

Toolbox for fatigue analysis of beam structures and its possible application to railways

Project Thesis at Master's programme in Civil Engineering

XAVIER UBACH ROLDAN

Department of Applied Mechanics
Division of Dynamics
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2007

PROJECT THESIS

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Cover: Figure with the studied notched steel I-profile, loaded with dynamic bending moment and axial force.

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ABSTRACT

In railways there are many structural elements submitted to fatigue loading, e.g. railway tracks, bridge structures or welds. Therefore, it can be of great importance to implement a computer tool able to estimate fatigue life for these elements.

In fatigue design of structures submitted to dynamic loading, several different methods can be employed. In the present thesis, four such methods are considered: Stress-Based approach (High Cycle Fatigue, HCF), Strain-Based approach (Low Cycle Fatigue, LCF), Linear Elastic Fracture Mechanics (LEFM) and finally a method for analysis of welded joints.

A toolbox in Matlab for fatigue analysis of railway steel structures is developed. The toolbox uses results from dynamic analysis, which provides histories of bending moments and normal forces. The toolbox derives the fatigue life of the investigated structure by use of the four mentioned fatigue design methods.

Finally, two main groups of calculations have been performed: in the first one, the four fatigue methods have been studied for a regular stress history. In a second group, irregular stress histories have been employed which should represent a more realistic loading of the structure.

Key words: Fatigue design, HCF, LCF, LEFM, welded joint.

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PREFACE

This thesis project was carried out during the spring of 2007 at the Department of Applied Mechanics, Chalmers University of Technology, Göteborg. It constitutes 12 ECTS credit units (equivalent to 8 Swedish credit units). The work also constitutes one of the two final degree projects of Civil Engineering at Universitat Politècnica de Catalunya UPC, in Barcelona.

First of all, I would like to express my gratitude to my supervisors Professor Roger Lundén and Doctor Anders Ekberg, who have both given me guidance and support during the work. I would also like to thank the staff at the Department of Applied Mechanics for their help. On the other hand, I am also very grateful to Professor Andrés López Pita for his constant help from Barcelona.

Finally, I would like to express my gratitude to Chalmers University of Technology for providing a profitable and pleasant time for me in Sweden.

Göteborg in May 2007

Xavier Ubach Roldan

NOTATION

This project thesis starts by defining suitable variables both for the structural problem and for the fatigue design problem. SI units are used. Figure 1 shows the orientation of the axes of the beam.

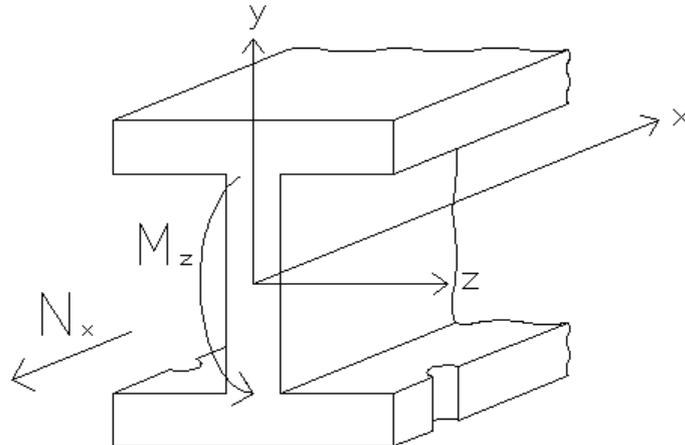


Figure 1. Sketch of the I-beam and the orientation of the axes

Roman upper case letters

M_z	Bending moment about z -axis (Nm)
N_x	Axial force in x -direction (N)
I_z	Moment of inertia about z -axis (m^4)
A	Cross-sectional area of I-beam (m^2)
E	Young's modulus of beam material (GPa)
S	Nominal (average) stress (MPa)
S_m	Nominal mean stress of a cycle (MPa)
S_a	Nominal stress amplitude of a cycle (MPa)
$S_{\max, \min}$	Maximum and minimum nominal stress in a cycle (MPa)
N	Number of cycles
N_e	Number of cycles to failure at the fatigue limit for Juvinall formulation
N_f	Number of cycles to failure
K_t	Stress concentration factor
K_f, K_f'	Fatigue notch factors

K	Stress intensity factor ($\text{MPa}\sqrt{\text{m}}$)
F	Shape (geometry) factor

Roman lower case letters

y	Position along y -axis for studied point on the profile (m)
m	Reduction factor, due to bending fatigue limit, load type, size (stress gradient) and surface finish
m'	Reduction factor, due to bending fatigue limit, load type, size (stress gradient) and surface finish
a	Crack length (m)
a_i	Initial crack length (m)
a_c	Critical crack length (m)

Greek upper case letters

ΔS	Stress range (MPa)
ΔK	Range of stress intensity factor ($\text{MPa}\sqrt{\text{m}}$)
ΔS_{TH}	Threshold stress range (MPa)

Greek lower case letters

σ	Normal stress at a point, or in a uniformly stressed member (MPa)
σ_u	Ultimate (fracture) stress (MPa)
σ_Y	Yield strength (MPa)
ε	Normal strain
$\varepsilon_{\text{max,min}}$	Maximum and minimum normal strain
ε_a	Strain amplitude

1 INTRODUCTION

The purpose of this project is to develop a toolbox for fatigue analysis of railway structures. The toolbox uses results from dynamic analysis of e.g. repeated loads caused by the train on the track or wind-induced and earthquakes vibrations affecting on a bridge, which provide histories of bending moments and normal forces. The toolbox then derives the fatigue life of the structure by employing different methods for analysis of fatigue.

Four different methods for analysis of fatigue have been implemented in the toolbox. For the theory, the textbook in Reference [1] has been the main reference and guide. The four employed methods for analysis of fatigue are as follows. *High Cycle Fatigue*: study of cyclic stresses leading to microscopic damage of the material eventually leading to structural failure. *Low Cycle Fatigue*: life estimation based on study of plastic deformations that may induce fatigue cracks. This procedure deals both with stresses and strains and analyzes them in parallel. *Linear Elastic Fracture Mechanics*: a structural part with an assumed initial crack is studied. The presence of cracks in structures or machine components is very common. It is therefore of main importance to know how the crack develops and when it will make the structure collapse. *Welded joints*: different categories of welds are considered and their response to cyclic loads is derived to predict the life of the joint.

The mentioned four methods for analysis of fatigue have been implemented in a Matlab code which is described in Chapter 3 of this report, and later shown in the Appendix. The code is then used to study fatigue of a steel I-beam. The studied cross-section of the beam is cyclically loaded with a bending moment and an axial force. The problem is treated as quasistatic (no inertia forces are included). Load histories are used as input to the problem. Afterwards, all necessary data is organized to proceed with the fatigue analysis. In Chapter 4, four combinations of load histories (two periodic and two non-periodic) are used and demonstrated. The examples show that the toolbox is able to calculate fatigue life for the four different methods. It provides plots of the input load sequences (stresses on a beam due to bending moment and axial force), as well as of the development of the crack length for the fracture mechanics analysis. Finally, in Chapters 5 and 6 concluding remarks, future developments and possible applications to railways of this thesis project are stated.

2 STATEMENT OF PROBLEM AND SOLUTION STRATEGY

In this chapter, the four methods for fatigue analysis employed in the thesis are formulated together with a strategy for building the toolbox. The governing equations from Reference [1] are stated. Moreover, additional theoretical reasoning has been extracted from the same book.

The toolbox is supposed to be used as a post-processor to a dynamic analysis which should provide histories of bending moments and axial forces acting in a cross-section of the beam. The loads are then converted to stresses as

$$S_x = \frac{M_z y}{I_z} + \frac{N_x}{A} \quad (1)$$

The study will focus on the lower flange of the profile, treating it as a plate submitted to an axial stress. It will also be considered that the plate is notched as shown in Figure 2.

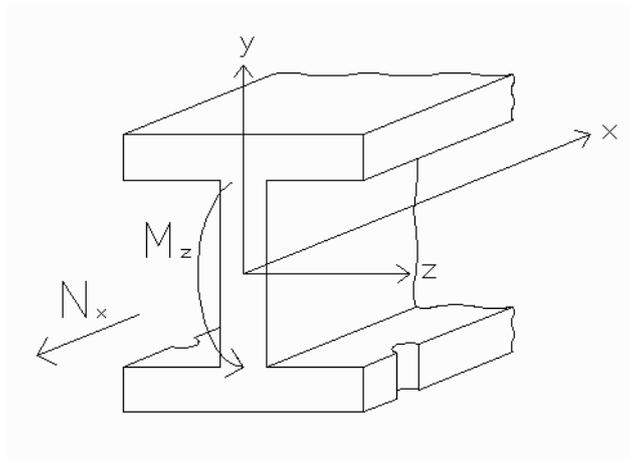


Figure 2. Sketch of the profile, with the notched lower flange

As the bending moment and the axial force are time-dependent functions, they result in stress histories that have to be organized in order to define the load cycles and their equivalent stress level. Such a reduction from load history to load cycles has been done by using the procedure called the *rainflow cycle counting* (RFC), which has been implemented in the initial part of the Matlab code. Each cycle is defined by two values: mean value and range (or amplitude, which is range divided by two).

2.1 High Cycle Fatigue

High Cycle Fatigue is also called Stress-Based Approach. The phenomenon behind this model is a process of material damage and failure due to cyclic loading. Repeated stresses in a given material, well below its ultimate strength initially lead to microscopic physical damage, and later to failure of the engineering component if the load keeps on acting. This process of damage is based on the nominal (average) stresses in the affected area of the element. The fatigue life is determined by considering mean stress effects, and by adjusting them for the effects of stress raisers like notches or holes. Stress-life curves (S-N) are employed to study the number of cycles to failure that specific load cycle/load level corresponds to.

In fatigue design and cyclic loading, a variety of equations have been proposed to take into account the effect of the mean stress. In this work the expression of *Smith, Watson and Topper* (SWT) is used. This is a very frequently employed relationship. It can be written in nominal (average) stresses as

$$S_{ar} = \sqrt{S_{\max} S_a} = \sqrt{(S_m + S_a) S_a} \quad (2)$$

Once the equivalent stresses are known, the S-N curves are used to evaluate the damage caused by each load cycle. There are some different methods to estimate fatigue life. In this project the procedure of *Juinall* (2000) is employed, see Figure 3.

The stresses S_{ar} , see lower curve in Figure 3, are evaluated thus obtaining a single value of the fatigue N_f life for each stress level. Cyclic loadings in

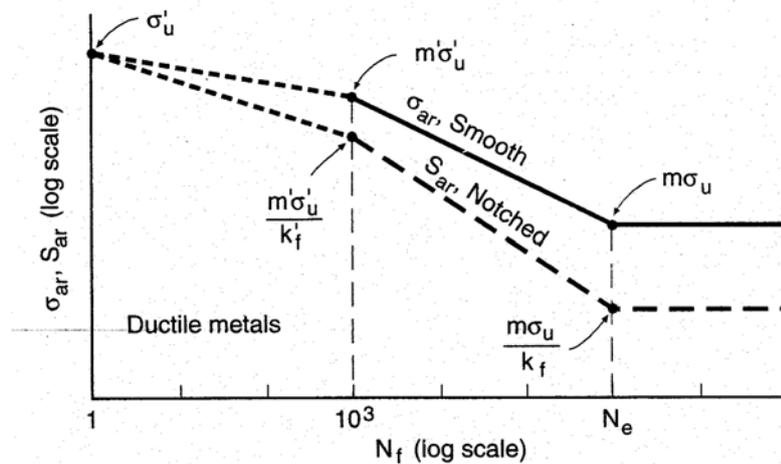


Figure 3. S-N Juvinall's curve. Figure from Reference [1].

practical applications are usually described by stress amplitudes that vary irregularly. As is explained at Reference [1], the *Palmgren-Miner* method is a linear damage accumulation rule, very easy to implement, that predicts the fatigue life N_f for an irregular load sequence with cycles $N_{f,i}$ (estimates how many times that sequence can be repeated)

$$N_f = \frac{1}{\sum_i \frac{1}{N_{f,i}}} \quad (3)$$

2.2 Low Cycle Fatigue

The Strain-Based Approach, involves a more thorough analysis of the local yielding that takes place at stress raisers during fatigue loading. Stresses and strains are analyzed at such locations and are then used as a basis for the derivation of fatigue lives. The Strain-Based Approach gives more accurate estimates than High Cycle Fatigue due to two main reasons. First, it considers a more thorough analysis of local plastic deformation. Second, it employs the local mean stress at the notch instead of the mean nominal stress on the component.

The LCF study starts by using *Neuber's rule* to obtain local strains at notched members. For an elastic, perfectly plastic material beyond the point of yielding the strains are calculated as

$$\varepsilon_{\max,\min} = \frac{(K_t S_{\max,\min})^2}{E\sigma_Y} \quad (4)$$

From this equation the strain amplitude is obtained by calculating half the difference between the maximum and minimum strain. After that, a strain-life relationship is used to acquire the number of cycles to failure for each value of amplitude strain. An equation with these features is often called *Coffin-Manson* relationship, and in this work it has been evaluated taking into account the mean stress effect according to *Smith, Watson and Tooper* explained above. The equation is

$$\sigma_Y \varepsilon_a = \frac{(\sigma_f')^2}{E} (2N_f)^{2b} + \sigma_f' \varepsilon_f' (2N_f)^{b+c} \quad (5)$$

and it contains four material constants: σ_f' , ε_f' , b and c , and all of them can be extracted from the tables in Reference [1]. The number of cycles to failure N_f for the strain amplitude ε_a can be obtained from equation (5).

After that the total fatigue life is obtained by using again the *Palmgren-Miner* damage accumulation rule.

2.3 Linear Elastic Fracture Mechanics

The presence of a crack in, for instance, a railway track can weaken it and cause failure. Cracks are very common in bridges and other civil engineering structures. Therefore, the study of fracture mechanics is of great importance in railways.

If the load applied to a member, containing an initial crack of length a_i , is too high, the crack will suddenly grow and cause a brittle fracture. To measure the severity of a situation with a crack, the *stress intensity factor* K is defined as a function of the crack length, the stress level and the geometry as

$$K = FS\sqrt{\pi a} \quad (6)$$

Negative values of K are referred to compressive stresses. These values are removed from the history due to their closure effect, meaning that only tensile load cycles lead to crack growth.

There are three modes of crack loading and they are denoted by I, II and III. In this project thesis only the first mode has been studied. These modes are called by the names "opening mode", "sliding mode" and "tearing mode", while the corresponding stress intensity factors are denoted K_I , K_{II} and K_{III} , see Figure 4.

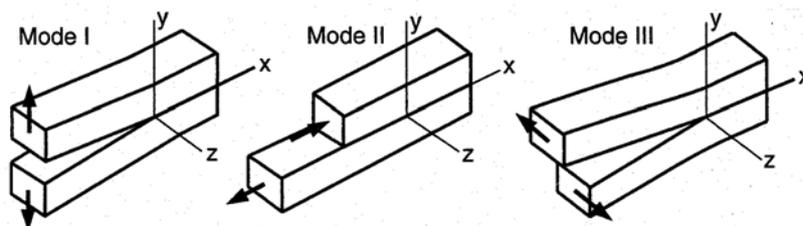


Figure 4. The three modes of crack loading. Figure from Reference [1].

To study the effect of a stress cycle on the crack growth is conventional to use the stress range ΔS when defining the range of stress intensity factor as

$$\Delta K = F\Delta S\sqrt{\pi a} \quad (7)$$

A given material will fail when the value of K arrives at the fracture toughness value K_c . This value is employed to obtain the critical crack length at which the structure will collapse, as

$$a_c = \frac{1}{\pi} \left(\frac{K_c}{FS_{\max}} \right)^2 \quad (8)$$

Engineering analysis of crack growth is often described by the relationship between cyclic crack growth rate da/dN and stress intensity range ΔK . The most frequently used equation is the so-called *Paris Law*, where C and m are material constants,

$$\frac{da}{dN} = C(\Delta K)^m \quad (9)$$

If the effect of mean stresses is to be considered, one must in each cycle account for the representative value of the R-ratio defined as the quotient between S_{\min} and S_{\max} . An increase in the R-ratio of the cycling loading causes larger growth rates for a given value of ΔK . Various relationships can be employed for characterizing the effect of the R-ratio on da/dN and ΔK , but particularly in this thesis the *Walker Equation* has been used. In combination with the *Paris Law* one obtains

$$\frac{da}{dN} = \frac{C_o}{(1-R)^{m(1-\gamma)}} (\Delta K)^m \quad (10)$$

where C_o , m and γ are material constants. The next step in the fatigue study is to derive the number of cycles to failure, in other words, the numbers of cycles that the material can withstand until the critical crack length is reached, followed by collapse. This derivation is here carried out by integration of equation (10) between the initial crack length a_i and the critical crack length a_c .

$$N = \int dN = \int_{a_i}^{a_c} \frac{(1-R)^{m(1-\gamma)}}{C_o(\Delta K)^m} da \quad (11)$$

Finally, another variable is studied: the crack length. Using equation (11) it is possible to derive the number of cycles that are needed to increase the crack length by 1 mm. Repeating this process from a_i to a_c we will study the evolution of the crack length as a function of load cycles.

2.4 Analysis of welded joints

Connections between structural components are often accomplished by welding, and a good example of this are the welds between tracks. Following the theoretical explanations in the textbook [1], S-N curves for use in structural design are here employed. An example for structural steel under tension and bending is given by the American Welding Society (AWS) defining curves as

$$\Delta S = \left(\frac{329C}{N_f} \right)^{0.333}, \quad \Delta S \geq \Delta S_{TH} \quad (12)$$

where the stresses are given as ranges, and C and ΔS_{TH} are constants depending on the weld category. Note that the formula derives the value of ΔS in dimensions of MPa. To choose appropriate category and hence S-N curve, the chart in the AWS code must be used.

Category *A* corresponds to plain structural steel without welds or other stress raisers. Category *B* applies for smooth stress concentrations, as for instance the simple longitudinal weld in Figure 5(a). Category *C* refers to more severe stress concentration, such as transverse welds in a sliced beam, see Figures 5(b) and 5(d). A case with a very severe stress concentration is represented by a transverse weld at the end of a plate as shows Figure 5(c). If the flange thickness is less than 20 mm, then category *E* is applied. Otherwise, the worst possible stress situation is described by category *E'*, see Figure 5.

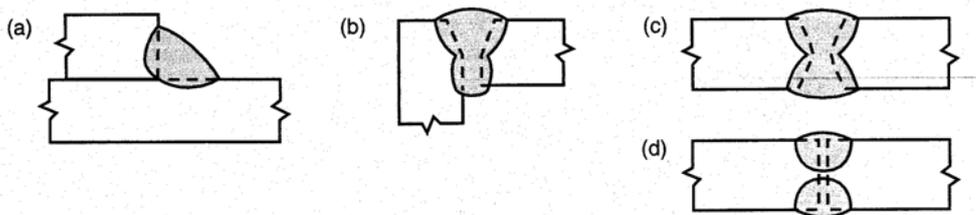


Figure 5. Sketch with typical welds that define the weld category. Figure from Reference [1].

3 DESCRIPTION OF MATLAB CODE

The main task in this project thesis has been to implement a Matlab code which follows the solution strategy described above. The structure of the program is displayed in Figure 6. A main file called *main.m* controls the entire toolbox and calls all the fatigue subroutines. Working together with this main file, there is another large routine, called *variables.m*, in which the user of the toolbox can introduce all the required input data. The file is prepared for an easy implementation of geometry, loading histories and fatigue parameters. In this file the user also decides the method of fatigue analysis to be performed: High Cycle fatigue, Low Cycle Fatigue, Linear Elastic Fracture Mechanics or analysis of welded joints. This file calls *functionKt.m* which provides the stress concentration factor K_t . The lower flange of the beam is assumed to behave as a plate with two notches at the edges. Thus, the K_t factor has been extracted from Figure 7.

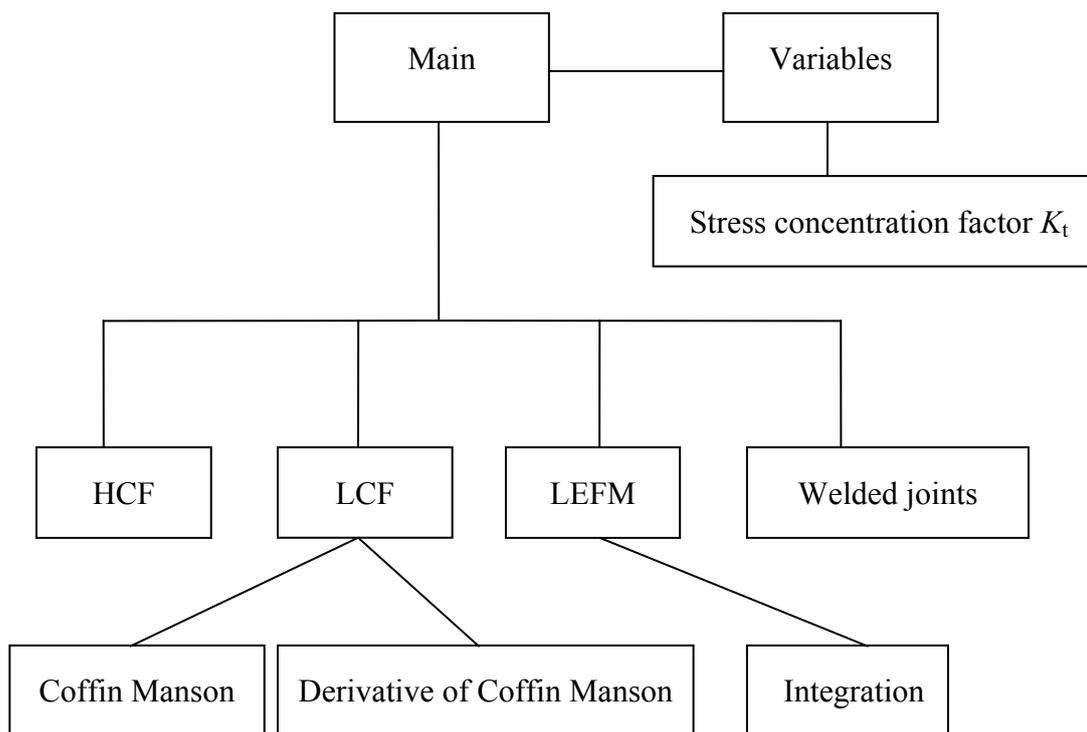


Figure 6. Diagram of the toolbox

Further, there are four sub-programs, *HCF.m*, *LCF.m*, *LEFM.m* and *weld.m*, to carry out the four mentioned fatigue approaches. Some of them work together with other functions that have been implemented in other files. In particular, at LCF the non-linear equation (5) has been solved by using Newton-Raphson method, in which the formulation of Coffin-Manson and its derivative is needed. The Newton-Raphson's method for a non-linear function $f(N)$ is

$$N_{i+1} = N_i - \frac{f(N_i)}{\frac{df}{dN}(N_i)} \quad (13)$$

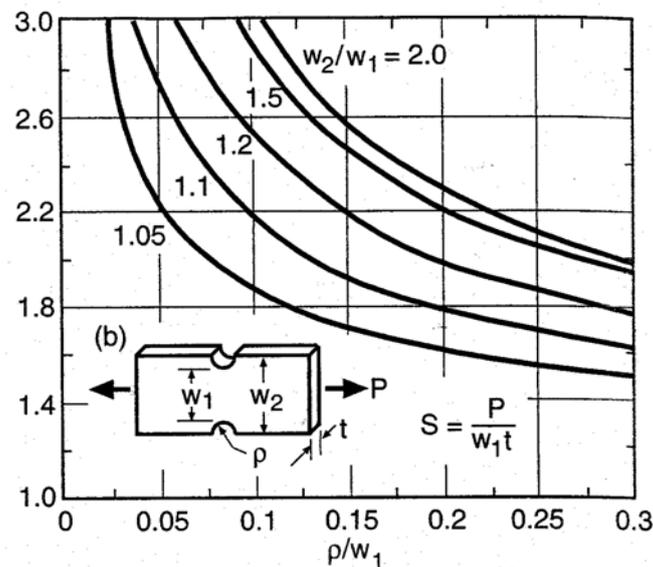


Figure 7. Elastic stress concentration factor for the notched plate.
Figure from Reference [1].

4 CALCULATION EXAMPLES

The loads acting on a beam structure can be of different natures, depending on the design and operation context. In this project, different histories of bending moments and axial forces are considered. Depending on its function, a structure may be submitted to different sequences of repeated loads, with higher or lower magnitudes. Hence, in order to build up a toolbox able to handle that variety of input loads, the program is planned to work with three different time functions.

The first time function is sinusoidal. The routine *variables.m* transforms this load history to a sequence of peaks and valleys by identifying points where the sign of the slope of the function changes. Hereafter, a new sequence of points is sent to the rain flow program. In Figure 8 an example of such a sequence is shown. The stress amplitudes are chosen to be at a level that will give a good demonstration of the fatigue models.

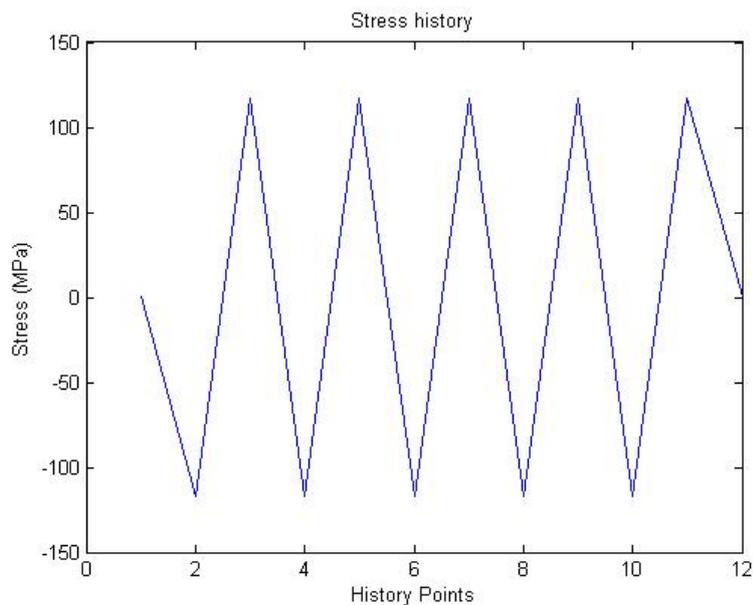


Figure 8. Load vector created by the toolbox from a sinusoidal load

However, it is observed that this time function gives zero mean stress. Combining a sinusoidal function for the bending moment with a constant axial force, a stress history with non-zero mean value is obtained, see Figure 9. Depending on the value of the constant, the mean stresses will be higher or lower, causing different fatigue behaviour of the structure and leading to two extreme cases; one where the entire cycles are in compressions and another one where cycles give only tensile stresses.

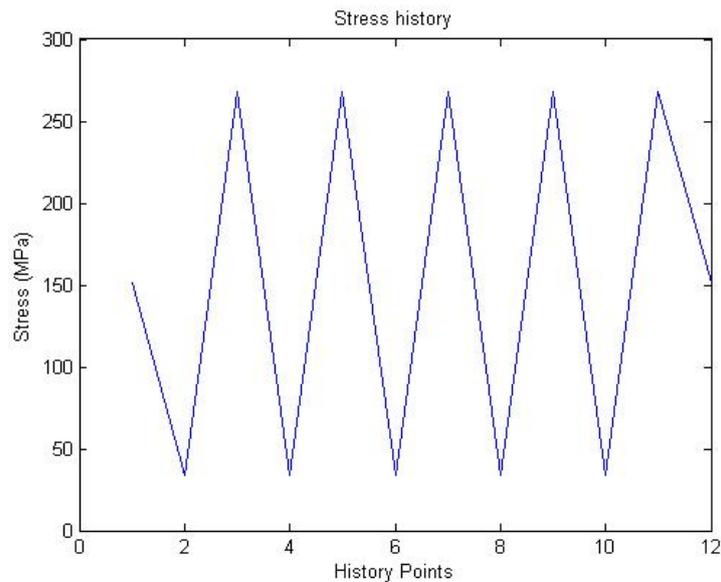


Figure 9. Load vector created by the toolbox from a sinusoidal + constant load

Finally, one last time function has been used to give a wider scope of the toolbox. Until now, we have only worked with periodic cycles, all of them giving the same stress range and the same mean value. With this third time function the aim is to present a variable amplitude loading, in this project also called non-periodic or irregular history of load cycles. The process has been to build up a load vector, representing values in a load history, with extreme values between +1.00 and -1.00 (as the sinus function does), but with a non-periodic sequence of loading. Using this time function for both bending moment and axial force we get a stress history as show in Figure 10.

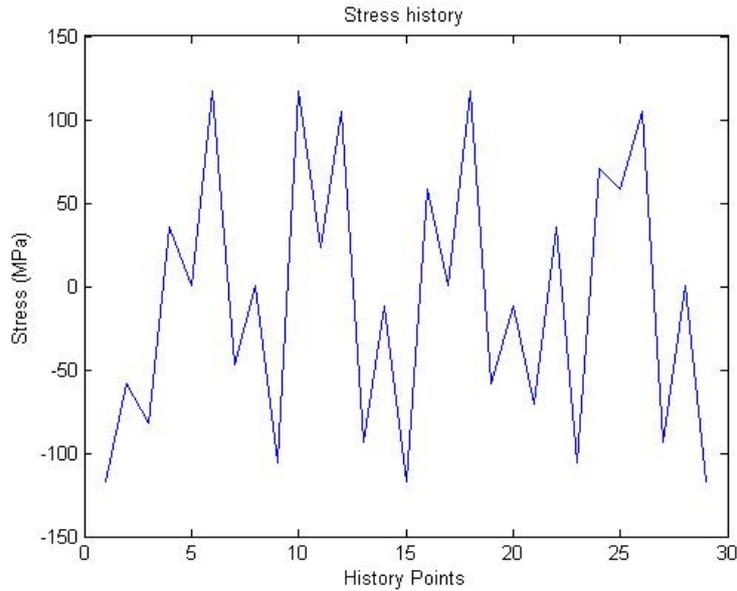


Figure 10. Non-periodic stress history created by the toolbox from the non periodic load vector

It is assumed that real cases of structural analysis will require this kind of non-periodic loadings, due to the large number of variables contributing to the problem and to the complexity of the behaviour when the structure is in service. Hence, these irregular stress histories will be used mainly for practical applications of the toolbox. However, some other more theoretical studies can also be needed, meaning that the sinusoidal time function will be useful. Finally, a constant axial force will move the mean stress up and down and will make it possible analyse the effect of mean stress on the fatigue life of the beam.

The employed non-periodic time function is given below. Values are described between +1.00 and -1.00 making the comparison with the sinus function straightforward.

Non-periodic load vector = [1 0.5 0.7 -0.3 0 -1 -0.5 -0.4 0.4 0 0.9 -1 -0.2 -0.9 0.8
0.1 1 -0.5 0 -1 0.5 0.1 0.6 -0.3 0.9 -0.6 -0.5 -0.9 0.8
0 1]

4.1 Sinusoidal bending moment, constant axial force

The bending moment $M_z = 3.00$ kNm has been used, giving a maximum stress $\sigma = 87.7$ MPa at the notch on the lower flange of the profile (nominal stress $S = 36.5$ MPa). The constant (i.e. time-independent) axial force $N_x = 100$ kN is applied giving a maximum stress $\sigma = 94.3$ MPa at the point of study (nominal stress $S = 39.3$ MPa).

Hence, the following loads have been combined to establish time functions.

$$\left\{ \begin{array}{l} M_z = 3.00 \text{ kNm} \times \sin(t) ; 0 \leq t \leq 10\pi \\ N_x = 100 \text{ kN} \end{array} \right.$$

For this combination of forces it is noted that the sinus function will give a stress amplitude effect, while the constant function will affect the mean stress value.

Figure 11 shows the sequence of loads after the data organization due to rain flow counting requests. It can be appreciated how the maximum nominal stress reaches almost 180 MPa while the minimum is slightly above zero, meaning that all load cycles will be purely tensile. This loading will produce a long fatigue life, since the maximum stress is far below the ultimate stress for the studied steel quality (RQC-100) is 1200 MPa. The closer the load level is to the ultimate stress, the less number of cycles will be resisted by the structure. On the other hand, for small loads the fatigue life will become infinite.

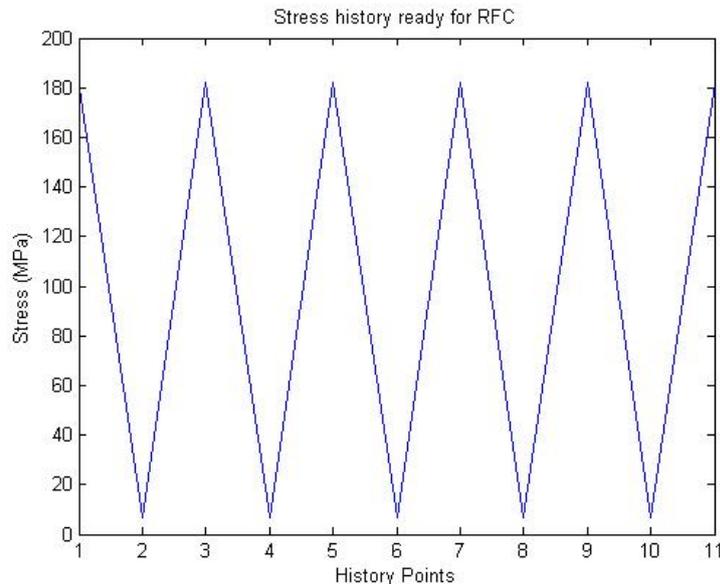


Figure 11. Stress history sequence

The fatigue lives when the beam is submitted to the load explained above are presented in Table 1, Loading case 1.

Table 1. Fatigue life in cycles to failure for all combinations of loads

Loading case	Load combination	Mz[kNm], Nx[kN]	HCF	LCF	LEFM	Welded joints
1	$M_z=3,00 \times \sin(t)$	$N_x=100$	115×10^3	$2,46 \times 10^6$	$35,3 \times 10^3$	305×10^3
2	$M_z=3,00 \times \sin(t)$	$N_x=50$	$1,38 \times 10^6$	123×10^6	112×10^3	305×10^3
3	$M_z=3,00 \times \sin(t)$	$N_x=100 \times \sin(t)$	$13,5 \times 10^3$	$24,2 \times 10^3$	$31,5 \times 10^3$	$34,1 \times 10^3$
4	$M_z=6,00 \times \sin(t)$	$N_x=200 \times \sin(t)$	232	360	$1,67 \times 10^3$	$4,26 \times 10^3$
5	$M_z=3,00 \times \text{irreg}(t)$	$N_x=100$	169×10^3	$4,17 \times 10^6$	$47,9 \times 10^3$	533×10^3
6	$M_z=3,00 \times \text{irreg}(t)$	$N_x=100 \times \text{irreg}(t)$	$19,8 \times 10^3$	$42,1 \times 10^3$	$38,5 \times 10^3$	$47,5 \times 10^3$

It is found that the strain-based approach gives a higher number of cycles to failure than the other methods. One reason can be that, as a result of the more detailed analysis of local yielding, this method gives improved life estimates. Also, LCF performs a better treatment of mean effects, studying them at local scale (at the notch), therefore giving a more accurate result. Later, when other load levels with zero mean stress will be accounted for, the difference between results of LCF and the other methods are found to be less important.

The effect of a lower mean stress is now studied. For instance, the axial force is reduced to half keeping the bending moment at the same level. Results are shown in Table 1, Loading case 2.

Loading with lower mean stress, obviously means a longer fatigue lives for all the methods, except for the analysis of welded joints. This fact agrees with the theory, since for the welded joint method only the stress ranges are affecting at the number of cycles to failure.

Linear Elastic Fracture Mechanics requires further explanations since the algorithm works with load cycles in a special way. Indeed, at LEFM compressive stresses are removed due to their closure effect, meaning that only tensile load cycles lead to crack growth. For this reason, if the mean value is modified towards to a certain compression level, there will be only limited tensile loads remaining, which would give infinite lives. This effect can be compared to a cyclic loading with a nearly zero mean stress and a small stress amplitude (small loads give infinite fatigue lives).

Finally, a study of the crack length as a function of load cycles is performed according to fracture mechanics analysis. The *LEFM.m* routine is prepared to give the number of cycles that the crack needs to increase 1 mm its length. After, this procedure is repeated from the initial crack length until the critical length, in which the beam collapses. The results of this analysis are shown in Figure 12, where crack length a and number of cycles N are plotted for the Loading case 1 ($M_z=3.00 \text{ kNm} \times \sin(t)$, $N_x=100 \text{ kN}$).

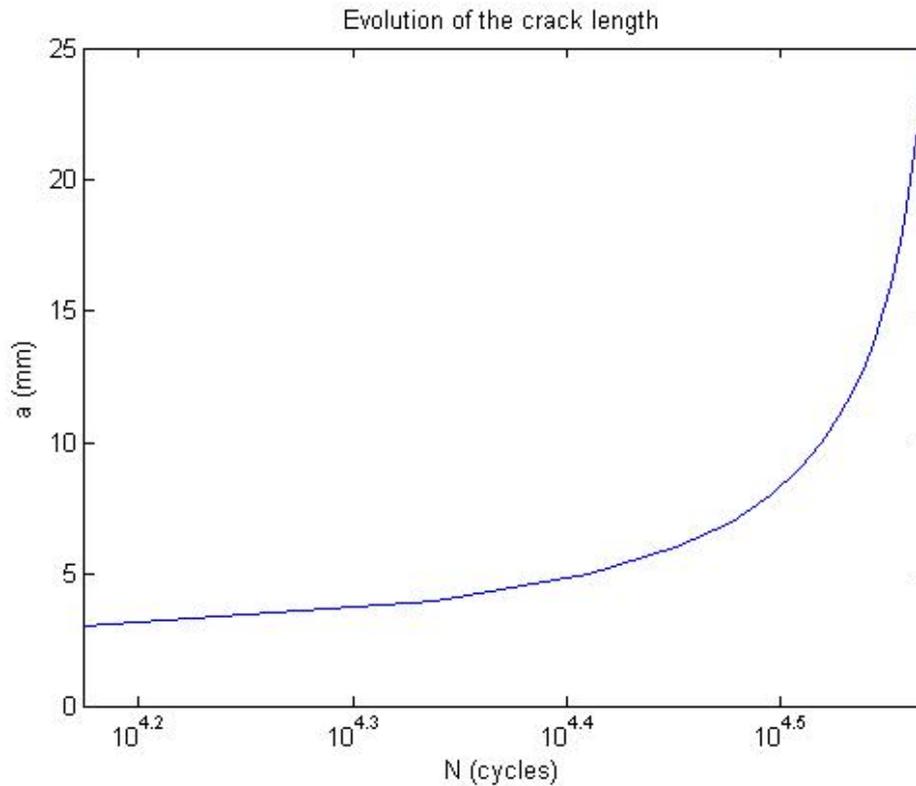


Figure 12. Evolution of the crack length for the Loading case 1.
Here, $10^{4.2} = 15.9 \times 10^3$ and $10^{4.5} = 31.6 \times 10^3$.

4.2 Sinusoidal bending moment and axial force

In this section the same values as before of bending moment (3.00 kNm) and axial force (100 kN) are used, but now both loads are sinusoidal.

$$\left\{ \begin{array}{l} M_z = 3.00 \text{ kNm} \times \sin(t) ; 0 \leq t \leq 10\pi \\ N_x = 100 \text{ kN} \times \sin(t) ; 0 \leq t \leq 10\pi \end{array} \right.$$

Considering two sinus functions the effect of mean stress disappears. On the other hand, both functions are contributing to the stress amplitude so that the resulting load will be a larger stress range than with zero mean stress.

Figure 13 shows the sequence of loads after the data organization due to rain flow counting. It can be appreciated now how mean stresses are around 0, while peaks and valleys are clearly extracted from a sinus of time function.

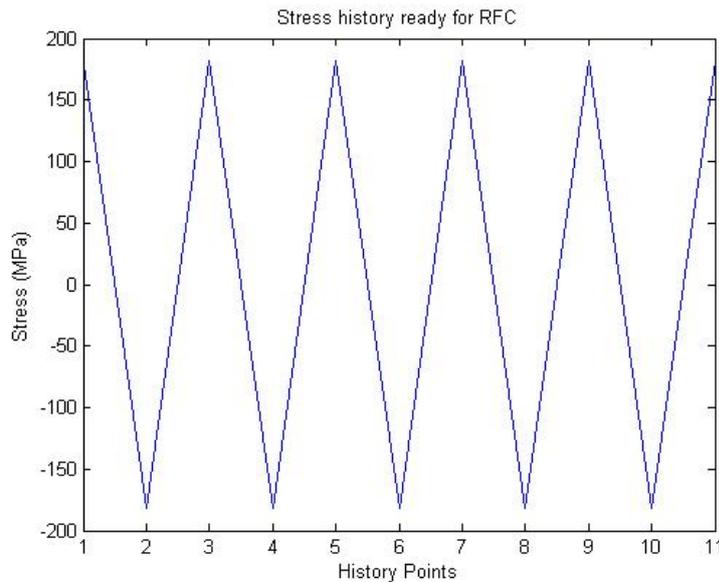


Figure 13. Stress history sequence

Fatigue life results are given in Table 1, Loading case 3. Only slight differences are now found between the methods. The strain-based approach is not more detailed now because there is no effect of the mean stress.

Modified load levels will alter the fatigue lives. Thus, higher stress amplitude gives lower number of cycles to failure and vice versa. For this case, there will not be significant differences between the fatigue methods since in all cases mean stresses are zero.

The case with resulting stress amplitude being double compared to the one calculated above is considered, by multiplying the bending moment and the axial force by two. The results are shown in Table 1, Loading case 4. By doubling the value of the bending moment and the axial force the stress range grows from 364 to 728 MPa. Translating this growth rate to fatigue lives, it can be said that they decrease by far more than proportional to the loading. As expected, there is a non-linear behaviour between fatigue life and stress range. This is the nature of the fatigue problem.

Finally, if we run the toolbox with bending moment and axial forces three times bigger, the stress range reaches the value of 1029 MPa. This value is very near to the ultimate stress (1200 MPa) of the profile's material, so only a very few cycles are performed before failure of the beam.

4.3 Non-periodic bending moment, constant axial force

Real dynamic loads are generally not behaving as regular oscillations. The sinusoidal load gives a theoretical treatment of fatigue loading analysis. Hence, non-periodic histories are used in the following to describe more realistic cyclic loads.

The idea has been to construct a load vector that can be compared with a sinus function having an argument between 0 and 10π . This last function gives 5 full cycles, so with the irregular vector the same number of large cycles should be performed. However, the irregular history gives many smaller peaks and valleys which, at the end, will make slightly smaller equivalent stress and therefore a little bit higher fatigue lives. In Figures 14 and 15 the two time vectors are shown.

While the axial force remains constant with the original value (100 kN), the irregular time vector has been multiplied by the bending moment (3.00 kNm), giving the following time functions.

$$\left\{ \begin{array}{l} M_z = 3.00 \text{ kNm} \times \text{irregular}(t) \\ N_x = 100 \text{ kN} \end{array} \right.$$

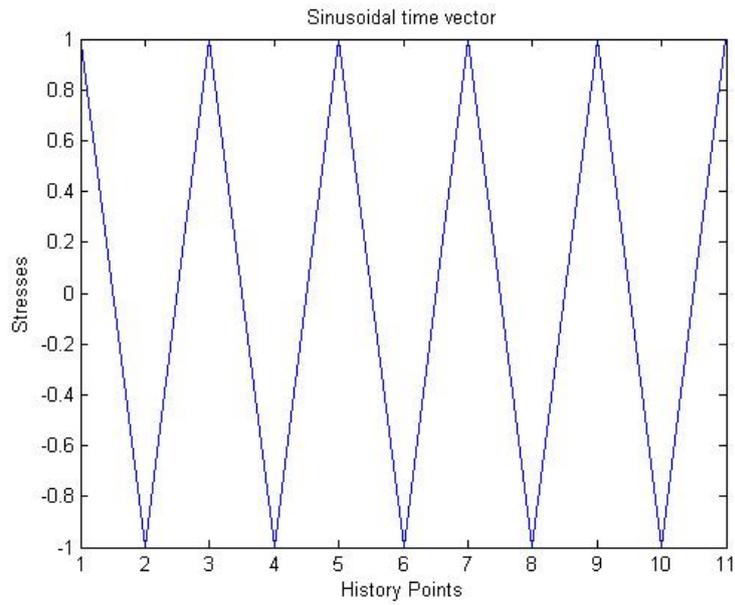


Figure 14. Sinus time vector oscillating between +1.00 and -1.00

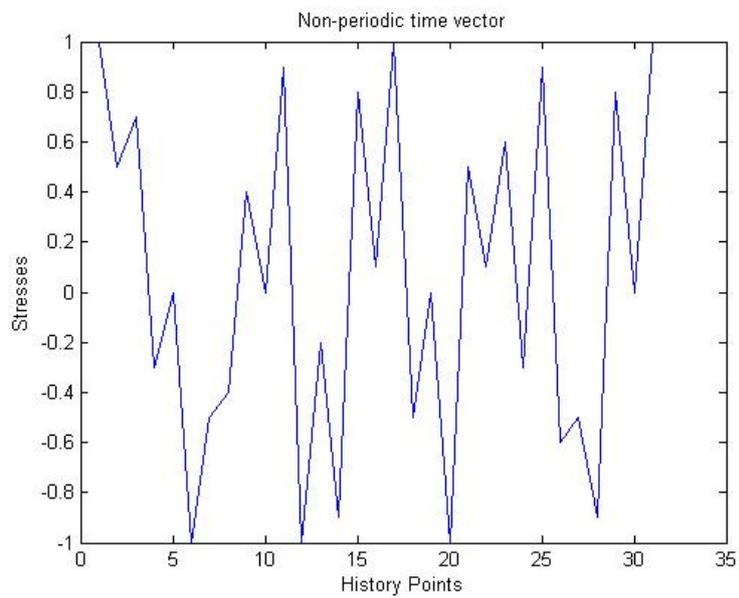


Figure 15. Non-periodic time vectors oscillating between +1.00 and -1.00

After the data organization performed by the rain flow cycle counting program, the load sequence will look like the irregular vector in Figure 10, but with other stress values. Now, the maximum nominal stress value is again 182 MPa while the minimum is around 0 MPa (similar extreme values at Figure 11). All the results are shown in Table 1, Loading case 5.

The obtained solutions are quite similar as the ones extracted from Section 4.1 (sinusoidal bending moment and constant axial force). Again, Low Cycle Fatigue is giving more accurate results due to a better study of local yielding near the notch.

This irregular method does not contribute so much to find new conclusions, so following the analysis in Section 4.1 no further explanations are needed. However, these irregular input stresses give a more realistic approach to cyclic loading, and are useful to show how the fatigue toolbox is able to work with irregular loading data.

4.4 Non-periodic bending moment and axial force

This combination of loads gives very similar results to Section 4.2. Here, both bending moment and axial force are irregular in time, although they are considered to follow the same time function and without a phase shift. Thus, a wider scope of the problem is considered. One last calculation has been performed, running the toolbox with the load values as follows.

$$\left\{ \begin{array}{l} M_z = 3.00 \text{ kNm} \times \text{irregular}(t) \\ N_x = 100 \text{ kN} \times \text{irregular}(t) \end{array} \right.$$

The plotting of this load combination would result in something similar to Figure 10, but with other values and a higher stress amplitude. The load combination can be understood as the behaviour of two non-periodic functions, both giving stress range and mean stress.

The results are shown in Table 1, Loading case 6. There are not big differences between the four methods because the mean values of the larger cycles are nearly zero. However, all kinds of other irregular stress histories can easily be implemented in the toolbox.

An example of the results from the *rainflow cycle counting* (RFC) is shown in Table 2. Loading case 6 has been employed to show stress ranges and stress

mean value for each cycle of the history. Stress history for Loading case 6 is shown in Figure 16.

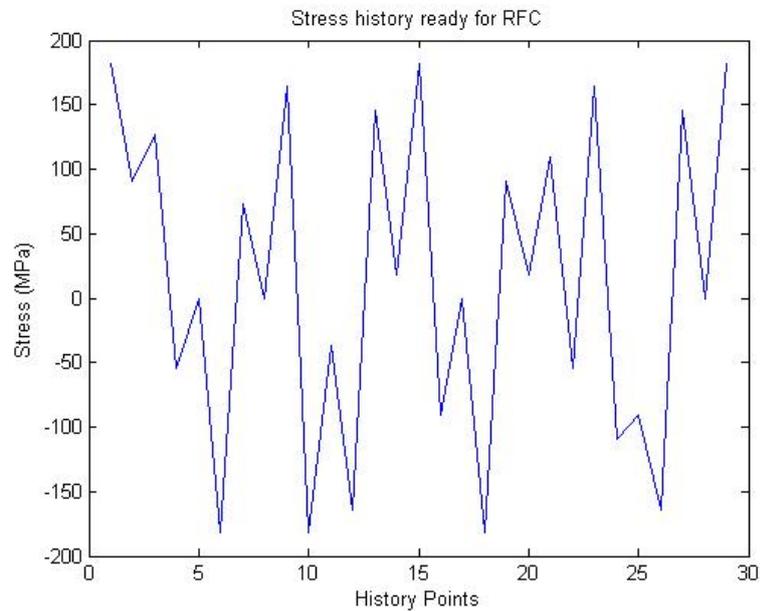


Figure 16. Stress history sequence for the Loading case 6

Table 2. Results from the RFC (stress mean values and stress ranges)

<u>Mean Value</u>	<u>Range</u>	<u>Mean Value</u>	<u>Range</u>
109,24	36,412	-45,515	91,029
-27,309	54,618	54,618	72,824
36,412	72,824	27,309	163,85
-9,1029	345,91	-100,13	18,206
-100,13	127,44	72,824	145,65
81,927	127,44	0	327,71
0	364,12	0	364,12

5 CONCLUDING REMARKS

In the present project thesis a toolbox for evaluating the fatigue life of railway structures is derived. Using histories of bending moments and axial forces the toolbox estimates life by use of four methods: High Cycle Fatigue (stress-based approach), Low Cycle Fatigue (strain-based approach), Linear Elastic Fracture Mechanics and analysis of welded joints.

Different types of load histories have been taken into account with the aim not only to model real situations for beam structures, but also for theoretical studies. Non-periodic loading histories have been employed to work with loads that are more close to realistic conditions. On the other hand, sinusoidal loads have been used for design under more theoretical conditions.

As a result of all these features, the toolbox becomes powerful for both design and verification of profiles, beams and other more complicated structures. More variables can be introduced into the toolbox, in order to make it better and more accurate. However, this is outside the scope of this thesis project.

The toolbox provides results that confirm theoretical knowledge on fatigue life. For instance, the bigger stress amplitudes that are applied on the beam, the less number of cycles will it resist. Hence, working with small stress amplitudes, infinite life is expected. On the other hand, keeping constant the stress amplitude the structure will resist less fatigue cycles to failure for high tensile mean stresses.

When the mean stress is nearly zero, all four methods of fatigue analysis estimate about similar number of cycles to failure. When the mean stress is non-zero, some differences appear between the fatigue methods. More precisely, in the strain-based approach (LCF), gives higher fatigue life due to the more accurate study of local yielding near the notch, and better treatment of local mean stress. Beside that, the analysis of welded joints is the only one that is not affected at all by a variation of the mean stress.

A last comment is given on Linear Elastic Fracture Mechanics. This method removes the influence of compressive stresses because they do not contribute to the crack growth. For this reason, if there are mainly compressive cycles, there will be only a limited number of tensile load cycles, giving very high fatigue lives compared to the other methods.

6 FUTURE DEVELOPMENT AND RAILWAYS APPLICATIONS

Fatigue design is of great importance for railway and other civil engineering structures. The present toolbox can be used for both design and analysis. Naturally, many future developments can be carried out from this project to give more detailed results and to create models that more closely relate to reality.

As to possible applications of this thesis project to the railway field we can state different usefulness for the toolbox:

- life estimates for tracks submitted to train loadings
- study of steel structures contained in railway bridges
- possible application to railway sleepers
- study of welded connections between tracks

As to future developments, other beam profiles should be considered, as well as different beam configurations, e.g. simply supported or cantilever beams loaded in different ways, with complex and irregular loadings. Having this project thesis as a basis, further extensions would not be difficult to implement.

This project also opens other possibilities of life estimates, not only for structural components but also for mechanical elements in the railway field.

It can also be of great value to make the toolbox able to model other materials as wood and concrete (bridges, sleepers). The project can be also extended to account for other positions of the notches. This would mean that different LEFM studies should be carried out making the toolbox more versatile. Also more advanced fracture mechanics studies could be employed.

A last improvement can be done regarding to theoretical formulations. The fatigue procedure for welded joints does not take into account the effect of the mean stress, which makes the analysis somewhat incomplete. A welded joint analysis working both with range and mean stress would give a more accurate approach of the fatigue process near the weld.

7 REFERENCE

- [1] Norman E.Dowling, *Mechanical behaviour of materials – Engineering Methods for Deformation, Fracture and Fatigue*. Pearson Prentice Hall, Upper Saddle River, 2007 (third edition, first edition 1993), 912 pp.

8 APPENDIX

Matlab code

main.m

```
clear all

[analysis_type,E,Kt,hist,means,Smax,Smin,ranges,Sequiv,fracture_stress,mm,
Kf,mp,Kfp,sigma0,sigma_f,b,epsilon_f,c,F,ai,ac,c0,m,gamma,Kmax,cc,
delta_Sth]=variables();

for i=analysis_type
    if i==1 %Stress analysis, High Cycle Fatigue
        fatigue_life_HCF=HCF(Sequiv,fracture_stress,mm,Kf,mp,Kfp)
    elseif i==2 %Strain analysis, Low Cycle Fatigue
        fatigue_life_LCF=LCF(Smax,Smin,E,Kt,sigma0,sigma_f,b,epsilon_f,c)
    elseif i==3 %Fracture Mechanics LEFM
        [fatigue_life_LEFM,crack_size]=LEFM(hist,F,ai,ac,c0,m,gamma,Kmax)
    elseif i==4 %Weld analysis
        fatigue_life_weld=weld(ranges,cc,delta_Sth)
    end
end
end
```

variables.m

```
function[analysis_type,E,Kt,hist,means,Smax,Smin,ranges,Sequiv,
fracture_stress,mm,Kf,mp,Kfp,sigma0,sigma_f,b,epsilon_f,c,F,ai,ac,c0,m,
gamma,Kmax,cc,delta_Sth]=variables()
```

```
analysis_type = [1 2 3 4]; % specify types of wanted analysis:
```

```
    % 1 HCF analysis
```

```
    % 2 LCF analysis
```

```
    % 3 Fracture Mechanics
```

```
    % 4 Welded members
```

```

%%%%%%%%%%
% Profile data: type IPN-100 %
%%%%%%%%%%

Iz=1710000; % moment of ineartia Mz(mm4)
A=1060; % area of the profile (mm2)
d=100; % (mm) depth of the profile
y=-50; % (mm)
w2=50; % (mm)
w1=40; % (mm)
ro=5; % notch radius (mm)
E=200000; % Young Modulus, steel (MPa)
Kt=factorKt(ro,w1,w2); % notch factor

%%%%%%%%%%
%%% Stress history vector %%%
%%%%%%%%%%
% LOAD OPTION 1 (SINUS HISTORY)
t=0:0.01:(10*pi); % steps of time
j=0;
for i=t
    j=j+1;
    Mz(j)=3000000*sin(i); % bending moment (N·mm)
    Nx(j)=-100000;%*sin(i); % axial force (N)
    loads(j)=-Mz(j)*y/Iz-Nx(j)/A; % nominal stresses acting on the beam
    (N/mm2)(MPa)
end

% LOAD OPTION 2 (IRREGULAR HISTORY)
% Mz=3000000*[1 0.5 0.7 -0.3 0 -1 -0.5 -0.4 0.4 0 0.9 -1 -0.2 -0.9 0.8 0.1 1 -
    0.5 0 -1 0.5 0.1 0.6 -0.3 0.9 -0.6 -0.5 -0.9 0.8 0 1];
% Nx=-100000*[1 0.5 0.7 -0.3 0 -1 -0.5 -0.4 0.4 0 0.9 -1 -0.2 -0.9 0.8 0.1 1 -0.5
    0 -1 0.5 0.1 0.6 -0.3 0.9 -0.6 -0.5 -0.9 0.8 0 1];

```

```

% loads=-Mz.*y./Iz-Nx./A;

% repeated values are eliminated from the load history
ir=0;
for i=1:length(loads)-1
    ir=ir+1;
    if ir<length(loads)
        if loads(ir)==loads(ir+1)
            loads(ir+1)=[];
            ir=ir-1;
        end
    end
end

% defining peaks and valleys
stress_history(1)=loads(1);
k=1;
for j=2:(length(loads)-1)
    if (loads(j)>loads(j-1)&loads(j)>loads(j+1))|(loads(j)<loads(j-1)&loads(j)<loads(j+1))
        stress_history(k+1)=loads(j);
        k=k+1;
    end
end

stress_history(k+1)=loads(length(loads));

% figure(1)
% plot(stress_history)
% title('Stress history')
% xlabel('History Points')
% ylabel('Stress (MPa)')

```

```

% we organize our load data to use the rainflow counting process
[c,i] = max(stress_history); % Maximum value

stress_history(length(stress_history))=[]; % we remove the last value

% built up of the new vectors x and y
y=stress_history(i:size(stress_history,2));
x=stress_history(1:i-1);
x=[x,y(1)];
history=[y,x]; % here the first and last point have the value of the highest peak

% some values at the end of the story can be repeated unnecessarily, so we
% remove them
ir=1;
for j=2:(length(history)-1)
    ir=ir+1;
    if (history(ir)>history(ir-1)&history(ir)<history(ir+1))|(history(ir)<history(ir-1)&history(ir)>history(ir+1))
        history(ir)=[];
        ir=ir-1;
    end
end

% figure(2)
% plot(history)
% title('Stress history ready for RFC')
% xlabel('History Points')
% ylabel('Stress (MPa)')

hist=history; % we export this vector to remove compressions at LEFM
analysis

```

```

%%%%%%%%%%
%%%%%%%%%% Rainflow Count Program %%%%%%%%%%
%%%%%%%%%%
j=2;
k=0;
while length(history)>1
    if abs(history(j+1)-history(j))>=abs(history(j)-history(j-1))
        means(k+1)=(history(j-1)+history(j))/2; % mean stress
        Smax(k+1)=max(history(j),history(j-1)); % Smax
        Smin(k+1)=min(history(j),history(j-1)); % Smin
        ranges(k+1)=abs(history(j-1)-history(j)); % stress range
        amplitudes(k+1)=ranges(k+1)/2; % stress amplitude
        if means(k+1)>=0
            Sequiv(k+1)=sqrt((means(k+1)+amplitudes(k+1))*amplitudes(k+1));
% nominal stress with mean stress effect SWT
        elseif means(k+1)<0
            Sequiv(k+1)=-
sqrt((abs(means(k+1))+amplitudes(k+1))*amplitudes(k+1));
        end
        k=k+1;
        history(j-1)=[];
        history(j-1)=[];
        j=2;
    else
        j=j+1;
    end
end

%%%%%%%%%%
% Data for HCF analysis - Juvinall %
%%%%%%%%%%
fracture_stress=1200; %sigma_u (MPa)
% bending fatigue limit factor me

```

```

me=0.5; % steels with fracture_stress smaller than 1460MPa
% Load type factor mt
mt=1.0; % bending and axial loading

% Size factor md
if d<50
    md=0.9;
elseif d>=50 & d<100
    md=0.8;
elseif d>=100
    md=0.7;
end

% Surface finish factor ms (1=polished,2=ground,3=machined)
surface_roughness_type=3; % we start with a machined surface as an example
if surface_roughness_type==1
    ms=1;
elseif surface_roughness_type==2
    sigma_u=[60 80 100 120 140 150 160 180 200 220 240 260]*6.894757; %
    MPa
    ms_ground=[0.9 0.9 0.9 0.9 0.9 0.9 0.89 0.87 0.84 0.80 0.75 0.72];
    ms=interp1(sigma_u,ms_ground,fracture_stress,'spline');% cubic spline data
    interpolation
elseif surface_roughness_type==3
    sigma_u=[60 80 100 120 140 150 160 180 200 220 240 260]*6.894757; %
    MPa
    ms_machined=[0.8 0.78 0.75 0.73 0.71 0.69 0.68 0.65 0.63 0.59 0.55 0.51];
    ms=interp1(sigma_u,ms_machined,fracture_stress,'spline'); % cubic spline
    data interpolation
end

alpha=10^(2.654e-7*fracture_stress^2-1.309e-3*fracture_stress+0.01103); %
    in millimetres

```

```

mm=me*mt*md*ms;
Kf=1+(Kt-1)/(1+alpha/ro);
mp=0.75; % due to axial loading (this value is more conservative than if we use
    mp=0.9 for bending)
Kfp=Kf;

%%%%%%%%%%
% Data for LCF analysis%
%%%%%%%%%%
% we start with a steel RQC-100 as an example
sigma0=778; % yied limit (MPa)

% data for Coffin Manson relationship table 14.1 Dowling
sigma_f=938;
b=-0.0648;
epsilon_f=1.38;
c=-0.704;

%%%%%%%%%%
% Fracture Mechanics (LEFM) analysis data%
%%%%%%%%%%
F=1.12; % shape factor
ai=0.002; % initial crack length (m)
ac=0.025; (m) it is considered that the beam will fail when the crack arrives to
    the half of the plate width, since we work with an I-beam

% constant values for a RQC-100 steel
c0=8.01e-14; % constant at Paris law (m/cycle)
m=4.24; % constant at Paris law
gamma=0.719; % constant at the Paris law
Kmax=150; % fracture toughness (MPa*sqrt(m))

```

```

%%%%%%%%%%
% Data for welded members analysis%
%%%%%%%%%%
%weld categories = [1=A,2=B,3=C,4=E,5=E']
weld_category=1; % for instance
if weld_category==1
    cc=2.5e10; % from the weld category (cycles)
    delta_Sth=166; % from the weld category (MPa)
end
if weld_category==2
    cc=1.2e10; % from the weld category (cycles)
    delta_Sth=110; % from the weld category (MPa)
end
if weld_category==3
    cc=4.4e9; % from the weld category (cycles)
    delta_Sth=69; % from the weld category (MPa)
end
if weld_category==4
    cc=1.1e9; % from the weld category (cycles)
    delta_Sth=31; % from the weld category (MPa)
end
if weld_category==5
    cc=3.9e8; % from the weld category (cycles)
    delta_Sth=18; % from the weld category (MPa)
end

```

factorKt.m

```

function Kt=factorkt(ro,w1,w2);
%w2/w1=2
% curve
ro_w1=[0.300 0.275 0.250 0.225 0.200 0.175 0.150 0.125 0.100 0.075 0.050
0.025];

```

```
Kt_data=[1.975 2.050 2.100 2.200 2.300 2.450 2.550 2.800 3.100 3.450 3.950  
4.900];
```

```
% interpolation
```

```
Kt(1)=interp1(ro_w1,Kt_data,ro/w1,'cubic');
```

```
% w2/w1=1.5
```

```
ro_w1=[0.300 0.275 0.250 0.225 0.200 0.175 0.150 0.125 0.100 0.075 0.050  
0.025];
```

```
Kt_data=[1.95 2.000 2.050 2.100 2.200 2.300 2.450 2.650 2.900 3.300 3.800  
4.500];
```

```
Kt(2)=interp1(ro_w1,Kt_data,ro/w1,'cubic');
```

```
% w2/w1=1.2
```

```
ro_w1=[0.300 0.275 0.250 0.225 0.200 0.175 0.150 0.125 0.100 0.075 0.050  
0.025];
```

```
Kt_data=[1.775 1.825 1.875 1.925 1.975 2.100 2.200 2.350 2.550 2.800 3.150  
3.800];
```

```
Kt(3)=interp1(ro_w1,Kt_data,ro/w1,'cubic');
```

```
% w2/w1=1.1
```

```
ro_w1=[0.300 0.275 0.250 0.225 0.200 0.175 0.150 0.125 0.100 0.075 0.050  
0.025];
```

```
Kt_data=[1.625 1.650 1.700 1.750 1.800 1.850 1.900 2.025 2.175 2.400 2.650  
3.200];
```

```
Kt(4)=interp1(ro_w1,Kt_data,ro/w1,'cubic');
```

```
% w2/w1=1.05
```

```
ro_w1=[0.300 0.275 0.250 0.225 0.200 0.175 0.150 0.125 0.100 0.075 0.050  
0.025];
```

```
Kt_data=[1.500 1.525 1.550 1.575 1.600 1.650 1.700 1.775 1.875 2.025 2.225  
3.000];
```

```
Kt(5)=interp1(ro_w1,Kt_data,ro/w1,'cubic');
```

```

% final interpolation
w2w1=[2.0 1.5 1.2 1.1 1.05];
Kt=interp1(w2w1,Kt,w2/w1,'cubic');

```

HCF.m

```

function fatigue_life_HCF=HCF(Sequiv,fracture_stress,mm,Kf,mp,Kfp);

% S-N Wöhler curve  $S=C1*N^{C2}$ , reduced by the parameters  $m,m',Kf,Kf'$ 
(Juvinall 2000)
C2=(log(mp*fracture_stress/Kfp)-log(mm*fracture_stress/Kf))/(log(1e3)-
log(1e6));
C1=(mp*fracture_stress/Kfp)/(1e3)^C2;

% evaluation of the number of cycles to failure for each stress component
N=(abs(Sequiv)/C1)^(1/C2);

% we remove values N beyond the Juvinall's maximum number of cycles
ir=0;
for i=1:length(N)
    ir=ir+1;
    if ir<length(N)
        if N(ir)>1e6
            N(ir)=[];
            ir=ir-1;
        end
    end
end

% evaluation of the fatigue life by Palmgren-Miner damage accumulation rule
fatigue_life_HCF=1./sum(1./N);

```

LCF.m

```
function
    [fatigue_life_LCF]=LCF(Smax,Smin,E,Kt,sigma0,sigma_f,b,epsilon_f,c)

% Neuber's rule
for i=1:length(Smax)
    if Smax(i)>=0
        epsilon_max(i)=((Kt*Smax(i))^2)/(E*sigma0);
    elseif Smax(i)<0
        epsilon_max(i)=-((Kt*Smax(i))^2)/(E*sigma0);
    end
    if Smin(i)>=0
        epsilon_min(i)=((Kt*Smin(i))^2)/(E*sigma0);
    elseif Smin(i)<0
        epsilon_min(i)=-((Kt*Smin(i))^2)/(E*sigma0);
    end
    strain(i)=1/2*(epsilon_max(i)-epsilon_min(i));
end

% number of cycles using the Coffin-Manson relationship
k=0;
for i=1:length(strain)

    maxiter=100; % maximum number of iterations in case of no convergence
    tol_N=1; % tolerance for the error of N (number of cycles)
    N_ini=1; % initial approximation for N
    for j=1:maxiter % Newton-Raphson iteration
        h=coffinmanson(N_ini,sigma0,strain(i),sigma_f,E,b,epsilon_f,c);
        l=coffderivative(N_ini,sigma_f,E,b,epsilon_f,c);
        N_new=N_ini-h/l;

        if abs(N_new-N_ini)<tol_N
```

```

    N(k+1)=N_new; % building up the N vector
    k=k+1;
    break
end
N_ini=N_new;
end
end

% evaluation of the fatigue life by Palmgren-Miner damage accumulation rule
fatigue_life_LCF=1./sum(1./N);

```

coffinmanson.m

```

function h=coffinmanson(N,sigma0,strain,sigma_f,E,b,epsilon_f,c)
h=(sigma_f)^2/E*(2*N)^(2*b)+sigma_f*epsilon_f*(2*N)^(b+c)-
sigma0*strain;

```

coffderivative.m

```

function l=coffderivative(N,sigma_f,E,b,epsilon_f,c)
l=4*(sigma_f)^2/E*b*(2*N)^(2*b1)+2*sigma_f*epsilon_f*(b+c)*(2*N)^(b+c-
1);

```

LEFM.m

```

function
[fatigue_life_LEFM,crack_size]=LEFM(hist,F,ai,ac,c0,m,gamma,Kmax)

```

% We convert compression values to zero

```
for i=2:(length(hist)-1)
```

```
    if hist(i)<0
```

```
        hist(i)=0;
```

```
    end
```

```
end
```

% repeated values are eliminated from the load history

```

ir=0;
for i=1:length(hist)-1
    ir=ir+1;
    if ir<length(hist)
        if hist(ir)==hist(ir+1)
            hist(ir+1)=[];
            ir=ir-1;
        end
    end
end

% RFC
j=2;
k=0;
while length(hist)>1
    if abs(hist(j+1)-hist(j))>=abs(hist(j)-hist(j-1))
        range_LEFM(k+1)=abs(hist(j-1)-hist(j)); % stress range
        R(k+1)=min(hist(j),hist(j-1))/max(hist(j),hist(j-1)); %R=Smin/Smax
        Smax(k+1)=max(hist(j),hist(j-1)); % Smax
        k=k+1;
        hist(j-1)=[];
        hist(j-1)=[];
        j=2;
    else
        j=j+1;
    end
end

vector_a=(ai:0.0001:ac);
% loop to get the number cycles to failure
for k=1:length(Smax)
    cycles=0;

```

```

for j=2:length(vector_a) % we calculate the number of cycles to failure as the
result of an integral
    a1=vector_a(j-1);
    a2=vector_a(j);
    area=abs(value(range_LEFM(k),R(k),a1,F,c0,m,gamma)+value(range_LEFM(k),R(k),a2,F,c0,m,gamma))*abs(a1-a2)/2;
    cycles=cycles+area;
end
N(k)=cycles;
end
fatigue_life_LEFM=1./sum(1./N);
crack_size=ac;

% evolution of the crack length as a function of load cycles, from ai to ac
size=(ai+0.001):0.001:ac;
h=0;
for g=size
    vector_a=(ai:0.001:g); % final crack size after each increment
    % loop to get the number cycles to failure
    for k=1:length(Smax)
        cycles=0;
        for j=2:length(vector_a) % we calculate the number of cycles to failure as
the result of an integral
            a1=vector_a(j-1);
            a2=vector_a(j);
            area=abs(value(range_LEFM(k),R(k),a1,F,c0,m,gamma)+value(range_LEFM(k),R(k),a2,F,c0,m,gamma))*abs(a1-a2)/2;
            cycles=cycles+area;
        end
        N(k)=cycles;
    end
    fatigue_life(h+1)=1./sum(1./N);
    h=h+1;
end

```

```
end
```

```
vector_a=vector_a*1000; % we change to mm  
fatigue_life=[0,fatigue_life] % we add the zero at the beginning  
figure(3)  
plot(log10(fatigue_life),vector_a)  
title('Evolution of the crack length')  
xlabel('log(N)')  
ylabel('a (mm)')
```

value.m (integration)

```
function f=value(range_LEFM,R,a,F,c0,m,gamma)  
f=[(1-R)^(m*(1-gamma))/(c0*(F*range_LEFM*sqrt(pi*a))^m)];
```

weld.m

```
function [fatigue_life_weld]=weld(ranges,cc,delta_Sth)
```

```
k=0;
```

```
for i=1:length(ranges)
```

```
    if ranges(i)>=delta_Sth
```

```
        N(k+1)=(329*cc)/ranges(i)^3;
```

```
        k=k+1;
```

```
    end
```

```
end
```

```
% If there is not any load value that cracks the beam we stop the weld study
```

```
% If there is one value or more, then we calculate the fatigue life
```

```
if k==0
```

```
    disp('There is not crack initiation next to the WELD, then the fatigue life will  
    be infinite')
```

```
    fatigue_life_weld=0;
```

```
else
```

```
fatigue_life_weld=1./sum(1./N);  
end
```