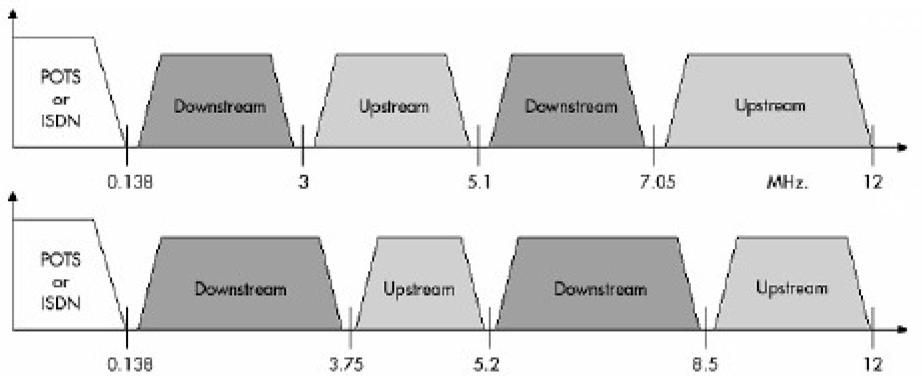


Figure 2. Band plan in the frequency range up to 12 MHz

2.4PSD mask

A VDSL modem shall confine the PSD of its transmit signal to be within the transmit PSD mask. Figure 3 shows two different PSD masks: M1 and M2. M1 is a PSD model compatible with ADSL and based on a Cabinet structure. M2 is the VDSL efficient model defined for T1E1.4 (ANSI standard group for VDSL) based on a Central Office structure. The latter is used in the simulations below.

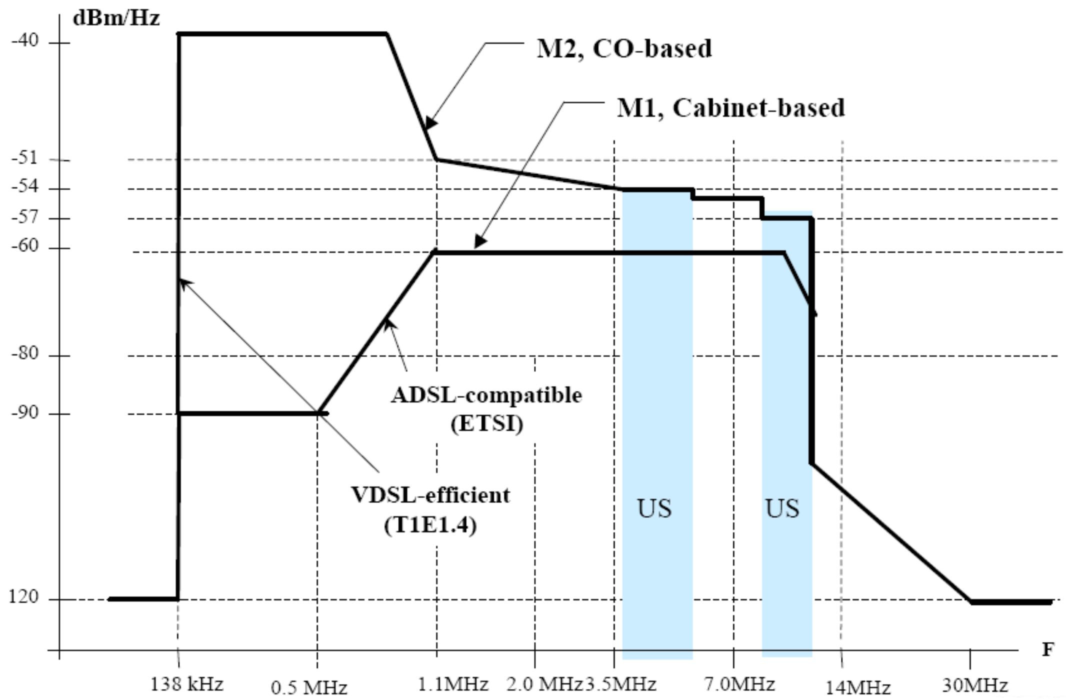


Figure 3. PSD masks

2.5 Reference models

Figures 4 and 5 show the VDSL reference model with its interfaces and its layered protocol model representation, respectively.

The γ interface allows the communication between the application and the transport-protocol-specific transmission convergence (TPS-TC) processing. γ-O is associated to the service provider end of the line and γ-R is associated to the customer end of the line.

The α interface is an application-independent interface between the TPS-TC and the physical medium-specific transmission convergence (PMS-TC) sublayers at the service provider side, and β is the same interface at the customer side of the line.

The VTO-O (or LT) is the VDSL transmission unit at the CO/ONU. The VTU-O converts digital data to and from the continuous-time physical-layer VDSL signals at the U2-O interface. The VTU.R (or NT) is the VDSL transmission unit at the remote location (CPE). The VTU-R performs the same functions as the VTU-O at the remote location of the U2-R interface. The VTU-R performs the termination of the VDSL modulation scheme, link control, and maintenance functions, and provides the logical β interface to CPE.

The service splitters separate VDSL signals from other services, which can be either POTS or ISDN signals. The U1 reference point refers to cooper pair media carrying composite signals; the U2 reference point specifies the VDSL modem ports.

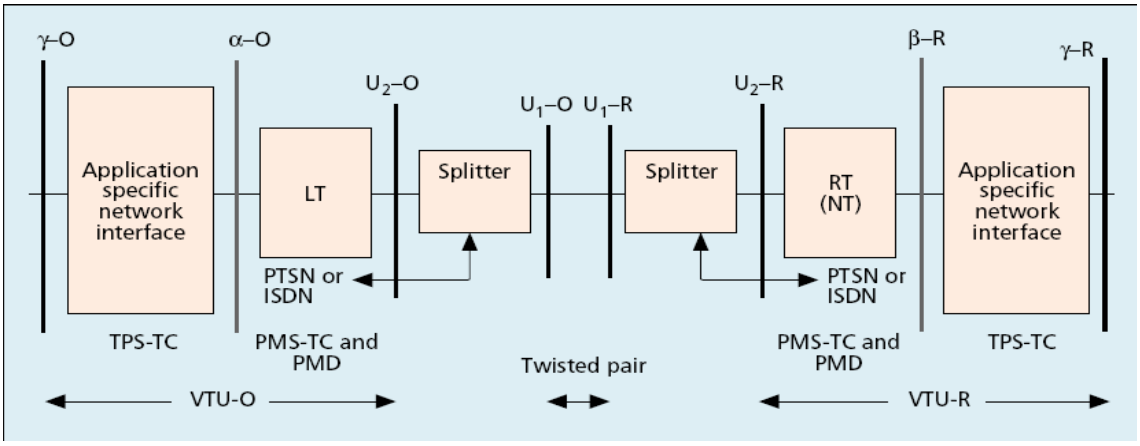


Figure 4. VDSL Reference Model – Interfaces -

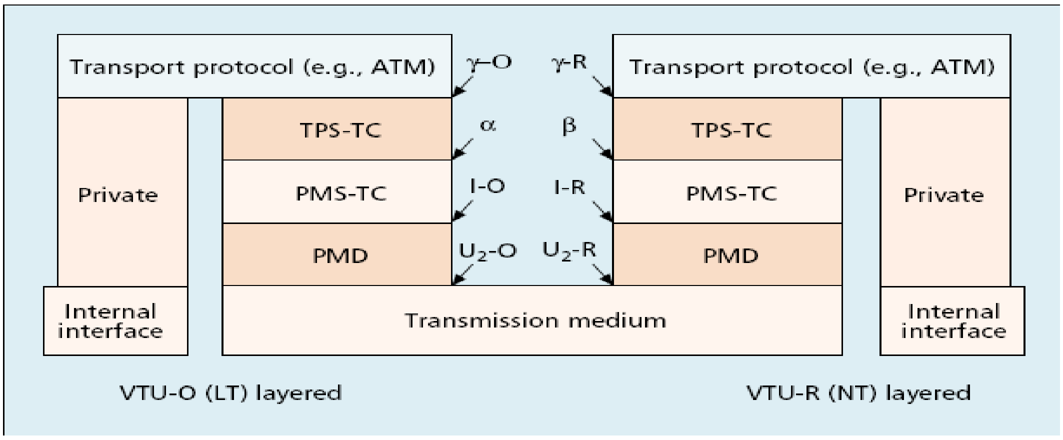


Figure 5. VDSL Reference Model – Layers -

2.6 OFDM modulation for VDSL

Orthogonal Frequency-Division Multiplexing (OFDM) is a modulation that segments a communications channel in such a way that many users can share it. Whereas TDM segments are according to time and CDM segments are according to spreading codes, OFDM segments are according to frequency. It is a technique that divides the spectrum into a number of equally spaced tones and carries a portion of a user's information on each tone. A tone can be thought of as a frequency, much in the same way that each key on a piano represents a unique frequency. OFDM can be viewed as a form of frequency division multiplexing (FDM), however, OFDM has an important distinctive property which is characterized by the fact that each tone is orthogonal with every other tone. FDM usually requires the existence of frequency guard bands between the frequencies to avoid them to interfere with each other. On the contrary, OFDM allows the spectrum of each tone to overlap, and because of the fact that they are orthogonal, they do not interfere with each other. By allowing the tones to overlap, the overall amount of spectrum required is reduced.

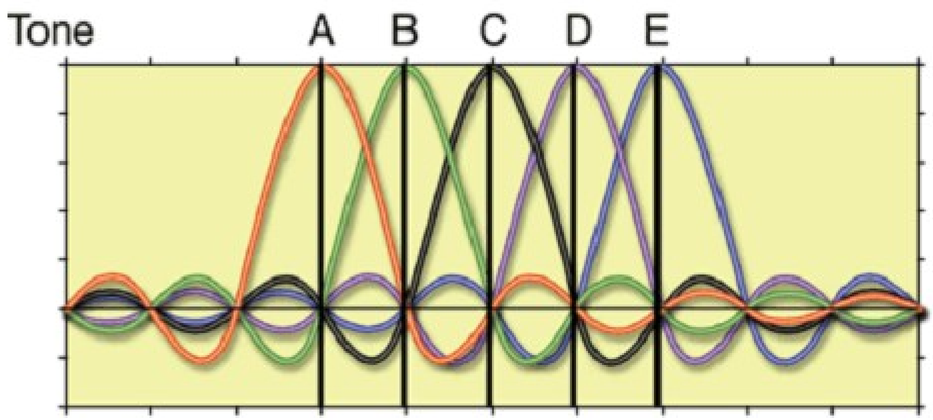


Figure 6 . OFDM Tones

OFDM is a modulation technique in the sense that it enables user data to be modulated onto the tones. The information is modulated onto a tone by adjusting the tone's phase, amplitude, or both. In the most basic form, a tone may be present or disabled to indicate a one or zero bit of information, however, either phase shift keying (PSK) or quadrature amplitude modulation (QAM) is typically employed. An OFDM system takes a data stream and splits it into N parallel data streams, each at a rate $1/N$ of the original rate. Each stream is then mapped to a tone at a unique frequency and combined together using the inverse fast fourier transform (IFFT) to yield the time-domain waveform to be transmitted.

OFDMA is multiple-access scheme derived from OFDM where an individual tone or groups of tones can be assigned to different users. Multiple users share a given bandwidth in this manner. When each user has information to send, the can be assigned a predetermined number of tones when they have information to send, or alternatively, a user can be assigned a variable number of tones based on the amount of information that they have to send. The assignments are controlled by the media access control (MAC) layer, which schedules the resource assignments based on the user demand.

To maintain orthogonality between tones, it is necessary to make sure that the symbol time contains one or multiple cycles of each sinusoidal tone waveform. This is usually the case, because the system is constructed in such a way that tone frequencies are integer multiples of the symbol rate 1/T, as is subsequently highlighted, where the tone spacing is 1/T. In the following expression of a sinusoidal signal,

$$s(t)=\sin m\omega t * sinn\,\omega t=\frac{1}{2}\cos(m-n)\,\omega t-\frac{1}{2}\cos(n+m)\,\omega t$$

if *m* and *n* are integers, then,

$$\int_{0}^{2\pi} s(t)=0$$

which means that there are integer number of periods fitting in the symbol duration. Seen in Figure 7 there are three tones over a single symbol period, where each tone has an integer number of cycles during the symbol.

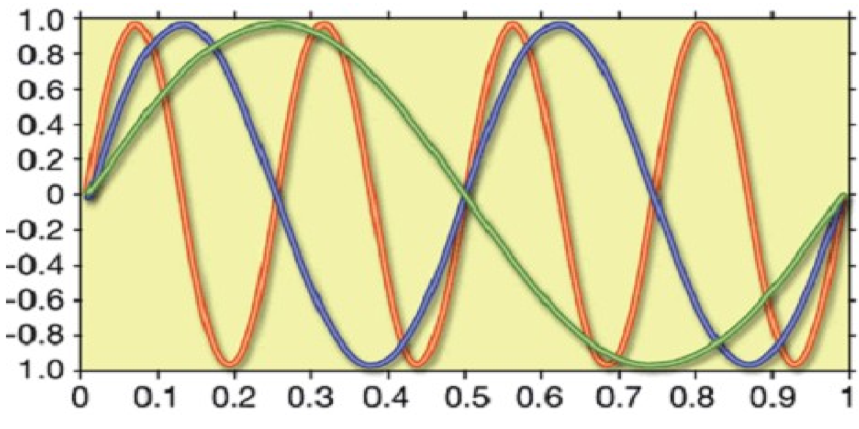


Figure 7 . Integer Number of Sinusoid Periods

In absolute terms, to generate a pure sinusoidal tone requires the signal start at time minus infinity. By adding a guard time, called a cyclic prefix, the channel can be made to behave as if the transmitted waveforms were from time minus infinite, and thus ensure orthogonality, which essentially prevents one subcarrier from interfering with another (called intercarrier interference, or ICI)

The cyclic prefix is actually a copy of the last portion of the data symbol appended to the front of the symbol during the guard interval. It is sized appropriately to serve as a guard time and to eliminate ISI. The fundamental trade-off here is that the cyclic prefix must be long enough to account for the anticipated multipath delay spread experienced by the system. The amount of overhead increases, as the cyclic prefix gets longer.

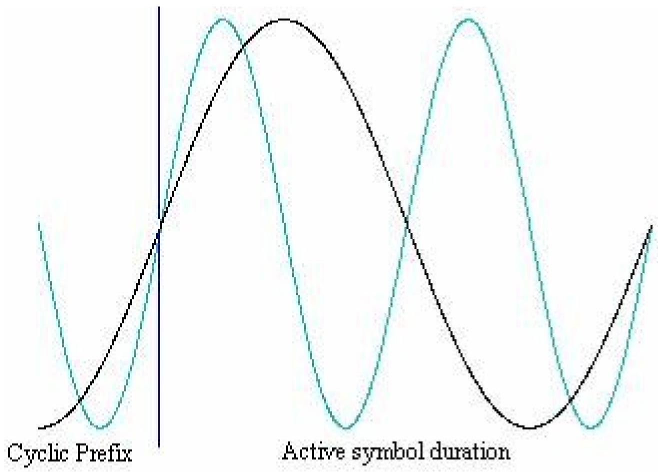


Figure 8. Cyclic Prefix vs OFDM symbol

3. MIMO (Multiple-Input Multiple-Output) in copper lines

3.1 Introduction

MIMO is the use of multiple transmitters and receivers in a communication. It has attracted a lot of attention in wireless communications, since it offers significant increases in data throughput without additional bandwidth or transmit power. However, it has been noticed that this advantages can also be achieved in wired communications as DSL technologies.

MIMO gets benefit of physical phenomena as multipath fading to increase the transmission rate and reduce error rate

There are three different main MIMO technologies:

- *Beamforming:* In beamforming, the same signal is emitted from each of the transmit antennas with appropriate phase weighting so that the signal power is maximized at the receiver input. The benefits of beamforming are the increase of the signal gain from constructive combining and the reduction of the multipath fading effect. In the absence of scattering, beamforming results in a well defined directional pattern, but in typical cellular conventional beams are not a good analogy. When the receiver has multiple antennas, the transmit beamforming cannot maximize simultaneously the signal level at all of the receive antenna and precoding is required. Note that precoding requires knowledge of the channel state information (CSI) at the transmitter.
- *Spatial multiplexing:* In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams, creating parallel channels for free. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher SNR. The maximum number of spatial streams is limited by the lesser in the number of antennas at the transmitter or receiver. Spatial multiplexing can be used with or without transmit channel knowledge.

- *Diversity coding:* This kind of techniques are used when there is no channel knowledge at the transmitter. In diversity methods a single stream (unlike multiple streams in spatial multiplexing) is transmitted, but the signal is coded using techniques called space-time codes. The signal is emitted from each of the transmit antennas using certain principles of full or near orthogonal coding. Diversity exploits the independent fading in the multiple antenna links to enhance signal diversity. Because of the fact that there is no channel knowledge, there is no beamforming or array gain from diversity coding.

3.2 Capacity in MIMO systems

3.2.1 Introduction

It is proved that the potential gains of multi-transmitters/receivers systems over SISO (Single-Input Single-Output) systems is rather large under independence assumptions for the fades and noises at different receivers.

In a linear model with *t* transmitters and *r* receivers where the transmitters are using a Discrete Multi-Tone modulation (as OFDM) the received vector *y* over one tone depends on the transmitted vector *x* as follows:

$$y_i = H_i x_i + n_i \qquad i = 1 \ldots \mathrm{N}_c$$

where N_c is the number of subcarriers, x_i is the vector of *t* transmitted signals on tone i, y_i the received signal vector, H_i is the *r* x *t* MIMO channel matrix and n_i is the noise containing Additive White Gaussian Noise (AWGN) and the external noise proceeding from the parallel lines.

The objective function for all the simulations in this paper is the capacity summed over the N_c tones

$$C(\Phi_i) = \sum_{i=1}^{Nc} \log_2 \left[det\left(I + \frac{(H_i \Phi_i H_i^H)}{N_i}\right) \right]$$

Here Φ_i is the covariance matrix of transmitted symbols $\Phi_i = E[x_i x_i^H]$ and N is the covariance matrix of the noise .

In order to achieve the maximum capacity it is necessary to find the optimal power allocation that will be different in every case of the three evaluated in this paper.

3.2.2Optimal PSD under total power constraint

The primal problem of finding optimal PSD's for a MIMO binder under a total power constraint P^{tot} is:

$$\max_{(\Phi_i)_i = 1 \ldots N_c} C(\Phi_i)_{(i=1 \ldots N_c)}$$

$$subject\ to \sum_{i=1}^{N_c} Trace(\Phi_i) \leqslant P^{tot}$$

$$\Phi_i \geqslant 0, i = 1 \ldots N_c$$

This is to be solved via its Lagrangian that decouples into a set of N_c smaller problem reducing, thus the complexity of the previous equation with a dual objective function with λ the Lagrange multiplier as follows:

$$F(\lambda) = \max_{(\Phi_i)_i = 1 \ldots N_c} \zeta(\lambda, (\Phi_i)_{(i=1 \ldots N_c)})$$

with

$$\zeta(\lambda, (\Phi_i)_{(i=1 \ldots N_c)}) = \sum_{i=1}^{N_c} \left(\log_2 [det(I + H_i \Phi_i H_i^H N_i^{-1})] - \lambda\, Trace(\Phi_i) \right) + \lambda P^{tot}$$

The dual optimization problem, then, is,

$$\underset{\lambda}{minimize}\ \ F(\lambda)$$

$$subject\,to\ \ \lambda \geqslant 0$$

Decomposing the noise as $N_i = L_i L_i^H$ and using the Singular Value Decomposition (SVD) $L_i^{-1} H_i = U_i D_i V_i^H$ as done in reference 1, leads us to the expression:

$$\Phi_i = V_i \left[\frac{I}{(\ln(2)\lambda)} - D_i^{-2} \right]^{+} V_i^H$$

which is the optimal power allocation that consists of finding the optimal Lagrange multiplier which meets the total power constraint using the closed form water-filling solution for MIMO systems.

3.2.3Optimal PSD under per-modem total power constraints

The primal problem of finding optimal PSD's for a MIMO binder under per-modem total power constraints P_j^{tot} is:

$$\max_{(\Phi_i)_i = 1 \ldots \mathrm{N}_c} C(\Phi_i)_{(i=1 \ldots \mathrm{N}_c)}$$

$$subject\ to \sum_{i=1}^{N_c} [\Phi_i]_{jj} \leqslant P_j^{tot} \quad \forall j$$

$$\Phi_i \geqslant 0, i = 1 \ldots \mathrm{N}_c$$

As before, this is solved as a dual objective function in order to find the Lagrange multipliers that make it optimal,

$$F(\Lambda) = \max_{(\Phi_i)_i = 1 \ldots \mathrm{N}_c} \zeta(\Lambda, (\Phi_i)_{(i=1 \ldots \mathrm{N}_c)})$$

with

$$\zeta(\Lambda, (\Phi_i)_{(i=1 \ldots \mathrm{N}_c)}) = \sum_{i=1}^{N_c} \left(\log_2[det(I + H_i \Phi_i H_i^H N_i^{-1})] - Trace(\Lambda \Phi_i) \right) + Trace(\Lambda\ diag(P_j^{tot}))$$

being Λ a diagonal matrix of Lagrange multipliers $diag(\lambda_1, \ldots, \lambda_N)$ and the dual optimization problem,

$$\begin{aligned} &\underset{\Lambda}{minimize} \quad F(\Lambda) \\ &subject\ to \quad \lambda_j \geqslant 0 \quad \forall\ j \end{aligned}$$

and, as previously mentioned, we use the SVD to get to the optimal power allocation with per-modem total power constraints:

$$\Phi_i = \Lambda^{(-1/2)} V_i \left[\frac{I}{(\ln(2))} - D_i^{-2} \right]^+ V_i^H \Lambda^{(-1/2)}$$

3.2.4Optimal PSD under per-modem total power constraints and spectral mask constraints

In this case the primal problem of finding optimal PSD's for a MIMO binder under per-modem total power constraints P_j^{tot} and spectral mask constraints is:

$$\begin{gathered} \underset{(\Phi_i)_i = 1 \ldots \mathrm{N}_c}{max} C(\Phi_i)_{(i=1 \ldots \mathrm{N}_c)} \\ subject\ to \sum_{i=1}^{N_c} [\Phi_i]_{jj} \leqslant P_j^{tot} \quad \forall\, j \\ [\Phi_i]_{jj} \leqslant \phi_i^{(mask,\, j)} \quad \forall\, i \quad \forall\, j \\ \Phi_i \geqslant 0, i = 1 \ldots \mathrm{N}_c \end{gathered}$$

with the objective function being the MIMO capacity summed over the N_c tones. In this case, the idea of dual decomposition by decoupling the primal problem into N_c smaller problems considering the per-modem total power constraints and the spectral mask constraints leads us to the dual objective function:

$$F(\Lambda, \tilde{\Lambda}_1, \ldots, \tilde{\Lambda}_{(N_c)}) = \underset{(\Phi_i)_i = 1 \ldots \mathrm{N}_c}{max} \zeta(\Lambda, \tilde{\Lambda}_1, \ldots, \tilde{\Lambda}_{(N_c)}, (\Phi_i)_{(i=1 \ldots \mathrm{N}_c)})$$

with

$$\zeta(\Lambda, \tilde{\Lambda}_1, \ldots, \tilde{\Lambda}_{(N_c)}, (\Phi_i)_{(i=1 \ldots N_c)}) = \sum_{i=1}^{N_c} \left(\log_2[det(I + H_i \Phi_i H_i^H N_i^{-1})] - Trace((\Lambda + \tilde{\Lambda})\Phi_i) \right)$$
$$+ Trace(\Lambda \; diag(P_j^{tot})) + \sum_{i=1}^{N_c} Trace(\tilde{\Lambda}_i \; diag(\phi_i^{(mask,j)}))$$

The Lagrange multipliers corresponding to the per-modem total power constraints are contained in the diagonal matrix $\Lambda = diag(\lambda_1 \ldots \lambda_N)$ the Lagrange multipliers corresponding to the spectral mask constraints for tone I are contained in the diagonal matrix $\tilde{\Lambda} = diag(\tilde{\lambda}_{i1}, \ldots, \tilde{\lambda}_{¿})$. The spectral mask for user j and tone i is represented for the diagonal matrix $diag(\phi_i^{(mask,j)}) = diag(\phi_i^{(mask,1)}, \ldots, \phi_i^{(mask,N)})$.

The dual optimization problem is:

$$\underset{\Lambda}{minimize} \;\; F(\Lambda)$$
$$subject\,t\,o \;\; \lambda_j \geqslant 0 \;\; \forall \; j$$

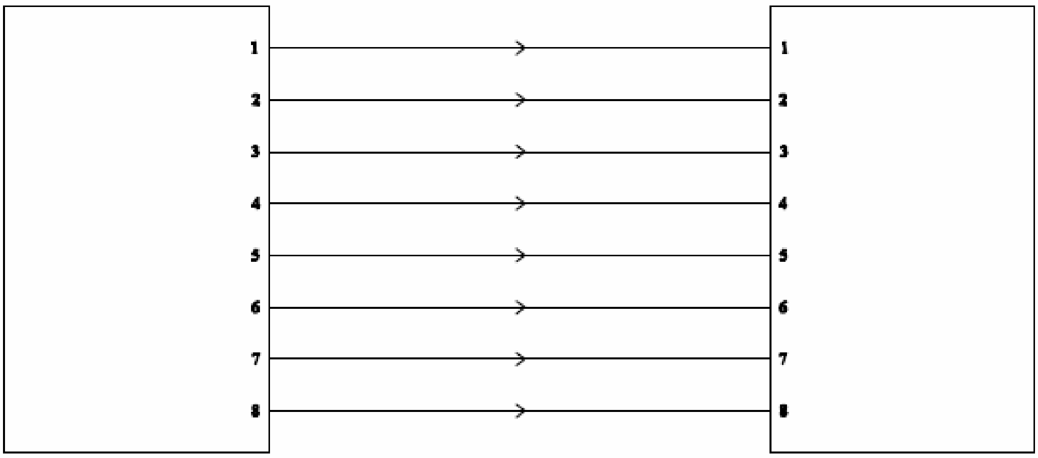
and, as said before, we use the Singular Value Decomposition to get to the optimal power allocation under per-modem total power constraints and spectral mask constraints:

$$\Phi_i = (\Lambda + \tilde{\Lambda}_i)^{(-1/2)} V_i \left[\frac{I}{(\ln(2))} - D_i^{-2} \right]^+ V_i^H (\Lambda + \tilde{\Lambda}_i)^{(-1/2)}$$

4. SIMULATIONS OF THE CAPACITY ON A MIMO SYSTEM USING VDSL TECHNOLOGY

4.1 Introduction

In this section we will evaluate the capacity on a VDSL transmission using a bonded system of 8 lines as seen illustrated:



We will define 4 different scenarios based on different kind of noise and different power constraints. The 4 different models are:

- Capacity under total power constraints with AWGN (Additive White Gaussian Noise).
- Capacity under total power constraints with AWGN and Alien Crosstalk.
- Capacity under per-modem total power constraints with AWGN and Alien Crosstalk.
- Capacity under per-modem total power constraints and spectral mask constraints with AWGN and Alien Crosstalk.

The FEXT model used as data transmitted through the system is an 8 lines transmission on 400 meters lines until 12 MHz applicable to VDSL2 standard.

All the simulations followed the band plan 998 based on ITU-T REC. G993.1 for the European region on the frequency range up to 12 MHz with subcarriers of 4,3125 Khz.

The modulation used is a Discrete Multi-tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channel.
The AWGN is fixed at -140 dBm/Hz for all simulations and the Alien Crosstalk comes from parallel lines also on a VDSL2 transmission.
The maximum transmit power P_j^{tot} is fixed at 14.5 dBm for each line. For the 8 lines this value is $P^{tot}=23.53\ dBm$.
The expression of the capacity will be the same in the 4 simulations:

$$C(\Phi_i)=\sum_{i=1}^{Nc}\log_2\left[det(I+\frac{(H_i\Phi_i H_i^H)}{N_i})\right]$$

There is also a simulation that evaluates the capacity of each of the single lines acting separately in order to see the gain of the MIMO system.
All the simulations were implemented in MATLAB.

4.2 Capacity under total power constraints with AWGN

In this simulation we took the FEXT model of 400 meters length lines and we evaluated over the water-filling proceeding in order to find the optimal PSD that maximizes the capacity of the whole MIMO system. In this case we only consider the noise of the channel itself as thermal noise being an Additive White Gaussian Noise (AWGN) of -140 dBm/Hz.
This is an almost idealized system as the noise is minimal which is caused by the fact that we do not consider other lines transmitting nor the restrictions imposed for the MIMO system and the VDSL technology.
The expression of the optimal power allocation is:

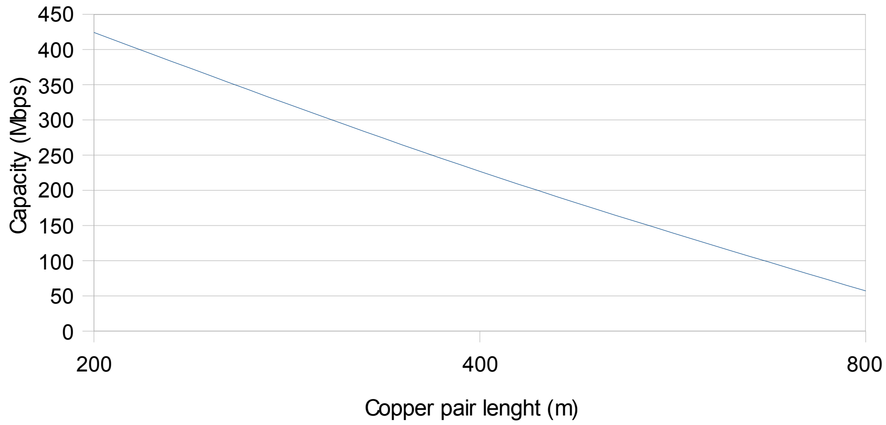
$$\Phi_i=V_i\left[\frac{I}{(\ln(2)\lambda)}-D_i^{-2}\right]^+V_i^H$$

Capacity in this case is evaluated in two different ways. First of all we calculate the capacity for 3 different length lines: 200 m, 400 m and 800 m on a bonded system with

8 lines. This allows us to see the effects of adding more lines to the MIMO system by calculating capacity in a system with 2 lines, 4 lines and 8 lines on a loop of 400 meters.

4.2.1 Capacity in different length lines

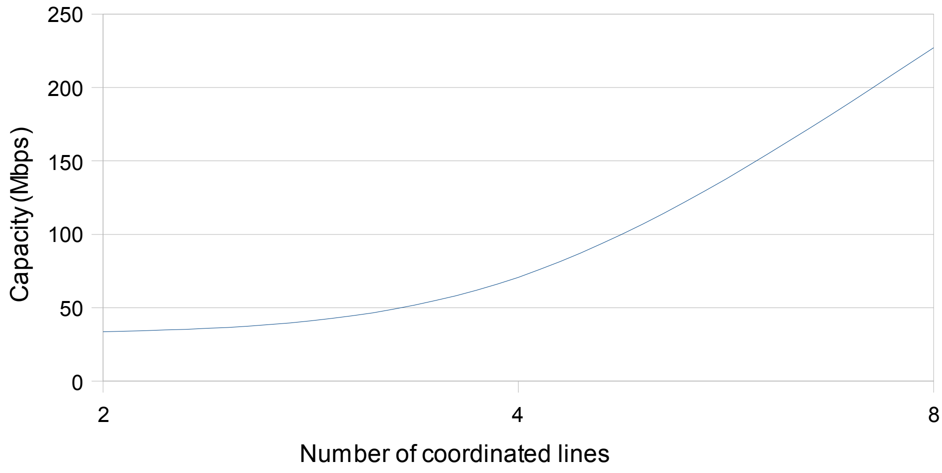
Length	Capacity
200	424,37
400	227,02
800	56,92



In this case, we can clearly see how capacity is badly affected when the copper lines are enlarged. In this particular case the decreasing is almost linear. The capacity in a cooper pair of 200 meters is over 400 Mbps but in a loop of 800 meters it barely gets the 60 Mbps. That gives a decrease factor of 8.

4.2.2Capacity with different number of lines

N. of lines	Capacity
2	33,63
4	70,8
8	227,02

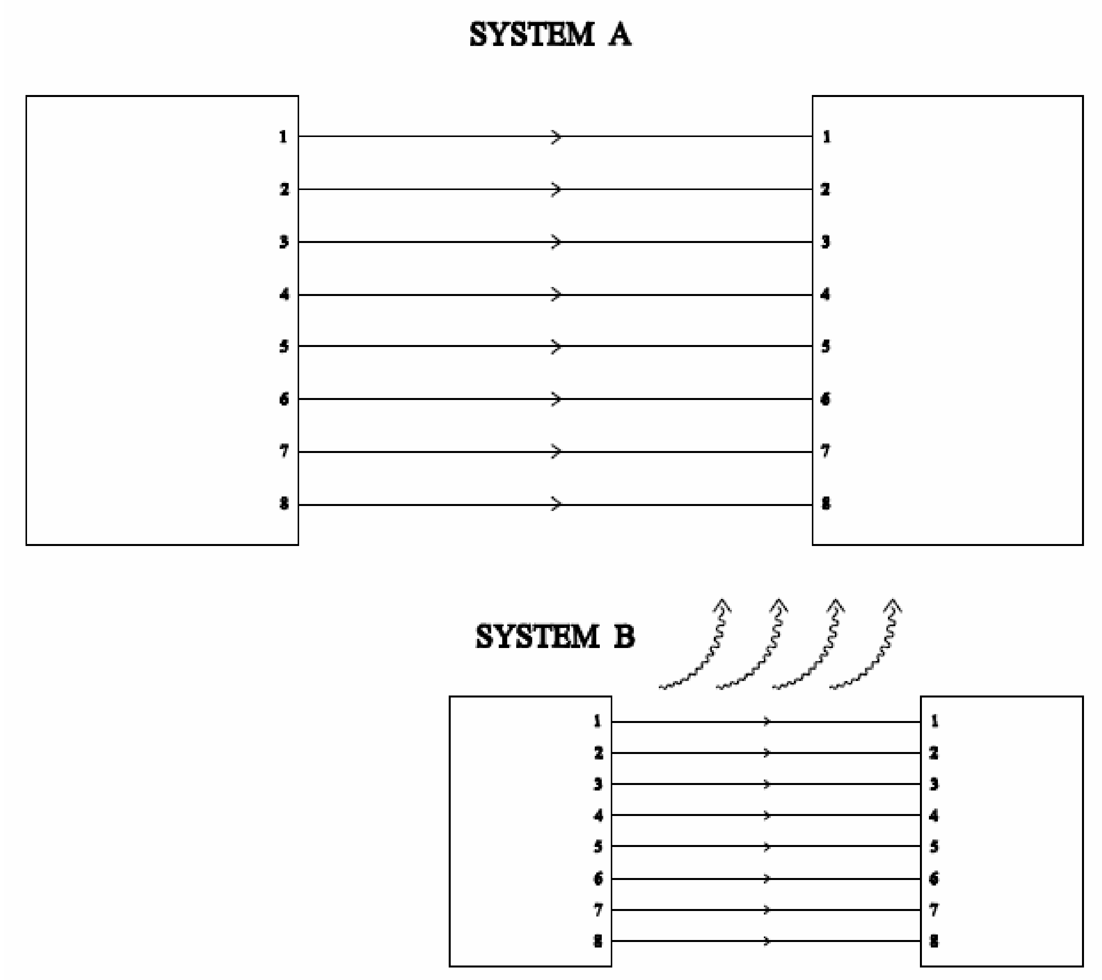


The total power constraint P_{tot} for the simulations with 2 and 4 lines is different to the previous value given of 23.53 dBm. The total power constraint for the case of 2 lines is 17.51 dBm and for the case of 4 lines is 20.52.

We can see the effects of increasing the number of lines in a MIMO system. Capacity increases exponentially its value as we add more lines. This is prove that bonding helps to increase the capacity of the whole system in front of the transmission of the single lines separately. For example, with a binder of 2 lines, each line is transmitting at 16,81 Mbps, with a 4-line binder the capacity of each line is 17,7 Mbps and with an system with 8 lines transmitting at the same time the capacity for each line is 28,4.

4.3 Capacity under total power constraints with AWGN and Alien Crosstalk

In this simulation we took the FEXT model of 400 meters length lines and we evaluated over the water-filling proceeding in order to find the optimal PSD that maximizes the capacity of the whole MIMO system. In this case we consider the noise of the channel itself as thermal noise being an Additive White Gaussian Noise (AWGN) of -140 dBm/Hz and an external noise coming from a parallel transmission with the same characteristics of the main transmission as follows:



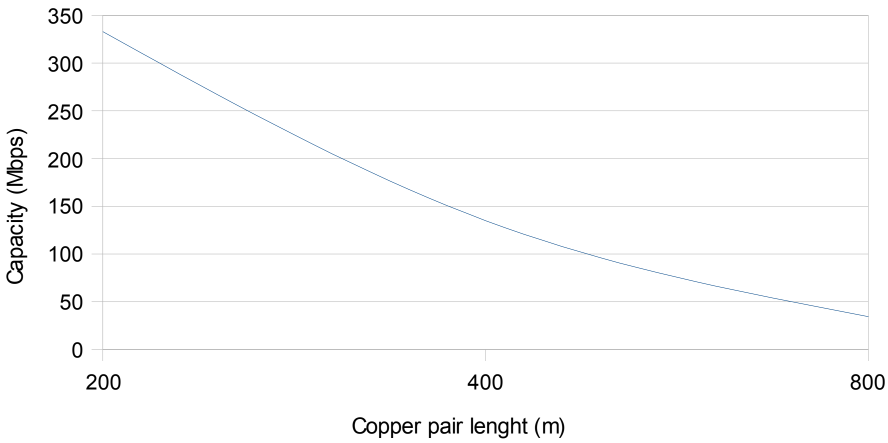
This scheme can also be applied for the next simulations with external noise.
The external noise is a $t_B \times r_B$ diagonal matrix of value $-60\text{dBm} \times diag(\Phi_{iB})$.
We introduce the -60 dBm as attenuation to a signal proceeding from another system.
The optimal power allocation, here, as before is:

$$\Phi_i = V_i \left[\frac{I}{(\ln(2)\lambda)} - D_i^{-2} \right]^+ V_i^H$$

In this case we will see how capacity is affected by the increase of the length of the lines evaluating it for 3 copper pair lengths: 200 m, 400 m and 800 m. The MIMO system is composed of 8 lines.
We also see the effects of bonding in capacity, increasing the number of lines acting coordinated so we evaluate capacity in MIMO systems of 2, 4 and 8 lines in a loop of 400 meters.

4.3.1Capacity in different length lines

Length	Capacity
200	333,07
400	135,01
800	34,52

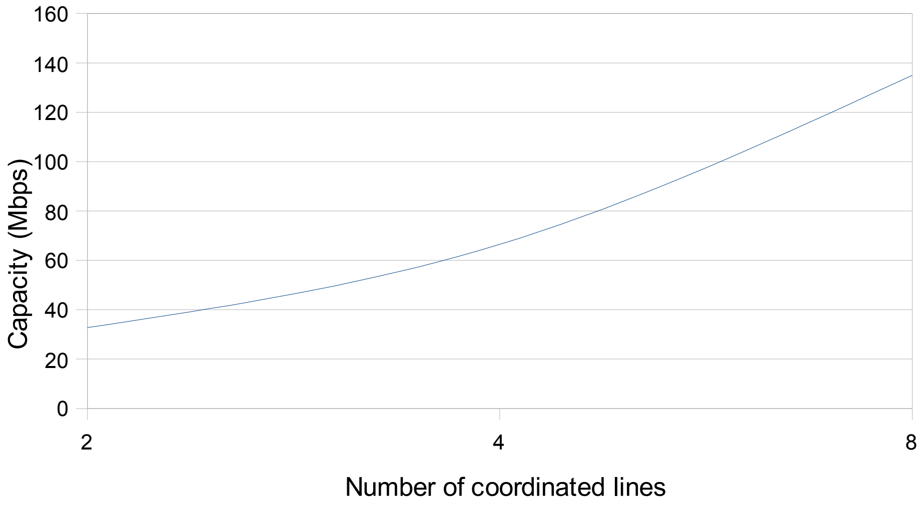


In general terms, the capacity is lower for each measure than in the previous case, being 333 Mbps the maximum capacity achieved on a very short loop of 200 meters, and 34.52 Mbps on the largest measure.

Capacity is now decreasing exponentially as we transmit to a further distance and the decrease factor is 10. So we can say that with an external noise affecting the transmission the effect of enlarging the loop is worse that when this noise does not exist.

4.3.2 Capacity with different number of lines

N. of lines	Capacity
2	32,79
4	66,54
8	135,01



Capacity here is also lower than in the previous case but we can see that the curve is less pronounced than in the last diagram, so we can deduce that alien noise affects negatively the increase of number of coordinated lines. However it is still positive the use of MIMO because with 2 lines transmitting, the capacity of each one of the lines is 16,4 Mbps, while with 4 lines, this capacity is 16,63 Mbps and with 8 lines, each line has a capacity of 16,87. We can say, then, that with an external noise affecting, the use of MIMO still benefits the whole system but in a lower degree.

4.4 Capacity under per-modem total power constraints with AWGN and Alien Crosstalk

A more realistic environment in MIMO systems with xDSL transmissions is to consider the power constraint of each modem of each line separately rather than a total power constraint for all the modems together as done before.

In this simulation we also took the FEXT model of 400 meters length lines and we evaluated over the water-filling proceeding in order to find the optimal PSD that maximizes the capacity of the whole MIMO system. We consider the noise of the channel itself as thermal noise being an Additive White Gaussian Noise (AWGN) of -140 dBm/Hz and alien noise coming from a parallel transmission with the same characteristics of the main transmission. Then the external noise is a $t_B \times r_B$ diagonal matrix of value $-60\text{dBm}\times diag(\Phi_{iB})$. We introduce the -60 dBm as attenuation to a signal proceeding from another system.

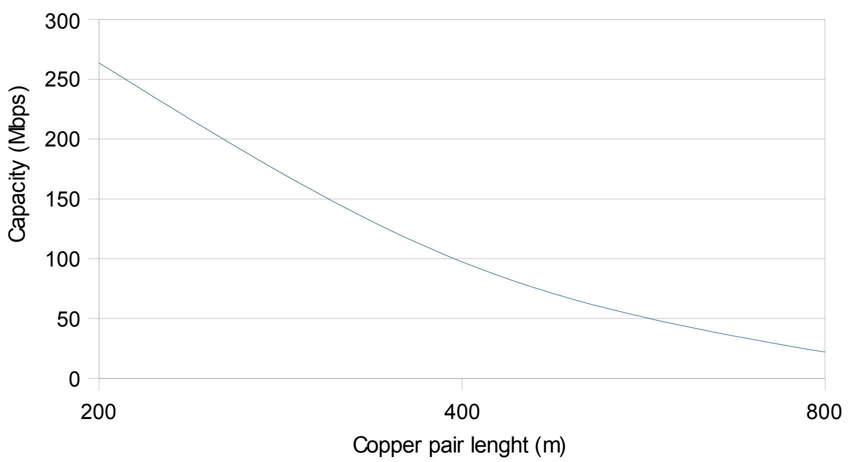
The optimal power allocation here is:

$$\Phi_i=\Lambda^{(-1/2)}V_i\left[\frac{I}{(\ln(2))}-D_i^{-2}\right]^+V_i^H\Lambda^{(-1/2)}$$

Capacity is evaluated in 3 different cooper pair lengths: 200 m, 400 m and 800 m with an 8-line MIMO binder. Then it is also evaluated in 3 different binders: 2 lines, 4 lines and 8 lines in a loop length of 400 meters.

4.4.1Capacity in different length lines

Length	**Capacity**
200	263,68
400	97,41
800	21,93

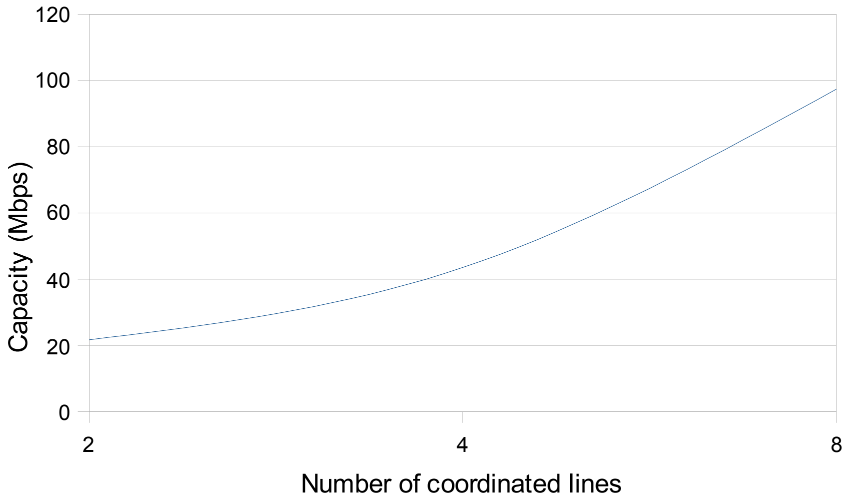


The maximum capacity achieved in this simulation for a loop of 200 meters is 263,68 Mbps which is almost the half of the capacity obtained in the first simulation.

Capacity also decreases exponentially, but here the decreasing factor is of 12 which means that the response of the MIMO system, taking into account each modem, is worse than the the two previous cases as the cooper pair is enlarged.

4.4.2Capacity with different number of lines

N. of lines	Capacity
2	21,68
4	43,48
8	97,41



The benefits of using MIMO are also clear in this graphic also; as we increase the number of lines the capacity grows exponentially. In contrast, this growth is smaller than the previous ones so we can deduce that the use of a power constraint for each modem affects negatively the MIMO binder and the capacity of the system. Here, for a binder of 2 lines, each line has a capacity of 10,84, for a binder of 4 lines, the capacity of each line is 10,87 and for a binder of 8 lines, each line can transmit at 12,17.

4.5 Capacity under per-modem total power constraints and spectral mask constraints with AWGN and Alien Crosstalk

Standards of xDSL technologies usually define spectral masks that each transmitter has to satisfy as well as the total power that each transmitter can transmit. In this simulation we will use an spectral mask equal for all the transmitters in the MIMO system at the same time that we keep on considering the power constraint of each modem separately. This is therefore a more realistic simulation.

We use the same FEXT model of 400 meters length lines and we evaluate it over the water-filling proceeding in order to find the optimal PSD that maximizes the capacity of the whole MIMO system. We consider the noise of the channel itself as thermal noise being an Additive White Gaussian Noise (AWGN) of -140 dBm/Hz and alien noise coming from a parallel transmission with the same characteristics of the main transmission.

Then the external noise is a $t_B \times r_B$ diagonal matrix of value . $-60\text{dBm} \times diag(\Phi_{iB})$ We introduce the -60 dBm as attenuation to a signal proceeding from another system.

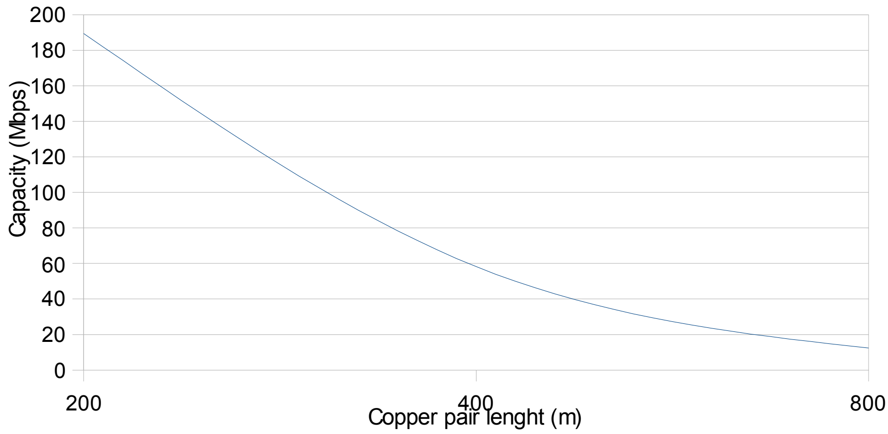
The optimal power allocation, here, as before is:

$$\Phi_i = (\Lambda + \tilde{\Lambda}_i)^{(-1/2)} V_i \left[\frac{I}{(\ln(2))} - D_i^{-2} \right]^+ V_i^H (\Lambda + \tilde{\Lambda}_i)^{(-1/2)}$$

Two different simulations are implemented as before. The first one through the consideration of different loop lengths of 200 m, 400 m and 800 m in a MIMO system of 8 lines. The second one considering 3 different MIMO binders of 2, 4 and 8 lines on a cooper pair length of 400 meters.

4.5.1Capacity in different length lines

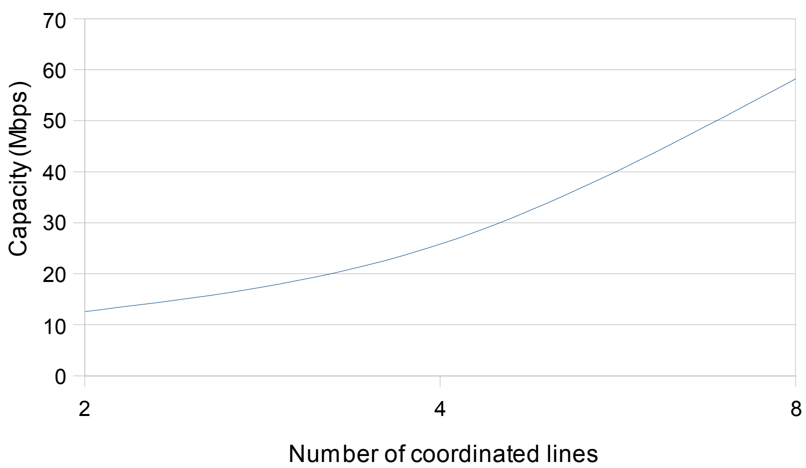
Length	Capacity
200	189,4
400	58,23
800	12,38



In this case, capacity decreases exponentially and very fast. In the shortest loop length of 200 meters the maximum capacity is 189,4 Mbps and with a decreasing factor of almost 15 the capacity reaches 12.38 Mbps on a cooper pair of 800 meters. We can say, therefore, that applying the spectral mask in the transmission affects negatively to the capacity of the system.

4.5.2Capacity with different number of lines

N. of lines	Capacity
2	12,6
4	25,81
8	58,23



Increasing the number of lines is clearly positive for the MIMO system as in the other simulations but this increase is less pronounced. For each line, the capacity is 6,3 Mbps, 6,45 Mbps and 7,27 Mbps for the binders of 2, 4 and 8 lines respecively. The increase of the capacity of each line is quite short here so we can conclude that the use of the spectral mask and the per-modem total power constraints affects badly to the system.

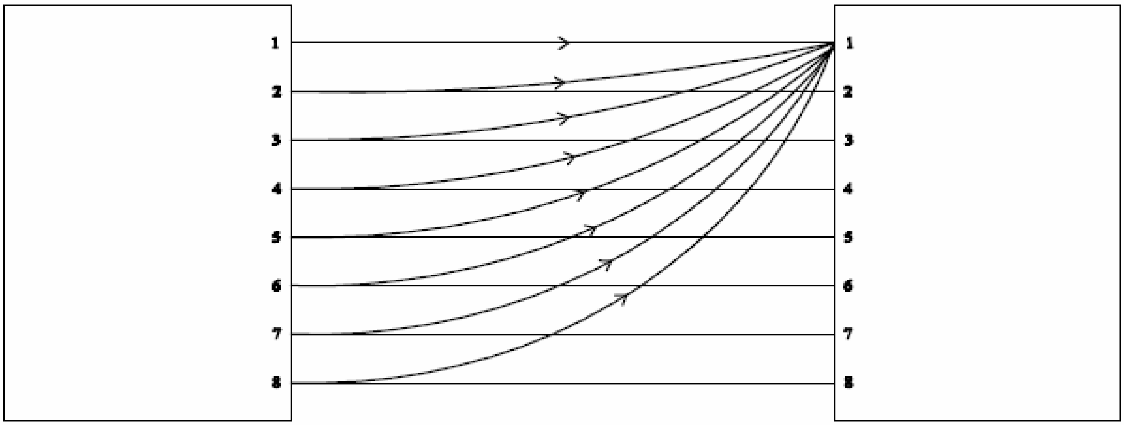
4.6 Capacity of the SISO system with AWGN and External noise

In this simulation we will see the behaviour of each of the 8 lines of the MIMO system separately in order to compare this capacities with the ones obtained using bonding. We will use the same FEXT model on loops of 400 meters long. The maximum transmit power for the lines is 14.5 dBm. The optimal PSD for each line is found through the water-filling proceeding in order to achieve tha maximum capacity for each line separately.

The capacity expression for each line is:

$$C(\Phi_i)=\sum_{i=1}^{Nc}\log_2\left[1+\frac{(H_i\Phi_i H_i^H)}{N_i}\right]$$

where H_i is the VDSL channel transfer function over the i tones, N_i is the sum of Additive White Gaussian Noise (AWGN), set at -140 dBm for each line, and the external noise originating from the parallel lines. In this particular simulation the external noise is directly the transmitting signals of the other lines as FEXT as represented in the following sketch:



Φ_i is the optimal PSD obtained after the following expression, being P the power of the channel and λ the maximum Lagrange multiplier found after a water-filling process over the i tones:

$$\Phi_i = \left[\frac{1}{(\ln(2)\lambda)} - P^{-2} \right]$$

The results of these simulations are shown in Table 1.

The capacities achieved are between 17.4 Mbps and 16,5 Mbps, the average is 16,86 Mbps. The sum of all the capacities of the single lines give a rate of 134,9 Mbps which is a far worse capacity than the one achieved using bonding.

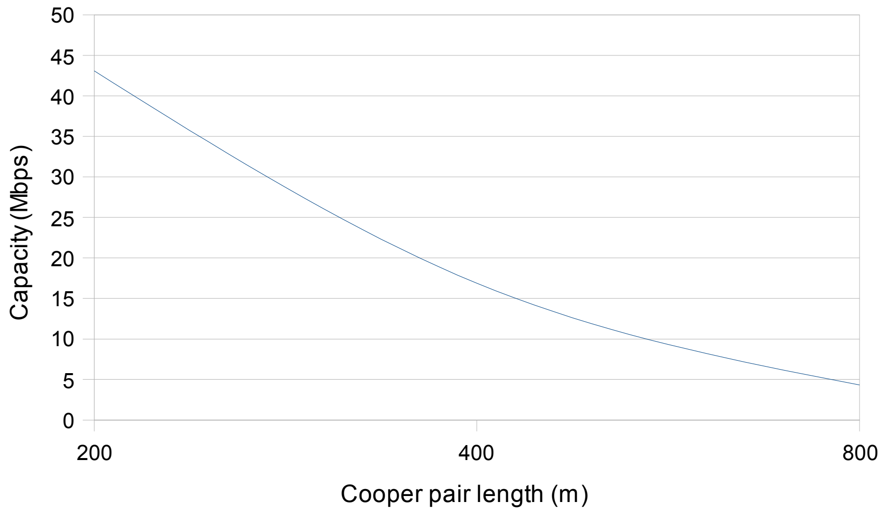
We can see the Lagrange multipliers obtained for each line as well as the noise level coming from the parallel lines which is, in all situations, very similar.

	PSD (dBm/Hz) ($x10^{-4}$)	λ ($x10^{4}$)	Ext. noise (dBm)	Capacity (Mbps)
Single line 1-1	-79,7460	9,6142	-119,6	16,7
Single line 2-2	-74,7620	10,1640	-119,5	16,8
Single line 3-3	-1,0885	7,7290	-118,2	16,9
Single line 4-4	-1,2361	7,3961	-119,6	17
Single line 5-5	-1,6737	5,3398	-116,7	16,7
Single line 6-6	-2,4494	3,7144	-116,5	16,5
Single line 7-7	-1,3143	6,8108	-118,9	17,4
Single line 8-8	-1,4900	5,5244	-116,9	16,9

Table 1. Single lines response

As in the previous simulations we will see the response of the lines in different loop lengths in order to take measures of the capacity with loops of 200 meters, 400 meters and 800 meters. These results are calculated with the average capacity of the 8 lines.

Length	**Capacity**
200	43,1
400	16,88
800	4,32



Although capacity is far lower than in the cases using MIMO we can see that large loops can affect it badly and that the decrease is also exponential.

4.7 Comparison of simulations

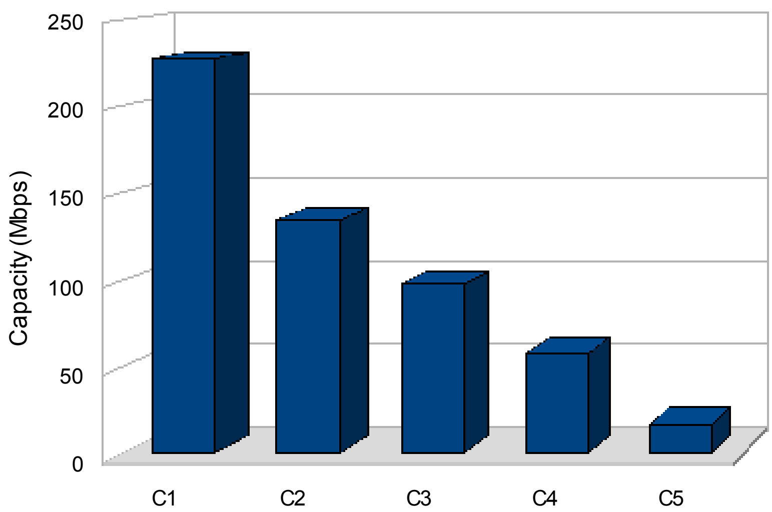
After seeing the results of the evaluation of capacity for each simulation separately we will now see them together in order to compare them and prove the benefits of MIMO in front of SISO, the effects of different kind of noise, the effects of different kind of power constraints and the effects of attenuation due to larger loops.

In these graphics:

- C1 is the Capacity under total power constraints with AWGN
- C2 is the Capacity under total power constraint with AWGN and External noise
- C3 is the Capacity under per-modem total power constraints with AWGN and External noise
- C4 is the Capacity under per-modem total power constraints and spectral mask constraints with AWGN and External noise

- C5 is the average Capacity of the SISO system under the total power constraint of the line with AWGN and External noise.

First of all, we will see the values of capacity for each simulation in a 400 meters loop and a 8-line binder (except for the SISO system C5).

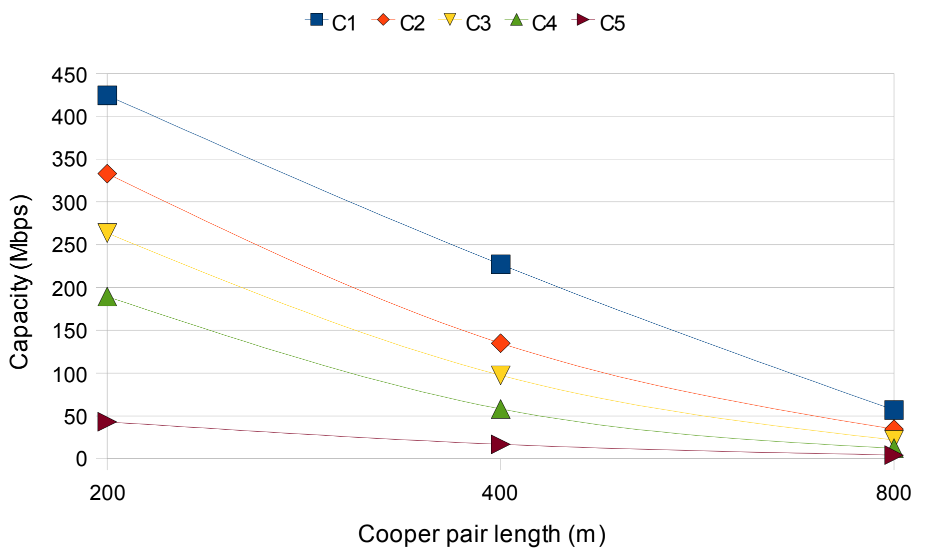


Each of the simulations we run was more restrictive compared to the previous ones. This is the cause of the shape of this last graphic. Between C1 and C2 we can see that there is a huge difference eventhough both of them are simulations of the capacity under total power constraints. The reason of this big step is because we introduced the external noise proceeding from a parallel transmission in C2 which is a much higher noise than the thermal one caused by the channel itself. We can say then, that the alien crosstalk affects seriously to the channel.

Between C2 and C3 we change from considering a total power constraint for all the modems together to considering each power constraint separately and between C3 and C4 we also add the spectral mask constraint. We can see in the graphic that these changes affect negatively to the capacity of the system but they help to make it more reliable.

Finally between C4 and C5 we find another big step. Here the pint is that we stop using MIMO and eventhough the SISO transmission does not use spectral mask constraints we can see that it is considerably below the previous value.

We now will compare the 5 simulations in an environment with different loop lengths: 200m, 400m and 800m. C1, C2, C3, C4 are MIMO systems with 8 tranmsitters and receivers and C5 is a SISO system.

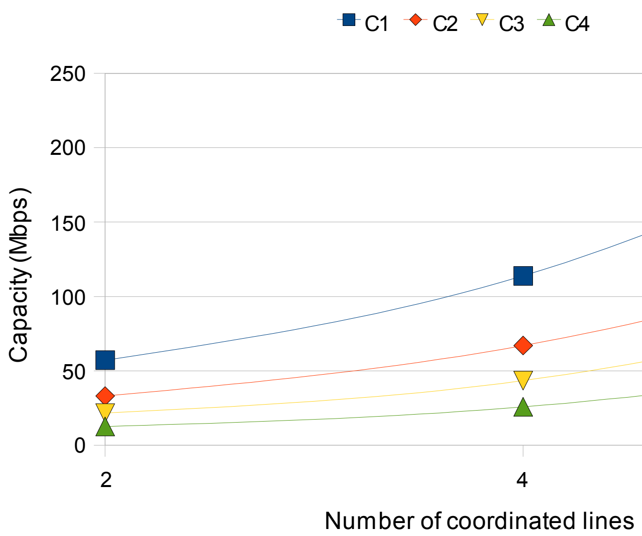


In this graphic we also can appreciate that each of the simulations is more restrictive respect the previous one but there are some details we could highlight. There is no doubt that increasing the length of the cooper pairs has an extremely bad effect in the capacity of the system. This is because of the very high frequencies used in VDSL that make attenuation increase quickly with the distance. We can also see that C1 has an almost linear decrease in contraposition with the other channels which have an exponential decrease due to the effect of external noise.

For C2, C3 and C4 we can see a very similar behaviour, starting from very high bit rates in short loops of 200 meters and ending below 50 Mbps when the lines are up to 800 meters.

Here, as before, we find a bigger difference in C5 that seems to be less affected for the increase of the length of the loop.

Finally, we will see the benefits of using MIMO increasing the number of lines. We will take measures with 2, 4 and 8 lines for loops of 400 meters. In this case, C5 will not appear, as it would be the value for C2 with only 1 line tranmitting because of the characteristics of the simulation (total power contraint with white noise and external noise).



The benefits of MIMO are reflected through this last graphic. We can see that as we increase the number of lines, for all simulations, the capacity grows exponentially. We could also say that as the noise and the power constraints are higher the effects of MIMO are reduced. We can see that on the graphic the line of each transmission is less slope than the previous one.

5. GENERAL CONCLUSIONS

After all these simulations we can say that we tested deeply a VDSL transmission using MIMO. The goals of this thesis were not so much testing the performance of VDSL but the performane of MIMO on a DSL system.

The truth is that everything is helping to achieve bit rates up to 400 Mbps. For example the huge frequency range of VDSL up to 30 MHz (in the simulations we only considered a maximum frequency of 12 MHz), the benefits of using a DMT modulation as OFDM which helps to reduce interferences and to get more profit of the frequency band, the transmission based on the Singular Value Decomposition of the transfer function and the water-filling proceeding in order to achieve the maximum capacity in every case. But what really makes the capacity to boast is the use of MIMO and as it has been proved, the larger the binder is, the better.

The simulations implemented in this paper were more restrective each time but also more real. For example, the results of the first simulation (Capacity under total power constraints with AWGN) is higher than the others but it can hardly be considered a real case because it only take into account a kind of white noise, something difficult in a real environment. On the contrary the last simulation has a considerably lower capacity but it is an example of a more real scenario due to the external noise affecting (apart from the white noise), the per-modem total power constraints that take into consideration the effect of each modem and the spectral mask constraints that uses the spectral mask defined on the VDSL2 standard in order to limit PSD.

In spite of this restrictions we can assure that the systems still get a real benefit of using MIMO technologies comparing to the simulations of the SISO systems. The throughput of a transmitter on a single line is far worse that the one achieved in MIMO systems, specially in short loops where the attenuation of high frequencies is not noticed. This length of the loops is one of the factors that affects most to DSL systems and with MIMO does not make a difference. The huge range of frequencies defined in VDSL2 can increase capacity but also the length of the twisted pairs gets very limited because of the attenuation at this high frequencies.

After all that, one thing is clear: MIMO systems are the next frontier for communicaitons over the Digital Subscirber Line to achieve rates that make possible the acces to video, voice and Internet over the same connection.

6. <u>REFERENCES</u>

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7. http://www.iec.org/online/tutorials/ofdm/topic04.html

7. ANNEXES

In this part you will be able to find the coding used to implement the simulations in Matlab. There is only the coding of the main simulations (Capacity under total power constraints, capacity under per-modem total power constraints and capacity under per-modem total power constraints and spectral mask constraints on a binder of 8 lines and a loop of 400 meters). The other material used to implement the variations of this main simulations is not included.

7.1 Coding for the capacity under total power constraint.

```
P=1;
n=2;
m=1;
Ptot=0;
I=eye(8,8);
x=(10^(23.53/10))/4312.5;
selfn=I*(10^(-14));
CN =((10^(-6))*(diag(diag(phi(i).matriz))+ selfn);
%Find Optimal PSD
while(abs(Ptot-x)>10^-9)
   Ptot=0;
   for i=1:2807
        [U,S,V]=svd((((CN)^(-1/2))*(H(i).matriz)));
        K=((I./(log(2)*P))-(S^(-2)));
        phi(i).matriz = (V*K*V');
        T = trace(phi(i).matriz);
        Ptot = Ptot + T;
    end
    if(Ptot-x<0)
        P=P/n;
        m=m*(1/2);
        n=n-m;
    end
    P=P*n;
    abs(Ptot-x);
end

C=0;
for i=1:2807

C = C + log2(det(I+(H(i).matriz*phi(i).matriz*H(i).matriz'*CN^(-1))));

end
```

7.2 Coding for the capacity under per modem total power constraints

```
P=[1 1 1 1 1 1 1 1];
n=[2 2 2 2 2 2 2 2];
m=[1 1 1 1 1 1 1 1];
Ptot= zeros(1,8);
I=eye(8,8);
x=(10^(14.5/10))/4312.5;
X=[x x x x x x x x];
selfn=I*(10^(-14));
T=[10^-9 10^-9 10^-9 10^-9 10^-9 10^-9 10^-9 10^-9];
CN =((10^(-6))*(diag(diag(phi(i).matriz)))+ selfn);
while(all(abs(Ptot-X)>T)==1)
    Ptot= zeros(1,8);
    for i=1:2807
        A=diag(P);
        [U,S,V]=svd((((CN)^(-1/2))*H(i).matriz));
        K=((I/log(2))-(S^(-2)));
        phi(i).matriz = ((A^(-1/2))*V*K*V'*(A^(-1/2)));
        Ptot = Ptot + [phi(i).matriz(1,1) phi(i).matriz(2,2)
                  phi(i).matriz(3,3) phi(i).matriz(4,4)
                  phi(i).matriz(5,5) phi(i).matriz(6,6)
                  phi(i).matriz(7,7) phi(i).matriz(8,8)] ;

    end
    for j=1:8
        if(all((Ptot-X)<0)==1)
            P(j)=P(j)/n(j);
            m(j)=m(j)*(1/2);
            n(j)=n(j)-m(j);
        end
        P(j)=P(j)*n(j);
    end
    abs(Ptot-X);
end

C=0;
for i=1:2807

 C = C + log2(det(I+(H(i).matriz*phi(i).matriz*H(i).matriz'*CN^(-1))))

end
```

7.3 Coding for the capacity under per-modem total power constraints and spectral mask constraints

```
P=[1 1 1 1 1 1 1 1];
n=[2 2 2 2 2 2 2 2];
b=[0 0 0 0 0 0 0 0];
Ptot= zeros(1,8);
I=eye(8,8);
x=(10^(14.5/10))/4312.5;
X=[x x x x x x x x];
selfn=I*(10^(-14));
T=[10^-9 10^-9 10^-9 10^-9 10^-9 10^-9 10^-9 10^-9];
Pq=[1 1 1 1 1 1 1 1];
nq=[2 2 2 2 2 2 2 2];
bq=[0 0 0 0 0 0 0 0];
nb_it=0;
CN =((10^(-6))*(diag(diag(phi(i).matriz))+ selfn);
while(all(abs(Ptot-X)>T)==1)
     A=diag(P);
     Ptot= zeros(1,8);
     for i=1:2807
        Pq=[1 1 1 1 1 1 1 1];
        nq=[2 2 2 2 2 2 2 2];
        bq=[0 0 0 0 0 0 0 0];
        nb_it=0;
        while nb_it<10
            nb_it=nb_it+1;
            L(i).matrix=diag(Pq);
            [U,S,V]=svd((((CN)^(-1/2))*H(i).matriz));
            K=((I/log(2))-(S^(-2)));
            phi(i).matriz = (((A+L(i).matrix)^(-1/2))*V*K*V'*((A
                                             +L(i).matrix)^(-1/2)));
                for j=1:8
                    switch i
                        case i<=32
                            if(phi(i).matriz(j,j)>(10^(-120/10)))
                                phi(i).matriz(j,j)=(10^(-120/10));
                            end
                            if(phi(i).matriz(j,j)-(10^(-120/10))<0)
                                bq(j)=bq(j)+1;
                                Pq(j)=Pq(j)/nq(j);
                                nq(j)=nq(j)-((1/2)^bq(j));
                            end
                            Pq(j)=Pq(j)*nq(j);
                        case 33<=i<=255
                            if(phi(i).matriz(j,j)>(10^(-40/10)))
                                phi(i).matriz(j,j)=(10^(-40/10));
                            end
                            if(phi(i).matriz(j,j)-(10^(-40/10))<0)
                                 bq(j)=bq(j)+1;
                                 Pq(j)=Pq(j)/nq(j);
                                 nq(j)=nq(j)-((1/2)^bq(j));
                            end
                            Pq(j)=Pq(j)*nq(j);
                        case 256<=i<=812
                            if(phi(i).matriz(j,j)>(10^(-52/10)))
                                 phi(i).matriz(j,j)=(10^(-52/10));
                            end
```

```
                              if(phi(i).matriz(j,j)-(10^(-52/10))<0)
                                   bq(j)=bq(j)+1;
                                   Pq(j)=Pq(j)/nq(j);
                                   nq(j)=nq(j)-((1/2)^bq(j));
                              end
                              Pq(j)=Pq(j)*nq(j);
                          case(813<=i<=2807)
                               if(phi(i).matriz(j,j)>(10^(-60/10)))
                                   phi(i).matriz(j,j)=(10^(-60/10));
                               end
                               if(phi(i).matriz(j,j)-(10^(-60/10))<0)
                                   bq(j)=bq(j)+1;
                                   Pq(j)=Pq(j)/nq(j);
                                   nq(j)=nq(j)-((1/2)^bq);
                               end
                               Pq(j)=Pq(j)*nq(j);
                      end
                  end
                  Ptot = Ptot + [phi(i).matriz(1,1) phi(i).matriz(2,2)
                                 phi(i).matriz(3,3) phi(i).matriz(4,4)
                                 phi(i).matriz(5,5) phi(i).matriz(6,6)
                                 phi(i).matriz(7,7) phi(i).matriz(8,8)] ;
          end
       end

     for j=1:8
         if(all((Ptot-X)<0)==1)
             b(j)=b(j)+1;
             P(j)=P(j)/n(j);
             n(j)=n(j)-((1/2)^b(j));
         end
         P(j)=P(j)*n(j);
     end
     abs(Ptot-X);
end

C=0;
for i=1:2807

C = C + log2(det(I+(H(i).matriz*phi(i).matriz*H(i).matriz'*CN^(-1))))

end
```