Simulation of Cavitation Processes

Author: Álvaro Jover Herrero

Director: Daniel Fuster Salamero

Institut Jean le Rond d'Alembert

Université Pierre et Marie Curie

June 2011

Acknowledgments

I would like to acknowledge all the people who have made this Master Project possible, without whom would have been harder to do it. First of all, I would like to thank all the people from University Pierre et Marie Curie and in particular to the people from Institute Jean le Rond d'Alembert for giving me the opportunity to work with them in a very interesting project. In particular I would like to thank Daniel Fuster for all his time, tips and explanations about cavitation. His enormous dedication and interest have been greater than I could have ever imagined. Without him this wouldn't have been possible. I am also grateful to Mr. Zaleski, who has also been helpful for me when I needed him. Finally I would like to thank all my family and close people of Barcelona who, even in the distance, have been a great support for me during these months.

Nomenclature

c : Speed of sound in the liquid (m/s).

d : Distance from the bubble location (m).

e : Internal energy (kg/m).

f : Body force vector.

h : Measure of the size of the support.

H_i : Enthalpy of the liquid at the interface (kJ/kg).

I^* : Potential induced by the N-1 surrounding bubbles at the location of the ith bubble.

kp : Polytropic coefficient.

Ma : Mach number.

N : Number of bubbles.

n_b : Number of moles inside the bubble.

P(t) : Pressure outside the bubble (Pa).

P_b(t) : Pressure inside the bubble (Pa).

P_g : Gas pressure (Pa).

P_l : Liquid pressure (Pa).

P_v : Vapor pressure (Pa).

R : Universal constant of ideal gases.

R_b : Bubble radio (m).

$\dot{R}_b$: Bubble interface velocity (m/s).

$\ddot{R}_b$: Bubble interface acceleration (m^2/s).

R_c : Cluster radio (m).

R_e : Bubble equilibrium radius (m).

T_b : Bubble temperature (K).

T_l : Liquid temperature (K).

u_b : Bubble velocity field (m/s).

u_l : Liquid velocity field (m/s).

V_b : Bubble volume (m).

v_l : Liquid cinematic viscosity (m/s).

x_i : Bubble location (m).

β : Cluster void fraction.

λ : Wavelength (m).

$\phi_{\infty,t}$: Potential created by the background flow and the induced by each bubble.

σ : Surface tension (N/m).

ρ_l : Density of the liquid (kg/m).

τ : Viscous stress tensor (kg/ms).

ω_b : Bubble natural frequency (rad/s).

ω_c : Cluster natural frequency (rad/s).

ξ : Population per unit liquid volume (m^{-3}).

$\Upsilon(d, h)$: Kernel function.

Contents

Chapter 1

Introduction

1.1 Motivation

Cavitation is the formation of empty cavities in a liquid by high forces and the immediate implosion of them. It occurs when a liquid is subjected to rapid changes of pressure causing the formation of cavities in the lower pressure regions of the liquid. Cavitation processes can produce high temperatures (10.000 °C) and pressures (1.000 atm) inside the bubbles and also generate micro jets produced by the asymmetric implosions of the bubbles. This phenomenon has been investigated during the last century in order to avoid its appearance because it has, in general, negative consequences for both naval and aeronautics industries (noise, loss of performance and component damages).

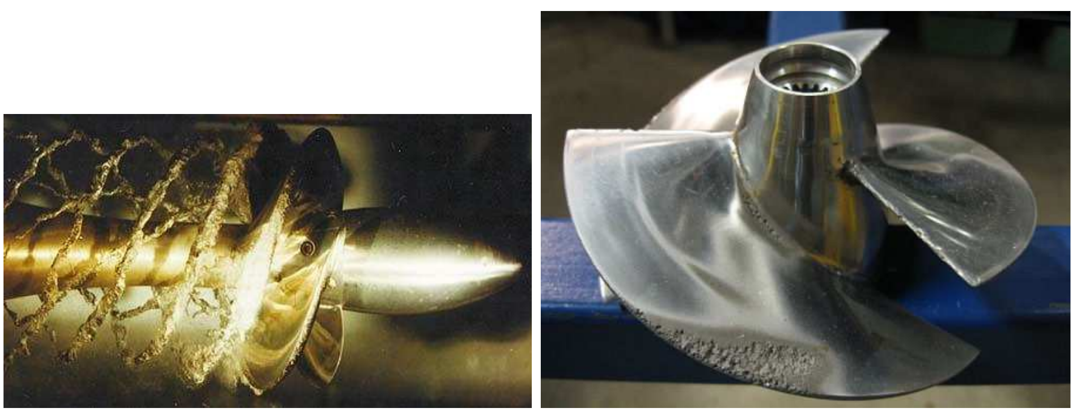

Figure 1.1: Cavitation in an helix

In the last years some applications based on cavitation have been developed and cavitation studies have been focused on how to control it. It is not difficult to find applications in different fields such as chemistry, biomedicine, cleanliness and military weapons.

The idea of this Master Project is to investigate the most relevant parameters controlling bubble clusters implosions. In order to achieve this, I have been working with a code, which can reproduce bubble implosions in a compressible liquid.

This project, funded by the oil company Total, is part of experimental studies, located in Saint Cyr, and computational investigation, currently undertaken at the hosting Institute Jean le Rond d'Alembert. The experimental facility consists of an apparatus which is able to inject air in to a fluid and later, and with the help of a piston, hit it to produce a pressure wave that induces cavitation. Two pressure transducers have been installed, one on the top of the chamber and one at the bottom in order to have exact measures of the pressure wave evolution. A high speed camera allows to record the bubble response. The experiments are controlled by a computer and an oscilloscope. The final goal of this study is to provide guidelines about the influence of different parameters on the conditions generated inside the chamber.

Figure 1.2: Experimental prototype

1.2 Basic equations

Cavitation involve two phases: gas and liquid. The different unknown variables for each phase are density, pressure, temperature and field of velocities.

The equations to solve these unknown variables are the basic conservation equations, namely:

- Conservation of mass.
- Conservation of momentum.
- Conservation of energy.

In addition we can use an equation of state to relate the thermodynamic quantities of each of the fluids.

1.3 Model

The model used here, follows the Continuum hypothesis. The continuum assumption, considers fluids to be continuous. That is, properties such as density, pressure, temperature, and velocity are taken to be well-defined at ”infinitely” small points, defining a REV (Reference Element of Volume). Properties are assumed to vary continuously from one point to another, and are averaged values in the REV. The fact that the fluid is made up of discrete molecules is ignored. The equations that we are going to use are:

- Liquid:
  - Continuity
$$\frac{d\rho_l}{dt} + \nabla(\rho_l \boldsymbol{u_l}) = \frac{\rho_l}{1-\beta}\left(\frac{d\beta}{dt} + \boldsymbol{u_l}\nabla\beta\right) \tag{1.1}$$
  - Momentum
$$\frac{d\rho_l \boldsymbol{u_l}}{dt} + \nabla(\rho_l \boldsymbol{u_l}\boldsymbol{u_l}) + \frac{\nabla P_l}{1-\beta} = \frac{\rho_l \boldsymbol{u_l}}{1-\beta}\left(\frac{d\beta}{dt} + \boldsymbol{u_l}\nabla\beta\right) + \frac{\nabla\tau}{1-\beta} \tag{1.2}$$
  - Energy

    Simplified as that temperature is constant and equal to the bubble.
$$T_l = T_b \tag{1.3}$$
  - State equation
$$P_l = P_{l_0} + c_l^2(\rho_l - \rho_{l_0}) \tag{1.4}$$
- Bubble: The equations for the gas phase are obtained under the assumption that the gas phase is well represented by discrete particles, each particle representing an spherical bubble that is explicitly tracked in the flow
  - We assume that velocities fields of the liquid and the bubbles are the same.
$$\boldsymbol{u_b} = \boldsymbol{u_l} \tag{1.5}$$
  - Continuity $\Longrightarrow$ Rayleigh-Plesset equation

– Momentum $\Longrightarrow$ Rayleigh-Plesset equation

– State equation

$$P_b V_b = nRT_b \tag{1.6}$$

The behavior of an isolated bubble is governed by the Rayleigh-Plesset's equation, which describes the dynamics of a single bubble where the far pressure is know, the equation is a mixture of the Continuity and Momentum equations. An entire demonstration could be found in [2].

$$\frac{P_b(t) - P(t)}{\rho_l} = R_b \frac{d^2 R_b}{dt^2} + \frac{3}{2}\frac{dR_b}{dt^2} + \frac{4v_l}{R_b}\frac{dR_b}{dt} + \frac{2\sigma}{\rho_l R_b} \tag{1.7}$$

The model proposes a variation of the Rayleigh-Plesset's equation to predict the bubble dynamics of a bubble cluster in a compressible liquid. The modified equation takes into account the bubble-bubble interaction effects, two aspects that are not considered in the original one. Details about how to obtain this formula can be obtained in [5].

$$\left(R_i\left(1 - \frac{\dot{R}_i}{c}\right)\right)\ddot{R}_i + \frac{3}{2}\dot{R}_i^2\left(1 - \frac{\dot{R}_i}{3c}\right) = (H_i + \phi_{\infty,t})\left(1 + \frac{\dot{R}_i}{c}\right) + \frac{R_i \dot{H}_i}{c} + I^* \tag{1.8}$$

The Laplace equation of vapor/gas bubbles links the pressure at the interface in both phases.

$$P_v + P_g = \frac{2\sigma}{R_e} + P \tag{1.9}$$

Finally the value of the void fraction, which is defined as the volume of gas per unit volume of mixture, is obtained as:

$$\beta(x,t) = \sum_{i=1} \frac{4}{3}\pi \bar{R}_i^3 \Upsilon(d,h) \tag{1.10}$$

Where h is a measure of the size of the support (a regulator parameter), $\bar{R}^3$ can be obtained from the probabilistic function of bubble radius distribution:

$$\bar{R}^3 = \int_0^\infty R_e f_e(R_e)\, dR_e \tag{1.11}$$

And d is the distance from the bubble location x_i:

$$d = |x - x_i| \tag{1.12}$$

The model in 2D defines a new distance along the 'z' direction, dz. In a general case this value can be a new degree of freedom, however with an isotropic distribution of bubbles we will consider the next formula is to be considered:

$$dz = C\left(\frac{4/3\pi R}{\beta}\right)^{1/3} \tag{1.13}$$

Where C is a constant that for the simulations in this Master Project will be equal to one.

Chapter 2

Numerical Results

A study of an isolated bubble and bubble clusters is presented in the next pages. A sinusoidal wave pressure impacts the bubbles in order to induce cavitation. The objective of this simulations is to study the influence of the different parameters in order to produce the most violent implosions.

2.1 Study of an isolated bubble

A sinusoidal wave is going to be used in order to excite the bubbles into a single frequency. The negative part of the wave will be the first to arrive to the bubble in order to make it grow, later the positive one will compress it and make it implode in a more violent way. The equation is as follows:

$$f(x) = -Ma * sin\left(\frac{2\pi x}{\lambda}\right) \tag{2.1}$$

Being:

$$Ma = \frac{\Delta P}{\rho_l c_l^2} \tag{2.2}$$

$$\lambda = \frac{2\pi}{\omega} \tag{2.3}$$

These two parameters modulate the intensity (2.2) of the pressure wave and the frequency (2.3).

The value of the radius tested here are:

R_1	10^{-3}
R_2	10^{-4}
R_3	10^{-5}

Figure 2.1 depicts the dynamic behavior of three different bubbles excited with the same wave ($\lambda = 2$ meters).

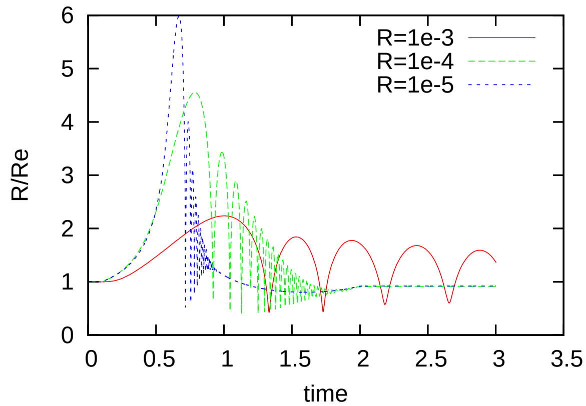

Figure 2.1: Behavior of three different radius with the same pressure wave

Firstly the bubble increases its size (until six times the initial one). After that, the positive part of the wave makes the bubble implode. It is interesting to see that for bigger bubbles the variation of radio is smaller and slower than for the smaller ones. This is due to different bubble's resonant frequencies. One bubble under its resonant frequency has a higher radio's variation and implode more violently.

2.1.1 Study of the method of solution

The code can work in two different ways. The first one is solving the equations of the model as were presented previously. The second one allows us to uncouple the bubble's equations from the liquid ones. The first option is considered the theoretical solution and the second one will only be used in order to do simulations more rapidly and with less precision. The R2's example is solved below in both ways to see the differences.

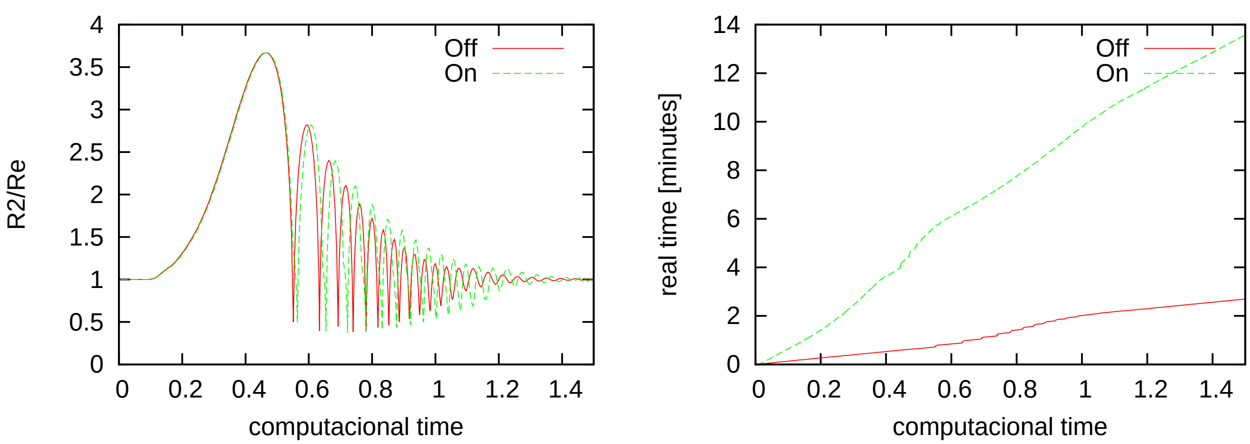

Figure 2.2: R2's evolution and computational time

Even being a simple example the evolution is not the same and in spite of an increase in the computing time, all the simulations will be solved with the equations coupled.

2.1.2 Resonant frequency of an isolated bubble

Finally a study of the resonant frequency of an isolated bubble will be done. For a given amplitude the maximum or peak response amplitude occurs at a frequency, $omega_b$, given by the minimum value of the spectral radius. The deduction of the equation can be found in [2].

$$\omega_b = 2\pi f_{res} = \left(\frac{3kp(P - P_v)}{\rho_l R_e^2} + \frac{2\sigma(3Kp-1)}{\rho_l R_e^3} - \frac{8v_l^2}{R_e^4} \right)^{-\frac{1}{2}} \tag{2.4}$$

The next table shows the values of the resonant frequencies of the bubbles that are used in the simulations.

R [m]	Resonant frequency [Hz]
10^{-3}	3246.67
10^{-4}	32621.34
10^{-5}	341265.83

For smaller radios the resonant frequency is higher. The maximum and minimum radius of these three bubbles according to the resonant frequency calculated with the formula (2.4) are shown.

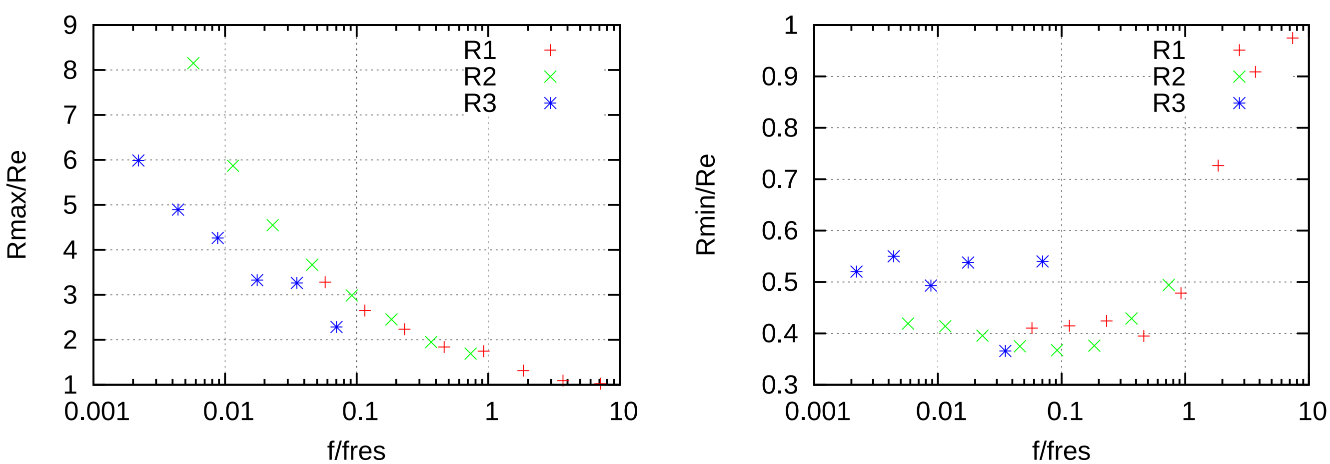

Figure 2.3: Rmax, Rmin: Isolated Patm

As proved by theoretical calculations, frequencies obtained with bigger radius are smaller than the ones with smaller radius. It is also very important to see that the most violent implosion are caused by the smallest radio. This changes completely in clusters.

2.2 Study of a cluster of bubbles in 2D

2.2.1 Choice of the grid

First of all it is important to make a comparison between different grids to see which one is the most suitable for making all the simulations. A comparison of four different grids will be done. The number of points in the 'x' direction will be 100, 200, 400 and 800. We will take the solution obtained with the grid of 800 points as the theoretical one. The data of the cluster and the wave pressure are:

R [m]	10^{-4}
R_{c_x} [m]	0.5
R_{c_y} [m]	0.5
β	10^{-3}
Number of bubbles	301572
Pressure	Patm

The evolution of the wave pressure in three different points is represented (one at the left of the cluster, one in the center and the last one at the right side). Pressures at the left side of the cluster is higher than in the right or in the center of the cluster because the own bubbles of the cluster have implode and have created new positives waves pressures. Some noise is also created at the right side of the wave. As Figure 2.5 presents, the amount of points is a significant variable in the final output solution.

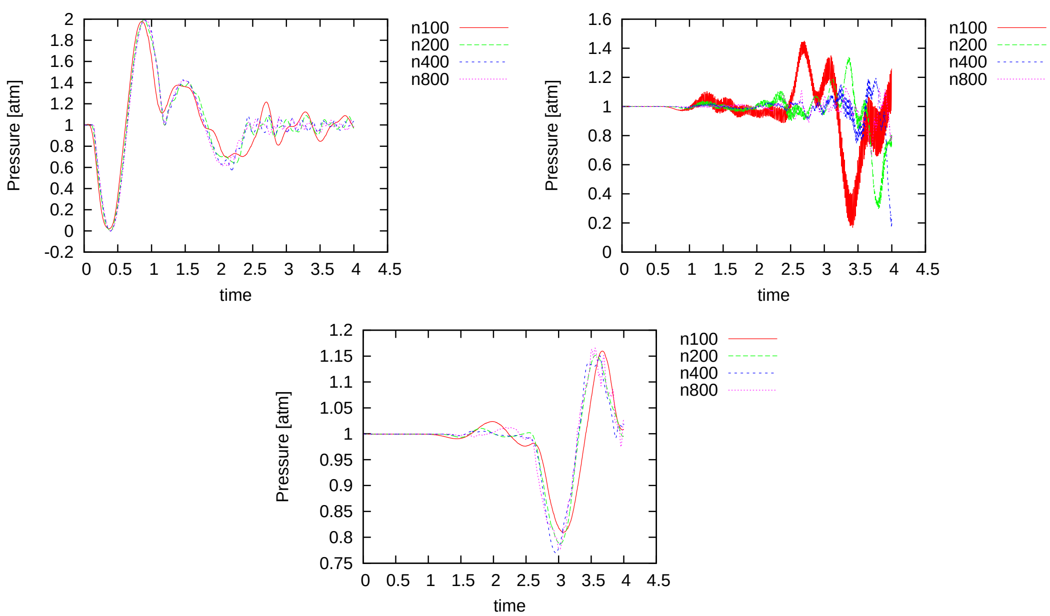

Figure 2.4: Pressure at the left, center at right side of the cluster

In the most accurate solution the minimum and maximum radios are located at the left side of the cluster due that in this site the pressures shown before were higher that the ones located at the right side or in the center of the cluster. This distribution is followed by the rest of the examples. In the left side we can observe the map of the maximum radius whereas in the right side we plot the minimum radios, which are a measure of the collapse intensity.

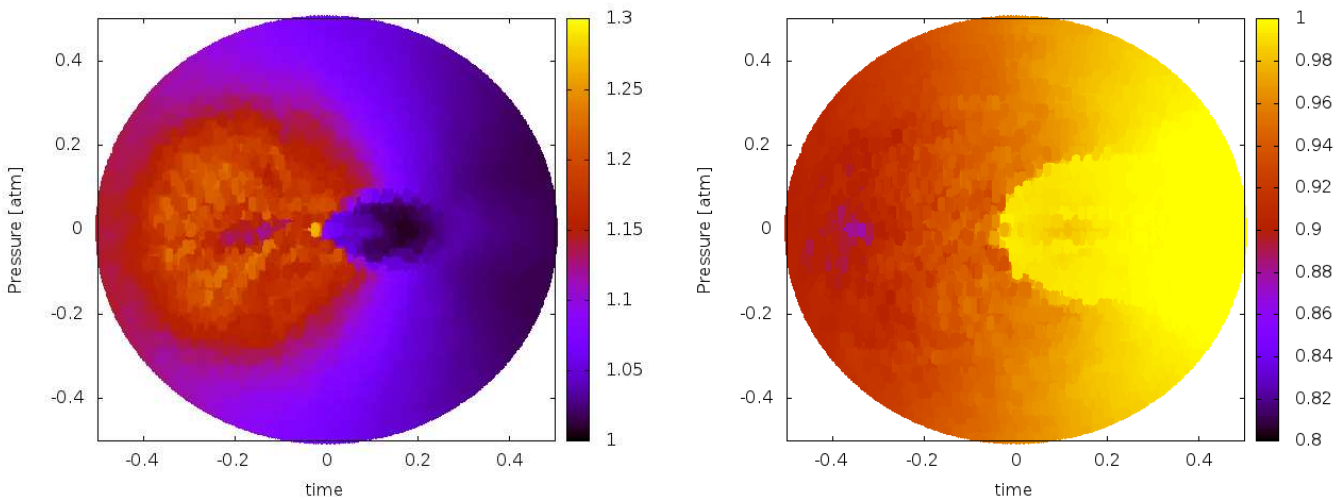

Figure 2.5: Maximum radios of the cluster and minimum radios of the cluster.

A graphic of the times of computation and the average void fraction are shown below:

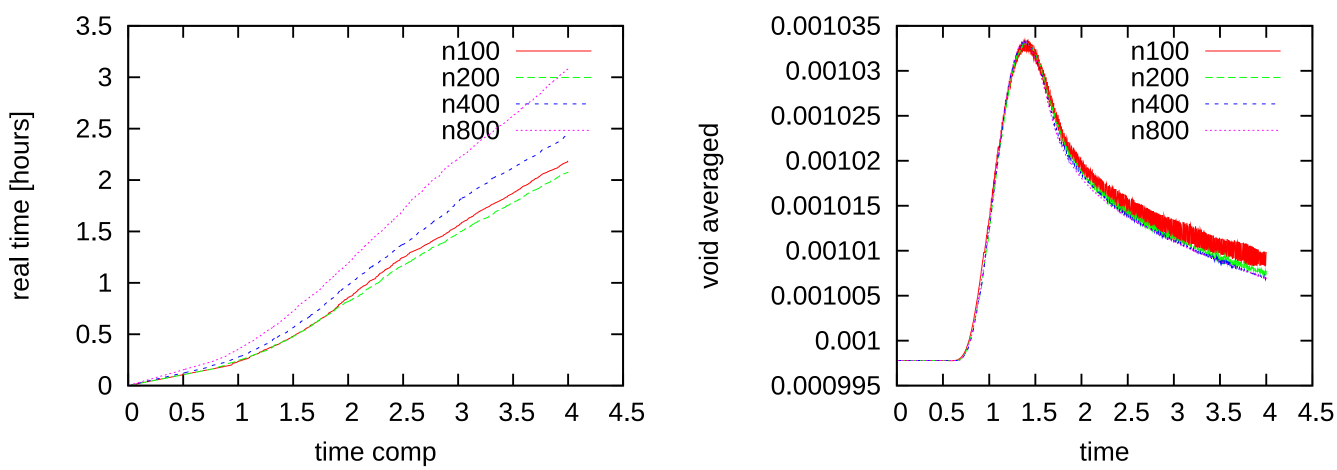

Figure 2.6: Computational time and averaged void averaged

There is a big difference between the times. Optimize the real time is a very important factor because some simulations can be running one hundred of hours more or less, so it's important to make the mesh as coarse as possible still preserving the quality of the solution. Finally the minimum and maximum radius in front of the type of mesh are shown. The solution tends to the theoretical one as more points are in the mesh.

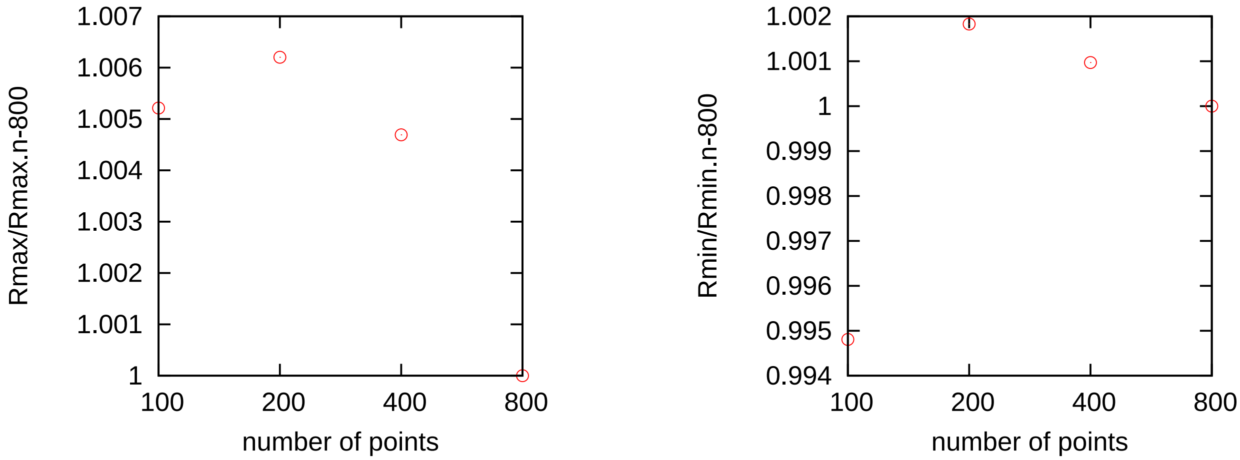

Figure 2.7: Rmax, Rmin in function of the theoretical solution

Thus these graphics we choose as an optimal mesh the one with 400 points in the 'x' direction. This mesh combines an accurate prediction to the maximum void fraction, still having a substantial save of computational time compared to the 800 points mesh. As a result of this at least eighty points per one meter of length should be used in order to do good simulations.

2.2.2 Choice of the depth

Even 2D simulations are indeed 3D a cylinder with a differential depth or dz is used. Five different widths will be compared. As we want to retain the void fraction equal to 10^{-6} when we increase the dz we are going to increase the number of bubbles too. Here is the data which is going to be used:

	dz [m]	number of bubbles
dz_1	0.161199	31
dz_2	1.61199	303
dz_3	16.1199	3023
dz_4	161.199	30219
dz_5	1611.99	302102

And here are evolution of the pressure in the three different points explained before.

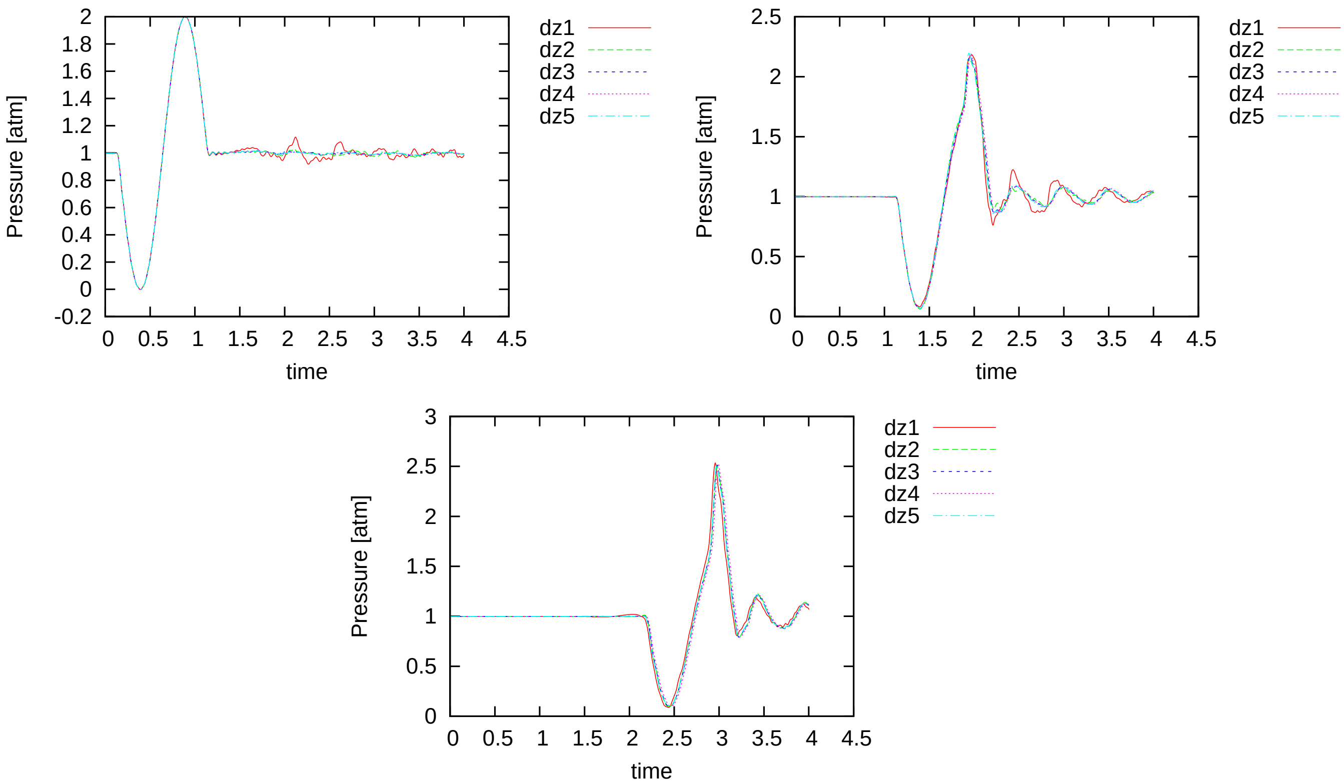

Figure 2.8: Pressure at the left, center and right side of the cluster

Finally times of computation and the averaged void fraction are shown.

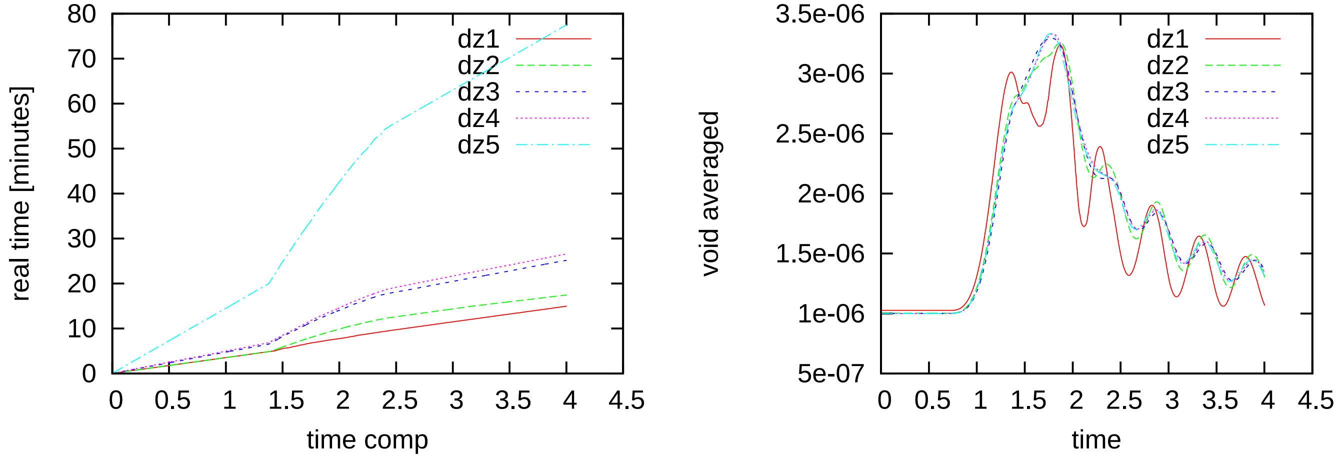

Figure 2.9: times, averaged void fraction

The five examples converge perfectly. The differences in the calculation times are due to the different number of bubbles in each case, taking more time as more bubbles are inside the cluster. So the conclusion is that for 2D simulations the width of the problem is not a relevant factor.

2.2.3 Resonant frequency of a cluster

In the next pages we discuss the influence of the frequency on the maximum and minimum radius. In addition, we investigate the relationship between the theoretical frequency of resonance and the ones that we obtain in the simulations will be presented. C.E. Brennen presents a formula in order to find the different resonant frequencies of the clusters. The formula is designed for 3D spheres. Here is presented the final one, more information about how to obtain the formula can be found [1].

$$\omega_c = \omega_b \left(1 + \frac{16\xi R_c^2 R_b}{(\pi(2n-1)^2}\right)^{-\frac{1}{2}} \tag{2.5}$$

Being :

$$\xi = \frac{3N}{4\pi(R_c^3 - NR_b^3)} \tag{2.6}$$

The simulations are done in 2D and ξ units are m^3 so the formula requires a variation in order to set the units. For this reason ξ becomes:

$$\xi = \frac{\beta}{\frac{4}{3}\pi R_b^3} \tag{2.7}$$

And the resonant frequency :

$$\omega_c = \omega_b \left(1 + \frac{16\beta R_c^2}{\frac{4}{3}\pi^2 R_b^2(2n-1)^2}\right)^{-\frac{1}{2}} \tag{2.8}$$

The parameter n in the formula is a dimensionless parameter which is used to obtain n different resonant frequencies values.

The graph of the formula for one of the radius and n = *1* is as follows:

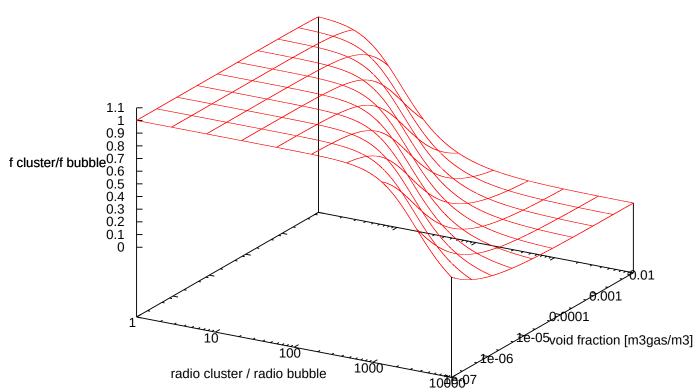

Figure 2.10: Brennen's formula for clusters with R1 and n = 1

This formula gets closer to the isolated one for small clusters and low void fractions whereas large differences are shown for large dense bubble clusters.

Monodisperse clusters

This section focuses on simulations of monodisperse clusters of bubbles with the three different. The void fraction is 10^{-6}, the radio of the cluster is equal to 0.5 meters and the pressure the atmospheric one. The rest of the data is:

Radio of the bubbles [m]	Number of bubbles	Cluster resonant frequency [Hz]	Bubble resonant frequency [Hz]
10^{-3}	3023	2843.19	3246.67
10^{-4}	37272	5821.86	32621.34
10^{-5}	302307	6188.86	341265.83

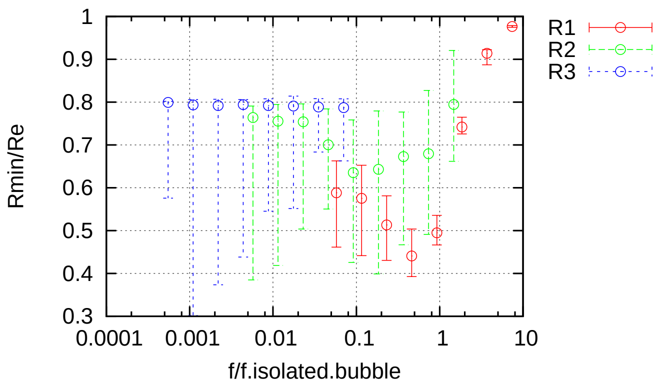

Figure 2.11: Rmin represented with isolated bubble's formula

In clusters, opposite to the behavior of isolated bubbles, the bubbles that have the most violent implosions are the bigger ones. The most intense implosions don't occur at the resonant frequency of isolate bubbles therefore the formula (2.4) doesn't seem to be representative of the systems behavior. Figure 2.13 and 2.14 show the results obtained using Brennen's formula for frequencies of clusters. In these examples we take *n=1* and *n=2*. The first case (*n=1*) will be the one used during this Master Project because is the one that brings a better approach of what is happening in the simulations.

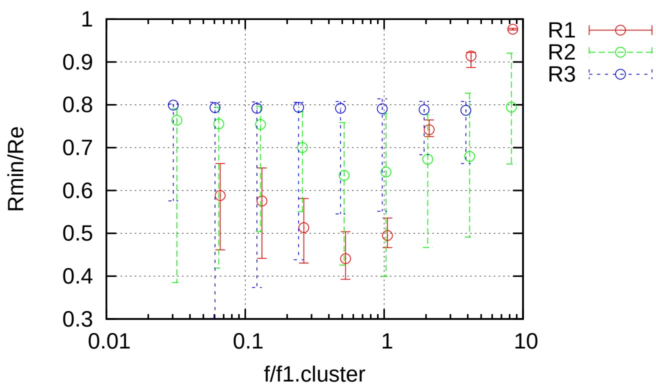

Figure 2.12: Rmin represented with Brennen's formula and n = 1, β=10^{-6}, atmospheric pressure, monodisperse

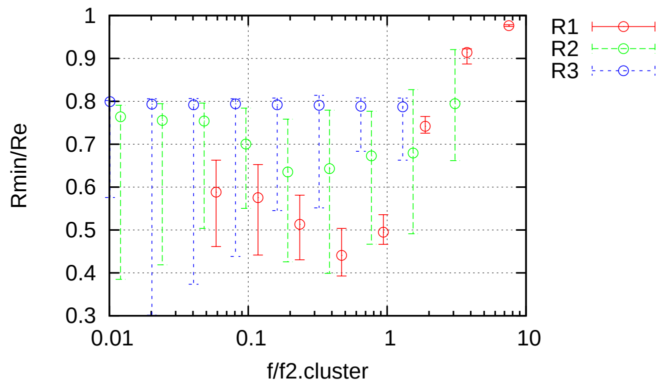

Figure 2.13: Rmin represented with Brennen's formula and n = 2, β=10^{-6}, atmospheric pressure, monodisperse

The first conclusion is that the implosions in the monodisperse clusters are stronger if we have bigger radius (R1). If we have little bubbles the effects of the implosions into the cluster are negligible as we can see in the cluster of R3 radius, where the evolution of the minimum radius is constant.

If we compare the behavior of an isolated bubble and the corresponding cluster we can easily see that for the same wave pressure the implosions are smaller for the cluster than for a single bubble. We can affirm that the implosions are damped by the own cluster.

In the resonant frequency of R1 (λ = 1 m) and R2 (λ = 0.5 m) high pressures are produced. On the left side there is the evolution of the wave for the R1's resonant frequency and on the right for the radio equal to R2.

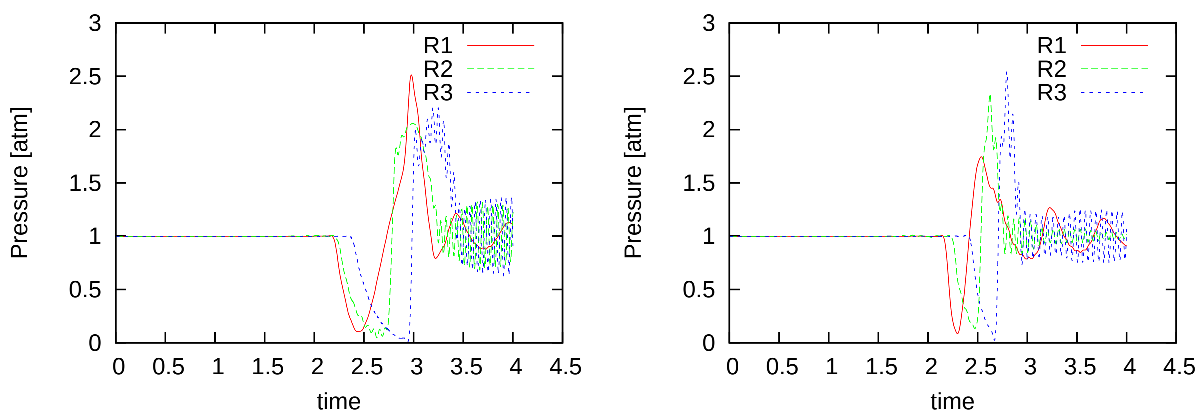

Figure 2.14: Pressures at the right side of the cluster for R1's resonant frequency (left) and R2's resonant frequency (right)

The pressure of R1 on the left graphic is higher than R2 because R1 is excited in its ,resonant frequency. On the right side happens the opposite because now the wave has a frequency closer to R2's resonant one.

Polydisperse clusters

Right now a study of a polydisperse cluster being the average radius equal to R2 will be done. The size of the cluster is the same that in the last examples. The resonant frequencies values do not change because the formula doesn't take into account if the clusters are monodisperse or polydisperse. First simulations with a void fraction equal to 10^{-6} at an atmospheric pressure will be done. The vertical line represented in all the graphics is the the resonant frequency found with the isolated bubble formula (2.4).

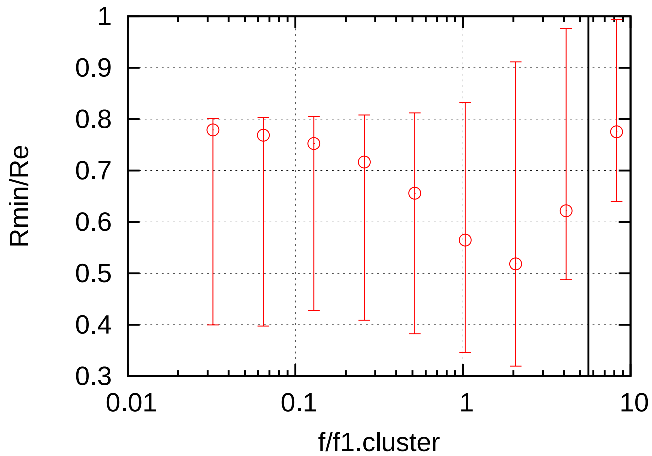

Figure 2.15: Rmin represented with Brennen's formula and n = 1, β=10^{-6}, atmospheric pressure, polydisperse

Brennen's formula for clusters is more precise than the one for isolated bubbles. It is interesting to compare the behavior between this polydisperse cluster and the monodisperse of the same radius.

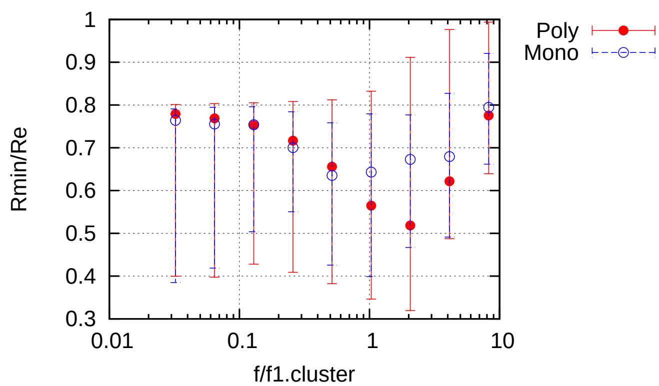

Figure 2.16: Representation of the Rmin of a monodisperse cloud and a polydisperse, β=10^{-6}, atmospheric pressure

The implosions are bigger in the polydisperse cloud than in the monodisperse one, being both of them in the same frequency approximately. For this reason the study will be focused on polydisperse clusters.

In the figure 2.17 the same polydisperse cluster but with a half of amplitude of the pressure wave is presented:

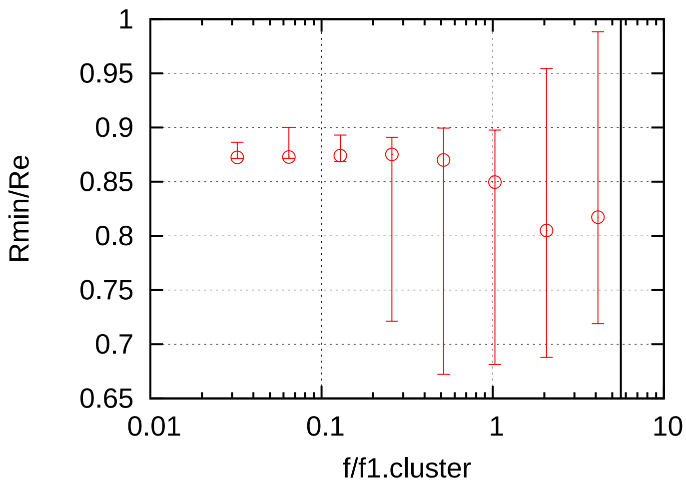

Figure 2.17: Rmin represented with Brennen's formula and n = 1, β=10^{-6}, 1/2 atmospheric pressure, polydisperse

It's interesting to compare the evolution of the cluster with different amplitudes of waves pressures.

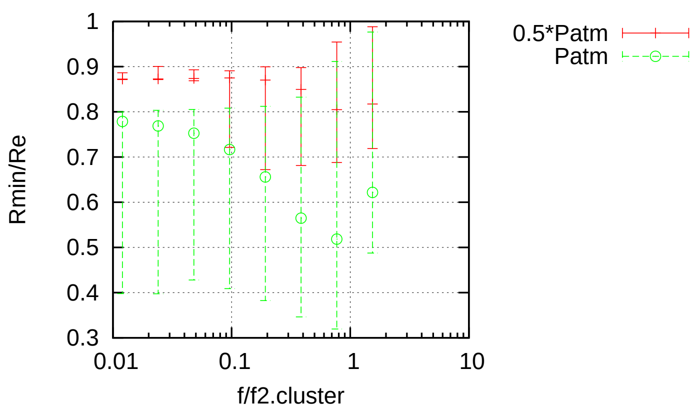

Figure 2.18: Representation of the Rmin of a polydisperse cluster subjected to two different amplitudes wave pressures, β=10^{-6}

The evolution of the clusters is the same for both pressures, being the implosions stronger with the higher amplitude. The important point is that the resonant frequency is the same for both examples so the amplitude is not a relevant point in the study of resonant frequencies.

Right now the void fraction is reduced and until 10^{-3}. In the Image 2.19 there is a monodisperse cluster under an atmospheric pressure.

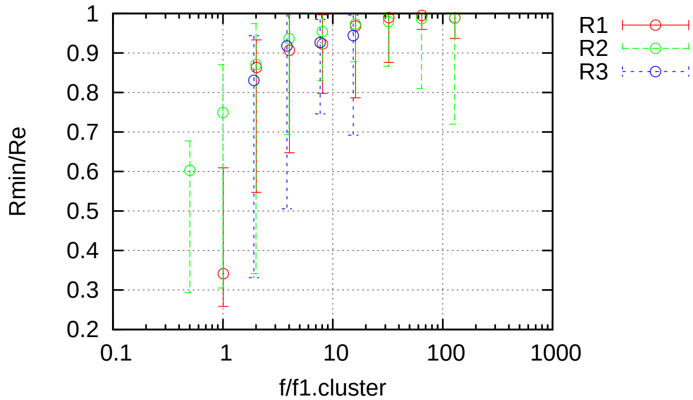

Figure 2.19: Rmin represented with Brennen's formula and n = 1, β=10^{-3}, atmospheric pressure, monodisperse

The implosions are really weak and it is difficult to find the resonant frequencies because they take place in very low positions. For this reason it's interesting to reduce the radio of the cluster in other to increase the resonant frequencies values. Remember that the resonant frequency of an isolated bubble of this size is 32621.34 Hz. The new data is:

Cluster's radio [m]	Type of cluster	Number of bubbles	Resonant frequency [Hz]
0.5	Monodisperse	301572	187.10
0.1	Polydisperse	13937	935.15
0.075	Polydisperse	7358	1246.47
0.05	Polydisperse	3299	1868.00

As smaller is the cluster less bubbles it has because the void fraction remains constant. A representation of the maximum and minimum radius for the three new clusters is shown below.

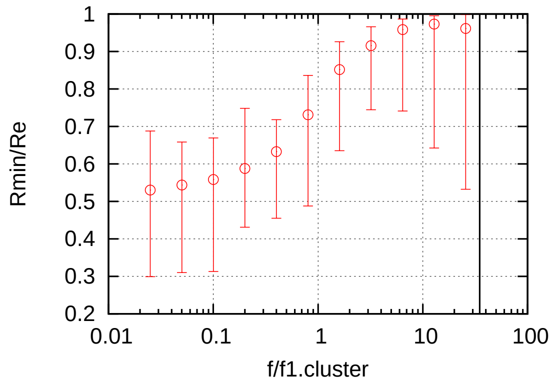

Figure 2.20: Rmin represented with Brennen's formula and n = 1, β=10^{-3}, atmospheric pressure, polydisperse, Rc = 0.1

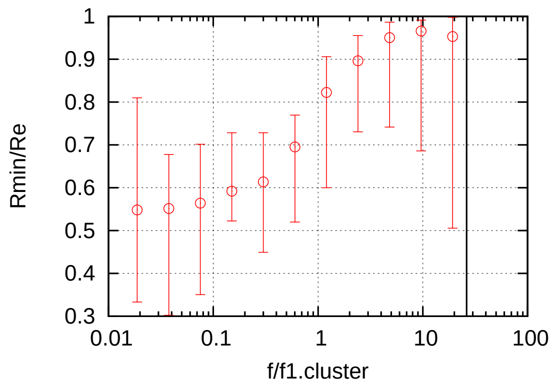

Figure 2.21: Rmin represented with Brennen's formula and n = 1, β=10^{-3}, atmospheric pressure, polydisperse, Rc = 0.075

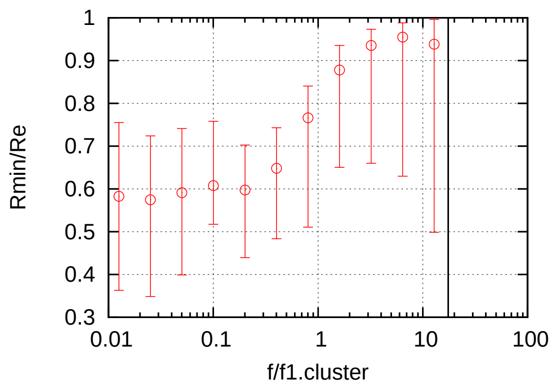

Figure 2.22: Rmin represented with Brennen's formula and n = 1, β=10^{-3}, atmospheric pressure, polydisperse, Rc = 0.05

It is interesting to represent the four different cluster's radios with the f/f.isolated.bubble and the f/f1.cluster frequency. As bigger is the cluster the resonant frequencies become lower. Even the code is not able to capture the resonances frequencies, Brennen's formula resembles an order of magnitude closer than the formula for isolated bubbles.

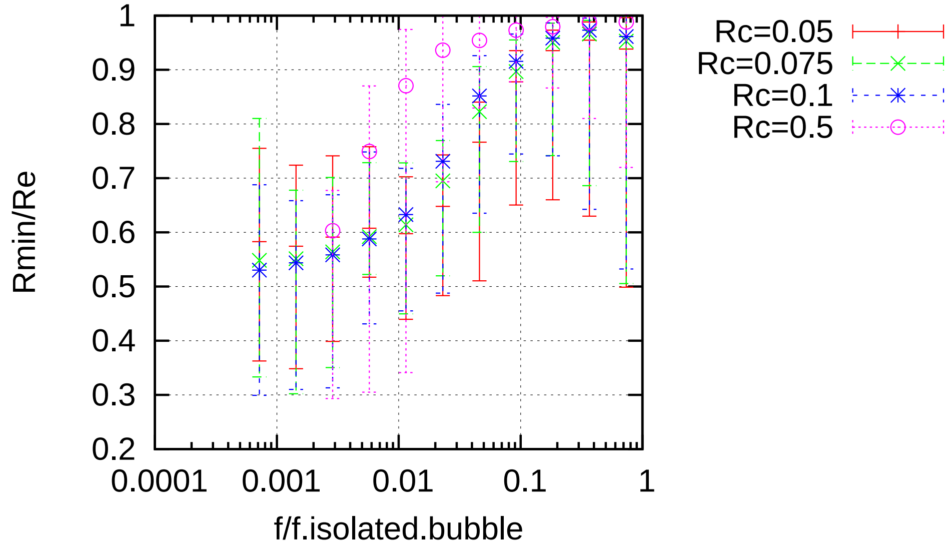

Figure 2.23: Rmin represented with isolated bubble's formula, β=10^{-3}, atmospheric pressure, Rc=(0.5, 0.1, 0.075, 0.05)

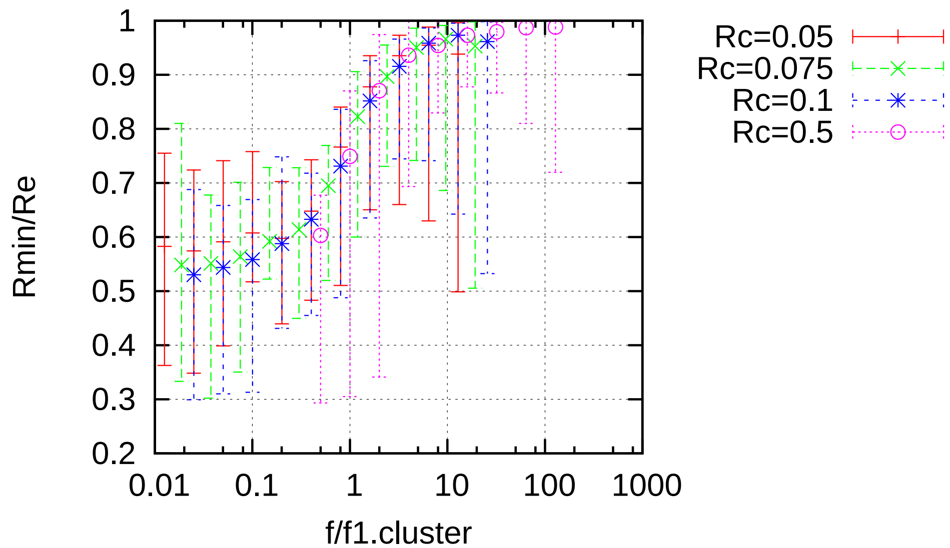

Figure 2.24: Rmin represented with Brennen's formula and n = 1, β=10^{-3}, atmospheric pressure, Rc=(0.5, 0.1, 0.075, 0.05)

The behavior of three different cluster with void fractions equals to 10^{-4}, 10^{-5} and 10^{-6} and atmospheric pressure are presented. The theoretical resonant frequencies are:

Void fraction	Type of cluster	Number of bubbles	Resonant frequency [Hz]
10^{-4}	polydisperse	37272	591.59
10^{-5}	polydisperse	37272	1868.00
10^{-6}	polydisperse	37272	5821.86

Remember that the resonant frequency of an isolated bubble of this size is 32621.34 Hz.

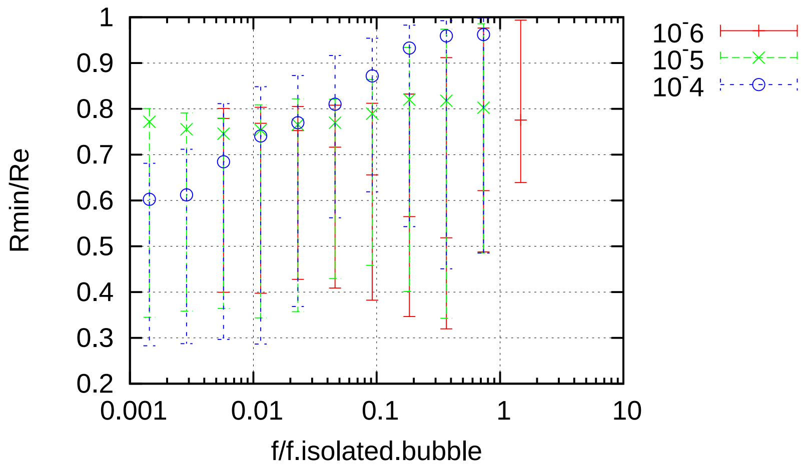

Figure 2.25: Rmin represented with isolated bubble's formula, $\beta = (10^{-4}, 10^{-5}, 10^{-6})$, atmospheric pressure, Rc = 0.5

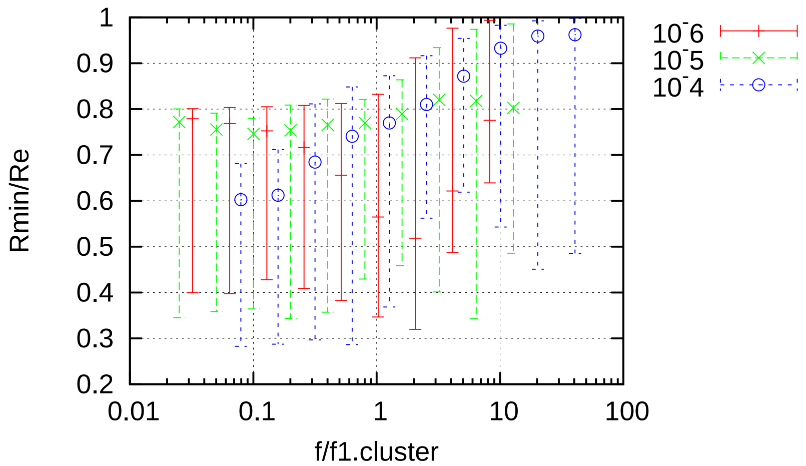

Figure 2.26: Rmin represented with Brennen's formula and n = 1, $\beta = (10^{-4}, 10^{-5}, 10^{-6})$, atmospheric pressure, Rc = 0.5

As the void fraction is increased, Brennen's formula gets far away of the simulation resonant frequency. Even this Brennen's formula is a couple of orders of magnitude closer than the isolated formula.

If the amplitude of the wave pressure is increased a twenty per cent (until 1.2 the atmospheric pressure) the graphics below are obtained.

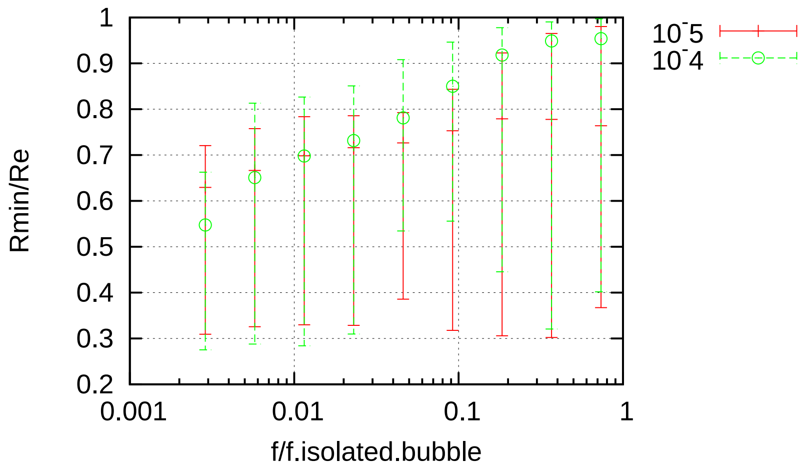

Figure 2.27: Rmin represented with isolated bubble's formula, $\beta = (10^{-4}, 10^{-5})$, 1.2 * atmospheric pressure, Rc = 0.5

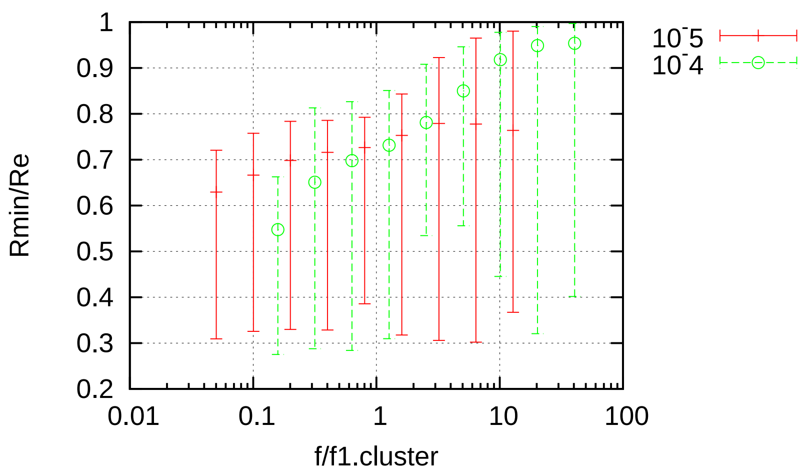

Figure 2.28: Rmin represented with Brennen's formula and n = 1, $\beta = (10^{-4}, 10^{-5})$, 1.2 * atmospheric pressure, Rc = 0.5

We are not able to capture the resonant frequencies again, which are at lower frequencies. May appear that the isolated bubble formula suits better than the Brennen one because the values of the x-axis scale are closer to one. The truth is that if we focus in the lower frequency points (which are the closest to the resonant frequencies) Brennen's formula is again a couple of orders of magnitude closer than the isolated formula.

Chapter 3

Conclusions

This Master Project presents a numerical study of isolated bubbles and clusters in compressible liquids.

First of all we have optimized the computational parameters of the code. The code works in a more accurate way with the equations of the gas and the liquid coupled despite the increase in the calculation time. If we want to have accurate results it's important to have more than 80 points per one meter of length. We have also conclude that the width of the cylinder of bubbles is not a relevant factor in the dynamic behavior of the bubble. Despite this we have to control this parameter because it controls the number of bubbles generated and, as a consequence of that; the computational time.

The single bubble study shows that the resonant frequency is the most important parameter in order to characterize the implosion process of the bubble. For high amplitudes, nonlinear regime, we have observed that violent implosions only occur for f $\leq f_{res}$. Smaller bubbles implode more violently, having a higher resonant frequency than the bigger ones. The resonant frequency formula works perfectly in the cases where we have been able to capture it.

Entering in the cluster's dynamic it's really relevant that smaller radios bring more violent implosions, which is opposed to the behavior of isolated bubbles. We have also proved that polydisperse clusters implode more violent than the monodisperse ones. If we focus in the resonant frequency of the clusters we have to said that the amplitude of the wave pressure is not relevant at all. Brennen's formula, which is thought for spheres, doesn't predict exactly what is the value of the resonant frequency in our cylinders. Even this it works better than the isolated one even in the hardest cases as high values of void fractions and with big cluster radios. Both formulas give similar results if we have small clusters and low void fractions.

Chapter 4

Bibliography

1 'On the acoustical dynamics of bubble clouds', L. d'Agostino and C.E. Brennen.

2 'Cavitation and bubble dynamics', C.E. Brennen.

3 'Hump code instructions', J. Franck.

4 'Liquid compressibility effects during the collapse of a single cavitating bubble', D. Fuster.

5 'Modelling and numerical simulation of the dynamics of liquid-bubble cavitating systems with chemical reaction processes', D. Fuster and Tim Colonius.

6 'Modeling bubble clusters in compressible liquids', D. Fuster.

7 'A user's guide to MPI', P. Pacheco.

8 'Instructions for the use of the grid generator and DNS solver', S. Pirozzoli.

Appendix A

Code Instructions

This code was created by Daniel Fuster in the California Institute of Technology. Here there is a short manual in order to do simple simulations. You can find more information about this code in [3], [7] and [8]. First of all is important to know that the code works always without units. There are three different reference values to achieve this, and are:

Length	l_c	1 m
Water density	ρ_{ref}	1000 kg/$m3$
Speed sound	c_{ref}	1500 m/s

For example, for introducing a pressure is needed to do this:

$$P_{code} = \frac{P_{real}}{c_{ref}^2 \rho_{ref}} \tag{A.1}$$

A.1 Isolated bubble

The code is implemented in Linux so the commands are only useful for this operative system.

1.rm -rf data

You have to delete the old folder of results.

2.mkdir data

That command creates a new folder.

3.gridgen Lx Ly Xc Yc 0

With this you create the mesh with the code is going to solve the simulation, being:

Lx, Ly : x and y distance.

Xc, Yc : center coordinates.

0 : uniform mesh.

4.hump

for running the program.

5. cp general.inp data

For copy the file into data.

6. cd data

   To enter inside of the folder data.

7. runextractvtk.sh # ti tf dt

   We use this command for create the files that Paraview is going to use for visualize the solution, being:

   #: number of processors.

   ti: initial time.

   tf: final time.

   dt: differential time.

8. cd vtr

   To enter in the folder where you have created the files.

9. paraview

   for view the simulation graphically.

10. cd ..

11.extractbubbles # > bubble#.dat

   You can create a file with the information of a bubble, being:

   #: number of the bubble that you want to extract.

12.gnuplot

   That command allows you to open a graphic drawing tool.

13.plot "./bubble#.dat" using *:*' with lines (points), being:

   * and *' the columns that you want to graph in x and y respectively.

A.2 Clusters

Now that singles bubbles have been explained let's focus on how to work with clusters.

A.2.1 How to create a cluster

The first difficulty is the creation of the cluster. The way to make them is like this:

1. cd input

2. vi bubblegen.inp

   This file contains the information of the cloud. You can find these parameters inside:

   β

   R_{c_x}

   R_{c_y}

   R_{c_z}

   $R_{averaged}$

   R_{min}

R_{max}

σ

3.a) cilinderbubblegenerator > bubbles.dat

That creates a cylinder of bubbles and safe them into the file bubbles.dat.

3.b) elipserdmbubblegenerator

With this you can create an ellipse. The bubbles are saved in the file fort.1 by default.

A.2.2 How to study specific points

For study the solution is important to focus and extract the information of some specific points. The way to do it is this:

1. cd input
2. vi probe.inp

   # : number of points that you want to study

   x y z
3. cd ..
4. cd data
5. extractprobes h * # > probe#.dat

   You extract the points that you have studied, being:

   *: the number of the processor.

   #: the number of the point that you want to extract.
6. sp "./bubblestats 0.dat" u *:*':*"" with points.

   For making a 3D graph.

   being:

   * , *', *" the columns that you want to graph in x, y and z respectively.