Performance Evaluation of Fairness
Adaptive Resource Allocation Algorithms for OFDMA Networks

Master of Science Thesis

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Abstract

The objective of this thesis is to implement and evaluate dynamic resource allocation algorithms that adjust the fairness of an OFDMA network. We consider the category of Rate Adaptive algorithms where the Base Station is assumed to transmit at full power and under this condition we try either to maximize the throughput of the system or to maximize the minimum rate of the users. The novelty within the implemented algorithms lies in the adaptive adjustment of the system fairness, an index that shows how fair the throughput of the system is split to the users. The adjustment is performed in general by reallocations of channels and power. Two approaches of adjusting the system fairness were implemented: The Fairness based Sum Rate Maximization with Proportionalities (FSRM-P) and the Fairness Based Max-Min Rate (FMMR). For each approach we initially formulate the optimization problem and then we evaluate three solutions. Because of the non-convex nature of the problems, the proposed novel solutions are iterative, heuristic and in general suboptimal. The evaluation is performed by considering the downlink of a single OFDMA cell serving a set of users which have different rate requirements. The algorithms whether an increase or decrease of the fairness is required, are able to meet the target. However this always comes at the cost of an opposite effect on throughput. Simulations results showed that for the same value of system fairness, the FSRM-P performs better in terms of throughput while the FMMR is able to be fairer with the users. Moreover, in most cases the FMMR achieves higher user satisfaction, however when the rate requirements of the users are increased the satisfaction drops to lower levels than the FSRM-P.
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<th>Description</th>
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<tbody>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>ADSL</td>
<td>Asynchronous Digital Subscriber Line</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<td>CSI</td>
<td>Channel State Information</td>
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<td>DVB-T</td>
<td>Digital Video Broadcasting</td>
</tr>
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<td>F-SRM-P</td>
<td>Fairness Sum Rate Maximization with Proportionalities</td>
</tr>
<tr>
<td>FMMR</td>
<td>Fairness Max-Min Rate</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>I-SRM-P</td>
<td>Iterative Sum Rate Maximization with Proportionalities</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>MA</td>
<td>Marginal Adaptive</td>
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<tr>
<td>MMR</td>
<td>Max-Min Rate</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Division Multiplexing</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Division Multiple Access</td>
</tr>
<tr>
<td>PA</td>
<td>Power Allocation</td>
</tr>
<tr>
<td>PLC</td>
<td>Power Line Communications</td>
</tr>
<tr>
<td>RA</td>
<td>Rate Adaptive</td>
</tr>
<tr>
<td>SA</td>
<td>Subchannel Allocation</td>
</tr>
<tr>
<td>SFI</td>
<td>System Fairness Index</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise ratio</td>
</tr>
<tr>
<td>SRM</td>
<td>Sum Rate Maximization</td>
</tr>
<tr>
<td>SRM-P</td>
<td>Sum Rate Maximization with Proportionalities</td>
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<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TTI</td>
<td>Transmission Time Interval</td>
</tr>
<tr>
<td>TU</td>
<td>Typical Urban</td>
</tr>
<tr>
<td>UFI</td>
<td>User Fairness Index</td>
</tr>
<tr>
<td>USI</td>
<td>User Satisfaction Index</td>
</tr>
<tr>
<td>Wi-Max</td>
<td>Worldwide Interoperability for Microwave Access</td>
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<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
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1. Introduction and motivation

Orthogonal Division Multiple Access (OFDMA) is a technique used in a variety of cable and wireless communications. These include Digital Video Broadcasting (DVB-T), Wireless Local Area Networks (WLAN, IEEE 802.11 a/g/n), Worldwide Interoperability for Microwave Access (Wi-Max, IEEE 802.16), Asynchronous Digital Subscriber Line (ADSL) and Power Line Communications (PLC). Although research on OFDMA dates back to 1960’s, the practical use of this technology became evident the last decade after advances in signal processors and power amplifiers. Recently OFDMA was incorporated in the Third Generation Partnership Project, in the 8th Release of the so-called Long Term Evolution project (LTE) which consists the last step towards the 4th generation of mobile radio technologies. In LTE the peak data rates are set to at least 100Mbps and 50Mbps for the downlink and uplink direction respectively thus providing high speed wireless internet and data network access able to meet the demands of increasing data to the users. In practice OFDMA is composed of a large number of closely spaced orthogonal subcarriers, the main advantage of which is ability to cope with frequency selective fading channels and Inter-Symbol Interference without any complex equalization filters.

1.1 Background

In principle no optimal resource allocation exists in a time-frequency varying wireless channel if a static resource allocation scheme is adopted over time (such as TDMA or FDMA). In this thesis by resources we mean the available channels and power of the system. The problem of optimal resource allocation to the users lead to a family of algorithms referred to as dynamic resource allocation algorithms that try to exploit the time and frequency dependent characteristics of the wireless channel in order to make an efficient use of them.

Two of the approaches that exist in the literature are the Rate Adaptive (RA) approach ([2], [4]) and the Marginal Adaptive (MA) approach [21], the objectives of which are different. In the MA approach, the objective is to satisfy the requirements of the users on the data rates and the bit error rates (BER) while keeping the overall transmit power of the BS at the minimum level. In the RA approach, the BS is assumed to transmit at full power and under this condition we try either to maximize the throughput of the system or to maximize the minimum rate of the users. In this thesis the RA approach is examined.

The RA approach which is targeted to maximize the throughput of the system will be referred in the following as Sum Rate Maximization (SRM) whether the RA approach which is targeted to maximize the minimum rate of the users as Max-Min Rate (MMR). A variant of the MMR is the Sum Rate Maximization with Proportionalities (SRM-P) which maximizes the throughput of the system by satisfying the required rate proportions of the users over the throughput of the system i.e. each user is allocated with a rate which corresponds to a certain proportion of the instantaneous throughput. Intuitively the resulting allocation of the rates between the SRM and MMR is contradictory. Assuming that the channel conditions remain almost the same,
in the SRM case some users will benefit from the good channel conditions while some others will get practically zero data rate. In the MMR case the heavy data users will be probably left unsatisfied for the sake of the weak users. In general, fairness among the users comes at the cost of reduced overall throughput, a trade off which is evident in the resource allocation algorithms.

1.2 Problem formulation

In general the optimization algorithms in resource allocation of OFDMA systems focus on the optimal management of the resources of the system. These include the allocation of the available channels to the users (referred to as subchannel allocation) and the allocation of the available power on the channels (referred to as power allocation). In both cases the objective is to perform optimal allocation in the sense that the objective of the SRM, MMR and SRM-P problems is satisfied. The formulation of these problems follows the one at optimization theory. An objective function which in the case of the RA approach may be the cell throughput is set to be maximized under a given set of constraints that describe the behaviour of the variables of the problem. In the RA resource allocation problems of the thesis the variables are the channel allocation matrix \( \rho_{k,n} \) and the power assigned to each channel/subchannel/subcarrier \( p_n \) where \( k \) is the user id/index and \( n \) is the channel id/index. The constraints in a sense formulate the domain of objective function.

\[
\begin{align*}
\max_{\rho, p} & \quad \text{Performance index } f(p, \rho) \\
\text{subject to} & \quad \text{constraints on } p \\
& \quad \text{constraints on } \rho
\end{align*}
\]

Figure 1-1 General formulation of the Rate Adaptive optimization problems

The domain of the function is generally not convex since in principle the variables that allocate the channels to the users are integers, 0 or 1, and they do not form a convex set (cf. Annex I, [3]). In fact, these kinds of problems belong to the category of combinatorial optimization and are NP-hard [1]. Solving such problems is very computationally demanding since no direct method applies that provides the solution to the problem (such as the Karush-Kun-Tucker conditions in the case of convex problems). In that sense every proposed solution is sub-optimal and in general tries to address the problem in one or more of the following ways:

- By relaxing one of the constraints: The integer constraint on the channel allocation matrix is relaxed allowing more than one user to share the same channel. In this case the problem is simplified and in the MMR case becomes convex [2] or may be simplified [1].
- By splitting the problem into a channel allocation problem and a power allocation problem: In this case, which is the most common approach, an iterative heuristic channel allocation algorithm follows a policy that satisfies
the constraints while meeting the objective of (1.1). Then as a second step a power allocation algorithm is applied which may be optimal [3], [4] or suboptimal based on reasonable assumptions [2].

- By applying heuristics: In this case, an iterative joint subchannel and power allocation algorithm is applied that meets the objective of the problem while satisfying the constraints [5].

A mixture of the above ways results in sub-optimal solutions of the problem (1.1).

Intuitively, someone could enumerate all possible channel allocation combinations and apply a brute force method to find the optimum solution for the subcarriers allocation problem. However, this method is intractable in systems with many users and many subcarriers as the order of complexity is $O(K^N)$ [4]. Therefore, suboptimal solutions are investigated, also in this thesis, which can guarantee a degree of optimality on the results. Although joint solutions by applying heuristics are more tractable since they deal at the same time with the two problems [5], in most cases it is difficult to find such solutions that meet the requirements of the problem (1.1). In order to further reduce the complexity of the algorithms, splitting the problem into two separate problems, a subchannel and a power allocation, is more tractable and is also followed here.

1.3 Thesis objectives

In the case of SRM the throughput of the system is maximized at the cost of low fairness among the users as some users are deprived from getting any resources if their channel conditions are bad. In the case of MMR the fairness among the users is generally high since the policy is targeted to always increase the minimum rate user however the resulting throughput of the system is low. In the case of the SRM-P the rate proportionalities are satisfied, the fairness is high, however the throughput is again low. The system fairness index [22] is a measure that captures the contradictory behavior of the system towards the users between SRM and MMR/SRM-P policies. In the case of SRM the system fairness is low and in the case of MMR and SRM-P high. The trade-off between fairness and throughput can be managed by adjusting the fairness of the system in order to have the opposite effect in throughput. For example to reduce the fairness in order to gain in throughput and vice versa.

Moreover, from an operator’s point of view, the system’s behavior between the two extremes of the SRM and MMR/SRM-P is not always desirable. A medium solution where the operator can adjust the level of system fairness between the two extremes seems more tractable and flexible. The trade-off between throughput and fairness can be managed by applying algorithms that adjust the system fairness between the two extremes as said before. The adjustment can depend on the operator’s general policy to the users of the system, for example it may depend on the percentage of gold class users. If the percentage of gold class users is low, the operator can be more fair with the user population thus adopting a policy with high fairness while in the scenario where the percentage of gold class users is high, to be less fair and adopt a policy with low fairness. The development of such algorithms is the scope of this thesis.
1.4 Thesis outline

The rest of the thesis is organized as follows:

In Chapter 2 we present the system model used for the development of the optimization algorithms. Initially the basic principles of OFDM systems are presented and then the simulation environment which is the background of the algorithm development. Specific reference is given to the assumptions of the model, the propagation environment, and the calculations of the system metrics and performance indexes.

In Chapter 3 we study the classical optimization algorithms that are found in the literature, the SRM, MMR and other specific variants of these. Initially we discuss the problem formulation and its objectives and then explain the algorithms that used to solve it. A comparison of the algorithms for different performance indexes (e.g. throughput, system fairness) is presented.

In Chapter 4 we follow the same philosophy to propose algorithms that are based to the ones in chapter 3 but can also manage to adjust the system fairness with different policies. The results of the comparisons in the performance indexes are given in the end of the chapter.

In the last chapter we summarize the conclusions of the thesis.
2. System Modeling

This chapter serves as an explanatory section before describing the implemented algorithms. It is divided into three parts. The first presents a general description of the Orthogonal Frequency Division Multiple Access (OFDMA) scheme. In the second part, a description of the implemented system model of the OFDM simulator is presented that it is the basis for the algorithm development of the thesis. In the third part are presented some additional key performance measures, the user fairness index (UFI) and the system fairness index (SFI), that also describe the behavior of the algorithms.

2.1 Principles of OFDM systems

A simplified OFDM system is shown in Figure 2-1 Simplified OFDM transmitter/receiver block diagram in 3GPP LTE OFDM is based on the transmission of many subcarriers orthogonal to each other at the same time (i.e. in parallel transmission). Consider a pool of $N_c$ subcarriers belonging to a greater system bandwidth $B$. Each one is separated by $\Delta f = B / N_c$ Hz or $\Delta f = 1 / T_s$ Hz where $T_s$ is the subcarrier duration time. Then consider an information sequence of symbols $a = \{a_0^m, a_1^m, ..., a_{N_c}^m\}$ which is placed in serial order and is to be modulated by the subcarriers at the time period $m \leq t < m+1$. The symbol time is $T_s$ as the subcarrier duration time, a characteristic which allows implementation of low complexity Fast Fourier Transform.

![Figure 2-1 Simplified OFDM transmitter/receiver block diagram in 3GPP LTE](image-url)
Initially a serial to parallel converter transforms the serial sequence \( a \) into a parallel one, in a way that each symbol is modulated by a subcarrier \( k \) in frequency. The complex baseband notation of the transmitted sequence is:

\[
x(t) = \sum_{k=0}^{N_c-1} x_k(t) = \sum_{k=0}^{N_c-1} a_k^m e^{j2\pi k f t}
\]

(2.1)

Where \( x_k(t) \) is the \( k^{th} \) modulated subcarrier at frequency \( f_k = k \Delta f \) and \( a_k^m \) is the modulated symbol. In this sense, during the time interval \( m \leq t < m + 1 \) an OFDM symbol is composed of \( N_c \) modulated symbols in parallel. The modulation symbols can belong to any modulation scheme (QPSK, 16QAM, 64 QAM). To illustrate the principle of OFDM transmission in time and frequency domain consider a 2-D grid where the horizontal axis is the time consisting of OFDM symbols which are transmitted over \( N_c \) subcarriers (frequency domain in vertical axis).

![Figure 2-2 Time-frequency grid of OFDM symbol transmission](image)

In Figure 2-2 a parallel sequence of \( N_c \) information symbols is modulated by \( N_c \) subcarriers at every OFDM symbol transmission period.

Now, assume that the signal in (2.1) is sampled every \( f_s = 1/T_s = N \Delta f \) Hz where \( N \) is chosen so that the Nyquist sampling theorem is satisfied (in general \( N \) may exceed \( N_c \)). The sampled version of the OFDM signal is then:

\[
x_n = x(nT_s) = \sum_{k=0}^{N_c-1} a_k^m e^{j2\pi k f t_n} = \sum_{k=0}^{N_c-1} a_k^m e^{j2\pi kn/N}
\]

(2.2)

which is the Inverse Discrete Fourier Transform (IDFT) of the modulation symbols \( a \). In the case where \( N = 2^n \) this process can be implemented with the Inverse Fast Fourier Transform (IFFT). Similar to the modulator, in the demodulator side a FFT process is implemented as shown in Figure 2-1.
Since a wireless channel allows multipath propagation, the orthogonality among the subcarriers is destroyed since there will be an overlap at the received signals over time. This increases the ISI and also creates interference among the subcarriers. A cyclic prefix is added after the symbol sequence with time length greater than the impulse response of the channel in order to avoid this phenomenon. The cyclic prefix is then removed at the receiver side.

Summing up, some characteristics of the OFDMA scheme are:

- A serial symbol stream is converted into a parallel one with much slower rate which is then modulated on a set of orthogonal subcarriers. The more subcarriers are used, the slower becomes the rate. The conversion along with the insertion of a cyclic prefix in the time domain with duration greater than the coherence time of the channel results in reduction/removal of the ISI and the adjacent channel interference.
- Therefore, the channel’s impulse response during an OFDM symbol can be considered as time invariant and since ISI and interference among channels are negligible, each subcarrier can be considered separately from the others.
- The transmitter/receiver implementation with IFFT/FFT modulators/demodulators has low complexity which simplifies their implementations.

More on OFDM systems can be found at [6],[7].

2.2 System model assumptions

Resource management algorithms try to answer the question of how to efficiently manage the available power and subcarriers of an OFDM system to the users in order to meet an objective of the allocation. To this end, a simplified case of users belonging/connected to a single BS gives a great insight on the overall performance of such algorithms. Even though interference from adjacent cells is not considered, the skeptic of the algorithms remains the same if someone would include into the calculations the intercell interference. A single cell multi user OFDM simulator was implemented in order to study the performance of the algorithms. Some general characteristics and assumptions of the simulator are:

- A single cell scenario with hexagonal area of coverage is considered. Therefore we do not consider inter-cell interference in the calculations and no handovers of the users with neighboring BSs take place.
- Only the downlink direction is examined where the Base Station (BS) transmits at full power of $P_{\text{max}}$ (W).
- Users have no mobility and have always data to transmit, i.e. users are static and a traffic model with full buffer is considered for each user.
- $K$ is the number of users that the BS serves where $k \in [1, K]$ is the user id/index. We do not consider new arrivals or departures of the users and K is fixed throughout the simulations.
- $N$ is the number of subcarriers/subchannels that are allocated in the system bandwidth $B(\text{Hz})$ where $n \in [1, N]$ is the channel id/index and $\Omega_k$ is the set of channels assigned to user $k$. This gives us a bandwidth per subchannel equal to $B/N(\text{Hz})$. 
• TTI is the *Transmission Time Interval* which is the basic time reference unit in the simulator. Scheduling and power allocation algorithms are executed at each TTI.

• The BS has perfect knowledge of the conditions of the channel of all users i.e. *perfect Channel State Information* (CSI) is considered.

2.2.1 Propagation environment – Path gains

The propagation environment is a typical urban wireless fading channel defined as in [8]. The path gain of user $k$ at subchannel $n$ is comprised of three terms:

- the *distance dependent* losses $L_{k}^{\text{dist}}$,
- the *slow/shadow fading* losses $G_{k}^{\text{sh}}$,
- the *fast/Rayleigh fading* losses $G_{k,n}^{\text{Ray}}$.

The path losses between the BS and the user are calculated by the summation in the dB domain:

$$G_{k,n} = L_{k}^{\text{dist}} + G_{k}^{\text{sh}} + G_{k,n}^{\text{Ray}} \text{ dB}$$  \hspace{1cm} (2.3)

where $L_{k}^{\text{dist}}$ is calculated using (2.4):

$$L_{k}^{\text{dist}} = 128.1 + 37.6 \log_{10}(d_{k}) \text{ dB}$$  \hspace{1cm} (2.4)

where $d_{k}$ (m) is the distance between the BS and the $k^{\text{th}}$ user. Since the users are not moving throughout the simulation this factor is fixed.

*Shadow fading* $G_{k}^{\text{sh}}$ is a zero-mean log-normal random variable with standard deviation $\sigma$ (db):

$$G_{k}^{\text{sh}} \sim \text{LogN}(0, \sigma)$$  \hspace{1cm} (2.5)

Since the users are not moving throughout the simulation this factor is also fixed. Rayleigh fading $G_{k,n}^{\text{Ray}}$ is implemented according to Jake’s model [13].

To visualize the channel conditions that a user experiences in such a propagation environment, a realization of the path gain across the subcarriers over many TTIs is plotted in Figure 2-3.
In general the deep fading occurrences at specific TTIs and subcarriers (e.g. at subcarrier 10 at TTIs 5 to 7) may not happen in a different user that is located elsewhere in the cell. The multi-user diversity over the channels is exploited by the dynamic resource allocation algorithm at each time instant. Considering subchannel and power resources used in OFDMA systems, the purpose of such algorithms becomes two-fold. Firstly, to identify at each TTI the channel conditions (e.g. the attenuation) for each user and assign the subchannels to the users according to a policy that tries to meet an objective (e.g. to maximize the throughput of the system). Secondly, given the previous subchannel allocation to apply a sophisticated power allocation over the subchannels that enforces the objective of the resource allocation.

### 2.2.2 Link adaptation and rate calculation

The well-known Shannon’s capacity formula gives a theoretical upper bound on the achieved data rate of the $k^{th}$ user at the $n^{th}$ subcarrier:

$$ R_{k,n} = \frac{B}{N} \log_2 \left( 1 + SNR_{k,n} \right) \text{ (b/s)} $$

Where $SNR_{k,n}$ is the received Signal to Noise Ratio (SNR) of the $k^{th}$ user at the $n^{th}$ subcarrier:

$$ SNR_{k,n} = \frac{P_n g_{k,n}}{N_0 B/N} $$

Where $N_0$ is the power spectral density of the thermal noise, $p_n$ is the transmitted power at the $n^{th}$ subcarrier and $g_{k,n}$ is the path gain.
A more accurate calculation in realistic scenarios is made by introducing a correction factor, the so called SNR gap [7], which is also used in the thesis. The SNR gap takes into account the specific Bit Error Rate (BER) requirements of the transmission concept. Assuming QAM detection and ideal phase detection the resulting SNR including the SNR gap is ([14]):

$$SNR_{k,n}^{\text{gap}} = \frac{1.5}{-\ln 5 \cdot BER} \cdot SNR_{k,n}$$  \hspace{1cm} (2.8)

By applying the SNR gap into the Shannon’s formula a more realistic value of the achieved date rate is calculated.

$$R_{k,n} = \frac{B}{N} \log_2 \left(1 + SNR_{k,n}^{\text{gap}}\right) \hspace{1cm} \text{(b/s)} \hspace{1cm} (2.9)$$

The spectral efficiency of the $k^{th}$ user at the $n^{th}$ subcarrier is then:

$$S_{k,n} = \log_2 \left(1 + SNR_{k,n}^{\text{gap}}\right) \hspace{1cm} \text{(b/s/Hz)} \hspace{1cm} (2.10)$$

By summing up (2.9) over all sub-carriers allocated to the $k^{th}$ user we derive the $k^{th}$ user’s rate:

$$R_k = \sum_{n \in \Omega_k} R_{k,n} \hspace{1cm} \text{(b/s)} \hspace{1cm} (2.11)$$

By summing up (2.11) for the users in the system we get the throughput of the system:

$$R_{\text{tot}} = \sum_{k=1}^{K} R_k \hspace{1cm} \text{(b/s)} \hspace{1cm} (2.12)$$

However, in this case we assume that all rates are possible which in principle is not true. In real systems a modulation type is chosen according to the value of the received SNR which results in a piece-wise mapping between the received SNR and a modulation type. The spectral efficiency is then $\log_2 (M)$ where $M$ is the chosen modulation type ([14]). In Figure 2-4 we plot the spectral efficiency versus the received SNR for the 3 scenarios to illustrate the approximations that are made for each case.
In this thesis the link adaptation is continuous meaning that we do not use any piece-wise mapping of the achieved SNR and the spectral efficiency and we calculate the rates directly from (2.9) and (2.11) using the SNR gap.

2.3 Performance indexes

2.3.1 User fairness index
The user fairness index is defined as [11]:

$$\phi_k = \frac{\tilde{R}_k}{\gamma_k}$$  \hspace{1cm} (2.13)

Where $\tilde{R}_k = R_k / \sum_{k=1}^{K} R_k$ is the normalized rate of the $k^{th}$ user, $\gamma_k$ is a constant proportion of the $k^{th}$ user’s rate over the system throughput. In other words $\gamma_k$ is the required proportion/ratio of the user rate over all the users rates, where $\gamma \in [0,1]$. Therefore describes *how fair the throughput will be split among the users*. In this sense, having different sets of requirements corresponds to different classes of users. For example the group of users with the highest requirement may belong to the *gold*-class while the group with the lowest requirement may belong to the *bronze*-class.

The index $\tilde{R}_k / \gamma_k$ shows how much close the user is from the requirement. In this way, if $\phi_k$ is 1 then the proportional ratio of the user is met i.e. $\tilde{R}_k = \gamma_k$. In general $\gamma$ may be any real positive number for which $R_1 / \gamma_1 = R_2 / \gamma_2 = \ldots = R_K / \gamma_K$ and may not be normalized as in (2.13) however the interpretation remains the same. In the thesis when referring to $\gamma_k$ we consider always the normalized ones. In any case, the general idea behind this index lies in the need to measure each user’s rate proportion/ratio in order to adjust it according to a policy. Note that this index does not give any insight on the satisfaction of the user rather than show, according to the requirements, how much the proportional ratio of the user is met.

2.3.2 Long term and short term user satisfaction
For a user $k$ the user satisfaction index (USI) is defined as [11]:

![Figure 2-4 Spectral efficiency using (2.10) and link adaptation with 6 modulation schemes](image)
Where $R_{req}^k$ is a constant rate requirement of user $k$. In principle is the minimum rate in bps that is required from the user. The user satisfaction index shows more “if” rather than “how much” the user is satisfied with the allocated rate. In general this index can refer either to the whole session of the user (long-term/session satisfaction) or to a specific time-window of the session, e.g. for time length of one TTI only, (short-term satisfaction). However the interpretation differs in the two cases. In the case of long-term/session satisfaction the index has the value 1 if on average the user got at least the minimum requirement and is 0 otherwise. By average we mean the average rate over the duration of the session. In the case of short term satisfaction the index shows if the user got the minimum rate requirement at the specific time window and therefore is associated more with the experienced satisfaction during the session. The short-term satisfaction shows the percent of time of the session the user was satisfied with the allocated rate.

2.3.3 System fairness index

Next, the system fairness index (SFI) is defined according to [10],[11],[22]:

$$\Phi = \frac{\left( \sum_{k=1}^{K} \varphi_k \right)^2}{K \cdot \sum_{k=1}^{K} \left( \varphi_k \right)^2}$$  (2.15)

Where $\varphi_k$ is the UFI based on (2.13). The interesting property of the index is that it is close to 1 when the user fairness indexes are almost equal. It is close to 0 otherwise. In simple words, this index shows how much the normalized user rates are equal to the requirements $\gamma$.

An outlook of the behavior of the SFI according to (2.15) is illustrated below with a simple example of 2 users in order to show the behavior of the index for different values of UFI’s. The example is simple but all possible combinations of UFI’s are plotted.
The result is that the more equal the UFIs are, the more the system fairness index approaches 1. Moreover the relation in (2.15) implies that the system fairness is bounded:

$$\frac{1}{K} \leq \Phi \leq 1$$  \hspace{1cm} (2.16)

The lower bound of (2.16) is achieved when in (2.15) holds that:

$$\varphi_k = \begin{cases} 1, & \text{for one user} \\ 0, & \text{for the rest} \end{cases}$$

To conclude, one can derive a simple rule of thumb when is trying to adjust the system fairness. To increase it the user fairness indexes have to be more equal while to decrease it, they have to be more unequal. Following this reasoning the system fairness can be adjusted in scheduling and/or power allocation algorithms as discussed later in chapter 4.
3. Classical algorithms description

In this chapter the background of the research work in the category of Rate Adaptive (RA) optimization algorithms is presented which form the basis of the proposed solutions. Initially the objective function of the problem is explained as well as the constraints that limit the behavior of the channel and power allocation variables. Having these defined, the method and philosophy of the solution is explained which in general follows the general guideline of solving RA problems as mentioned in the Introduction. As a second step, the algorithms for channel and power allocations are presented explaining how they satisfy the peculiarities of the individual problems. The reader may also refer to the given literature for further details on the algorithm implementation. In the last part of the chapter the results from simulations are presented in order to compare and draw conclusions of the performance of the different algorithms in several performance indexes.

3.1 The Max Min Rate - MMR

3.1.1 Problem formulation

The objective of the classical Max Min Rate algorithm is to provide a fair share of the overall system throughput across the users by maximizing the minimum of all user rates. The system fairness is expected to be high since equalization among the user rates is performed. The problem is formulated as in (3.1), [2]:

\[
\max_{p, \rho} \min_k \sum_{n=1}^{N} \rho_{k,n} R_{k,n}
\]

subject to
\[
\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad \text{C1}
\]
\[
p_{k,n} \geq 0 \quad \forall k, n \quad \text{C2}
\]
\[
\rho_{k,n} \in \{0,1\} \quad \forall k, n \quad \text{C3}
\]
\[
\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k, n \quad \text{C4}
\]

Where \( \rho \) is the channel allocation matrix and \( \rho_{k,n} \) is 1 if the \( n^{th} \) channel is assigned to the \( k^{th} \) user and 0 otherwise. \( P_{\text{max}} \) is the maximum transmitted power from the BS and \( p_{k,n} \) is the allocated power of the \( n^{th} \) sub-channel allocated to \( k^{th} \) user.

C1 limits the total transmitted power to \( P_{\text{max}} \).
C2 limits the transmitted powers to be non negative.
C3 dictates that there is no sharing of any channel among users. A constraint that destroys the convexity of the domain of the objective function since \( \rho_{k,n} \) is an integer variable (cf. Annex I).
C4 implies that each sub-channel is assigned only to one user at a time.
Although the problem is not convex (c.f. Annex I), the authors in [15] transform the problem into a convex one by relaxing the constraint that each channel is allocated to only one user. By relaxing the constraint C3 in (3.1) the new convex problem is formulated as follows:

$$\max_{p,\rho} \min_k \sum_{n=1}^{N} \rho_{k,n} R_{k,n}$$

subject to

$$\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}}$$

$$p_{k,n} \geq 0 \quad \forall k,n$$

$$\rho_{k,n} \in (0,1] \quad \forall k,n$$

$$\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n$$

(3.2)

Where $\rho_{k,n}$ can now be interpreted as the sharing factor of user $k$ at sub-channel $n$. Note that for cases for which $K \ll N$, $\rho_{k,n}$ approaches 0 or 1 since few users compete for many channels ([2]).

However, the computational complexity of the new convex problem is still high. To overcome this burden, the problem is split into a channel and a power allocation problem. The channel allocation is performed to meet the objective and then an equal power allocation of $P_{\text{max}} / N$ per sub-channel is performed, [2].

### 3.1.2 Algorithm description

The subchannel allocation is performed as follows: each user is assigned the best channel and then for the user with the minimum rate, the best channel is allocated. This procedure continues until all channels have been allocated. The rationale behind this is that the highest priority in sub-channel allocation is given to the user with the minimum rate so far. Note that in this step, the rate calculations assume equal power allocation. In Algorithm 3-1 the algorithm for subchannel allocation is shown [2].
1. Set: \( R_k = 0 \) for all \( k = [1: K] \),
   \( \Omega = \{1, 2, \ldots, N\} \)
2. For \( k = 1: K \)
   a) Find \( n: G_{k,n} > G_{k,j} \) for all \( j \in \Omega \)
   b) Update \( R_k \), exclude \( n \) from \( \Omega \)
3. While \( \Omega \neq \emptyset \)
   a) Find \( k: R_k \leq R_i \) for all \( i, 0 \leq i \leq K \)
   b) For the found \( k \)
      Find \( n: G_{k,n} \geq G_{k,j} \) for all \( j \in \Omega \)
   c) Update \( R_k, \Omega_k \), exclude \( n \) from \( \Omega \) and update

Algorithm 3-1 Channel allocation algorithm of the classical Max Min Rate algorithm in [2]

The algorithm is sub-optimal in the sense that addresses the objective of the problem in a heuristic way. Next, a simple equal power allocation step is added of \( \frac{P_{\text{max}}}{N} \) per subcarrier which is proven that provides almost the same results as by solving problem (3.2) [2].

3.1.3 Another approach

3.1.3.1 Problem formulation

Another approach is to form the objective of the Max Min Fairness (MMR) to maximize the minimum of the ratio \( \frac{R_k}{\gamma_k} \) while having the same constraints as in the case of the classical Max Min Rate described before. It differs from the classical one in the sense that now the algorithm is adapted to satisfy the proportional constraints \( \gamma_k \) which in principle may be different. The problem is formulated as follows:

\[
\begin{align*}
\max_{\rho, \rho_k} \min_k \frac{\sum_{n=1}^{N} (\rho_{k,n} R_{k,n})}{\gamma_k} \\
\text{subject to} \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} &\leq P_{\text{max}} \\
p_{k,n} &\geq 0 \quad \forall k, n \\
\rho_{k,n} &\in \{0, 1\} \quad \forall k, n \\
\sum_{n=1}^{N} \rho_{k,n} &\leq 1 \quad \forall k, n 
\end{align*}
\]

(3.3)

The same set of constraints apply in (3.3) as in the classical MMR (3.1).
In the case that $\gamma_k$ are equal the problem is the same as the classical MMR. Problem (3.3) is also not convex and the problem is again split into a channel allocation problem and a power allocation problem.

3.1.3.2 Algorithm description

The channel allocation policy follows the same rationale as in the case of the classical MMR in (3.1). The only change is that instead of looking at the pure/raw rate of the users, the algorithm looks at the ratio $R_k / \gamma_k$. The algorithm is described as follows:

1. Set: $R_k = 0$ for all $k = [1: K]$,  
   $\Omega = \{1,2,...,N\}$
2. For $k = 1: K$
   a) Find $n: G_{k,n} > G_{k,j}$ for all $j \in \Omega$
   b) Update $R_k$, exclude $n$ from $\Omega$ and update
3. While $\Omega \neq \emptyset$
   a) Find $k: R_k / \gamma_k \leq R_i / \gamma_i$ for all $i, 0 \leq i \leq K$
   b) For the found $k$
      Find $n: G_{k,n} \geq G_{k,j}$ for all $j \in \Omega$
   c) Update $R_k, \Omega_k$, exclude $n$ from $\Omega$ and update

Algorithm 3-2 Channel allocation algorithm of the MMR algorithm

The interesting feature of the algorithm is that the user with the minimum ratio of $R_k / \gamma_k$ has always the priority to select the best channel. This way, the fairness is increased at each channel allocation. On the other hand the channels are not allocated to the users that gain the most throughputs out of it and therefore the increase in overall throughput is limited. This trade-off between fairness and throughput is more explained in section 3.2.

After channel allocation, equal power allocation of $P_{\text{max}} / N$ among the channels is applied as in the classical MMR algorithm.

3.2 The Sum Rate Maximization - SRM

3.2.1 Problem formulation

The objective of the Sum Rate Maximization (SRM) approach is to maximize the throughput of the cell under the same constraints as explained in (3.1). Although the cell throughput is maximized, this policy benefits only the users close to the BS with good channel gain and therefore the resulting system fairness is low.
The set of constraints is the same as in the case of the classical Max Min Rate approach of (3.1). Problem (3.4) is not convex and in [4] the problem is split into a channel allocation problem and a power allocation problem.

3.2.2 Algorithm description
Initially the subchannel allocation is performed by assigning each channel to the user with the highest path gain on that subchannel. This policy excludes in principle users with poor channel gains and it is possible that many of them will not be assigned any channels. Then after subchannel allocation, the power allocation across each of the user’s subchannels is performed by applying the water-filling policy. Water-filling in general maximizes the rates of the users given the constraint on the available power ([4], [15], [16]). The reader can refer to [17] for further details about the SRM approach.

3.3 The Iterative Sum Rate Maximization with Proportionalities – I-SRM-P

3.3.1 Problem formulation
The MMR and SRM algorithms described in sections 3.2 and 3.3 exhibit contradictory philosophies in the sense that they try to address the problems of maximizing the minimum rate of the users and maximizing the total throughput of the cell, however without considering any rate proportionalities of the users. In [18] the authors try to maximize the throughput of the system while satisfying specific rate proportionalities among the users. We refer to this solution as Iterative Sum Rate Maximization with Proportionalities (I-SRM-P). The I-SRM-P problem is formulated as follows:

\[
\begin{align*}
\text{max} & \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} R_{k,n} \\
\text{subject to} & \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad \text{C1} \\
& \quad p_{k,n} \geq 0 \quad \forall k,n \quad \text{C2} \\
& \quad \rho_{k,n} \in \{0,1\} \quad \forall k,n \quad \text{C3} \\
& \quad \sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n \quad \text{C4}
\end{align*}
\]
Where the additional constraint C5 expresses the required rate proportions among the users. In this sense the philosophy of the algorithm is to keep the proportionalities among the users by satisfying the ratio \( \gamma_k = \gamma_1 : \gamma_2 : \cdots : \gamma_K \) given the instantaneous throughput of the system. Therefore it achieves high fairness among all users however without guaranteeing that the minimum rate requirements of the users are always met.

When all \( \gamma_i \) terms are equal, the objective function in (3.5) is similar to the objective function in the classical MMR problem (3.1), since maximizing the sum capacity while making all \( R_k \) terms equal is equivalent to maximizing the worst user’s rate. So, the MMR approach is a special case of (3.5).

The problem is also non convex (cf. Annex 1) and is also split into a channel and a subcarrier allocation problem where the channel allocation tries to meet the objective while satisfying the constraints of the problem.

As a second step, an iterative power allocation is performed so that the normalized rate \( \tilde{R}_k \) approaches the required rate proportionality \( \gamma_k \) and therefore enforcing the fairness among users. The idea of the power allocation is to start with equal power allocation among the subchannels and then iteratively perform reallocations of small fragments of power among the subcarriers.

### 3.3.2 Algorithm description

The channel allocation policy tries to efficiently maximize the throughput of the system while achieving the fairness constraint C5 of (3.5). The criterion is a modified MMR subchannel allocation which aims at maximizing the minimum proportional rate ratio of the users. Initially the number of subchannels per user is calculated by

\[
N_k = \left\lceil \gamma_k \tilde{H}_k N \right\rceil \quad (3.6)
\]
Where $\tilde{H}_k = H_k / \sum_{k=1}^{K} H_k$ is the normalized path gain of user $k$ and $H_k = \sum_{n=1}^{N} H_{k,n} / N$.

The skeptic behind this factor is that each user should be assigned channels that are proportional to the required proportional ratio and the current channel conditions. However, this would be true in the case where all the users exhibit on average the same channel conditions thus the resulting normalized path gain would only depend on the fast fading component. In this case, the factor can capture the different instantaneous channel conditions of the users and be used in (3.6). Since in our simulated environment the position of the users is uniformly distributed in the cell, on average the path gain mainly depends on the distance dependent component. Instead (3.7) is applied to calculate $N_k$.

$$N_k = \lceil \gamma_k N \rceil$$  \hspace{1cm} (3.7)

The summation of $N_k$ maybe larger than the system channels $N$ but after performing the scheduling only $N$ channels are allocated:

```
Algorithm 3-3 Channel allocation algorithm of the Iterative Sum Rate Maximization with Proportionalities, I-SRM-P [18].

1. Set: $S = \emptyset$, $R_k = 0$ for all $k = [1 : K]$, $\Omega = \{1,2,\ldots,N\}$, if $N(S) \neq K$
   a) Find $k,n : G_{k,n} > G_{k,j}$ for all $j \in \Omega$
      If $k \notin S$
   b) Update $R_k, \Omega_k$, exclude $n$ from $\Omega$
      $S = S + \{k\}$

3. While $\Omega \neq \emptyset$
   a) Find $k : R_k / \gamma_k \leq R_i / \gamma_i$ for all $i, 0 \leq i \leq K$
   b) For the found $k$
      Find $n : G_{k,n} \geq G_{k,j}$ for all $j \in \Omega$
      If $N_k > 0$
      c) For the found $n$
         Update $R_k, \Omega_k$, exclude $n$ from $\Omega$
         Else
         $S = S - \{k\}$
   End
End
```

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The principle behind the allocation is for the user with the minimum user fairness index, \( R_k / \gamma_k \), to use the subchannels with high path gain as much as possible. An interesting property for the scheduling is that the number of channels that each user is allowed to obtain is predetermined according to the value of the required rate proportion \( \gamma \). This property allows users with higher rate proportionalities requirements to obtain more channels than the users with lower ones.

For a given channel allocation matrix \( \rho_{k,n} \) the problem in (3.5) is reformulated as

\[
\text{max} \quad \sum_{k=1}^{K} \sum_{n \in \Omega_k} R_{k,n} \\
\text{subject to} \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad \text{C1} \\
p_{k,n} \geq 0 \quad \forall k, n \quad \text{C2} \\
\Omega_1, \Omega_2, \ldots, \Omega_K \text{ are disjoint} \\
\Omega_1 \cup \Omega_2 \cup \ldots \cup \Omega_K \subseteq \Omega \\
R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K \quad \text{C5}
\]

The feasible set of (3.7) is still not convex (cf. Annex I) and optimal solutions of (3.7) make use of the Lagrangian optimization technique in order to generate a set of non-linear equations which are then solved by an iterative algorithm such as the Newton Raphson ([3]). However, in this case the power allocation problem is not solved always as the solutions of the non-linear equations require that the users possess channels with high channel gain. Nonetheless, the problem in [3] is greatly simplified by assuming high SNR conditions and performing a subchannel allocation similar in the one in Algorithm 3-2. In this case, after scheduling it holds that the number of allocated channels are proportional to the proportional fairness indexes ([19],[3]):

\[
N_1 : N_2 : \ldots : N_k \approx \gamma_1 : \gamma_2 : \ldots : \gamma_K
\]

Then, relax the constraint C5 of (3.7) to:

\[
R_1 : R_2 : \ldots : R_K \approx N_1 : N_2 : \ldots : N_K \quad (3.9)
\]

i.e. assume that the ratio of the allocated rates is almost the same as the ratio of allocated channels. Then, it follows that \( N_k \approx \gamma_k \) and the non-linear set of equations become linear which can be easily solved. Then, having calculated the power assigned to each user, the problem reformulates into maximizing the rate of each user given the constraint on the power of the user that has been found previously. Solving this kind of problem results in a waterfilling method ([3], [16]). Further details are found in [17].

However, in [18] the power allocation is performed with an iterative method until the maximum difference between \( \bar{R}_k \) and \( \tilde{\gamma}_k \) is less than a predetermined threshold error.
The idea is to initially start with an equal power allocation among the subchannels of $P_{\text{max}} / N$ and then reallocate a small fragment of power, $dp$, from the user with the maximum proportional rate to the user with the minimum one. In this way fairness among the users is gradually achieved over each iteration. The choice of the power step $dp$ is a trade-off between complexity and accuracy on the fairness. A fixed power step is used. The reason for choosing the condition of the iterative loop to be $\tilde{R}_k - \tilde{\gamma}_k$ is to control/track the normalized rate constraint deviation:

$$D = \frac{\sum_{k=1}^{K} |\tilde{R}_k - \tilde{\gamma}_k|}{\max_k (|\tilde{R}_k - \tilde{\gamma}_k|)}$$

(3.10)

The proposed power allocation algorithm is performed as follows:

1. Initial power allocation
   $$p_{n} = P_{\text{max}} / N \; \forall \; n \in N,$$

2. Rate proportionality tracking
   While $\max_k (|\tilde{R}_k - \tilde{\gamma}_k|) > \epsilon$,
   a) $k_{\text{max}} \leftarrow \arg \max_{k \in K} (R_k / \gamma_k)$
   b) $k_{\text{min}} \leftarrow \arg \min_{k \in K} (R_k / \gamma_k)$
   c) $p_{\text{max}} (n_{\text{min}}) \leftarrow p_{\text{max}} (n_{\text{min}}) - dp$
   d) $p_{\text{max}} (n_{\text{max}}) \leftarrow p_{\text{max}} (n_{\text{max}}) + dp$

c) $n_{\text{min}} \leftarrow \arg \min_{n \in A} (\Delta R_{\text{dec}} / \Delta p)$
   d) $n_{\text{max}} \leftarrow \arg \max_{n \in A} (\Delta R_{\text{inc}} / \Delta p)$

d) Update $R_{\text{max}}, R_{\text{min}}$

Algorithm 3-4 Power allocation for the Iterative Sum Rate Maximization with Proportionalities
algorithm [18]

The proposed scheduling/subcarrier algorithms in [18] have $O(KN)$ complexity for the scheduling part and $O(N \log(N))$ complexity for the power allocation.

3.4 Results

In this section we present the results obtained by simulations in order to compare the MMR, SRM and I-SRM-P algorithms according to the performance indexes. For the
case of MMR the modified MMR as described in 3.1.3 was used which is suitable for different rate proportion requirements. The performance indexes are:

- The system *throughput* which is calculated according to (2.12).
- The system *fairness* which is calculated according to (2.15).
- The long and short term user *satisfaction* which are calculated according to (2.13).
- The average CPU time needed to perform the algorithms.

In most cases the performance is measured with respect to the number of users in the system. However, in order to examine the behavior of a user in comparison with the others, some performance indexes are plotted with respect to the user number/id. These indexes are:

- The *normalized rate* of each user which is calculated as
  \[ \tilde{R}_k = \frac{R_k}{\sum_{k=1}^{K} R_k} \]  
  (3.11)

- The *deviation* of the normalized rates according to (3.10)

### 3.4.1 Simulation parameters

In the following table, the simulation parameters that were used for the simulations in this Chapter and the following are shown:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells</td>
<td>1 hexagonal</td>
</tr>
<tr>
<td>Maximum BS transmission power, $P_{\text{max}}$</td>
<td>1 W</td>
</tr>
<tr>
<td>Cell radius, $R$</td>
<td>500 m</td>
</tr>
<tr>
<td>Mobile Terminal speed</td>
<td>no mobility/static, uniformly distributed in the cell</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Number of sub-carriers, $N$</td>
<td>192</td>
</tr>
<tr>
<td>Sub-carrier bandwidth, $B/N$</td>
<td>15 KHz</td>
</tr>
<tr>
<td>Path loss, $G_{\lambda,n}$</td>
<td>using (2.3)</td>
</tr>
<tr>
<td>Log-normal shadow fading std. dev. $\sigma$</td>
<td>8 dB</td>
</tr>
<tr>
<td>Fast-Rayleigh fading</td>
<td>Typical Urban (TU), [8]</td>
</tr>
<tr>
<td>AWGN power per subcarrier</td>
<td>-123.24 dBm</td>
</tr>
<tr>
<td>BER requirement</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Link adaptation</td>
<td>Continuous using Shannon’s capacity formula with SNR gap (2.10)</td>
</tr>
<tr>
<td>Transmission Time Interval (TTI)</td>
<td>0.5 ms</td>
</tr>
<tr>
<td>Traffic model</td>
<td>Full buffer</td>
</tr>
<tr>
<td>Simulation period</td>
<td>0.5 s</td>
</tr>
<tr>
<td>Number of independent simulation runs</td>
<td>100</td>
</tr>
</tbody>
</table>

*Table 3-1 Simulation parameters*
In order to compare the classical algorithms among them, we also consider that the requirements for the users in terms of proportional ratios, \( \gamma_k \in \{1/7, 2/7, 4/7\} \) and rate requirements, \( R_{\text{req}}^k \in \{128, 256, 512\} \) Kbps follow the probability mass function:

\[
\begin{array}{|c|c|}
\hline
\text{Probability Mass Function} & \gamma_k, R_{\text{req}}^k (\text{Kbps}) \\
\hline
0.5 & \{1/7\}, \{128\} \\
0.3 & \{2/7\}, \{256\} \\
0.2 & \{4/7\}, \{512\} \\
\hline
\end{array}
\]

Table 3-2 Probability Mass Function of the user rate proportionalities and rate requirements

### 3.4.2 System fairness and throughput versus the number of users

In figures 3.5 and 3.6 we plot the achieved system fairness and throughput of the MMR, SRM and I-SRM-P algorithms with respect to the users in the system in order to see which one performs better.

![Graph of system fairness versus load of users for the SRM, MMR and I-SRM-P algorithms](image)

Figure 3-1 System fairness versus load of users for the SRM, MMR and I-SRM-P algorithms
The SRM algorithm maximizes the throughput by allocating each channel only to the user with the best channel condition. As a result users that suffer from poor channel conditions will not be allocated any channels and remain unscheduled. The resulting system fairness is low since no consideration is taken of the rate requirements of the users.

Since in the MMR the priority is always given to the user with the minimum proportional rate the system fairness is high. However, the fairness is decreased as the number of users increase since the channels remain the same. The more the users, the more channels are needed to satisfy the rate proportionalities, $\gamma$, for all of them. Since the number of channels is the same, a decrease of the system fairness is expected. Moreover, this policy provides general low throughput.

In the I-SRM-P algorithm not only the rate proportionalities of the users are considered but also the maximization of the throughput. Since the power allocation is iterative and targeted in satisfying the proportionalities, the system fairness is high, close to 1, in all load scenarios. The throughput is comparable with the MMR case.

It is evident from the figures that the optimization algorithms provide a trade-off between system fairness and throughput.

3.4.3 Long and short term satisfaction
In figures 3.7 and 3.8 we compare the long and short term user satisfaction of the SRM, MMR and I-SRM-P algorithms against the load of users in order to show how they perform. The long term satisfaction shows if on average the users got the minimum rate requirement. Short term satisfaction looks specifically at each TTI and checks also if the users got the minimum rate requirement.

Long term and short term satisfaction generally exhibit the same behavior. The SRM is more sensitive with an increasing load and drops for the case of 16 users to nearly the half compared with the case of 4 users.
The SRM algorithm exhibits generally a low satisfaction over the load of users since only few users with good channel conditions are benefited.

The MMR algorithm considers the users with the minimum fairness index and exhibits high user satisfaction which decreases with an increasing load of users. Same behaviour is noticed for the I-SRM-P. One should keep in mind that the I-SRM-P and MMR algorithms satisfy only the rate proportionalities of the users but not their rate requirements. The rate proportionalities only guarantee a fair split of the instantaneous throughput and high system fairness. For this reason we expect that if the users’ rate requirements increase, the satisfaction will drop radically even though the system fairness will be high. In figure 3.9 and 3.10 the rate requirements of the users have been multiplied by five in order to exhibit this behaviour. The satisfaction for the I-SRM-P and MMR drops drastically in levels even lower than the SRM. This radical decrease in the satisfaction is not observed in the SRM which is able to satisfy few users.
Figure 3-5 Long term user satisfaction with 5 times more rate requirements

Figure 3-6 Short term user satisfaction with 5 times more rate requirements
3.4.4 CPU time

A metric for the complexity of the algorithms is the computational time that they require to perform the channel and power allocation steps. In Table 3-3 the average required CPU time for the SRM, MMR and I-SRM-P algorithms is shown.

<table>
<thead>
<tr>
<th>Load of users</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRM</td>
<td>$3 \times 10^{-3}$ s</td>
<td>$3 \times 10^{-3}$ s</td>
<td>$3 \times 10^{-3}$ s</td>
<td>$3 \times 10^{-3}$ s</td>
<td>$3 \times 10^{-3}$ s</td>
</tr>
<tr>
<td>MMR</td>
<td>$8 \times 10^{-3}$ s</td>
<td>$8 \times 10^{-3}$ s</td>
<td>$8 \times 10^{-3}$ s</td>
<td>$8 \times 10^{-3}$ s</td>
<td>$8 \times 10^{-3}$ s</td>
</tr>
<tr>
<td>I-SRM-P</td>
<td>$19.5 \times 10^{-2}$ s</td>
<td>$17.9 \times 10^{-2}$ s</td>
<td>$20.5 \times 10^{-2}$ s</td>
<td>$21.2 \times 10^{-2}$ s</td>
<td>$24.3 \times 10^{-2}$ s</td>
</tr>
</tbody>
</table>

Table 3-3 - Average CPU time versus the load of users for the SRM, MMR and I-SRM-P algorithms

The SRM and MMR are independent from the load of users. This is true since the channel allocation steps depend on the number of channels and the power allocation steps are not iterative. The iterations in the power allocation in the I-SRM-P algorithm require one order of magnitude higher computational time to be performed with respect to the SRM and MMR. Moreover, in the I-SRM-P when the number of users increases the computational time also increases since the iterations which are needed to satisfy the user fairness’s increase also.

3.4.5 Normalized rates and deviation

In Figure 3.11 we plot the normalized rates for the SRM, MMR and I-SRM-P algorithms for a scenario of 16 users. The required rate proportion of the $k^{th}$ user, $\gamma_k$, is denoted by the variable $\Gamma$. The purpose of the comparison is to look how each algorithm performs at each user.
The SRM assigns resources only to few users which are the ones that have the best channel conditions. Most of the users remain without any channel assignment. The I-SRM-P performs better than the MMR as it seems to be more close to the required rate proportionalities.

A performance measure that shows how much the required rate proportionalities are satisfied is the rate deviation in (3.10). The less the deviation, the more users satisfy the proportionalities.

In table 3-4 we show the resulting deviation of the MMR and I-SRM-P algorithms and see that the I-SRM-P is more efficient in satisfying the proportionalities.

<table>
<thead>
<tr>
<th></th>
<th>4 users</th>
<th>7 users</th>
<th>10 users</th>
<th>13 users</th>
<th>16 users</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMR</td>
<td>2</td>
<td>3,12</td>
<td>4,12</td>
<td>7,09</td>
<td>8,08</td>
</tr>
<tr>
<td>I-SRM-P</td>
<td>2</td>
<td>2,52</td>
<td>2,92</td>
<td>4,5</td>
<td>5,16</td>
</tr>
</tbody>
</table>

Table 3-4 – Deviation of the normalized rates of the MMR and I-SRM-P algorithms with respect to the load of users

The I-SRM-P provides the lowest deviation for all the loads of users. Depending on how small is set the error at the power allocation of the I-SRM-P algorithm (Algorithm 3-4) one can achieve more or less deviation. In the considered scenarios the error is set to:

$$\max_{\hat{x}} \left( \left| \tilde{R}_x - \tilde{\gamma}_x \right| \right) \leq 5 \times 10^{-3}$$

And proves to be sufficient to provide system fairness almost one and less deviation than the MMR case.
4. Proposed algorithms description

In this chapter the proposed algorithms in the category of Rate Adaptive (RA) optimization algorithms is presented. The basis of the development of the new algorithms is the Max Min Rate (MMR) and Iterative Sum Rate Maximization based on proportionalties (I-SRM-P) algorithms already presented in chapter 3. The main focus in this chapter is to use the aforementioned classical approaches as a first step and then concentrate on finding ways of increasing and decreasing the system fairness to meet a target value while meeting the objective of the specified optimization problem.

Initially a general framework for adjusting the system fairness index is presented in order to show the three possible ways/variants of how the adjustment can be made, the subchannel reallocation only (SA), the power reallocation only (PA) and the combination of the SA and PA (JOINT). Depending on the objective function (MMR or I-SRM-P) the new optimization problems are formulated by adding one extra constraint of meeting the system fairness target (Fairness based MMR, FMMR and Fairness based I-SRM-P, FSRM-P). For each of the FMMR, FSRM-P and their variants (SA, PA, JOINT) the proposed heuristic algorithms are presented explaining how they satisfy the peculiarities of each of the objectives. In the last part of the chapter the results from simulations are presented in order to compare and comment the performance of the new algorithms.

4.1 Framework for adjusting the system fairness

A key characteristic of the SRM and MMR as well as I-SRM-P algorithms is that the allocation of channels and power result in either low fairness of the system however with high throughput (case of SRM) or high fairness of the system but with low throughput (cases of MMR and I-SRM-P). In this thesis is addressed the problem of adjusting the system fairness between the aforementioned extremes for the cases of MMR and I-SRM-P\(^1\). Intuitively by increasing or decreasing the fairness, the system throughput is expected to have the opposite effect due to the trade-off between fairness and throughput mentioned in chapter 3. An adjustment of the fairness of the system enables the operator to be flexible with the operational point in the fairness-throughput domain. The adjustment is performed by adding a new constraint in the classic problem formulation of the MMR and I-SRM-P as will be stated later in the chapter. In all cases the walkthrough followed is by applying heuristic algorithms in the channel and power allocation steps that can guarantee that the fairness target is met. The adjustment is performed by using channel reallocations and/or possible power reallocations.

Classical channel allocation
In all cases initially a channel allocation is performed by assuming equal power allocation as in the case of the MMR and I-SRM-P algorithms (as described in chapter 3), thus achieving a system fairness which is in principle high. However this value

\(^1\) The reader may refer to reference [17] for the adjustment in the case of SRM
may be less or greater than the system fairness target \( \Phi' \). \( \Phi \) is the System Fairness Index (SFI) and is calculated according to (2.16) as:

\[
\Phi = \frac{\left( \sum_{k=1}^{K} \varphi_k \right)^2}{K \cdot \sum_{k=1}^{K} (\varphi_k)^2}
\]

(4.1)

Where

\[
\varphi_k = \frac{\bar{R}_k}{\gamma_k}
\]

(4.2)

is the User Fairness Index (UFI) and \( \gamma_k \) are the set of normalized predetermined values that assure proportional fairness among users.

Subchannel reallocation only – SA only

After the classical channel allocation a second channel reallocation step is performed with a policy such that the system fairness is gradually increased or decreased depending on the fairness achieved in the classical channel allocation and the target until \( \Phi_{\text{SA}} \approx \Phi' \). The resulting fairness due to the channel reallocations may be above or below the target because is selected the allocation which is closer to the target. In figure 4-1 starting from \( \Phi_{\text{classical}} \) and the fairness target \( \Phi' \) we may increase or decrease the fairness. The power allocation remains equal.

Power reallocation only – PA only

In this case after the classical channel allocation, instead of performing reallocations of channels, a power reallocation step is performed different than equal power allocation to increase or decrease the fairness as shown in figure 4-2. By reallocating amounts of power it is possible to meet the fairness target with accuracy. After this step it holds that \( \Phi_{\text{PA}} = \Phi' \).
Joint channel and power reallocations – Joint

This case is a mixture of the SA only and PA only cases. The fairness may be increased or decreased with the SA only policy. After the SA only policy it holds that $\Phi^\text{SA} \approx \Phi'$ and as a next step the PA only algorithm is performed so that $\Phi^\text{Joint} = \Phi'$. This Joint case we is shown in figure 4-3.

Remarks:
Another way is to perform the channel reallocations until the system fairness is below the target. Then the power allocation should be performed to increase the system fairness up to the target. However in general this cannot be accomplished. For example consider the case with 4 users in the system initially with high fairness and after channel reallocations in order to reduce it, only 2 of them are assigned channels. In this case according to (2.15) the system fairness even by equalizing the rates of the remaining 2 users can be at most:
In general, if the channel reallocations result in a fairness level lower than the target, the power reallocations may not reach the target.

In table 4-1 the general framework is presented with the three aforementioned variants:

<table>
<thead>
<tr>
<th>Subchannel reAllocation only (SA)</th>
<th>1st step Channel allocation</th>
<th>2nd step Channel reallocation</th>
<th>3rd step Power reallocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[\Phi^\text{sched} \Phi'] or [\Phi^\text{sched} \Phi']</td>
<td>[\Phi^\text{SA} \approx \Phi']</td>
<td>-</td>
</tr>
<tr>
<td>Power reAllocation only (PA)</td>
<td>[\Phi^\text{sched} \Phi'] or [\Phi^\text{sched} \Phi']</td>
<td>-</td>
<td>[\Phi^\text{PA} = \Phi']</td>
</tr>
<tr>
<td>Subchannel and Power reAllocation – (Joint)</td>
<td>[\Phi^\text{sched} \Phi'] or [\Phi^\text{sched} \Phi']</td>
<td>[\Phi^\text{SA} \approx \Phi']</td>
<td>[\Phi^{\text{joint}} = \Phi']</td>
</tr>
</tbody>
</table>

Table 4-1 General framework for adjusting the system fairness index

**Fairness adaptive implementation**

The implemented algorithms are able to detect if the fairness target is below or above the one resulting from the initial 1st channel allocation step and apply the corresponding policies in order to increase or decrease the fairness and meet the target. It is to be noticed that channel reallocation is more effective in adjusting the fairness since the users exchange channels while power reallocations are less effective since the users exchange a quantity of power among their allocated channels.

In the following a description of each algorithm is given.

### 4.2 Proposed algorithms

#### 4.2.1 Fairness based Max Min Rate Adaptive - FMMR

The objective of the *Fairness Based Max Min Rate* adaptive (FMMR) algorithm follows the rationale of the classical MMR approach with the modification that the system fairness is adjusted with the incorporation of the proportional fairness constraints into the objective function. The problem is formulated as follows:
\[
\max_{p, \rho} \min_k \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} \left( \frac{R_{k,n}}{\gamma_k} \right)
\]

subject to \( \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \) C1
\( p_{k,n} \geq 0 \quad \forall k,n \) C2
\( \rho_{k,n} \in \{0,1\} \quad \forall k,n \) C3
\( \sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n \) C4
\( \Phi = \Phi' \) C5

(4.3)

Where in (4.1) \( \rho \) is the channel allocation matrix and \( \rho_{k,n} \) is 1 if the \( n^{th} \) channel is assigned to \( k^{th} \) user and 0 otherwise. \( P_{\text{max}} \) is the maximum transmitted power from the BS and \( P_{k,n} \) is the power at the \( n^{th} \) sub-channel allocated to \( k^{th} \) user.

For the cases that \( \gamma_k \) are equal, the objective function in (4.3) is similar with the objective function in the classical MMR case:

\[
\max_{p, \rho} \min_k \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} R_{k,n}
\]

subject to \( \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \) C1
\( p_{k,n} \geq 0 \quad \forall k,n \) C2
\( \rho_{k,n} \in \{0,1\} \quad \forall k,n \) C3
\( \sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n \) C4
\( \Phi = \Phi' \) C5

(4.4)

However, since (4.3) incorporates also (4.4) we will use (4.3) as a reference in the following. In (4.3):

C1 limits the total transmitted power to \( P_{\text{max}} \).
C2 limits the transmitted powers to be non negative.
C3 dictates that there is no sharing of any channel among users. A constraint that destroys the convexity of the domain of the objective function since \( \rho_{k,n} \) is an integer variable (c.f. Appendix I).
C4 implies that each sub-channel is assigned only to one user at a time.
C5 is the target SFI.

Problem (4.3) is not convex as in the case of MMR and splitting the problem into a channel allocation and a power allocation problem is a tractable way to solve it sub optimally.
4.2.1.1 The Fairness Max Min Rate Adaptive with Subchannel ReAllocation - FMMR-SA

In order to implement the strategies to adjust the fairness one has to keep in mind that fairness is reduced by making the UFIs $\varphi_k$ in (4.2) more unequal and satisfy at the same time the objective of the function before deciding for which channel reallocations to perform. In order to increase it, the UFIs have to become more equal. In problem (4.3) the objective is to maximize the minimum of the rates. Starting from the classical MMR channel allocation policy that achieves high system fairness, the reallocation step follows the principle shown in algorithm 4-1. Initially the classical channel allocation step achieves a system fairness of $\Phi^{\text{classical}}$. Then, a channel reallocation step is applied in order to meet roughly the system fairness target depending on whether the system fairness has to be increased or decreased. The reallocations are performed until the resulting system fairness meets the target with the highest accuracy as possible. The resulting system fairness may be above or below the target.

In order to adjust the system fairness by channel reallocations the algorithm in Algorithm 4-1 is implemented:

<table>
<thead>
<tr>
<th>1. Channel allocation</th>
<th>Classical MMR channel allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $\Phi^{\text{classical}} &gt; \Phi'$</td>
<td>If $\Phi^{\text{classical}} &lt; \Phi'$</td>
</tr>
<tr>
<td>2. Channel reallocation</td>
<td>While $\Phi &gt; \Phi'$</td>
</tr>
<tr>
<td>a)</td>
<td>Find $k_{\text{max},2} : 2^{\text{nd}} \max \left( \frac{R_k}{\gamma_k} \right)$</td>
</tr>
<tr>
<td>b)</td>
<td>Find $n : G_{\text{max},2,n} \leq G_{\text{max},2,j}$ for all $j \in \Omega_{\text{max},2}$ exclude $n$ from $\Omega_{\text{max},2}$, update $R_{\text{max},2}$,</td>
</tr>
<tr>
<td>c)</td>
<td>Find $k_{\text{max},1} : \max \left( \frac{R_k}{\gamma_k} \right), \Omega_{\text{max},1} = \Omega_{\text{max},1} + {n}$, update $R_{\text{max},1}$</td>
</tr>
<tr>
<td>d)</td>
<td>Update $\Phi$</td>
</tr>
<tr>
<td>e)</td>
<td>If $\Phi &lt; \Phi'$, Pick allocation closest to $\Phi'$</td>
</tr>
<tr>
<td>end</td>
<td>end</td>
</tr>
</tbody>
</table>

Algorithm 4-1 Subchannel reAllocation policy to adjust the System Fairness in the FMMR

Note that in step 1 in the channel reallocation policy if $\gamma_k$ are equal, the behavior of the algorithm is the same is the case of the classical MMR.

In the case that we need to decrease the system fairness and make the user fairness indexes (UFIs) more unequal, one way is to assign to the user with maximum proportional rate more channels to increase his rate even more. In this way we make
the UFIs more unequal and the system fairness is decreased. A question rises about which user we subtract channels from and also from which channel of his subset. We pick the user with the 2nd maximum proportional rate because we are sure that in this way we do not reduce the rate of the minimum user thus not working against the objective of the problem. We pick the worst channel of him because we minimize the impact on the loss of overall rate thus keeping again the minimum proportional rate to a maximum as much as possible. Moreover, the UFIs become unequal more quickly in comparison with the case that we were always choosing for example the 3rd or the 4th user. After several channel reallocations it is possible that the 2nd best rate user will change. Step e) is performed in order to select the channel allocation closest to the target.

Remarks:
The system fairness is reduced by increasing the rate of the best user only. Moreover, channels are subtracted from the 2nd best which may change over the reallocations. Overall, everyone is loosing rate in favor of the best which increases his UFI at each reallocation thus becoming the only one that benefits from the reallocations.

In the case that we need to increase the system fairness while maximizing the minimum proportional rate it is straightforward to select the worst channel of the best users and assign it to the worst user. In this way the UFIs become more equal and the fairness is increased.

4.2.1.2 The Fairness Max Min Rate with Power Allocation only - FMMR-PA

Another way to adjust the fairness to the target value is to perform only allocations of power in a way that the UFIs are more unequal or equal depending whether we need to increase or decrease the fairness. Algorithm 4-2 depicts the framework for meeting the system fairness target exactly. The algorithm is iterative until the system fairness target is met up to a value defined by the error $\epsilon$. Note that the objective is to maximize the minimum proportional rate. To make the UFIs more unequal and reduce the fairness while keeping the minimum rate user intact, we perform power reallocations from the 2nd best to the best user (the 2nd best may change after some reallocations). In this way, as in the FMMR-SA policy, we enforce only the rate of the best user thus decreasing the fairness. We do not consider the worst user because in this way we reduce more the minimum rate thus we would work against the objective of the function.
Start with equal power allocation of \( P_{\text{max}} / N \) per subcarrier

<table>
<thead>
<tr>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( \Phi^{\text{classical}} &gt; \Phi' )</td>
<td>Find ( k_{\text{max},2} : 2^{\text{nd}} \max \left( \frac{R_k}{\gamma_k} \right) )</td>
</tr>
<tr>
<td></td>
<td>Find ( k_{\text{max},1} : \max \left( \frac{R_k}{\gamma_k} \right) ),</td>
</tr>
<tr>
<td></td>
<td>( \Delta R_{\text{dec}} \leftarrow R_{\text{max},2} \left( p_{\text{max},2} \right) - R_{\text{max},2} \left( p_{\text{max},2} - \Delta p \right) )</td>
</tr>
<tr>
<td></td>
<td>( n_{\text{min}} \leftarrow \arg \min_{n \in A} \left( \Delta R_{\text{dec}} \right) )</td>
</tr>
<tr>
<td></td>
<td>( \Delta R_{\text{inc}} \leftarrow R_{\text{max},1} \left( p_{\text{max},1} \right) + \Delta p - R_{\text{max},1} \left( p_{\text{max},1} \right) )</td>
</tr>
<tr>
<td></td>
<td>( n_{\text{max}} \leftarrow \arg \max \left( \Delta R_{\text{inc}} \right) )</td>
</tr>
<tr>
<td>If ( \Phi^{\text{classical}} &lt; \Phi' )</td>
<td>Find ( k_{\text{min}} : \min \left( \frac{R_k}{\gamma_k} \right) )</td>
</tr>
<tr>
<td></td>
<td>Find ( k_{\text{max}} : \max \left( \frac{R_k}{\gamma_k} \right) ),</td>
</tr>
<tr>
<td></td>
<td>( \Delta R_{\text{dec}} \leftarrow R_{\text{min}} \left( p_{\text{max}} \right) - R_{\text{min}} \left( p_{\text{max}} - \Delta p \right) )</td>
</tr>
<tr>
<td></td>
<td>( n_{\text{min}} \leftarrow \arg \min_{n \in A} \left( \Delta R_{\text{dec}} \right) )</td>
</tr>
<tr>
<td></td>
<td>( \Delta R_{\text{inc}} \leftarrow R_{\text{min}} \left( p_{\text{min}} + \Delta p \right) - R_{\text{min}} \left( p_{\text{min}} \right) )</td>
</tr>
<tr>
<td></td>
<td>( n_{\text{max}} \leftarrow \arg \max \left( \Delta R_{\text{inc}} \right) )</td>
</tr>
</tbody>
</table>

**Algorithm 4-2 Power reallocations policy to increase or decrease the system fairness in the FMMR**

In Algorithm 4-2 we perform reallocations between the channel with the minimum decrease in rate and the channel with the maximum increase in order to achieve the greater throughput as possible.

### 4.2.1.3 The Fairness Max Min Rate with Joint Subchannel and Power Allocation - FMMR-Joint

In this case the channel reallocation step is performed as described in the FMMR-SA policy such that \( \Phi > \Phi' \) depending on whether the system fairness has to be increased or decreased. In order to reduce the system fairness up to the target value with high accuracy, a second power reallocation step as in the case of the FMMR-PA is applied which decreases the system fairness by making the UFIs more unequal. This case is a combination of algorithms 4-1 and 4-2.

### 4.2.2 The Fairness based Sum Rate Maximization with Proportionalities – FSRM-P

The objective of the Fairness Based Sum Rate Maximization (FSRM-P) follows the rationale of the problems that try to maximize the throughput of the system (e.g. the Iterative Sum Rate Maximization with Proportionalities, I-SRM-P in chapter 3) with
the additional constraint that the System Fairness has to be adjusted to a target value $\Phi'$:

$$\max_{\rho,\varphi} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} R_{k,n}$$

subject to

$$\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad C1$$

$$p_{k,n} \geq 0 \quad \forall k,n \quad C2$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k,n \quad C3$$

$$\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n \quad C4$$

$$\Phi = \Phi' \quad C5$$

The constraints C1-C5 are the same as in the FMMR case in (4.3). The philosophy to adjust the system fairness is the same as in the FMMR problem discussed previously with the change that the channel reallocations have to guarantee the maximization of the throughput at the same time. The rate proportionalities are still being considered but cannot be met due to the new constraint C5.

4.2.2.1 The Fairness based Sum Rate Maximization with Proportionalities with Subchannel reAllocation – FSRM-P-SA

In this case we are interested in decreasing the system fairness and maximizing the throughput at the same time only with channel reallocations. One way is to find the user with the worst ratio $R_k / \gamma_k$ and subtract his worst channel in terms of path gain. This channel is then assigned to a user with best path gain on that channel who has also not given any channel in previous reallocations. This is done to avoid any ping pong effects among the users. Following this policy we make the UFIs more unequal and decrease the system fairness. We are also sure that the resulting throughput is the maximum possible thus meeting the objective function.

To increase the fairness while maximizing the throughput we select from the user with the best $R_k / \gamma_k$ his worst channel in terms of path gain and assign it to the user with the best path gain on that channel.

In Algorithm 4-3 the FSRM-P-SA algorithm is shown:
1. Channel allocation

<table>
<thead>
<tr>
<th>I-SRM-P channel allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $\Phi^{\text{classical}} &gt; \Phi'$</td>
</tr>
<tr>
<td>If $\Phi^{\text{classical}} &lt; \Phi'$</td>
</tr>
</tbody>
</table>

2. Channel reallocation

<table>
<thead>
<tr>
<th>While $\Phi &gt; \Phi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>While $\Phi &lt; \Phi'$</td>
</tr>
</tbody>
</table>

a) Find $k_{\text{min}} : \min \left( R_k / \gamma_k \right)$

b) Find $n : G_{k_{\text{max}}} \leq G_{k_{\text{max}}} - j \forall j \in \Omega_{k_{\text{min}}}$

<table>
<thead>
<tr>
<th>exclude $n$ from $\Omega_{k_{\text{max}}}$, update $R_{k_{\text{max}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find $n_{\text{max}} : G_{k_{\text{max}}} \leq G_{k_{\text{max}}} - j \forall j \in \Omega_{k_{\text{max}}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>exclude $n_{\text{max}}$ from $\Omega_{k_{\text{max}}}$, update $R_{k_{\text{max}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find $G_n \geq G_j \forall j \notin \Omega_{k_{\text{max}}}$</td>
</tr>
</tbody>
</table>

| $\Omega_{k_{\text{max}}} = \Omega_{k_{\text{max}}} + \{n\}$, update $R_{k_{\text{max}}}$ |
| $\Omega_{k_{\text{max}}} = \Omega_{k_{\text{max}}} + \{n\}$, update $R_{k_{\text{max}}}$ |

c) $\Omega_{k_{\text{max}}} = \Omega_{k_{\text{max}}} + \{n\}$, update $R_{k_{\text{max}}}$

d) Update $\Phi$

<table>
<thead>
<tr>
<th>Update $\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>End</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>e) If $\Phi &lt; \Phi'$,</td>
</tr>
</tbody>
</table>

| Pick allocation closest to $\Phi'$ |
| Pick allocation closest to $\Phi'$ |

Algorithm 4-3 Channel reallocation to increase or decrease the system fairness in the FSRM-P

The algorithm ends up in a system Fairness which is the closest to the target.

4.2.2.2 The Fairness Sum Rate Maximization with Proportionalities and Power Allocation – FSRM-P-PA

Another approach to reduce the system fairness is by performing only power reallocations. As in the case of the FMMR-PA we aim at adjusting the system fairness by performing only power reallocations. The objective is again to maximize the throughput of the system while adjusting the system fairness.

In Algorithm 4-4 the power allocation algorithm to decrease the SFI up to the target is shown but also the policy to increase the fairness:
1. Initially Equal Power allocation

If $\Phi^{sched} > \Phi'$

2. While $|\Phi - \Phi'| > \varepsilon$

   $k_{max} \leftarrow \arg \max_k \left( R_k / \gamma_k \right)$
   $K' = K - \{k_{max}\}$

   a) $\Delta R_{dec} \leftarrow R(p_{k'}(n)) - R_k(n_{\gamma_k}) - \Delta p$
      $n_{min} \leftarrow \arg \min_k (\Delta R_{dec})$
      $\Delta R_{inc} \leftarrow R(p_{k_{max}}(n) + \Delta p) - R_k(n)_{\gamma_k}$
      $n_{max} \leftarrow \arg \max_k (\Delta R_{inc})$
      $p_{K'}(n_{min}) \leftarrow p_{K'}(n_{max}) - \Delta p$
      $p_{K_{max}}(n_{max}) \leftarrow p_{K_{max}}(n_{\gamma_k}) + \Delta p$

   b) Update $R_{K'}, R_{K_{max}}, \Phi$
   c) Update $R_{K'}, R_{K_{max}}, \Phi$

End

If $\Phi^{sched} < \Phi'$

2. While $|\Phi - \Phi'| > \varepsilon$

   $k_{max} \leftarrow \arg \max_k \left( R_k / \gamma_k \right)$
   $K' = K - \{k_{max}\}$

   a) $\Delta R_{dec} \leftarrow R(p_{k_{max}}(n)) - R_k(n_{\gamma_k}) - \Delta p$
      $n_{min} \leftarrow \arg \min_k (\Delta R_{dec})$
      $\Delta R_{inc} \leftarrow R_k(n_{\gamma_k}) - R_{K'}(n_k) - \Delta p$
      $n_{max} \leftarrow \arg \max_k (\Delta R_{inc})$
      $p_{k_{max}}(n_{min}) \leftarrow p_{k_{max}}(n_{max}) - \Delta p$
      $p_{K'}(n_{max}) \leftarrow p_{K'}(n_{\gamma_k}) + \Delta p$

End

Algorithm 4-4 Power reallocation to increase or decrease the system fairness in the FSMR-P

To decrease the system fairness we try to make the proportional rates more unequal but also at the same time maximize the throughput. To this end, we subtract power from the channel with the worst decrease in rate and add it to the channel of the best user with the maximum increase in rate. In this way the best user increases his rate at all times and the UFIs become more unequal thus decreasing the system fairness.

To increase the fairness, we make the proportional rates more equal by subtracting power from the user with the best $R_k / \gamma_k$, from one of his channels that has the lowest decrease in rate. This power is then assigned to one of the remaining channels in the system that have the highest increase in rate.

4.2.2.3 The Fairness Sum Rate Maximization with Proportionalities and Joint Subchannel and Power Allocation – FSRM-P-JOINT

In this case initially a channel allocation step as described in the FSRM-P-SA policy is performed. Then, the channel reallocation is performed again depending on whether the system fairness has to be increased or decreased. The channel reallocation is performed as in Algorithm 4-3 but without step c) in order to obtain fairness above the target for the same reason as mentioned in the FMMR-Joint case. After this step it holds that $\Phi > \Phi'$. Then in the second step, power reallocation is performed as in the FSRM-P-PA by applying Algorithm 4-4 so that $\Phi = \Phi'$. 

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4.3 Results

In this section we present the results of the FMMR and FSRM-P algorithms and the variants that were implemented. As discussed in Chapter 3, the classical MMR and I-SRM-P algorithms provide high system fairness and are used in this section as a reference for comparison with the proposed algorithms.

Initially for each algorithm that adjusts the system fairness (FMMR, FSRM-P) we compare the variants (i.e. adjustment with subchannel reallocation only (SA), with power reallocation only (PA) and the joint (JOINT)) by setting the system fairness target $\Phi_f$ to 0.6. The purpose of this comparison is to choose the most efficient one in terms of meeting the fairness target, achieving higher throughput and requiring the least computational time. After having selected the variant of each policy (FMMR and FSRM-P), we compare the policies between them and comment on their behavior.

4.3.1 Comparison of SA, PA and Joint

4.3.1.1 System fairness and throughput for the SA, PA and Joint

In Figures 4-8 and 4-9 we plot the achieved system fairness of the FMMR and FSRM-P algorithms with respect to the number of users for the different policies:

- with subchannel reallocation only (SA)
- power reallocation only (PA)
- joint (JOINT)

Also the achieved system fairness of the corresponding classical algorithm is shown.

![Graph showing system fairness for FMMR SA, PA, JOINT and classical MMR cases](image-url)
All the policies starting from a high value of system fairness perform adjustment of the system fairness to the target of 0.6. With satisfactory accuracy for all user loads the target is met.

Assume the scenario where a user is initially allocated few channels and due a channel reallocation this user loses one of them. It is possible that the loss of the channel will cause significant reduction in the user rate and as a consequence of this reduction the resulting system fairness may also change significantly. This behavior makes it difficult to meet exactly a fairness target while performing channel reallocations. This is the reason why in figures 4-4 and 4-5 the SA policy slightly diverges from the target. On the other hand during the power allocation step only small amounts/steps of power are reallocated which cause a small change in the rate of the user and therefore in the resulting fairness. For this reason the cases of PA and the JOINT which include a power reallocation step meet the target with satisfactory accuracy.

In figures 4-6 and 4-7 we plot the achieved throughput versus the load of users of the system in order to compare the variants among them and see which one has better performance.
For all user loads in the FMMR the JOINT achieves higher throughput while the PA the worst. In the FSRM-P the JOINT performs almost the same as the SA for all user loads.

In general the trade-off between system fairness and throughput is noticed by comparing the throughput of the classical algorithms which have higher system fairness close to 1. The loss in system fairness is translated in gain in throughput.

Moreover, it is evident that in both the FMMR and FSRM-P cases the algorithms with power allocation only, (PA), exhibit the worst performance in terms of throughput.
Initially in the FMMR-PA case the 1st channel allocation step was performed according to the MMR channel allocation policy which allocates channels always to the user with the minimum proportional rate \( R_k / \gamma_k \) assuming equal power allocation among the channels. These users usually suffer from bad channels conditions with respect to the rest and are allocated many channels after this step. As a second step the channel allocation remains the same and the fairness is reduced only by performing reallocations of power from the 2nd best user in terms \( R_k / \gamma_k \) to the best. However, a continuous small increase in power in the channels of the best user does not increase the rate so much since the Shannon’s formula is not linear with respect to the power. In contrast with the 2nd best user who loses significant rate due to the loss of power in one of his channels. Therefore, the reallocation of power is inefficient with respect to the SA or JOINT case. Reallocation of the channels among the users to reduce the system fairness is more efficient in achieving higher throughput than in the PA case.

**4.3.1.2 Long and short term user satisfaction**

Long and short term satisfactions provide an insight on how the algorithms perform in terms of the user satisfaction as described in chapter 2. In figures 4-8 and 4-9 we plot
the long and short term satisfaction for the SA, PA and JOINT cases for the FMMR and I-SRM-P algorithms, respectively. The classical I-SRM-P and MMR are also plotted as a reference.

Figure 4-9 Long and short term user satisfaction for the FSRM-P case

The power allocation only policy exhibits the worst user satisfaction in all cases. The SA and JOINT have a steady user satisfaction and almost equivalent for the considered rate requirements both in the long term and short term metrics.

The classical MMR in figure 4-8 achieves higher long and short satisfaction even if the system fairness is higher in comparison with the FMMR variants (set to the target of 0.6). For the MMR case a drop in satisfaction happens since it can guarantee a strict satisfaction of the rate proportionalities with system fairness around 0.9 but cannot guarantee the rate requirements of the users as described in chapter 3. In the FMMR case by adjusting the fairness to a lower value the objective is to maximize the minimum rate proportionality at all times by allocating channels only to the best user thus some loss in satisfaction is observed.

The classical I-SRM-P in figure 4-9 achieves fairness close to 1 but the resulting long term satisfaction drops rapidly with an increasing load of users. The crossings due to this reduce in satisfaction with the other cases happens for fewer users if the rate requirements of the users are increased. The drop is also observed in the short term satisfaction if the rate requirements are increased also. This drop in satisfaction happens since I-SRM-P can only guarantee a very strict guarantee of the rate proportionalities with system fairness close to 1 but cannot guarantee the rate requirements of the users as described in chapter 3. Therefore when the number of users or the rate requirements increase one should consider reducing the system fairness in order to keep the long term satisfaction high. In the case of short term user satisfaction the FSRM-P SA and JOINT always perform better than the I-SRM-P without any crossings. In conclusion when using the I-SRM-P there is a trade off between system fairness and user satisfaction.
4.3.1.3. CPU time
Since the algorithms are iterative, the required computational time provides an insight on the complexity and the required iterations that are needed to achieve the fairness target. In table 4-2 we show the average CPU time required for the FMMR and FSRM-P and their variants in order to meet the fairness target of 0.6 for the case of 4 and 16 users.

<table>
<thead>
<tr>
<th></th>
<th>FMMR</th>
<th></th>
<th>FSRM-P</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SA</td>
<td>JOINT</td>
<td>PA</td>
<td>SA</td>
</tr>
<tr>
<td>4 users</td>
<td>0.047s</td>
<td>0.095s</td>
<td>0.376s</td>
<td>0.022s</td>
</tr>
<tr>
<td>16 users</td>
<td>0.028s</td>
<td>0.121s</td>
<td>0.282s</td>
<td>0.032s</td>
</tr>
</tbody>
</table>

Table 4-2 Average required CPU time for the FMMR and FSRM-P policies

Though the SA and JOINT cases exhibit almost the same complexity, the PA case requires approximately one order of magnitude more CPU time to achieve the target since it tries to meet the target only with reallocations of power. Channel reallocations are proven to be more effective in adjusting the system fairness.

4.3.1.4 Conclusions
To simplify the comparison between the different algorithms (FMMR, I-SRM-P) we select from the previous discussion one variant for each algorithm. The JOINT case in the FMMR and FSRM-P cases is selected which meets with accuracy the fairness target, has a high throughput, requires a comparable computational time as the SA case and provides high satisfaction. Moreover, comparing with the classical I-SRM-P algorithm it is shown that the user satisfaction is kept higher when the number of users is increased. From now on when referring to the FMMR or the FSRM-P we will mean the JOINT versions.

4.3.2 Comparison of the FMMR-JOINT and FSRM-P-JOINT
4.3.2.1 Throughput versus system fairness and normalized rates
From the previous discussion we select the JOINT case which meets with accuracy the fairness target, has a high throughput and requires a satisfactory computational time. We consider a broad range of system fairness’ and plot the system throughput with respect to the classical algorithms (SRM, I-SRM-P and MMR) which are used as the reference for the comparison. The number of users is always 16.
Figure 4-10 Throughputs versus system fairness for the FMMR and FSRM-P algorithms

One should keep in mind that for the system fairness holds that:

$$\frac{1}{K} \leq \Phi \leq 1$$

and therefore none of the algorithms can provide a system fairness index lower than \(\frac{1}{K}\) where \(K\) are the number of users.

In figure 4-10 the throughput versus the system fairness is plotted. For low system fairness’s there is a clear distinction on the performance in terms of throughput of the algorithms. It is noticeable that it is inefficient to use any algorithm for fairness values below the SRM. The SRM may achieve a very low fairness value of around 0.1 but with the highest throughput of the system.

For high system fairness’s the algorithms seem to coincide in the results even though different policies are applied for each one of them.

On the region of low fairness the SRM algorithm achieves the highest throughput since the objective is to maximize only the rate.

The MMR achieves high fairness but the throughput is lower since the objective is to maximize the minimum rate.

Since the FMMR-joint is adaptive on the system fairness target that we set, it can perform in the whole range of system fairness. Starting from high fairness, the FMMR reaches the target that is set. If the fairness target is higher than the MMR, the FMMR applies the channel reallocation policy to increase the fairness followed by a power allocation step to meet the target. If the target is set lower, then the channel reallocation step decreases the fairness and a following power allocation step adjusts...
the value to the target. While decreasing the fairness, the FMMR does not gain much throughput since the objective is to maximize the minimum rate.

The I-SRM-P achieves fairness almost 1 by a channel allocation and a subsequent step of power allocation as described in chapter 3.

Same philosophy applies for the FSRM-P case with the difference that the objective is to maximize the rate while decreasing the system fairness. If the fairness target is higher than the fairness level provided by the I-SRM-P, the FSRM-P applies a channel allocation step followed by a power allocation step to increase the system fairness. If the target is set lower, then a channel reallocation step decreases the fairness and a following power reallocation step adjusts the value to the target. While decreasing the fairness, the FSRM-P gains throughput since the objective is to maximize the rate. For the majority of fairness targets the achieved throughput is, more than 2 times greater than in the FMMR case.

The FSRM-P cannot reach the fairness levels of the I-SRM-P since during the channel reallocation step it reallocates channels only to the users that have the best path gain. Thus the users with bad path gains and rates are not benefited from the reallocation and therefore the system fairness is unable to reach 1 as in the I-SRM-P. A subsequent power reallocation step is applied but again due to lack of channels of the worst users the power reallocation step is extremely time-consuming as it will be seen later in Table 4-3. For the same reason the FMMR cannot reach values close to 1.

In figure 4-15 we plot the achieved normalized rates for the case of 16 users for the FMMR and FSRM-P case for three different fairness targets: 0.2 0.5 and 0.8. The simulation seed was kept fixed between the system fairness indexes and among the algorithms. The rate requirements are met more when the system fairness is even higher and close to 1. However here the purpose is to see how the normalized rates are different for the FSRM-P and FMMR case while the system fairness is decreasing from high to low values.
Starting from the case of high fairness target of 0.8 we can see how the achieved normalized rates are close to the normalized rate requirements. When the fairness is decreasing to 0.5 or even more to 0.2 the FMMR tries to maximize the user with minimum normalized rate by assigning channels from the 2\textsuperscript{nd} best user to the best thus protecting the minimum normalized rate user. The best user is user number 16 where the proportional rate requirement is maximum. This becomes greater as the fairness is reduced since this user remains the same over the reallocations. On the other hand, in the FSRM-P case where the throughput is maximized, when the fairness is decreasing from 0.8 to 0.2 only the users with the best path gains on the reallocated channels are benefited. These users are the ones with general good channel conditions. In the considered scenario these users are number 2 and 7. In order to reduce the system fairness in such low values it is possible that some users will have zero rates.

The two policies exhibit different behavior with respect to the users. On the one hand the FSRM-P maximizes the throughput in favor of the users with good channel conditions but at the cost of neglecting many users’ requirements which happen to be in worse channel conditions. On the other hand the FMMR reduces also the system fairness but in a way that the users requirements are preserved with respect to the case with high system fairness. However the resulting throughput as shown in figure 4-10 is approximately 2 times less.

4.3.2.2 Short term user satisfaction versus system fairness

In figure 4-12 we compare the short term user satisfaction against the system fairness for the FMMR and FSRM-P algorithms for the scenario of 16 users and 3 sets of rate requirements; the second is the reference case while the first is 1/4\textsuperscript{th} of the second and the third is 3 times more. The classical algorithms are plotted as a reference.
The FMMR algorithm for the case with low rate requirement exhibits higher satisfaction since special care is taken for the worst users in terms of $R_k / \gamma_k$ and the system throughput seems sufficient to satisfy the rate requirements of the users. However, if the rate requirements of the users are increased, the FMMR is unable to meet them and the satisfaction decreases. The FSRM-P on the other hand since it is able to reallocate the channels and the power between the users that have the most benefit in rate achieves higher satisfaction.

Same behavior holds also for the long term satisfaction.
In figure 4-13 is shown the long term and short term user satisfaction per group of users with respect to the system fairness for the FMMR and FSRM-P cases. Since there is no specific handlings per group in most cases the satisfactions are the same. However, group 1 with the least rate requirements of the FMMR case exhibits higher long and short term satisfaction with respect to group 2 or 3. This is due to the fact that it is improbable during channel reallocations the users of this group to be 2nd best in terms of rate proportionalities and therefore to lose channels. On the other hand, in the FSRM-P the users that lose channels are the ones with the worst rate proportionalities, i.e. users that have been allocated few channels and therefore are the ones that lose channels thus exhibiting lower satisfaction than the groups 2 or 3.

In figure 4-14 is shown the plot of the long term and short term satisfaction with respect to the system fairness for the three different groups. Since the FMMR and FSRM-P policies do not differentiate the groups when channels are reallocated, the performance per group is the same. However, based on the results of figures 4-13 and 4-14 it is expected that if the rate requirements are reduced the groups of the FMMR will start exhibiting higher satisfaction than the groups of the FSRM-P.

4.3.2.3 Average CPU time

One important aspect of the performance of the algorithms is the required time to adjust the system fairness to the target value especially in the extreme cases of very high or very low fairness. This is important since in general the algorithms start from a high system fairness with the classical MMR and I-SRM-P algorithms and adjust the fairness to the target through iterations. For this reason in the following table the required CPU time for the JOINT is shown for several targets and different number of users. Notice that for 4 users the target 0.2 cannot be met and for 16 users the
theoretical limit is 0.0625. The considered extreme cases of target equal to 0.08 and 0.999 are also shown.

<table>
<thead>
<tr>
<th></th>
<th>FMMR</th>
<th>FSRM-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 users</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F = 0.08</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>F = 0.2</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>F = 0.5</td>
<td>0.118</td>
<td>0.093</td>
</tr>
<tr>
<td>F = 0.8</td>
<td>0.14</td>
<td>0.093</td>
</tr>
<tr>
<td>F = 0.999</td>
<td>-</td>
<td>0.093</td>
</tr>
<tr>
<td>10 users</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F = 0.08</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>F = 0.2</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>F = 0.5</td>
<td>0.107</td>
<td>3.6</td>
</tr>
<tr>
<td>F = 0.8</td>
<td>0.179</td>
<td>0.179</td>
</tr>
<tr>
<td>F = 0.999</td>
<td>-</td>
<td>0.027</td>
</tr>
<tr>
<td>16 users</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F = 0.08</td>
<td>0.25</td>
<td>13.2</td>
</tr>
<tr>
<td>F = 0.2</td>
<td>0.076</td>
<td>5.2</td>
</tr>
<tr>
<td>F = 0.5</td>
<td>0.104</td>
<td>5.2</td>
</tr>
<tr>
<td>F = 0.8</td>
<td>0.54</td>
<td>0.185</td>
</tr>
<tr>
<td>F = 0.999</td>
<td>0.4</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>13.2</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Table 4-3 Average CPU time in seconds for the FMMR and FSRM-P for different system fairness targets

The FMMR exhibits more or less the same order of magnitude computational time to perform the adjustment for all the targets with the exception of the extreme cases.

where the order of magnitude increases by 1. To be able to reach extreme values of system fairness requires more iterations since the reallocations gradually become more inefficient as the fairness reaches extreme values. Especially in the case of target of 0.08 and 0.2 the policy of assigning channels only to the best user proves to be time efficient in comparison with the case of the FSRM-P where for 0.08 and 0.2 the required time is two orders of magnitude greater. In the latter the channels are allocated to the user with the best path gain on the channel, a search which requires more time to be done.

In figure 4-15 the horizontal axis shows the iteration number and the vertical axis the corresponding achieved system fairness for 1 run only. Two cases are displayed; the FMMR-SA and FSRM-P-SA. The figure shows the evolution of the system fairness with the iterations. Initially the channel reallocations prove to be very efficient in decreasing the fairness; however, as the algorithm evolves and the channels are assigned to users with the best path gain, the decrease becomes less efficient. To reduce the fairness in low values while maximizing the throughput, it is probable that some users will lose all their channels in this reallocation process. The remaining users will exchange the remaining channels, but the resulting change in their rates will not be efficient to reduce even further the system fairness. Notice that the target of
0.08 cannot be met always with channel reallocations only and a subsequent step of power allocation in the JOINT case is performed to meet the target.

4.3.2.4. CDF of the rate proportionalities over the required rate proportion for 16 users

The empirical CDF of the rate proportionalities ($R_{\text{norm}}/\gamma$, eq. 2.13) is plotted in figure 4-16 in order to show how the policies while decreasing the fairness “destroy” the rate proportionalities. All user groups are considered.

![CDF of the rate proportionalities](image)

**Figure 4-16 CDF of the rate proportionalities ($R_{\text{norm}}/\gamma$) for the FSRM-P and system fairness targets 0.2 0.5 and 0.8**

Initially the I-SRM-P achieves high system fairness close to 1 and the ratio of the normalized rates over the required rate proportionalities ($R_{\text{norm}}/\gamma$) are close to 1 for all users. Then, while decreasing the fairness the ratios are diverged from the unity. The user who has the worst ratio loses channels and the one who has the best path gain on that channel obtains the channel and increases his ratio. The system fairness is gradually reduced in this way. For $F = 0.2$ we notice that 80% of the users decreased their ratio in favor of the other 20% that increased it. The latter are the users with the best path gains on the reallocated channels.

In figure 4-17 we plot the CDF for the FMMR case.
Initially the MMR also achieves high system fairness close to 0.9 and the ratio of the normalized rates over the required rate proportionalities ($R_{norm}/\gamma$) are close to 1 for all users with a small divergence. Then, while decreasing the fairness the users with the 2nd best ratio lose channels in favor of the user with the best ratio. For $F = 0.2$ we notice that more than 80% of the users decreased their ratio in favor of the rest less than 20% which achieved ratios up to 12. It is clear that the FMMR while decreasing the fairness benefits few users.

In order to see how the FMMR and FSRM-P change the distribution of the rate proportionalities we plot in figure 4-18 the CDF for three system fairness targets: 0.8, 0.5 and 0.2.

One should keep in mind that the FMMR starts from the classical MMR and reduces the system fairness while the FSRM-P starts from the classical I-SRM-P. By comparing the CDF of the FMMR for system fairness of 0.8 and 0.5 we can see that the users still have rate proportions close to one. This is true since in the policy we assign channels from the 2nd best in terms of rate proportion to the best that always increases his rate. The increase in the rate can be observed on the upper part of the CDF which increases values as the system fairness is reduced from 0.8 to 0.5 and 0.2. While reducing the system fairness to low values of 0.2 the reduction of the rate happens only to the 2nd best user who may change with the channel reallocations. The lower rate users are protected and objective function of the FMMR to maximize the minimum proportional rate user is satisfied.

On the other hand for the FSRM-P starting from system fairness of 0.8 we can see already that the rate proportions divert from unity although the system fairness is high. This is true since the worst channel of the user with the worst rate proportion is reallocated to the user with the best path gain on that channel according to the rate maximization policy of the problem. By performing this policy some users end up without any channels as it can be seen for the cases of system fairness 0.5 and more 0.2. While the system fairness is reduced more, few users that have the best path gains
on the reallocated channels are obtaining the channels and increase their rates as it is observed in the upper parts of the CDF.

![CDF Graphs](image)

**Figure 4-18 CDF of the rate proportionalities ($R_{\text{norm}}/\gamma$) for system fairness targets 0.2, 0.5 and 0.8**

From figure 4-18 it is also evident that the throughput in the FSRM-P is greater than the FMMR in accordance with the throughput-fairness figure 4-10. For example for fairness of 0.8 with the FSRM-P, 60% of the users achieved proportional rates more than their requirements and also higher than the FMMR case. This is true since in the FSRM-P the reallocated channels are given to users with the best path gains on the channels while the users with bad channels conditions are not allocated any channels (e.g. around 10% and 30% of the users for system fairness's of 0.5 and 0.2 respectively). As it can be seen in the FMMR, even for low fairness of 0.2 the users with the worst rate proportionalities are protected.
5. Conclusions

The main motivation of this thesis is to find possible ways of adjusting the fairness of an OFDMA system by exploiting the channel and power allocation steps that are conducted in typical resource management tasks. To this end, a first part of the thesis is devoted in implementing some existing/classical algorithms already in the literature by comparing them in order to stress their performance peculiarities and advantages mainly in terms of achieved system throughput and fairness. The second part is devoted in implementing a set of heuristic algorithms that adjust the fairness of the system to a predetermined target. These algorithms are then compared with the classical ones.

All of the classical algorithms include initially a channel allocation step and a subsequent power allocation step which may be in the simplest case an equal power allocation across the channels. The objective of the algorithms is to maximize the system throughput (SRM), to maximize the minimum user rate (MMR) and to maximize the system throughput under the constraint of satisfying the rate proportionalities \( R_{\text{norm}}/\gamma \), eq. 2.13) of the users (I-SRM-P). Simulations of the downlink of a single cell with different classes of users were performed to compare the performance.

The results are indicative of the trade-off between the system throughput and fairness and show the specific behaviour of the algorithms towards the classes of users. The SRM as it just tries to maximize the system throughput:
- Shows no merit for any specific user class.
- Only the users with the best channel conditions are allocated channels keeping silent the rest.

The MMR, by maximizing the minimum user proportional rate:
- Provides a fair distribution of the system throughput to the users keeping the user satisfaction at high levels.

The I-SRM-P is the one that strictly satisfies the rate proportionalities of the users and therefore:
- Has the highest system fairness, close to 1.
- However at the cost of reduced throughput and increased computational time.

The possibility of adjusting the system fairness was studied in chapter 4. The rationale of the adjustment is to enable the operator to set the system fairness at a predetermined target and not at the default extremes which are implied by the classical algorithms. Since the classical algorithms meet different objectives, the implemented algorithms are focused in adjusting the fairness by following the same objectives. To this end, a fairness based algorithm was implemented in accordance with a classical one. The FMMR based on MMR and the FSRM-P based on I-SRM-P. The adjustment in all cases is dynamic meaning that the algorithms depending on the instantaneous system fairness which results from the classical ones, increase or decrease the fairness to meet the target. Different normalized user rate distributions can lead to the same system fairness (c.f. figure 2-5 for the case of 2 users). The resulting distribution depends on the applied policy of the FMMR and FSRM-P.

For each proposed algorithm (FMMR, FSRM-P), three varieties are implemented to meet the fairness target. Only channel reallocation, only power reallocation and both/joint performed in consecutive steps. The results showed that:
• performing jointly the adjustment, the fairness target is met with greater accuracy and at comparable time and it was selected for the rest of the comparisons.

Simulations showed that:
• the FSRM-P performs better in terms of throughput with respect to the FMMR across all system fairness targets.
• The CDF of the rate proportionalities shows that in the FSRM-P case it is more likely for users with bad channel conditions to get zero rate while in the FMMR those users are protected.

The algorithms were also examined in the context of user satisfaction over the whole duration of the simulation but also for every transmission time interval in order to obtain an insight from the user's point of view. The behaviour of the two types of satisfaction found to be the same. Simulations showed that:
• Depending on the users’ rate requirements higher satisfaction is achieved either with the FMMR or with the FSRM-P across all system fairness target values.

For low rate requirements with respect to the system throughput it was found that:
• the FMMR may achieve user satisfaction close to 100% for all system fairness values.

However, increasing the rate requirements:
• The satisfaction of the FMMR drops rapidly and in this case the FSRM-P proves to provide more satisfaction to the users.

Even though in the algorithms no specific handling per group is done, trying to differentiate the satisfaction among the groups of users, it was found that:
• The lowest rate requirements group of the FMMR has higher satisfaction than the rest groups, a behaviour which is not true in the FSRM-P.

In terms of required computational time it was found that:
• Adjusting the system fairness to extreme values is generally inefficient especially for the FSRM-P case in which the order of magnitude is two times more than the FMMR.
Appendix I - Convexity analysis of the Rate Adaptive algorithms

Definition of a convex function:
For an objective function to be convex, the domain of the function must be a convex set.
If the domain is a convex set then the 2nd order derivative of the objective function must be positive ([20] p. 71).

Definition of a convex set ([20], p.23)
A set $S$ belonging to $\mathbb{R}^n$ dimensional space is convex if for each $x, y \in S$ (in the simplified case $x$ can be just one channel and $y$ also a channel), $\lambda \in [0,1]$ it holds that $\lambda x + (1 - \lambda)y$ belongs to set $S$ again. That is any line segment between 2 points of the set $S$ belongs to the set $S$ again.

Non convexity of Rate Adaptive problems
In general, since the set $S$ in the general formulation of the Rate Adaptive problems is sets of channels, the domain of the objective function is not a convex set (i.e. the objective function is a summation over channel set selection which is not convex, [2]). In particular, the channel allocation variable in an RA problem denoted as $\rho_{k,n}$ is:

$$\rho_{k,n} = \begin{cases} 0, \text{channel } n \text{ not assigned to user } k \\ 1, \text{channel } n \text{ assigned to user } k \end{cases} \quad \text{(A.1)}$$

which is a binary integer variable. The problems that deal with such channel allocation variables are integer programming problems and are generally NP-hard and non-convex. There have been proposed several methods to overcome this complexity.

In the Max Min Rate case, ([2]), the initial non-convex problem is:

$$\max \min_{\rho,\rho'} \sum_{n=1}^{N} \rho_{k,n} R_{k,n}$$
subject to

$$\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad \text{C1}$$

$$p_{k,n} \geq 0 \quad \forall k, n \quad \text{C2}$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k, n \quad \text{C3}$$

$$\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k, n \quad \text{C4}$$

(A.2)
Which then is reformulated into a convex one by relaxing the constraint C3 that the channels can only be assigned to one user (this is done by adding an auxiliary variable, convexity is also proved in [3] - Appendix I).

\[
\max_{p,p'} \min_k \sum_{n=1}^{N} \rho_{k,n} R_{k,n} \\
\text{subject to} \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad (A.3)
\]

\[p_{k,n} \geq 0 \quad \forall k,n\]

\[\rho_{k,n} \in (0,1] \quad \forall k,n\]

\[\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n\]

The new channel allocation variables then have the form \(\rho_{k,n} = (0,1]\) and the domain of the objective function becomes convex. In that sense by performing only this relaxation which is justified on cases when \(K \ll N\), the optimal solution can be found but still requires lot of computational effort. Therefore, the authors in [2] propose a suboptimal channel allocation and an equal power allocation which is proven that has results comparable with the case of solving the relaxed convex problem.

In the cases where a proportional fairness constraint is added the previous would be true if the new constraint implied by the equalities \(R_1 / \gamma_1 = R_2 / \gamma_2 = \ldots = R_K / \gamma_K\) were linear.

\[
\max_{p,p'} \sum_{n=1}^{N} \rho_{k,n} R_{k,n} \\
\text{subject to} \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} \leq P_{\text{max}} \quad C1
\]

\[p_{k,n} \geq 0 \quad \forall k,n\]  

\[\rho_{k,n} \in \{0,1\} \quad \forall k,n\]  

\[\sum_{n=1}^{N} \rho_{k,n} \leq 1 \quad \forall k,n\]  

\[R_1 : R_2 : \ldots : R_K = \gamma_{1}^{\text{req}} : \gamma_{2}^{\text{req}} : \ldots : \gamma_{K}^{\text{req}} \quad C5\]

The non-linearity of the equalities forms a domain of the objective function which is not convex. A general approach is then to linearize the non-linear constraints however there is a trade off between satisfaction of the constraints and improvement of the objective. Moreover these solutions are still computationally complex to solve.

In every problem with proportional fairness constraints, even if the problem is split in an initial step of channel and a subsequent step of power allocation, the resulting problem of power allocation is still non-convex because of the non-linearities in C5.
In [3], after the channel allocation the Lagrange multipliers technique is used for the
determination of the optimal power allocation among users which yields the optimal
power allocation. In general the Lagrange multipliers technique can be used in non-
convex problems. The resulting Lagrangian function is always a concave function
even if the initial problem is not ([20], p. 216) and yields a lower bound on the
optimal value ([20] (5.15) and Figure 5.2). Only when the initial/primal problem is
convex and Slater’s condition holds then the duality gap is zero meaning that the
lower bound is the same as the optimal value. Otherwise weak duality holds and the
lower bound is less than the optimal value.

Another application of the Lagrange multipliers is on the classical Sum Rate
Maximization (SRM) problem in [4]. The SRM problem is again split into a channel
and a power allocation problem. After a channel allocation which is proven to be
optimal, the power allocation solution yields by applying the Lagrangian method. The
derived solution is the so called waterfilling method (proved in [20] p.245, [4]).
References


