11. THE MAXIMUM WAVE HEIGHT PARADOX

11.1. Introduction

As explained in Chapter 5, the observations of Longuet-Higgins (1980) and Holthuijsen (2007) show that the theoretically estimated significant wave height from the spectrum should be reduced by a factor of 5%-10% in order to properly represent the real value. Moreover, some observations also show that the scaled Rayleigh distribution agree with the observed one (see Figure 5.3). This discrepancy can be expected since the Rayleigh distribution (R), and therefore H_{m_0} (calculated as $4\sqrt{m_o}$), is derived with some assumptions (linear theory, narrow-band spectrum...) which in reality do not hold.

What it was surprising is that in some observations the estimation of the maximum wave crest agrees with the real value (see Figure 11.1Figure 11.2). In this estimation, instead of the Rayleigh distribution, the distribution of Cartwright & Longuet-Higgins (1956) is used, which considers the maxima of all local maximum crest (see Chapter 9). However, the difference with the Rayleigh distribution is less than 2% for N >30 approx., which is less than the 5-10% found discrepancy in the significant wave height. Therefore, at first sight, it seems that the extreme crest heights do not seem to suffer from the same scale discrepancy as the wave heights.

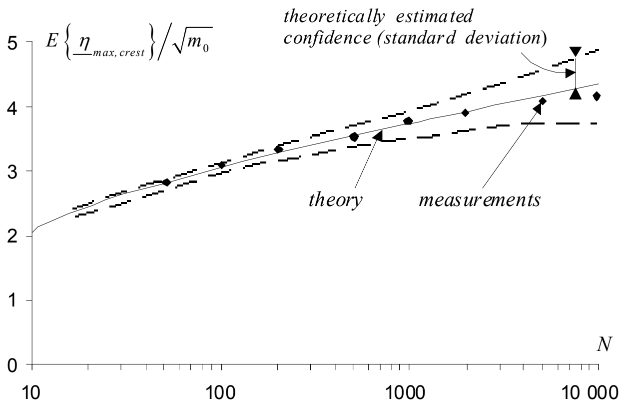

Figure 11.1. The observed and theoretically estimated expected value of the maximum crest height in a duration of N waves (Cartwright, 1958)

11.2. Possible reasoning

The paradox was meant to be resolved by considering the nonlinear effects and more exactly the Rayleigh- Edgeworth distribution (RE) with includes a parameter named BFI (see Chapter 6). Figure 11.2 compares the R and RE pdf (the wave height has been standardised by the significant wave height calculated as $4\sqrt{m_o}$). The expectations were that the wave height was overpredicted by the linear theory due to the wide-band spectrum, including the assumption $H \approx 2\eta_{crest}$. Nonlinearities presumably enhanced the extreme waves, counteracting this scaling (as previously mentioned, the Rayleigh-Edgeworth distribution also assumes a narrow band spectrum).

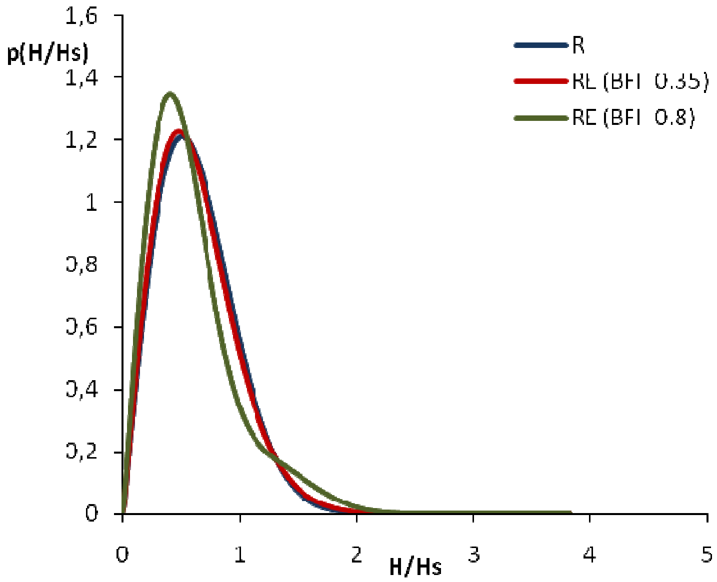

Figure 11.2 Comparison of Rayleigh distribution (R) with Rayleigh Edgeworth (RE) for two different values of BFI.

With the RE distribution, the probability of higher and low wave heights is enhanced whereas in the mid range wave height it is reduced. The estimated maximum wave height becomes higher and the significant wave height practically remains the same (it is slightly higher). In addition, the higher the BFI, the more pronounced such effects are, but with a larger number of waves, the estimated maximum have heights for R and RE become closer (see Table 11.1). Therefore, by using the RE distribution, it is expected to find more or less the same discrepancy between estimations and observations in both significant wave height and maximum wave height. In such a case, there would be overprediction but it would be consistent in the sense of being present in both parameters and perhaps only a scale factor should be applied.

Table 11.1 Comparison R-RE

Relation RE/R		BFI=0.35	BFI=0.8
H_{mean}		0.99	0.95
H_s		1.015	1.026
$E(H_{max})$	N=1,000	1.072	1.175
	N=10,000	1.078	1.160
	N=100,000	1.078	1.145

For BFI=0.35, the percentage of enhancement of the maximum wave height by the RE distribution (7%) is more or less the same as the discrepancy found in the significant wave height (5-10%) between observations and the Rayleigh distribution.

However, after the analysis of the data in the following chapters, the scope of this study has been changed and it seems that the discrepancy is between wave height and wave crest and not between significant and maximum wave height.