Chapter 5

The elastoplastic saturated static cone model

5.1 Shortcoming of the available literature and related studies

An extensive literature research was performed in order to evaluate the available analytical models and their suitability in this particular case, concluding:

> **Problem**: There is a gap in the literature when it comes to analytical modeling of pseudostatic tests in saturated sand, thus, accounting for the generation and dissipation of excess pore pressures.

Most models either assume penetration in sand is fully drained or, when considering undrained behavior, it always relates to clay. No modeling of undrained sand response has been carried out yet, therefore, no insight into the response of saturated sand under fast (pseudostatic or dynamic) penetration has been acquainted for. The only model explicitly pseudostatic, the UPM method, does not consider the presence of water. One of the main objectives of this research is then to choose and adapt one of the available models to the case.

5.1.1 Evaluation and proposal

There are static and dynamic models available. Dynamic theories have been developed for elasticity. Excess pore pressure generation is, however, related to volumetric strain and plastic deformation. This means that the soil must certainly be modeled as an elastoplastic material. Combining plasticity with dynamics is far more complicated than this thesis allows for. Consequently, the analytical model will be under static loading. This hypothesis should not be so incorrect as the thesis involves pseudostatic tests; precisely, the direct correlation with static results is the most powerful feature of this kind of tests, thus, stress-wave theory should be possibly neglected. Still, it could be possible to intrinsically introduce some somehow 'dynamic' portion. For instance, the cone geometry of Wolf [21], [22], [23], [24], which was defined due to the wave propagation patterns, can be used as input model geometry, while keeping the calculations static. If the results of the static model were not satisfactory, indicating that dynamic calculations are necessary, the analytical model could be easier modified to account for that.

Cavity expansion is the most extended model for pile penetration and supposes elastoplasticity but it's fully static. Although Verruijt [25] developed a dynamic solution for cavity expansion, once more the solution is only elastic. Also static cavity expansion could be an option for a first contact. Nevertheless, Wolf's cone would account for some dynamic behavior, even if it were only in its definition, thus it seems an interesting choice. Cavity expansion has been used for estimating pore pressures, but only in clay. No parallel formulas are yet available for sand, so the ones of clay could be extrapolated or looking back at the theory of cavity expansion for granular materials, the stress variations could be deduced and related to excess pore pressures. Anyway, the power of cavity expansion, that was its widespread application, is weakened if one needs to modify and extrapolate the previous related studies.

Proposal:Not being able to find a fully satisfying solution, the analytical model will be derived from an adaptation for the Wolf theory.

Besides, as the objectives point towards the pore pressures and these will mainly be generated from compression at the pile tip, focusing in this area only seems appropriate. The excess pore pressures will be calculated following the premises of Randolph and Wroth [26], using the same concepts: for undrained case, excess pore pressures are only generate in the plastic zone and are due to mean total stress changes. Also for the consolidation, similar boundary conditions will be applied and the generated pore pressure distributions will be used as initial condition too .

5.2 Model definition

5.2.1 Main features

The core concept of the cone model rests in the idealization of the zone beneath the pile toe as a truncated cone with geometry easy to determine. In the simplest case of a cone in an homogeneous half-space its geometry may be defined quite straightforward if the pile characteristics and soil properties are known. In this case the last infinitesimal layer of the pile can be idealized as a massless disk supported on a free-space with a load applied in its center that will lead to stresses in the soil half-space and that act on an area that increases linearly with depth. In the cone theory axisymmetry is considered. Moreover, the cone is regarded as a rod (bar) with the displacement pattern over the cross section determined by the corresponding value on the axis of the cone. Strength of materials is applied, namely, plane sections remain plane. The domain of the soil half-space outside the cone is disregarded.

The principal idea of the analytical model can then be summarized:

It is an axisymmetrical plane model that deals with the soil under the pile tip. It has the shape of a truncated cone. On top of it, the pile is neglected and only the last layer is assimilated to a massless loaded disk. The soil outside the cone and above it are not considered, as well as the shaft of the pile.

Note that in every defining feature intrinsic limitations are being introduced. By now

MODEL		Reasons for its possible suitability	Reasons for its refusal	CHOICE
1.Statnamic model (UPM)		-It was especifically conceived to model pseudostatic tests	-Explicitly neglects excess pore pressure generation	-NOT appropriate
2. Stress wave theory	2.1.Smith	-Pseudostatic test is still a dynamic one→stress-wave theory is applicable	-Parameters lack physical significance -Doesn't include excess pore pressures	-NOT appropriate
2. Stress wave theory	2.2.Nguyen		-Parameters lack physical significance -Doesn't include excess pore pressures	-NOT appropriate
2. Stress wave theory	2.3.Base		-Doesn't include excess pore pressures, difficult to include	-NOT appropriate
2. Stress wave theory	2.4.Shaft		-Doesn't include excess pore pressures, maybe could be included, but the calculation would be too complicated	-NOT appropriate
3. Cavity expansion	3.1. Cylindrical	-It is a very good model, widely used in literature, to represent pile penetration -Has been used to evaluate excess pore pressures in clay	-Represents pile shaft, not suitable for zone beneath the tip -Difficult application dynamics	-NOT appropriate
3. Cavity expansion	3.2. Spherical		-Difficult application dynamics	-Could be a good option but:previously only used in clay→extrapolate to sand? -NOT preferred
4. Cone model (Wolf)		-Can be assimilated to a simple rheological model→Physical significance -Simple and accurate -Flexible, offers the possibility to manipulation -Intrinsecally accounts for some dynamic effects (p.e. geometry), even if not explicited in the calculation	-NOT yet been used to predict excess pore pressure→but can be adapted! -Only zone under pile tip→OK if interested in excess pore pressure prediction (compression)	**-PREFERRED CHOICE**

NOTE: There is NO model that completely fullfills our requirements, namely:
Explicitly predict generation of excess pore pressures generated in saturated sand due to pseudostatic loading.
Then:

The CONE model of Wolf will be adapted

Figure 5.1: Available analytical models evaluation

the main one is the fact that the shaft of the pile and consequently the soil around it are neglected. This may be questioned when considering failure of the pile, thus the ultimate capacity, as the shaft resistance is left out of the model. Still, the model has been developed to reproduce the generation of excess pore pressures. As the most significant part of the excess pore pressures are generated under the tip due to compression, the assumption can be accepted.

5.2.2 Assumptions and limitations

Once the principal idea of the model has been defined, it is time to look more in-depth to its characteristics. The current analytical model is a simplification of the real case. Hence, some assumptions are made and will probably introduce a strong limitation in the results applicability. The most remarkable ones are:

- Static problem. No dynamics are considered.
- The soil is considered as an homogeneous fully saturated perfectly linear elastic-plastic material.
- Excess pore pressures are only generated in the plastified area.
- Fully undrained loading. Once the loading process is finished, consolidation starts.
- Unidimensional axial deformation

The main limitations introduced by assumptions are related below as well as other hypothesis:

- The dynamics of the pseudostatic test are neglected. The cone model of Wolf was developed for elasticity and dynamics. However, plasticity and dynamics is too complicated. As the dynamics in the pseudostatic test are low-frequency and, if it is true that the results can be directly assimilated to the static ones, it can be a good first approach a static analysis. No wave theory is introduced by now. Precisely the best fit to this would have been a translational cone in vertical motion. Besides, the force is supposed to act as a single pulse whereas experimentally reboundings have been seen to occur when it's dropped on the pile head.

- It is assumed fully undrainend loading. Sand, as a granular material, is mostly supposed to behave as drained. Nevertheless, as the pseudostatic is a fast test, it is possibly the case, and actually experimentally it has been so, that excess pore pressures are generated. Thus, the behavior will be assumed to be fully undrained. Further in the calculations chapter, the correctness or not of this hypothesis will be discussed. By now, the excess pore pressures can be estimated assuming that, in undrained conditions, the mean effective stress remains constant. The excess pore pressure is equal to the change in mean total stress in the plastified zone. Outside the plastified zone, the hypothesis states fully elastic behavior and no permanent deformation, and so excess pore pressures, if generated there, are immediately dissipated and not taken into account.

- The cone is in a half-space, the soil domain is infinite. This is not true, the bottom of the tank acts like an underlying rigid rock. In addition to the decay of amplitudes as the wave propagates in the simple cone, we should include the reflections at rock interface and on the free surface, presenting the echo constants.

- The last layer of the pile is simulated as a massless disk on the surface of a half-space. In reality this disk is embedded in a full-space,double-cones should be used with Green's function analysis.It is not possible to extend the concept of a single semi-infinite truncated cone to an embedded foundation, the concept of the double cone needs to be preferred.

- Considering only the last infinitesimal layer of the pile is not exact. The pile should be modeled as a sandwich of embedded disks

- In the case of saturated soil analyzed as a single-phase medium, the Poisson's ratio is essentially 1/2 due to the near incompressibility of the water filling the pores. Vertical motions with this ranges of Poisson's ratio might need to introduce two special features:

  1. The appropriate wave velocity dominating the radiation damping is twice the shear-wave velocity and not the dilatational-wave velocity.
  2. A trapped mass $\Delta M{=}\mu\rho r_0^3$ with $\mu = 2.4\pi(\nu - 1/3)$ is introduced.

  . As a first approach we will not take into account this observations.

If any other assumptions are made while calculating the model, these will be explained in the next chapter and the limitations introduced will be evaluated.

5.2.3 Procedure

In general terms, any analytical model that pretends to simulate the generation of excess pore pressures and its effect in the pile bearing capacity, if the loading is undrained, is meant to have three objectives, and steps in the calculation:

1. Determine excess pore pressure generated during loading

2. Evaluate the consolidation process after loading. Hence, in principle it is assumed that there is no consolidation during loading and it only occurs once the loading process is finished

3. Study how the generation of excess pore pressures affects the pile bearing capacity.

A particular model has been defined, though. To achieve the three main purposes, the steps to perform in more detail are enumerated below. The coming chapter includes all the calculations of the model:

1. Calculate the extent of the plastic zone. It needs two sub-steps:

   (a) Model of an elastic cone

   (b) Model of an elastoplastic cone

2. Calculate the generated excess pore pressure

3. Generate a consolidation model and calculate it

4. Check the results and the hypothesis of undrained loading

5. Estimate pile bearing capacity under current conditions.

Before starting the calculations, the model need to be defined quantitatively. The next sections explain the stresses that will be used as input and the geometry.

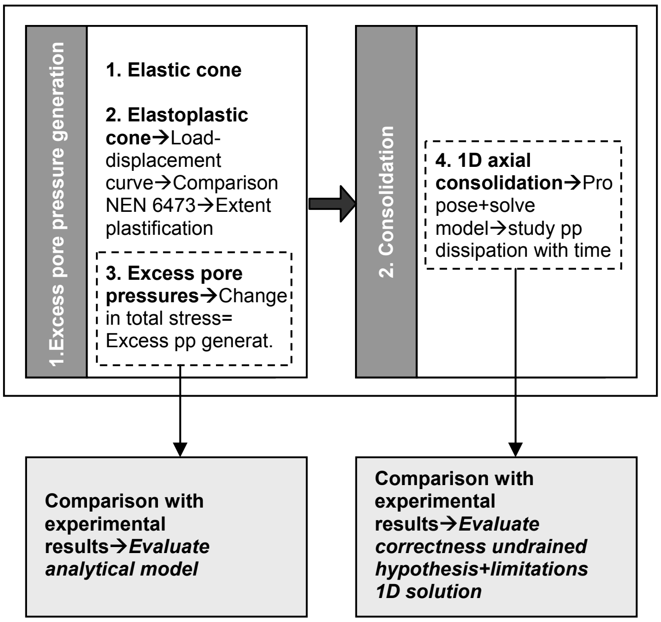

Figure 5.2: Steps to perform in analytical model

5.3 Force that reaches the soil

The dropping mass on the pile head is known. Using dynamics theory the force on the pile head can be obtained and how it propagates through the pile, including dissipation due to shaft friction. Finally, the force that reaches the soil can be calculated.
In fact, one can represent the loading device with the following rheological model. Take into consideration that the pile acts as a damper.

Dijkstra[5] estimated for the same pile and sand a force that reached the pile toe, and so the soil of **12 kN**. This value will be used for the calculations.

5.4 Construction of the cone

In the case of a semi-truncated cone in an homogeneous half-space, the defining features are:

1. Top radius:it is the radius of the pile
2. Cone height: determined by the embeddement depth and the tank depth
3. Aspect ratio: determines the opening angle of the cone. It is a function of the soil properties.

The two first properties are known. Just remains undetermined the opening angle. Generally it has been seen to range between 12^o for failure and 45^o.The (apex) aspect ratio (z_0/r_0) follows for each degree of freedom from matching the static-stiffness coefficient of the disk with radius equal to the pile radius to that of the corresponding cone. For the translational disk in vertical motion, the static stiffness is defined:

$$k_s = \frac{4Gr_0}{1-\nu} \tag{5.1}$$

And for the translational cone:

$$K = \rho c^2 A_0 / z_0 \tag{5.2}$$

This leads to an aspect ratio:

$$\frac{z_0}{r_0} = \frac{\pi}{4}(1-\nu)\left(\frac{c}{c_s}\right)^2 \tag{5.3}$$

Note that z_o/r_0 depends only on ν. For translational cone in vertical motion deformation occurs axially and the waves propagate with the velocity of a dilatational wave ($c = c_p$). This velocity is a function of the properties of the soil:

$$c = c_p = \sqrt{\frac{E}{\rho}} = \sqrt{2\frac{G}{\rho}\frac{1-\nu}{1-2\nu}} \tag{5.4}$$

Similarly, the velocity of the shear waves:

$$c_s = \sqrt{\frac{E}{\rho}\frac{1-2\nu}{1-\nu}} = \sqrt{\frac{G}{\rho}} \tag{5.5}$$

Properly substituting these values into the expressions one could end up with the definition of the cone's geometry.

5.5 Conclusion

The analytical model can be summarized with the following figure. where α is the opening

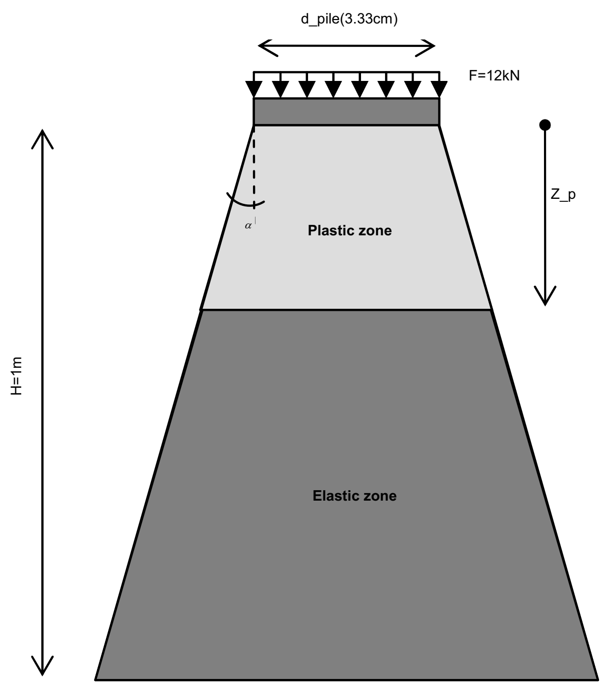

Figure 5.3: Analytical model geometry

angle. It depends only on the Poisson ratio. Yet, there is a lack of reliable soil properties data. A standard value of 40^o will be assumed. z_p is the extent of the plastification. It has to be determined to be able to calculate the generated excess pore pressure. The other parameters are inputs derived from the experimental tests in the calibration chamber.