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A Fairness-Oriented Interference-Balancing Scheme for Cooperative Frequency Hopping Ad Hoc Networks

Master Thesis

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Erklärung

Ich versichere hiermit, dass ich die vorliegende Arbeit selbständig und ohne unzulässige fremde Hilfe angefertigt habe. Die verwendeten Literaturquellen sind im Literaturverzeichnis vollständig zitiert.

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Abbreviations

Acronym	Meaning
AHC	average Hamming correlation
AP	access point
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase-shift keying
CDF	cumulative distribution function
CDMA	code-division multiple access
CS	coordinated scheme
DES	Data Encryption Standard
DSSS	direct sequence spread spectrum
e.g.	*exempli gratia*
FFHSS	fast frequency hopping spread spectrum
FH-CDMA	frequency hopping code-division multiple access
FH-SSMA	frequency hopping spread spectrum multiple access
FHSS	frequency hopping spread spectrum
FOS	fairness-oriented scheme
FSK	frequency-shift keying
GF	Galois field
GSM	Global System for Mobile Communications
i.e.	*id est*
ISI	intersymbol interference
LAN	local area network
LCM	least common multiple
MAC	medium access control
MHC	maximum Hamming correlation
OFDM	orthogonal frequency division multiplexing
OP	outage probability
OSI	Open Systems Interconnection
PDF	probability density function
PHY	physical layer of the OSI model
PMF	probability mass function
PN	pseudonoise
PPP	Poisson point process
PSD	power spectral density

Acronym	Meaning
PSK	phase-shift keying
QoS	quality of service
QPSK	quadrature phase-shift keying
RS	random scheme
RS code	Reed-Solomon code
SFHSS	slow frequency hopping spread spectrum
SINR	signal to interference plus noise ratio
SIR	signal to interference ratio
TDMA	time division multiple access

“Nothing is to be preferred befor justice.“

Socrates

1 Introduction

1.1 Motivation

It may quite frequently occur in an ad hoc network that the density of users in a given area becomes relatively high during some period of time. At the same time, network resource scheduling is not done regarding these moments with a higher load, but it is done according to average parameters of the network in order to have a trade-off solution that reaches the highest possible *quality of service* (QoS) while keeping the assigned resources for it as close to be fully exploited as possible. As a result, it may happen that the number of users in a network willing to transmit information is higher than the available resources in that network.

On the other hand, in ad hoc networks, the *frequency hopping spread spectrum* (FHSS) multiple access technique is widely used in the radio interface. In FHSS, the total available bandwidth is partitioned into a certain number of frequency channels having the same bandwidth. In time domain, there is also a division into time intervals. This thesis is focused on the case in which the time intervals have the same duration, called *hop period*, and specifically on the case where the hop periods for all transmitters are synchronous. Then, in every hop period, every transmitter is been assigned some of those channels to transmit. It means that every transmitter will transmit through some frequency channels, and after the hop period, the transmission will hop into some other channels; and this will happen recursively after every hop period. When, in every hop period, every assigned frequency channel has been assigned to one (and no more) transmitter, then, there is *orthogonality* among users or, in other words, the *hopping sequences* or *codes*, according to which channels are assigned to each user in every hop, are *orthogonal*.

When the hopping sequences are orthogonal, there is no interference among the users of the system because no transmitter coincides with any other in the same frequency channel at the same time (namely, there are no collisions). Orthogonality can always hold when the number of transmitters is less or equal to the number of available frequency channels in the FHSS system. However, when this condition is no longer satisfied, i.e., when there are more transmitters that may simultaneously transmit than available frequency channels, orthogonality is no longer preserved and, so, interference occurs unavoidably. This is likely to happen in a scenario of an eventual increase of the spatial density of the nodes within a certain region, as the one described above.

When this situation takes place, then some questions concerning the design of the hopping sequences arise: Assuming that interference among transmitters is impossible to avoid, which transmitters will collide with which ones? How will they collide along both time and frequency? How can the hopping sequences be designed in order to guarantee a *fair* or, more precisely, an *equitable* distribution of the interference across all users, while maintaining the minimum possible number of collisions? This thesis aims to answer all these questions, specially the last one, which sumarises to a large extent the main goal of the thesis.

1.2 Overview of ad hoc networks and frequency hopping spread spectrum

1.2.1 Ad hoc networks

A wireless ad hoc network is made up of some devices that spontaneously set links among them during some time. According to [1], "an ad hoc network is a collection of autonomous nodes that communicate in a decentralized fashion without relying on a pre-established infrastructure or on a control unit." This means that nodes do not need an *access point* (AP) or a *base station* (as in mobile cellular networks) to communicate, but they forward information to each other. Moreover nodes can move, so, the *topology* of the network can change. As a consequence, ad hoc networks are one of the types of network where QoS is more difficult to guarantee because the network state has to be dynamically estimated. Guaranteeing a certain level of QoS has to do directly with the resources allocation. This fact represents a challenging problem and it has motivated an extensive research in the recent years. In this context, the present thesis tackles the interference balancing across nodes according to its frequency allocation from the point of view of fairness.

Wireless ad hoc networks first appeared in the decade of the 70's and they have evolved progressively until the present time, specially rapidly during the recent years. As asserted in [2], not needing pre-existing infrastructure make ad hoc networks attractive for some reasons: they are very suitable for the wide current assortment of wireless personal devices to communicate, for home applications, in schools, hotels or conferences; they are ideal for emergency situations where normal communications have been interrupted, for instance in disaster areas (after an earthquake or a hurricane); they are a good alternative for developing countries where there is not an important infrastructure; and the fact of not using infrastructure also makes wireless ad hoc networks have lower costs.

1.2.2 Spread spectrum signals and frequency hopping spread spectrum

As stated in [29], spread pectrum (SS) signals, first created purely for military purposes, have the main characteristic that the total bandwidth they use for digital information transmission is several times larger than the individual user rate of transmitted information through this bandwidth. This loss of spectral efficiency provides on the other hand some desirable benefits when the channel conditions are adverse in terms of interference, detections or interceptions. Spread pectrum signals were originally designed for mitigating or suppressing the interference effects of a jammer or a broadband interferer; for making the signal undetectable by hiding it under the noise level by means of a spectral expansion of the energy (this is for the case of DSSS); and for avoiding interceptions of unintended listeners. There are two basic spread spectrum techniques used in digital communications systems: the *direct sequence spread spectrum* (DSSS) modulation and the *frequency hopping spread spectrum* (FHSS) modulation.

Spread spectrum also allows *multiple-access* (MA) communications, i.e., several users are multiplexed along the same total bandwith at the same time. This configuration is known as *code-division multiple access* (CDMA). However, despite of this naming, the multiplexion in code (that is, explioting the code domain diversity) just happens, strictly speaking, in the case of DSSS. Whereas in FHSS there is time domain overlapping but frequency domain multiplexing among all users in a system, in DSSS there is overlapping in both time and frequency domains, and demultiplexing the signal of every user can be achieved just by code domain diversity. In spite of this fact, both types of spread spectrum techniques use *pseudorandom* or *pseudonoise* (PN) sequences (or codes), thanks to which they can exhibit so desirable properties like the ones exposed above. Besides, the mentioned loss of spectral efficiency inherent in a single-user spread spectrum system may be largely reduced in the multiple-access configuration.

Apart from being robust against jamming interference, detections and interceptions (this last case also known as *eavesdropping*), and apart from allowing multiple-access, spread spectrum signals display a good performance against either *intersymbol interference* (ISI) or *cross-interference* among the users of the system. The intersymbol interference is due to the multipath propagations in the channel, that is, the addition to the transmitted signal of scaled and delayed replicas of it. By means of a RAKE receiver, the spread spectrum technique allows the coherent detection when multipath components occur. All these desirable features make the spread spectrum system attractive not only in the military field, where it achieved a widespread use, but it also occuppies a prominent position in civil applications. For instance, in communications, either the second and third generations of cellular systems, on the one hand, and the second generation of wireless LANs, on the other, are based on spread spectrum. Other examples, in which specifically FHSS is used, are Bluetooth (which operates in the unlicensed $2,4$ GHz band with 80 frequency bands of 1 MHz width) or GSM (where there is a combination of *time division multiple acces* (TDMA) techniques and *slow frequency hopping* SS (SFHSS)); see references [3] and [30]. But spread spectrum is also used in other fields such as location and timing acquisition, because of its wide bandwidth.

1.3 Related work: a comparative overview

Interference in wireless ad hoc networks is a crucial capacity-limiting factor. It has direct effects in significant indicators of the transmission efficiency of a network such as the overall *throughput*, the *spectral efficiency* or the *outage probability*. In this context, extensive research with the aim of enhancing network efficiency by mitigating interference has been done in recent decades, specially intense in recent few years. The approaches and developed strategies are multiple and diverse, mostly focusing on the PHY and MAC layers.

1.3.1 Hopping sequences design

In FHSS wireless networks, a key element which determines interference situation is the design of the hopping sequences. Regarding this point, there is much developed theory in different approaches. One of these approaches is based on the *Hamming correlation* and specifically on the *average Hamming correlation* (AHC) and on the *maximum Hamming correlation* (MHC) as important performance measures. The Hamming correlation between two hopping sequences is the number of *hits* between these sequences for a given shift between them. See the definition of the Hamming correlation, as well as the definition of the AHC and of the MHC, for instance, in [4]. Many contributions present new families of hopping sequences displaying some desirable properties in terms of AHC and MHC. In [5], a method of construction of codes with good properties in the MHC for the atucorrelation and for the cross-correlation based on cubic functions and *finite fields theory* or *Galois fields theory* is presented. This solution is first extended in [6] to a scheme with combinations of cubic, quadratic and linear generating functions over finite fields $GF(p)$. This scheme is further extended in [7] to the general case of polynomial generating functions over finite fields $GF(p)$. This allows much larger families of hopping sequences than the previous two solutions and also outperforms them considerably in terms of the trade-off between maximum Hamming auto- or cross-correlation and the familiy size. In [4] some frequency hopping sequences sets with optimal AHC and low MHC are presented as well as the construction of some of them based on cyclotomy. In [8] optimal sequences also based on cyclotomy are derived. Other methods for the construction of sequences with optimal or good Hamming correlation properties are derived in [9], [10], [11], [12] and [13]. The two last of these papers are based respectively on the ideas of *few-hit zone* and *no-hit zone*, and the last one also on the concept of *cognitive radio*.

Nevertheless, all these strategies that aim to obtain hopping sequences with good Hamming correlation properties, in fact, do not tackle the problem of achieving a fairness-oriented interference distribution. Although the MHC guarantees a maximum number of collisions for every user to some other user, the Hamming correlation approach does not pay attention on the total number of collisions to each of the other users that a given user may experience. Therefore, it is not investigated neither if this number of total collisions during a hopping sequence is equitably balanced across all users. Instead, the Hamming cross-correlation and autocorrelation give rather insight in terms of inter-symbol interference (ISI) in multipath propagation scenarios, because they indicate the number of hits between two delayed sequencies for a given delay. Moreover, the Hamming correlation approaches are maybe more meaningful in *multihop-networks*, in which nodes can interfere only subsets of nodes in their coverage area, which is not the case studied in the present work, in which all nodes see each other, as it will be specified in section 3.1. Moreover, guaranteeing upper bounds for MHC and AHC, as said, do not guarantee an upper bound to the number of collisions that every user will bear per hopping sequence and, so, it is also unknown how many collisions will occur across the network in every hop. Therefore, in the case of interest in this work of having more transmitters in the network than available frequency channels, all the reviewed approaches may be suboptimal in terms of interference level because more total collisions per hop than the minimum possible may take place. Another point to bear in mind is the fact that hopping sequences are not synchronous in general in the mentioned literature, whereas the scheme presented in this work needs synchronous FHSS.

On the other hand, some research focused on the analysis of the randomness properties of the hopping sequences also has been done. As said in section 1.2.2, it is of high importance for sequences to achieve a random appearance in order to avoid jamming or interception. [14] and [15] are two examples on this approach. They describe two methods based on *chaotic map* for designing hopping sequences whith good correlation properties while being *pseudorandom* sequences. Another example of designing frequency hopping sequences with a random appearance while having good correlation properties is given by [16]. This example is based on a kind of cyclic code, the *RS code*, related to GF theory. In [17], the robustness against interceptions or jamming interferers is achieved by means of the *triple Data Encryption Standard* (DES) block cipher in the design of the hopping sequences. As a remark, it is convenient bearing in mind that the schemes already mentioned in the previous paragraph also derive pseudorandom hopping sequences, but their statistical features are not studied, but only the Hamming correlation properties.

In contrast to the above mentioned approaches, [18] and [19], both based on *orthogonal frequency division multiplexing* (OFDM), present and analyse a scheme which allows a collision-free scenario while preserves the inherent anti-jamming and anti-interception features of conventional FHSS sytems. In other words, with this scheme, orthogonal hopping sequences are designed while maintaining pseudorandomness. Thus, if the aim of the approaches mentioned in the previous paragraph is to find pseudorandom sequences with good correlation properties, the approach of the two latter examples aims to find pseudorandom sequences whithout any collision among them. This means that with this scheme there cannot be more transmitters in the network than available frequency channels, as explained in section 1.1. In contrast, in all other approaches above mentioned there can be more transmitters than available frequency channels; as many transmitters as available hopping codes that every approach provides. However, the number of total collisions per hopping sequence that each user may experience can be extremely high if the number of users is high, and at the same time this number of collisions can be considerably unbalanced across all users. On this account, all those approaches are not valid solutions for the problem stated in section 1.1. Furthermore, for the case when the number of transmitters in the network is lower than the available frequency channels, [18] and [19] give the optimal solution because there is no interference. All the other above mentioned solutions are in general suboptimal, because the derived sequences are not necessarily orthogonal. This fact implies a decrease of spectral efficiency. On the other hand, also regarding the spectral efficiency, in [20] a FHSS new modulation which also encodes the user information with the hopping sequences is described. This scheme allows either increasing the total number of transmitters in a network or their transmission rate, keeping a collision-free scenario.

1.3.2 Multiple access protocols for wireless networks performance enhancement

Besides, there is much literature that deals with increasing of the throughput in wireless networks by designing medium access protocols or schedulings aimed to reduce or to remove the mutual interference among users. In [21], adaptive antennas are exploited in order to remove interference and, therefore, with the same number of frequency channels more transmissions can be carried out successfully. The approach in this paper uses concepts of *graph theory* to represent the network topology. In [22] and [23], two *medium access control* (MAC) protocols based on collision-avoidance handshakes are presented and analysed. In both protocols a dedicated common channel listened by all users is set in order to exchange synchronization information. The approach of these solutions differs from the one of this thesis in the point that they avoid interference by making users wait if necessary using higher-level mechanisms (MAC protocols). Finally, a general perspective of proposed MAC protocols for enhancing the performance of multiple acces wireless networks is given by [24]. In this review, protocols are classified according to some criteria such as how many transcievers per host are required, how many rendezvous are required, who selects the channel to communicate, the use of a common control channel, the type of synchronisation, the use of carrier-sensing, etc.

1.3.3 Distributed resource scheduling strategies for wireless networks

As seen in section 1.3.2, in interference-limited multiple acces ad hoc networks, interference allocation solutions require distributed algorithms and node cooperation. As a consequence, these solutions may add a large *overhead* in the system which implies a decrease of the network efficiency. In [25], the complexity of two proposed scheduling algorithms is surveyed. At the expense of being suboptimal (but arbitrarily close to the optimal solution), these algorithms manage to considerably reduce the computational complexity. On the other hand, in [26] some strategies for resources allocation, adaptive transmission and power control are presented and analysed in terms of the overall network capacity as well as in terms of the computational complexity and receiver-to-transmitter feedback. In some cases, the given conclusions for cellular networks are also valid for ad hoc networks if each cell is considered to contain only one node. Key issues for this thesis such us fairness in network scheduling or the consideration of the *signal to interference plus noise ratio* (SINR) as a significant measure parameter are also regarded.

1.3.4 Conclusion

As a conclusion, it has been seen that the existing research on the topic of multiple access FHSS ad hoc networks does not focus specifically on balancing interference across the users. Fairness is mentioned in few cases and no fairness-oriented approaches have been developed to date. Specifically, in the reviewed literature on hopping sequence design, the goal is to obtain good Hamming correlation properties, which is not directly connected to the problem of interest in this work, as discussed in section 1.3.1. Neither in the reviewed research on wireless networks protocols for efficiency enhancement the above posed problem is raised.

1.4 Abstract

This work tackles the problem stated in section 1.1 by finding a method for designing frequency hopping sequences that achieve at the same time a fair interference allocation across all users and an optimal interference reduction. The proposed scheme is optimal in terms of interference level because the overall number of collisions that take place in every hop is the minimum possible. At the same time, this fairness-oriented approach distributes the collisions among nodes in such a way that the variance of the interference power seen by the nodes is considerably reduced. Thereby, this scheme achieves a much more equitable interference balancing across the nodes. The results displayed by the fairness-oriented scheme in terms of fairness are much better than those ones displayed by the two reference studied schemes for both slow and fast frequency hopping.

2 Preliminaries

2.1 Probability theory

In this section, some basic elements on probability theory that will be used in the analysis section are provided. The given definitions are taken from [31] and [32].

Continuous random variables

Cumulative distribution function (CDF): Let $X \in \mathbb{R}$ be a random variable. The *cumulative distribution function* of X is defined as

$$F_X(x) \triangleq \mathbf{P}\left\{X \leq x\right\} \ .$$

Probability density function (PDF): If $X \in \mathbb{R}$ is an absolutely continuous random variable and $F_X(x)$ is its CDF, the *probability density function* of X is defined as

$$f_X(x) \triangleq \frac{\partial}{\partial x} F_X(x) \ .$$

i-th moment of a continuous random variable: Let X be a continuous random variable and let $f_X(x)$ be its PDF. Then,

$$m_i \triangleq \int_X x^i \, f_X(x) \, dx$$

is the *i-th moment* of X, with $i = 1, 2, \ldots$, if the integral is absolutely convergent.

i-th central moment of a continuous random variable: With the same assumptions than in the previous definition,

$$\mu_i \triangleq \int_X (x - m_1)^i \, f_X(x) \, dx$$

is called the *i-th central moment* of X, with $i = 1, 2, \ldots$, if the integral is absolutely convergent.

In this thesis, the first moment, m_1, known as the *expectation*, and the second central moment, μ_2, known as the *variance*, will play a central role, and they will be denoted as $\mathbf{E}\{\cdot\}$ and $\mathbf{Var}\{\cdot\}$, respectively. This assertion concerns also discrete random variables.

Expectation of a function of a random variable: Let X be a continuous random variable with PDF $f_X(x)$ and $g(X)$ any function of X. Then,

$$\mathbf{E}\{g(X)\} = \int_X g(x)\, f_X(x)\, dx \ .$$

Discrete random variables

A discrete random variable X is determined by the values it can take $x_1, x_2, \ldots, x_n$ (whose number is finite or countable) and by the probabilities $p_k = \mathbf{P}\{X = x_k\}$ with which X takes these values.

Then, for a discrete random variable, the definition of its CDF, given above, leads to the formula

$$F_X(x) = \sum_{x_k \leq x} p_k \ .$$

Probability mass function (PMF): If X is a discrete random variable with possible values $x_1, x_2, \ldots, x_n$, its *probability mass function* is

$$f_X(x_k) \triangleq \mathbf{P}\{X = x_k\} \ .$$

i-th moment of a discrete random variable: Let X be a discrete random variable, then,

$$m_i \triangleq \sum_k x_k^i\, p_k$$

is the *i-th moment* of X, with $i = 1, 2, \ldots$, if the series converges absolutely.

i-th central moment of a discrete random variable: For the same discrete random variable X as in the previous definition,

$$\mu_i \triangleq \sum_k (x_k - m_1)^i\, p_k$$

is the *i-th central moment* of X, with $i = 1, 2, \ldots$, also if the series converges absolutely.

Bernoulli distribution: A discrete random variable X is said to be *Bernoulli distributed* if it takes two values, x_1 and x_2, with probabilities p_1 and $p_2 = 1 - p_1$, respectively.

Binomial distribution: A discrete random variable X is said to be *binomially distributed with parameters* n *and* p if it takes the possible values $k = 0, 1, \ldots, n$ with probabilities

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \ . \tag{2.1}$$

Expectation of a product of two random variables: If two random variables X and Y are independent, then

$$\mathbf{E}\{X\,Y\} = \mathbf{E}\{X\}\,\mathbf{E}\{Y\} \ . \tag{2.2}$$

This result is valid either for continuous or discrete random variables.

2.2 Stationary Poisson point processes

The reader will find in this section supplementary background information on sationary Poisson point processes. This topic concerns the network model used in this work. However, the information provided in this section is more than the strictly necessary for understanding the following sections and just some items are later used. The aim of including here this information is to provide some introductory basics on the topic of stationary Poisson point processes and Neyman-Scott point processes. Most of the theory in this section has been taken from [33]. In the field of stochastic geometry, a *measure* is a real-valued function, defined on families of sets, which has the properties of additivity and positivity. Let A be a certain set belonging to a compact geometrical body in the space, and A_1 and A_2 two disjoint subsets of A such that $A = A_1 \cup A_2$; that is, A_1 and A_2 form a partition of A. Then, the set function φ is said to be *additive* if $\varphi(A) = \varphi(A_1 \cup A_2) = \varphi(A_1) + \varphi(A_2), \quad \forall A_1, A_2$. It is useful mathematically that this kind of functions display a stronger property, the σ-*additivity* property. A set function φ is σ-additive when, given a set A which can be divided into a countable union of disjoint subsets, then

$$\varphi(A) = \varphi\left(\bigcup_i A_i\right) = \sum_i \varphi(A_i). \tag{2.3}$$

It can be shown that measures are naturally defined in some systems of sets known as σ-*algebras.* A σ-algebra is a system $\mathcal{X}$ of subsets of a ground set X that satisfies

1. $X \in \mathcal{X}$,
2. if $A \in \mathcal{X}$, then $A^c \in \mathcal{X}$, where A^c is the complement of A,
3. if $A_1, A_2, \ldots \in \mathcal{X}$, then $\bigcup_{K=1}^{\infty} A_k \in \mathcal{X}$.

There is a very important example of σ-algebra, which is interesting for describing the network model presented in this thesis, known as *Borel sets*. The family $\mathcal{B}^n$ of Borel sets of $\mathbb{R}^n$, an n-dimensional Euclidean space, is defined as the smallest σ-algebra on $\mathbb{R}^n$ that contains all the open subsets of $\mathbb{R}^n$. In other words, it contains all the subsets of $\mathbb{R}^n$ that can be constructed from open subsets by means of the basic set operations (countable union, countable intersection and relative complement) and limits. Borel sets constitute a very large σ-algebra and since it includes all closed sets, it includes necessarily all compact sets. On the other hand, a set X, together with a σ-algebra $\mathcal{X}$ of subsets of X, form a *measurable space*. At the same time, a function $f : X \to \mathbb{R}$ is defined to be $\mathcal{X}$-*measurable* if for all Borel sets B belonging to $\mathcal{B}^1$ the inverse image of B, $f^{-1}(B) = \{x \in X \mid f(x) \in B\}$, belongs to the σ-algebra defined by X.

As stated in [33], a random *point process* (or *point-pattern*) Φ can be regarded in two ways: from one perspective, as a random set $\Phi = \{x_1, x_2, \dots\}$; from another, as a random counting measure. From this second perspective, the random measure $\Phi(B)$ applied to a Borel set B is equal to the number of points of the point process Φ within the set B. This measure is a locally finite σ-additive Borel measure whenever the random point pattern is locally finite (and it will be so considered in this thesis). Also because of having locally finite random point-patterns the set Φ will always be a closed set (this is a direct consequence of the property of *completeness* of real numbers).

A *stationary* Poisson point process (stationary PPP) Φ is characterised by the two basic following properties:

1. The statistical distribution of the random variable *number of points of* Φ in a bounded Borel set B has a Poisson distribution, with mean value $\lambda\nu_n(B)$, where λ is an scalar constant and $\nu_n(B)$ is the *Lebesgue measure*.

   The Lebesgue measure is a locally finite measure defined in general in the measurable space formed by the set $\mathbb{R}^n$ and the σ-algebra $\mathcal{B}^n$ as

   $$\nu_n(B) = (v_1 - u_1) \cdot \ldots \cdot (v_n - u_n), \tag{2.4}$$

   which would be the hypervolume of B, when $B = [u_1, v_1] \times \ldots \times [u_n, v_n]$. From this result, the ν_n-measure can be also deduced if applied in the particular case of sets of elementary geometrical bodies such as spheres, cylinders, toruses, and so on. If $n = 3$, the Lebesgue measure indicates their volume, and if $n = 2$ it indicates their area. In this thesis, the studied networks are all assumed to be in a flat surface, so they are contained in the plane $\mathbb{R}^2$. Thus, $\nu_n(B)$ will always represent the area of B.

   On the other hand, the Poisson probability density function (PDF) of Φ is the discrete function

   $$f_\Phi(k) = \frac{\mu^k e^{-\mu}}{k!}, \qquad k = 0, 1, 2, \dots, \tag{2.5}$$

where

$$\mu = \lambda \nu_n(B) = \mathbf{E}\{\Phi(B)\} \tag{2.6}$$

is the mean value of Φ for all bounded borel sets B. Hence, if $\nu_n(B)$ is the area of B, λ is the expected density of points of Φ in B. λ is the characteristic parameter of the stationary Poisson point process and it is known as its *density* or *intensity*. In the following it is assumed that λ is a positive and finite real number.

2. The second fundamental property of a stationary Poisson point process is called *Independent scattering* property or also *completely random* or *purely random* property. It states that the j random variables defined as the number of points of Φ in j disjoint Borel sets are independent random variables, for any j.

Some other properies of stationary Poisson point processes which are derived from these two fundamental properties are:

I. Properties 1 and 2 have as a consequence that a stationary Poisson point process Φ of intensity λ can be completely determined by λ.

II. Another direct consequence of properties 1 and 2 is that for a stationary Poisson point process Φ, since, for j disjoint Borel sets $B_1, \ldots, B_j$, the corresponding random variables $\Phi(B_1), \ldots, \Phi(B_j)$ are independent Poisson random variables with the corresponding means equal to $\lambda\nu_n(B_1), \ldots, \lambda\nu_n(B_j)$, then, the following joint probabilities are

$$\mathbf{P}\{\Phi(B_1) = k_1, \ldots, \Phi(B_j) = k_j\} = \frac{\lambda^{k_1+\ldots+k_j}(\nu_n(B_1))^{k_1} \cdot \ldots \cdot (\nu_n(B_j))^{k_j}}{k_1! \cdot \ldots \cdot k_j!}\, exp\left(-\lambda \sum_{i=1}^{j} \nu_n(B_i)\right), \tag{2.7}$$

which is the product of the individual probabilities.

III. This property is known as *motion-invariance*. A point process is motion-invariant if it has the properties of *stationarity* and *isotropy*.

a. A point process $\Phi = \{x_i\}$ is a stationary point process if the translated process $\Phi_\delta = \{x_i + \delta\}$ has the same distribution than $\Phi \quad \forall \delta \in \mathbb{R}^n$.

b. A point process $\Phi = \{x_i\}$ is an isotropic point process if any rotated process $\mathbf{r}^\Phi = \{\mathbf{r}x_i\}$ has the same distribution than Φ.

Since the characteristic parameter λ is invariant under translation and rotation, it follows from properties number 1 and 2 that a stationary Poisson point process is stationary and isotropic, i.e., it is motion-invariant.

This property means that the distribution of the points remains statistically constant along the plane at any direction because the expectetd points density is the same everywhere, it does not depend neither on where the coordinates origin is placed nor on the orientation of the cartesian coordinates axes. This implies that, for instance, if a network cluster is defined within a circular area, in that cluster there will be circular simetry regarding the stationary Poisson point process that models the distribution of the points.

2.2.1 Neyman-Scott processes

As stated above, the Neyman-Scott point processes are a particular case of *Poisson cluster processes*. Poisson cluster processes are obtained by applying a *homogeneous independent clustering* to a stationary Poisson process. The *clustering* operation consists in replacing every point x of a given point process Φ_p by a cluster N^x of points. N^x are also point processes, each of them having a finite number of points. The points of Φ_p are known as *parent points* and the clusters' points are the corresponding *daughters* of every parent point. The result of the clustering operation is the *cluster point process* Φ

$$\Phi = \bigcup_{x \in \Phi_p} N^x. \tag{2.8}$$

Two considerations are generally assumed. The first is that points of different clusters do not coincide, i.e., $N^x \cap N^y = \varnothing$ if $x \neq y$. The second is that Φ is locally finite.

Consider now a parent point process Φ to be stationary with a density λ_p, and consider the clusters N^x to be of the form

$$N^{x_i} = N_i + x_i \qquad \forall x_i \in \Phi_p. \tag{2.9}$$

Since the clusters N^{x_i} form a family of independent, identically distributed, finite point sets with a distribution independent of the parent point process Φ_p, because so are the clusters N_i, this is a *homogeneous independent clustering*. Then, if the parent point process Φ, apart from being stationary, is also a Poisson process, the resulting process Φ is said to be a *Poisson cluster process*. Independently of the form of Φ_p, this new process Φ has, then, a density

$$\lambda = \lambda_p \bar{c} \tag{2.10}$$

where $\bar{c}$ is the mean value of the number of points in a cluster N_i, by means of which the clustering has been build.

If a Poisson cluster process Φ does not include the parent points but only the daughters, then, Φ is a *Neyman-Scott process.* A Neyman-Scott process is always stationary and if the scattering distribution of N_i is isotropic, then Φ is also isotropic.

3 System Model

3.1 Network geometry

The ad hoc netwok model used in this work is defined by a *binomial point process of* N_T *points*, denoted by $\Phi_{W(N_T)}$. A binomial point process consists of N_T independent points $\Phi_{W(N_T)} = \{\xi_1, \ldots, \xi_{N_T}\}$ which are uniformly distributed within a compact set $W \subset \mathbb{R}^n$. In oder words, when one fixes a number of points of a process to an arbitrary N_T and, at the same time, one distributes these points independently and uniformly in a bounded region, then the resulting process is a binomial point process. For the present model, the compact set W has been chosen to be a disk in the plane of radius R. An example of one realisation of a binomial point process distributed in a disk is given in Fig. 3.1.

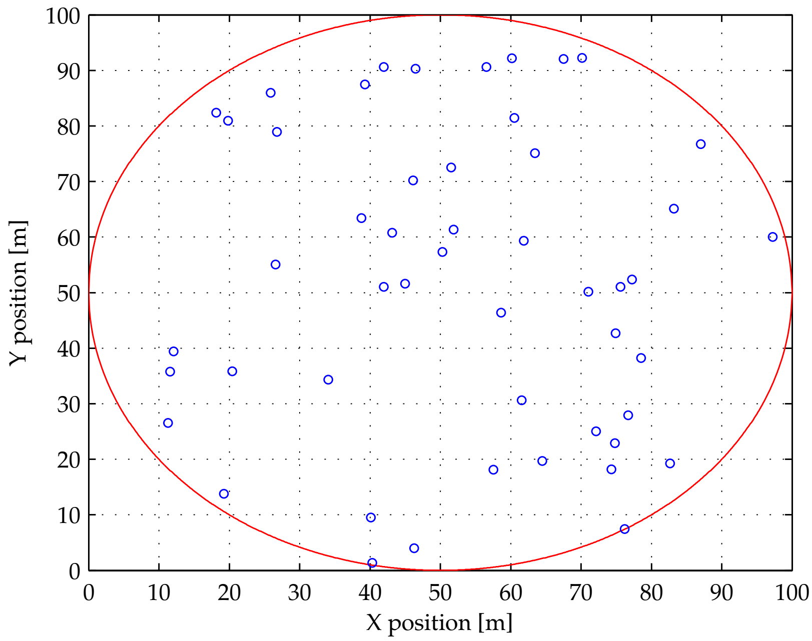


Figure 3.1: Network model, an example: $N_T = 50$ nodes distributed according a binomial point process throughout a circle with $R = 50$ m.

This network can be considered as a close neighborhood of a clustered network. A good example of point pattern to model clustered networks is a Neyman-Scott point process. The definition of a Neyman-Scott process is stated in 2.2.1 and an example is depicted in Fig. 3.2.

It can be observed that nodes are gathered in neighborhoods. Each of these neighborhoods

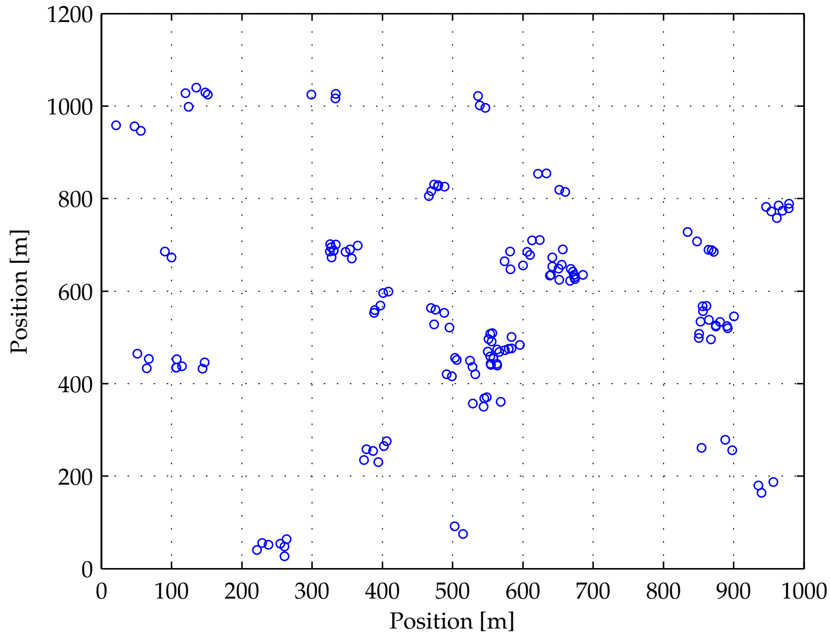


Figure 3.2: A realisation of a Neyman-Scott point process.

can be the ad hoc network depicted in Fig. 3.1, under two assumptions regarding the Neyman-Scott process Φ: First, The clusters' distribution is defined by a binomial point process of N_T points. An second, the density λ_p of points (nodes) of the parent Poisson point process Φ_p is sufficiently low so that the probability of the respective delimiting circles of two neighborhoods to intersect is negligible. In other words, it is assumed that the circles of any pair of neighborhoods never intersect and, so, they are isolated. Furthermore, it is assumed that λ_p is so low that the coverage areas of users of different neighborhoods do not intersect. The notation used in the previous lines is introduced in section 2.2.1.

3.2 Network topology

It is assumed that all nodes in the ad hoc network are placed within the coverage range of all other nodes and all of them have the same coverage radius. This means that the disk W has to have a radius R at least two times smaller than the coverage radius of the nodes. Fig 3.3 illustrates that this condtion has to be satisfied. This kind of ad hoc networks are called *single-hop* ad hoc networks. This means that all possible links among the nodes in the network do exist. Therefore, if the ad hoc network is represented in terms of a *communication graph* $G = (V, E)$, where $V = \{\xi_1, \ldots, \xi_{N_T}\}$ is the set of nodes, or *vertices*, and $E = \{e_{ij}\}, 1 \leq i < N_T,\ 1 < j \leq N_T,\ i < j$, is the set of links between these nodes, or *edges*, G is a *complete graph*. Fig. 3.4 displays an example of a complete graph.

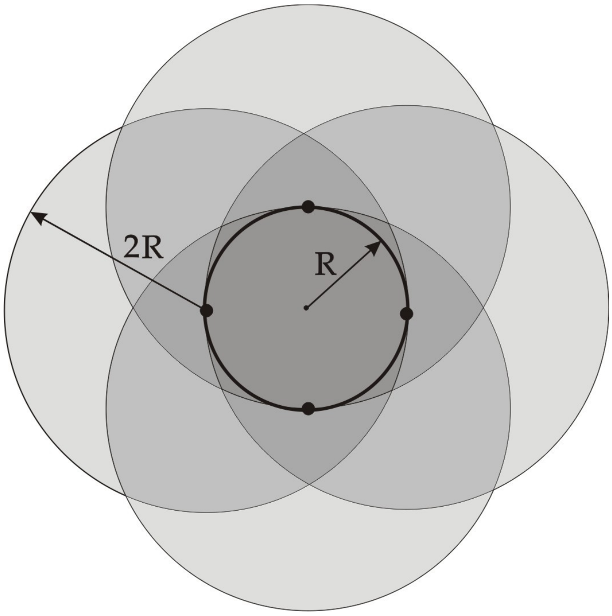


Figure 3.3: Nodes on the border of W can still see each other if their coverage radius is at least $2R$.

3.3 The frequency hopping spread spectrum system

In FHSS the total available bandwidth W is divided into M contiguous nonoverlapped frequency channels or slots with equal bandwidth B. Hence,

$$\frac{W}{M} = B \,, \tag{3.1}$$

where B is the inverse of the symbol period

$$B = \frac{1}{T} \,. \tag{3.2}$$

Thus, the set of central frequencies of every of these intervals is given by

$$\mathcal{F} = \{f_1, \ldots, f_M\} \,. \tag{3.3}$$

Every transmitter in the system will use one of these frequency channels during a time period T_h called *hop period*. After T_h, every user will hop into another frequency channel. Thereby, in every hop period, all users will hop synchronously into sets of frequency channels by shifting their carrier frequencies. This thesis focuses on synchronous FHSS. Hence, if

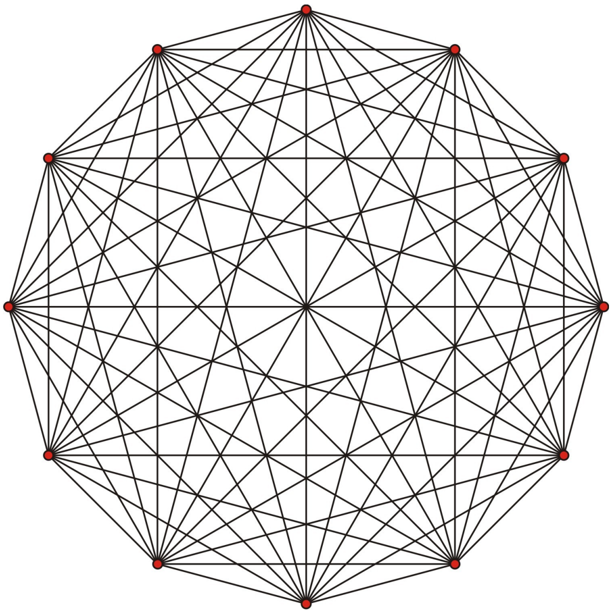


Figure 3.4: Complete graph with 12 vertices.

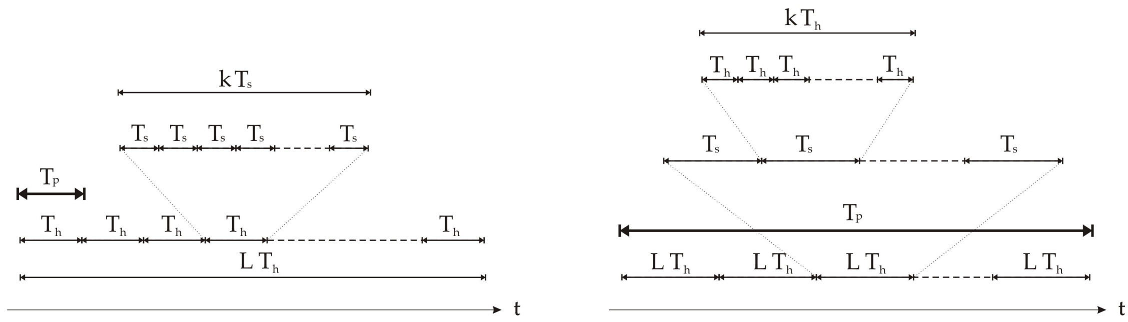
the total number of transmitters in the FHSS system is $N \leq M$, and in every hop all users' channels are different, then, from the overall FHSS received signal

$$s(t) = \sum_{i=1}^{N} s_i(t) \tag{3.4}$$

every user's signal $s_i(t)$ can be entirely recovered in the receiver (if the noise effect is neglected). When the previous condition occurs (regarding always that all users are synchronised) it is said that users have *orthogonal hopping sequences* or *codes*, and any collision among any system users will not happen. So, this case constitutes a *collision-free* scenario. Then, the performance of the multiple acces FHSS system (also known as FH-CDMA or FH-SSMA) from the point of view of one user is the same than in the single user FHSS system.

3.4 Slow hopping and fast hopping spread spectrum

As previously stated in section 3.3, at every hop period T_h, every user transmits through a certain frequency slot, and after this period the transmitting slot for every user is changed. According to [30], the symbol duration T_s of the transmitted signal can be either several times smaller than the hop period, $T_h = k\,T_s$, or several times times greater, $T_h = \frac{T_s}{k}$, for a certain $k \in \mathbb{N}$. These two situations are known, respectively, as *slow hopping* and *fast hopping* FHSS (also SFHSS and FFHSS). In this thesis, the reference time to analyse the performance of the studied schemes is the average packet transmission time T_p. Acoording to this reference time, the slow hopping case will take place when the hop period is equal to the packet transmission time, namely, $T_h = T_p$, and fast hopping will occur when T_p will be equal to the time of an integer number of hopping sequences, i.e., when $T_p = m\,(L\,T_h)$, where $m \in \mathbb{N}$. The reference time T_p has certainly the same value regardless of which of both modes has been chosen. Defining slow and fast hopping in this second manner is indeed equivalent to doing it according to the first manner if, for the case of fast hopping, the average number of hops per packet $L\,m$ is a multiple of the average number of symbols per packet $S_p = \frac{T_p}{T}$, as depicted in Fig. 3.5.



(a) Slow hopping.

(b) Fast hopping.

Figure 3.5: Slow and fast hopping cases, with the constant reference times T_p and T_s.

In the fast hopping case, frequency diversity is achieved for every symbol transmission because one symbol is transmitted through different frequency slots during time intervals of T_h. This fact provides to the fast hopping approach the benefit versus slow hopping that every symbol is protected against frequency-selective fading in the channel and spectral nulls. And, although in the present model the channel response is assumed to be flat over the whole bandwidth, in reality FFHSS outperforms SFHSS in this point.

On the other hand, the slow hopping case in turn is divided into two cases: *static slow hopping* and *dynamic slow hopping*. In static slow hopping, it is supposed that the number of interferer transmitters N out of the total number of users in the network N_T is not known and, so, a hopping sequence will be assigned to every of the N_T nodes, even if some of them are not transmitting in a certain moment. For the analysis, as it can be seen in section 6.1.3, the probability of one user to be a transmitter during a sequence transmission is denoted by p. Therefore $N = p\,N_T$. In contrast, in the case of dynamic slow hopping, it is already *a priori* known in every sequence transmission which N users out of the N_T are transmitting. The same thing happens in the case of fast hopping.

3.5 Transceiver architecture

In the following, it is assumed that all nodes in the ad hoc network have the same transmitter, with the same architecture and the same features (such as the sensitivity), and they all transmit with the same transmitted average[1] power P_T. Hence, given any two nodes in the network, it is the same saying that one is in the coverage range of the other than vice versa. Transmitters are supposed to have omnidirectional antennas. The used modulation is not specified in the system model. Any modulation compatible with FHSS such as FSK, PSK, BPSK, QPSK, etc. is valid.

3.6 Adaptability, distributed intelligence and cooperation

As asserted in [34], "[...] building efficient wireless ad hoc networks (hybrid or standalone) requires integration of several different techniques, and adaptivity. This leads fundamentally to issue that network must be *aware of the changes in its environment* and it must be capable of *adapting to changes in optimized and cooperative manner*". Indeed, in two of the three schemes studied in the following there is the need of adaptability.

Due to *mobility* of nodes, ad hoc networks have frequently a *dynamic topology*. This means that the architecture of the network (namely, which nodes -and links- form the network graph G) changes in time because nodes can move. As a consequence, neighborhoods can experience the arrival of new nodes while they can lose some of the former nodes. So, the number of nodes N_T in a neighborhood may not remain constant and, in particular, neighborhoods can even disappear while new neighborhoods can arise. This means that the static model presented in section 3.1 has to be updated periodically and it will be assumed to be valid during this period. Thereby, N_T has to be updated at every period. The resulting samples of updating the value of N_T for any of the neighborhoods correspond to a Poisson distribution.

The immediate question to solve is, then, to determine a suitable bound for the updating period. For such purpose, two considerations have to be previously regarded. Firstly, it is supposed to have a *low-mobility* ad hoc network, in which the speed of nodes is low. And secondly, it is considered that the change of position of nodes is so slow that during the time for transmitting several packets it can be neglected because all nodes in a neighborhood can still see each other forming a complete graph G. Thus, the two schemes that are adaptive out of the three studied schemes, the FOS and the CS (described in sections 5 and 4.2 respectively), have to adapt periodically. Between these two schemes, the FOS is the most restrictive concerning the updating period: so as to meet the objective of fairness interference allocation, there is a minimum time requirement, namely, at least a whole hopping sequence has to be completed befor the scenario changes. And the worst case, when a sequence transmission takes more time is in the slow hopping case. Hence, it is considered that the updating period is always greater than the hopping sequence transmission time.

The FOS and the CS will follow two adaptation algorithms, Ξ and Ω respectively. The necessary input data for Ξ and Ω will be recursively updated. These inputs are in both

[1]From here on, the word "average" will be omitted because in this work the instantaneous power is never considered.

cases which nodes belong to the ad hoc network (and implicitly how many they are) at every sampling time. Thus, at a cursory glance, it is clear that any protocols and techniques that implement Ξ or Ω need somehow the cooperation among nodes. Ξ or Ω are, evidently, *distributed algorithms*, involving some OSI model layers. Cooperation techniques and distributed algorithms in ad hoc networks are presented and widely discussed in [34] and in [35]. The *computational complexity* and the *overhead* costs will be decisive on the design of such protocols for them to be feasible in terms of loss of efficiency.

3.7 Medium access

It is assumed that nodes do not listen to the channel for transmitting but they simply transmit whenever they want. So, there are no collision avoidance mechanisms. When nodes transmit, they simply acces the medium according to the assigned frequency hopping sequences. Therefore, the medium acces method is Slotted ALOHA. Since ad hoc networks do not use preexisting infrastructure, there is not neither a base station nor an access point. The periodic adaptation of the FOS and the CS can be carried out by electing a central node or *cluster head*.

3.8 Propagation model and channel features

First, it is assumed to have the simplified case in which all nodes transmit the same average transmit power $P_{Ti} = 1W$, which is unitary. Under this assumption, the *power allocation problem* (see [26]), existent in the general case, is here circumvented. Second, given two arbitrary nodes ξ_i and ξ_j, the assumed model for the *power decay* or *path-loss* between them is

$$P_{Ri}(r) = P_{Ti}\Lambda(r) \ , \tag{3.5}$$

where

$$\Lambda(r) = \frac{1}{(r+\varepsilon)^{\alpha}} \ . \tag{3.6}$$

In (3.5), P_{Ti} is the average transmit power by user ξ_i, P_{Ri} is the average received power by user ξ_j from user ξ_i and $\Lambda(r)$ is the total channel decay factor. In (6.3), r is the distance between ξ_i and ξ_j, ε is a constant real number and α is the path-loss exponent, also constant, whose value can be chosen in the range $2 \le \alpha \le 5$, as indicated in [35]. If $\varepsilon = 0$, this propagation model is the same than the one presented in [35], which is widely accepted to be an accurate model close to reality. However, when $r < 1$ this model degenerates because it implies having a higher received power than the transmit power, and when $r \to 0$ the received power tends to infinity. So, this *far field* model does not fit for a transmitter too close to the receiver and, thus, the ε term has been added at the model. Furthermore, in the following ε is assumed to be equal to one, since this value gives the most realistic performance to the model by continuity, i.e.,

$$\lim_{r \to 0} P_{Ti}(r+1)^{-\alpha} = P_{Ti} \; . \tag{3.7}$$

And, at the same time, the chosen model approximates to the model proposed in [35] for large distances:

$$\lim_{r \to \infty} (r+1)^{-\alpha} - r^{-\alpha} = 0 \; . \tag{3.8}$$

With regard to α, this parameter indicates the channel conditions, which depend on the environment. When $\alpha = 2$, a free space propagation model (where the transmit power density decays only because of the propagation of the spherical wave itself) is implemented, whereas, when $\alpha = 5$, a very severe shadow fading channel (e.g., an urban scenario) is performed. α, as well as ε, is considered to have the same value for all users and, thus, it is supposed that the shadow fading in the direct line of propagation between two nodes is constant, it does not depend on the position of the nodes. In other words, it is assumed that the attenuation conditions for the direct path wave are homogeneous within the disk. Furthermore, it is assumed that these conditions are constant in time, so, the channel is not time variant. On the other hand, different spectral attenuation may occur for different frequency channels. However it is not considered in the present model, and all frequency channels are supposed to introduce the same attenuation, equal to 0 dB for simplicity. Apart from this, multipath fading will not be considered. It is assumed that the multipath propagation effects produce a delay on the replicas which is so much lower than the symbol duration T that it can be neglected. That is, the channel coherence time is lower enough than T, that the *inter-symbol interference* (ISI) can be omitted.

3.9 Problem formulation

As preliminarily exposed in section 1.1, when the number of users that may transmit $N \in \mathbb{N}$ in a wireless ad hoc network is strictly greater than the total number of frequency channels $M \in \mathbb{N}$ in the FHSS system that supports the radio interface, then, orthogonality is no longer held in users's hopping sequences. Hence, users may collide between them unavoidably in every hop, that is, in every hop some frequency channels will be perforce assigned to more than one user and, therefore, users will interfere each other. This is saying that, if $\mathcal{F} = \{f_1, \ldots, f_M\}$ is the set of M central frequencies of every channel (or *carriers*) and $\mathcal{D}_i = f_i \subset \mathcal{F}$[2] is the carrier assigned to transmitter i in a given hop, then, for any carrier allocation across the transmitters it will always happen that $\mathcal{D}_i \cap \mathcal{D}_j \neq \varnothing$ for some transmitters ξ_i, ξ_j, $i \neq j$. Thus, in every hop at least $2(N-M)$ transmitters will be involved in a conflict, i.e., at least $2(N-M)$ transmitters will be interfered. A transmitter is interfered by another transmitter when the first one transmits information to a given receiver through a certain frequency channel and the second one transmits through the same channel at the same time.

As a result of the carrier frequencies assignation across the nodes in every hop, and assuming from here on that every node has at its disposition only one frequency channel per hop, a carrier frequencies sequence $\{f_{\xi_i}^{(n)}\}$ will be assigned to every node ξ_i. n denotes the n-th hop in the sequence. The total sequence lenght L is its periodicity, namely,

$$\left\{f_{\xi_i}^{(n)}\right\} = \left\{f_{\xi_i}^{(n+L)}\right\} \qquad \forall n \; . \tag{3.9}$$

Let $\mathcal{H}$ be the set of all possible M^L hopping sequences and $\mathcal{A}$ the set of all possible transmitters. Then, the carriers assigning can be formulated as an application $\Sigma : \mathcal{A} \longrightarrow \mathcal{H}$. Likewise, the N assigned sequences to all nodes can be represented in an illustrative manner through matrix notation. The sequences matrix is

$$\mathcal{M} = \begin{bmatrix} f_{\xi_1}^{(1)} & f_{\xi_1}^{(2)} & \cdots & f_{\xi_1}^{(L)} \\ f_{\xi_2}^{(1)} & f_{\xi_2}^{(2)} & \cdots & f_{\xi_2}^{(L)} \\ \vdots & \vdots & \ddots & \vdots \\ f_{\xi_N}^{(1)} & f_{\xi_N}^{(2)} & \cdots & f_{\xi_N}^{(L)} \end{bmatrix} . \tag{3.10}$$

In $\mathcal{M}$ rows represent the carriers sequences of every user and columns represent the carriers allocations across all users in every hop. That is, the vertical direction represents the users dimension and the horizontal direction is the time dimension.

A lightening example of the posed problem is given by the simple case of an ad hoc network with $N=5$ transmitters with $M=4$ frequency channels in the FHSS radio interface. As seen, if transmitters in this network were up to 4, a collision-free design of the hopping sequences is possible:

$$\mathcal{M} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \\ f_2 & f_3 & f_4 & f_1 \\ f_3 & f_4 & f_1 & f_2 \\ f_4 & f_1 & f_2 & f_3 \end{bmatrix} . \tag{3.11}$$

[2] Note that whereas in the case $N \leq M$ more than one carrier in $\mathcal{F}$ can be eventually assigned to the same node, when $N < M$ it is assumed that every node will be assigned only one carrier. In general, the subset of the M_i subcarriers assigned to the i-th user in the n-th hop of the sequence is denoted by $\mathcal{D}_i = \{f_{n,i_1}, \ldots, f_{n,i_{M_i}}\}$.

In $\mathcal{M}$ in (3.11) every column has all frequencies of $\mathcal{F}$ without repeating any of them. Thus, there are no collisions (there is no interference) in any hop and the hopping sequences are said to be orthogonal. However, when a 5-th transmitter is added to the network, since in every hop all frequencies are already assigned to the other users, with 5 users there will be collisions unavoidably. In every hop at least $2(N-M)=2$ transmitters will be interfered.

Besides, a useful parameter for describing the situation in the network is

$$\gamma = \frac{N}{M}\,, \qquad 0 < \gamma \leq 2\,, \tag{3.12}$$

the quotient between the number of transmitters and the number of frequency channels.

4 Reference Schemes

In this section two representative schemes of the current solutions for the problem formulated in section 3.9 are presented. On the one hand, as seen in section 1.3, many strategies have been recently developed in designing FHSS sequences which hop in a random or pseudorandom manner. This approaches will be here represented by the scheme defined in section 4.1. In this scheme, nevertheless, in contrast to some reviewed pseudorandom schemes, the frequency channels are assumed to be independent and identically distributed with uniform distribution for all nodes in every hop. On the other hand, a simple approach to the problem would be to extend orthogonal hopping sequences to the case $N > M$ by copying already used sequences and assigning them to the new users in the network at the expense of seriously disserving only some nodes. These two schemes will be the reference schemes to be compared to the new scheme presented in section 5.

4.1 The random scheme (RS)

With this scheme a node pseudorandomly tunes to a frequency from $\mathcal{F}$ in every hop. For any node, in each hop, any frequency has the same probability to be used. Besides, for a given node in a certain hop, the used frequency is independent either on the frequencies used by the other nodes in the same hop or on which frequency has used the same node in the previous hops (there is no memory). Therefore, the probability that a given transmitter ξ_k tunes to an arbitrary $f_i \in \mathcal{F}$ for the n-th hop is

$$\mathbf{P}\left\{f_{\xi_k}^{(n)} = f_i\right\} = \frac{1}{M} \ . \tag{4.1}$$

This means that, in a given hop, one transmitter can be interfered by a number of transmitters between 0 and $N-1$. The discrete random variable N_I *number of interferers* to a given node in a given hop has a binomial distribution. Therefore, its PMF is equal to

$$f_{N_I}(k) = \binom{N-1}{k} \frac{1}{M^k} \left(\frac{M-1}{M}\right)^{N-1-k} , \qquad k = 0, \ldots, N-1 \ . \tag{4.2}$$

Thus, the expectation and the variance of N_I are given by

$$\mathbf{E}\left\{N_I\right\} = \frac{N-1}{M} \tag{4.3}$$

and

$$\mathbf{Var}\{N_I\} = \frac{(N-1)(M-1)}{M^2} \tag{4.4}$$

respectively.

Accordingly, with this scheme the allocation of the frequency channels will not be orthogonal in general, also for the case $1 < N \leq M$. In this case the probability of having an orthogonal distribution of carriers in one hop is

$$\prod_{i=0}^{M-1} \frac{M-i}{M}\ , \tag{4.5}$$

which tends to zero as M tends to infinity. The sequence length L is considered to be infinite. Thus it is assumed that there is no periodicity in the hopping pattern and, therefore, the RS is considered to have fully random sequences instead of pseudorandom sequences. Apart from that, this scheme is not adaptive, i.e., the assigning of sequences to the nodes does not depend on the number of nodes in the network. On the contrary, the strategy is the same whether $1 < N \leq M$ or $M < N \leq 2M$. Consequently, this approach gererates few overhead because there is no need of cooperation between nodes, neither to detect new nodes within the area of the ad hoc network nor to distribute the hopping sequences with coordination.

4.2 The coordinated scheme (CS)

By contrast to the RS, the CS distinguishes between the cases $1 < N \leq M$ and $M < N \leq 2M$. When $1 < N \leq M$ nodes are assigned the carrier frequencies orthogonally. It is not relevant for this thesis how this scheme reaches orthogonality. As an example, one simple solution would be to generate a sequence $\{f_{\xi_1}^{(n)}\} \in \mathcal{H}, \quad 1 \leq n \leq M$, of length M for user ξ_1 which would consist, for instance, in putting all the carriers ordered

$$\left\{f_{\xi_1}^{(n)}\right\} = \left\{f_{\xi_1}^{(1)} = f_1, f_{\xi_1}^{(2)} = f_2, \ldots, f_{\xi_1}^{(M)} = f_M\right\}. \tag{4.6}$$

Then, the assigned sequence to the second user ξ_2 would be $\{f_{\xi_1}^{(n)}\}$ shifted one position, and the assigned sequence to the third user ξ_3 would be $\{f_{\xi_1}^{(n)}\}$ shifted two positions, and so on. Thus, for the k-th user, the assigned sequence would be

$$\left\{f_{\xi_k}^{(n)}\right\} = \left\{f_{\xi_1}^{(n+(k-1)) \pmod M}\right\}. \tag{4.7}$$

The matrix representation of the sequences of this solution then would be

$$\mathcal{M} = \begin{bmatrix} f_1 & f_2 & f_3 & \cdots & f_{M-1} & f_M \\ f_2 & f_3 & f_4 & \cdots & f_M & f_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ f_M & f_1 & f_2 & \cdots & f_{M-2} & f_{M-1} \end{bmatrix} \tag{4.8}$$

This example is the generalization of the one given in section 3.11. Of course, on the one hand, any permutation of the rows of $\mathcal{M}$, that is, any permutation of the sequences across the users, or, on the other hand, any permutation of the columns of $\mathcal{M}$, would also be an orthogonal solution. Another example of an orthogonal solution for the case $1 < N \leq M$ would be a pseudorandom orthogonal assigning, such as the one presented in [18].

Independently of the assigning Σ done in the situation $1 < N \leq M$, the characteristic feature of this scheme is the assigning Σ for the case $M < N \leq 2M$. First, it is assumed that for the case $N = M$ an orthogonal assigning has been done. As said in section 3.6, the adaptive distributed algorithm for implementing this scheme is denoted by Ξ. According to Ξ, the value of N has to be updated periodically. When in one of the updating iterations the case $M < N \leq 2M$ is confirmed, then M of the N transmitters are selected and they are assigned orthogonal sequences $\{f_{\xi_i}^{(n)}\}$, $i = 1, 2, \ldots, M$, like it would be done for the case $N = M$. Then, $N - M$ different sequences of these M already assigned sequences are also assigned to the $N - M$ remaining transmitters. Thus, with this scheme, when $M < N \leq 2M$, in the ad hoc network there are $M - (N - M) = 2M - N$ transmitters which never will be interfered by the others in the whole hopping sequence length, and there will be $2(N - M)$ nodes that in every hop of the sequence will be interfered each of them by one node, the same in a whole sequence. That is, the $2(N - M)$ interfered nodes in every hop interfere among them in pairs in such a way that each of them collides in every hop with the same node. This is like saying that there are the same pairs, the same collisions, in every hop. Consequently, the assigning Σ in this scheme is not injective for the case $M < N \leq 2M$, as different users transmit with the same hopping sequence. Once the scheme has been described, it comes out to be obvious that in the situation where $N = 2M$ all nodes collide in every hop between them. For this reason this value is the upper boundary for N, because for a greater N some users would perforce collide to more than one other user in every hop. Consequently, in this case the interference level is too high and, therefore, the QoS becomes poor. Besides, it is also important to mention that with this scheme the sequency length L is always equal to M, independently of whether N is greater or not than M.

A solution to the example problem given in section 3.9 with this scheme would be characterised by the matrix

$$\mathcal{M} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \\ f_2 & f_3 & f_4 & f_1 \\ \mathbf{f_3} & \mathbf{f_4} & \mathbf{f_1} & \mathbf{f_2} \\ f_4 & f_1 & f_2 & f_3 \\ \mathbf{f_3} & \mathbf{f_4} & \mathbf{f_1} & \mathbf{f_2} \end{bmatrix}. \tag{4.9}$$

As depicted in the matrix in (4.9), the third and the fifth rows have the same values. This means that the third (ξ_3) and the fifth (ξ_5) transmitters have the same hopping sequence and, therefore, they will collide in every hop. The two colliding nodes could have been any pair of nodes of $\{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5\}$. If in a given iteration of Ξ, $N = 6$, then a possible solution with this scheme would be represented by the matrix

$$\mathcal{M} = \begin{bmatrix} \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_4} \\ f_2 & f_3 & f_4 & f_1 \\ f_3 & f_4 & f_1 & f_2 \\ \underline{f_4} & \underline{f_1} & \underline{f_2} & \underline{f_3} \\ \underline{f_4} & \underline{f_1} & \underline{f_2} & \underline{f_3} \\ \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_4} \end{bmatrix}. \tag{4.10}$$

Hence, as depicted in (4.10), the collisions will occur between users ξ_1 and ξ_6 and between users ξ_4 and ξ_5.

5 The Fairness-Oriented Scheme (FOS)

5.1 Description

The present scheme is a fairness-oriented solution to the problem formulated in section 3.9. The basic aim of this approach is to achieve that, after a certain number of hops, all users have experienced the same number of collisions while keeping the minimum number of collisions per hop. That is to say, this scheme pursues an equitable interference allocation across all users, as well as it pursues at the same time to do it optimally in terms of the interference level seen by each user. As seen in section 3.9, when $N > M$ at every hop there are perforce at least $2(N - M)$ nodes involved in a collision. With this scheme, as well as in the CS, described in section (4.2) and in contrast to the RS, described in section 4.1, the number of collided nodes per hop is exactly the minimum possible $2(N - M)$. This means that the average interference energy per time unit (i.e., the average interference power) that an arbitrary user ξ_i will have is the minimum possible. Apart from this, as in 4.2, in the present scheme N will always be upper bounded by $2M$, so, the case $M < N$ in fact is always the case $M < N \leq 2M$. A value of N above $2M$ is not considered because it means that all transmitters collide in every hop and in some hop they may collide to more than one other user.

The strategy for achieving the purpose stated above consists in designing the hopping sequences in such a way that, for the case $M < N \leq 2M$, after a whole hopping sequence is completed, every node ξ_i has collided once (and no more) to each of the rest of the nodes. It is assumed that any node will not collide with more than one node in the same hop period so that collisions will always be by pairs of nodes. As a consequence, in every hop there will be $Q_c = N - M$ collisions of two nodes. On the other hand, if every node ξ_i has to collide to the other $N - 1$ nodes in a whole hopping sequence, there will be

$$Q = \binom{N}{2} = \frac{N!}{(N-2)!\,2!} = \frac{N(N-1)}{2} \tag{5.1}$$

total possible collisions. All of them have to take place during the transmission time of a whole hopping sequnece. Thus, since all possible collisions have to be distributed across all hops belonging to a hopping sequence and in every hop Q_c collisions happen, the required sequence length for this scheme is

$$L_r = \frac{Q}{Q_c} = \frac{\frac{N(N-1)}{2}}{N-M} = \frac{N(N-1)}{2(N-M)} = \frac{N-1}{2(1-\frac{M}{N})} \ . \tag{5.2}$$

In general, L_r may not be an integer, so, there are two possibilities. If L_r is an integer, its value is directly set as the sequence length $L = L_r$. Otherwise, L_r cannot be the sequence length because hopping sequences $\{f_{\xi_i}^{(n)}\}$ have an integer length by definition. In this case, let ρ be the remainder of $\frac{Q}{Q_c}$,

$$\rho = (Q \pmod{Q_c}) \ . \tag{5.3}$$

Then, if for example the chosen sequence length was $L = \lceil L_r \rceil$ (where $\lceil\cdot\rceil$ denotes the smallest of the greater integers), the ρ remaining conflicts would be placed in the hop number $\lceil L_r \rceil$, the last hop of the sequence. But, since in every hop Q_c conflicts have to occur, $Q_c - \rho$ conflicts which have already been chosen in some previous hops of the sequence would have to be added to this last hop of the sequence. This would mean that $Q_c - \rho$ collisions would happen twice during the transmission of a hopping sentence, whereas the remaining $Q - (Q_c - \rho)$ possible collisions would happen just once. Thus, this solution would not be a completely equitable solution because each of the $2(Q_c - \rho)$ nodes involved in the $Q_c - \rho$ repeated conflicts would experience $(N-1) + 1 = N$ collisions per hopping sequence, whereas the other nodes would experience $(N-1)$ collisions per hopping sequence. Evidently, this difference becomes less significant as N (and M) increases and, in terms of interference level, the greater is N, the closer is this solution to a completely fair solution. Besides, it is possible to resolve this problem. The proposed strategy to this end is to set the sequence length as

$$L = \mathrm{LCM}\{\rho, Q_c\}\, L_r \ , \tag{5.4}$$

where $\mathrm{LCM}\{\rho, Q_c\}$ is the *least common multiple* of ρ and Q_c. In such way,

$$LQ_c = kQ, \qquad k \in \mathbb{N} \ . \tag{5.5}$$

In fact, $k = \mathrm{LCM}\{\rho, Q_c\}$. Therefore, in every sequence transmission period LT_h every node will collide to each of the other nodes the same number of times k. That is, every node will have the same number $k(N-1)$ of collisions per sequence. Yet, from here on, without loss of generality in the results, the sequence length will be always considered to be $L = L_r$ for simplicity.

Example 1:

Considering the simple case given as an example in section 3.9, assume $N = 5$ and $M = 4$. So, $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$. Then, a possible solution according to this scheme would be the one represented in 5.6.

$$\mathcal{M} = \begin{bmatrix} \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_4} & f_1 & f_2 & f_3 & f_4 & f_1 & f_2 \\ \mathbf{f_1} & f_3 & f_4 & f_1 & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_4} & f_1 & f_2 & f_3 \\ f_3 & \mathbf{f_2} & f_1 & f_2 & \mathbf{f_2} & f_4 & f_1 & \mathbf{f_2} & \mathbf{f_3} & f_4 \\ f_4 & f_1 & \mathbf{f_3} & f_3 & f_4 & \mathbf{f_3} & f_2 & \mathbf{f_2} & f_4 & \mathbf{f_1} \\ f_2 & f_4 & f_2 & \mathbf{f_4} & f_3 & f_1 & \mathbf{f_4} & f_3 & \mathbf{f_3} & \mathbf{f_1} \end{bmatrix}. \tag{5.6}$$

Observe that in every column of $\mathcal{M}$, that is, in every hop, $N_c = 2(N - M) = 2$ nodes collide (written in bold type) while the sequences of the other nodes are orthogonal in that hop and observe that, after the sequence length $L = \frac{N(N-1)}{2(N-M)} = 10$ hops, all possible $Q = \frac{N(N-1)}{2} = 10$ pairs of nodes have collided.

Example 2:

If now, for the same ad hoc network consisting of $N = 5$ nodes, the FHSS sytem has $M = 3$ frequency channels $f_i \in \mathcal{F} = \{f_1, f_2, f_3\}$, then, one possible solution according to this scheme is the one depicted in 5.7.

$$\mathcal{M} = \begin{bmatrix} \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_1} & f_2 \\ f_2 & \underline{f_1} & \underline{f_2} & \mathbf{f_1} & \mathbf{f_3} \\ \underline{f_1} & \mathbf{f_2} & f_1 & \underline{f_3} & \mathbf{f_3} \\ \underline{f_1} & \underline{f_1} & \mathbf{f_3} & f_2 & \underline{f_1} \\ \mathbf{f_1} & f_3 & \underline{f_2} & \underline{f_3} & \underline{f_1} \end{bmatrix}. \tag{5.7}$$

Note that in this solution $f_1^{(1)} = f_5^{(1)}$, $f_3^{(1)} = f_4^{(1)}$, $f_1^{(2)} = f_3^{(2)}$, $f_2^{(2)} = f_4^{(2)}$, $f_1^{(3)} = f_4^{(3)}$, $f_2^{(3)} = f_5^{(3)}$, $f_1^{(4)} = f_2^{(4)}$, $f_3^{(4)} = f_5^{(4)}$, $f_2^{(5)} = f_3^{(5)}$ and $f_4^{(5)} = f_5^{(5)}$. So, as in the previous example, all possible $Q = \frac{N(N-1)}{2} = 10$ pairs of nodes have collided within $L = \frac{N(N-1)}{2(N-M)} = 5$ hops, with $N_c = 2(N-M) = 4$ nodes involved in a collision in every hop (and, consequently, with $Q_c = (N-M) = 2$ collisions per hop) and one node free of collisions in every hop. Also observe that in this second example the sequence length L is half of the one in *Example 1* because in this example the number of collisions per hop is two times greater. This sequence length variability will be important in the following when the analysis results will be discussed.

Example 3:

A third simple example is required to elucidate the case when Q is not a multiple of Q_c. Suppose now that there are $N = 6$ transmitters and $M = 4$ available frequency channels. Consequently, $Q = 15$ and $Q_c = 2$ and its division $L_r = 7,5$ does not yield an integer.

Using (5.3) one has $\rho = 1$. Thus, $\text{LCM}\{\rho, Q_c\} = Q_c = 2$. Thus, the assigned sequence length will be $L = 15$. A possible solution is depicted in (5.8).

$$\mathcal{M} = \begin{bmatrix} \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_3} & \mathbf{f_4} & \mathbf{f_1} & f_2 & f_3 & \mathbf{f_4} & \mathbf{f_2} & \mathbf{f_2} & \mathbf{f_2} & \mathbf{f_4} & f_2 & f_1 & f_3 \\ \mathbf{f_1} & \underline{f_3} & f_4 & f_1 & f_2 & \mathbf{f_3} & f_4 & \underline{f_3} & \mathbf{f_2} & f_3 & f_4 & \underline{f_3} & \mathbf{f_1} & \mathbf{f_2} & \mathbf{f_2} \\ \underline{f_3} & \mathbf{f_2} & f_1 & \underline{f_3} & f_3 & \underline{f_4} & \mathbf{f_1} & \mathbf{f_4} & f_1 & f_4 & f_3 & \underline{f_3} & \underline{f_3} & \underline{f_4} & \underline{f_1} \\ \underline{f_3} & f_1 & \mathbf{f_3} & \underline{f_3} & \underline{f_4} & \mathbf{f_3} & \underline{f_2} & f_2 & \underline{f_3} & \mathbf{f_2} & \underline{f_1} & f_2 & \mathbf{f_1} & \underline{f_4} & f_4 \\ f_4 & \underline{f_3} & \underline{f_2} & \mathbf{f_4} & \underline{f_4} & f_1 & \mathbf{f_1} & f_1 & \underline{f_3} & \underline{f_1} & \mathbf{f_2} & f_1 & \underline{f_3} & f_3 & \mathbf{f_2} \\ f_2 & f_4 & \underline{f_2} & f_2 & \mathbf{f_1} & \underline{f_4} & \underline{f_2} & \underline{f_3} & f_4 & \underline{f_1} & \underline{f_1} & \mathbf{f_4} & f_4 & \mathbf{f_2} & \underline{f_1} \end{bmatrix} \tag{5.8}$$

In the three examples above, $\mathcal{M}$ depicts the nodes that collide in every hop. However, there is more than one solution to the channel distribution across the nodes in every hop once the collisions distribution has been decided. These examples represent only one possible solution each. So far, the present scheme has been described only from the perspective of which nodes have to collide in every hop and how, but it has not been said in which channels have to happen these collisions and which channels have to be assigned to the transmitters that does not collide in a given hop. This would be the second step of hooping seuqneces design for the present scheme. Whereas the design of the distribution of the collisions across the nodes decides the interference balancing in the network, the channel distribution across the nodes in each hop determines the anti-jamming and interception avoidance capacity of the system. Therefore, this work is focused on the collisions allocation and not to the second part. As a guidance, maybe one suitable approach for achieving a high anti-jamming and interception avoidance capacity should be based on the solution proposed in [18]. This solution is optimal for the case $N < M$ because it derives pseudorandom orthogonal hopping sequences, i.e., there is not any collision among users while the obtained sequences display the desired anti-jamming and anti-interception properties; the same strategy should be somehow extended for the case $M < N \leq 2M$.

The present scheme, along with the CS, is an adaptive solution. As previously stated in section 3.6, the implementation of this scheme has to follow a recursive algorithm Ω. This algorithm has to check periodically, how many nodes make the ad hoc network up in every checking instant. For this, cooperation amongst nodes is required. Another input information to Ω, apart from N_T, is which are the N nodes that are transmitting, and these nodes have to be identified by assigning each a number. This number will be valid while the model is considered to be static. Hence, in every checking instant there are two possible cases. For the case $N \leq M$, Ω designs orthogonal hopping sequences following some method. And for the case of interest for this thesis $M < N \leq 2M$, Ω implements the strategy presented above in the present section. On the other hand, if the time interval between two updating instants of Ω is a multiple of the time of transmission of a hopping sequence, then, with this scheme the complete equity is achieved. Otherwise, since the last hopping sequence would not be completed, there would be some users that would be at a disadvantage with respect to the other users because they would experience one more collision during a hopping sequence than the others. If the number of consecutive hopping sequences transmitted during the time interval between two updating instants of Ω is high, then the difference of one collision more becomes negligible. From here on this condition will be assumed to be satisfied.

As a final remark, the *a priori* benefits of this scheme can be briefly highlighted as follows. This scheme corresponds to the optimal case in terms of interference level and it does not need to introduce more interference as a compensation for the equitable interference distribution across all nodes; on the contrary, it achieves total equity while it keeps interference at the minimum possible level.

5.2 Proof of existence

There is an immediate problem which directly arises from the description given in section 5.1: assuming that $L_r = L \in \mathbb{N}$, is it possible to group the Q total possible pairs of colliding nodes into sets of Q_c pairs of nodes in such a way that in any of the L sets any node does not appear in more than one pair? In other words, is it possible to make appear all Q collisions in L hops satisfying at the same time that in every hop the Q_c collisions involve N_c different nodes (i.e., nodes collide at most once per hop)?

It is important that a solution to this problem exist, otherwise it would not be possible to make nodes collide by pairs in every hop but it would perforce have to occur that in some hops nodes would have to collide in larger groups. This means that in some hop U nodes would hop into the same frequency channel, with $2 < U \leq N_c$. Thus, there are $\frac{U(U-1)}{2}$ different collisions among the U nodes, and

$$\frac{U(U-1)}{2} > \frac{U}{2} \tag{5.9}$$

if $U > 2$, which is true by hypothesis. (5.9) implies that more than Q_c collisions would occur in a hop because not all the N_c nodes collide in pairs. As previously seen, Q_c is the minimum possible number of collisions per hop (always in the case $M < N \leq 2M$). If each user experiences more collisions per hop than the minimum possible, then the FOS will not be optimal in terms of the interference level. Apart from the perspective of a single hop, this can also be seen from the perspective of one sequence. If more than Q_c collisions occur in some hops of the sequence, then the sequence length L will not be equal to $\frac{N(N-1)}{2(N-M)}$ but it will be shorter. Consequently, with the FOS each node will collide with the rest of the nodes (i.e., each node will experience $N-1$ collisions) during a shorter period of time (because the sequence length is shorter). Thus, the average interference power that each node will see is greater. As an example, suppose an ad hoc network with $M = 3$ and $N = 5$. Suppose than in the n-th hop, the vector with the carriers assigned to each user is

$$\begin{bmatrix} f_{\xi_1}^{(n)} \\ f_{\xi_2}^{(n)} \\ f_{\xi_3}^{(n)} \\ f_{\xi_4}^{(n)} \\ f_{\xi_5}^{(n)} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_2 \\ f_3 \end{bmatrix} . \tag{5.10}$$

So, the transmitter ξ_1 is not interfered and the rest of transmitters are interfered by another transmitter each. Hence, there are two sets of $U = 2$ colliding nodes, $\{\xi_2, \xi_4\}$ and $\{\xi_2, \xi_4\}$. Consequently there are $Q_c = 2$ collisions and $N_c = 4$ colliding transmitters, which collide *only with another transmitter and no more.* However, if the carriers are assigned in the manner

$$\begin{bmatrix} f_{\xi_1}^{(n)} \\ f_{\xi_2}^{(n)} \\ f_{\xi_3}^{(n)} \\ f_{\xi_4}^{(n)} \\ f_{\xi_5}^{(n)} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_3 \\ f_3 \end{bmatrix} , \tag{5.11}$$

there is just one set of $U = 3$ colliding nodes with $\frac{U(U-1)}{2} = 3 > Q_c = 2$ collisions. Therefore, although now there are two transmitters which do not collide, ξ_1 and ξ_2, each of the rest of the transmitters collide *with two other transmitters.* So, comparing both cases, one concludes that if in the first case four transmitters are collide with one other transmitter each and in the second case three transmitters collide with two other transmitters each, the expected number of collisions per hop per user is lower in the first case. In fact, the first case is the optimal case from this perspective, because it spreads the collisions to more frequency channels.

The above stated problem can be formulated mathematically as follows. Let the $N \times (N-1)$ matrix $\mathcal{N}$, denoted by

$$\mathcal{N} \triangleq \begin{bmatrix} \xi_1 & \xi_1 & \cdots & \xi_1 \\ \xi_2 & \xi_2 & \cdots & \xi_2 \\ \vdots & \vdots & \ddots & \vdots \\ \xi_N & \xi_N & \cdots & \xi_N \end{bmatrix} , \tag{5.12}$$

a matrix with $N-1$ equal columns built up with the N transmitters of the network. Let now $\mathcal{C}_{ij}$ be a subset of two elements $\xi_i, \xi_j \in \mathcal{B}$, where $i \in \{1, \ldots, N-1\}$, $j \in \{2, \ldots, N\}$, $i < j$; and let $\mathcal{C}$ be a set of all $Q = \frac{N(N-1)}{2}$ possible different $\mathcal{C}_{ij}$, i.e., $\mathcal{C} = \{\mathcal{C}_{ij}\}$.

Definition: An *edge coloring* of a graph is an assignment of colors to all edges of a graph in such a way that any adjacent edges do not have the same color.

Lemma 1: The minimum possible required number of colors (known as the *chromatic index* and here denoted by κ) used in an edge coloring of a complete graph $G = (V, E)$ with N vertices is equal to $N-1$, if N is even, and it is equal to N, if N is odd and $N \geq 3$.

Proof: Find it in [27].□

Example: In Fig. 5.1 an edge coloring of a complete graph of $N = 5$ vertices using the minimum possible number of different colors ($\kappa = 5$) is displayed.

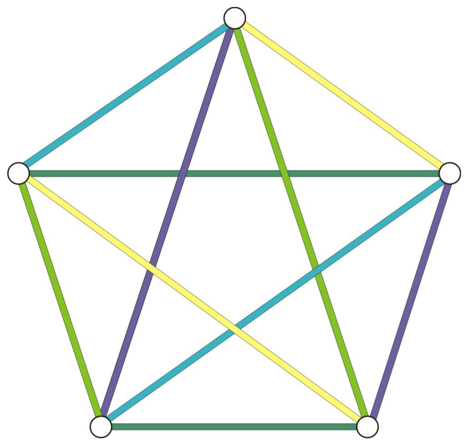


Figure 5.1: Edge colored complete graph with 5 vertices.

Lemma 2: If an edge coloring with the minimum required number of different colors κ is done to a complete graph G with an even number N of vertices, then, each color is assigned to $\frac{N}{2}$ edges.

Proof: Assuming that N is even, for any vertex ξ_i of G there are $N-1$ edges that share this vertex. Therefore, all these $N-1$ edges have to have a different color in an edge coloring. On the other hand, if N is even, then $\kappa = N-1$, which means that if the edge coloring is done with the minimum required number of different colors κ, all vertices have all the $N-1$ colors in their adjacent edges. □

Corollary 1: It is possible to make an edge coloring to a complete graph G with an even number N of vertices with L different colors and with the same number of edges Q_c to each color, for the particular case where $\frac{N}{2}$ is multiple of Q_c.

Corollary 2: If $\frac{\frac{N}{2}}{Q_c} = \eta \in \mathbb{N}$, then $L = \eta\, \kappa$.

Lemma 3: It is possible to make an edge coloring to a complete graph G with an even number N of vertices with L different colors and with the same number of edges Q_c to each color, for any $Q_c \in \mathbb{N}$, $1 \leq Q_c \leq \frac{N}{2}$, for Q multiple of Q_c.

Proof: From *Lemma 2* it can be inferred that, for any given color of an edge coloring using κ different colors, all the N vertices of G have to be joined two by two through the $\frac{N}{2}$ edges of that color, because, by virtue of *Lemma 1*, one, an only one, edge of each color has to be connected to each vertex. This fact is illustrated by Fig. 5.2.

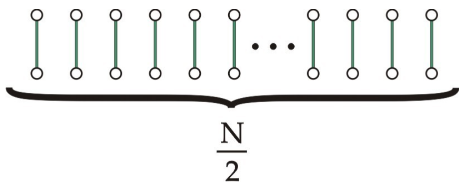


Figure 5.2: Representation of the $\frac{N}{2}$ edges of G with the same color.

Then, if the edges belonging to any other given color are added to the pairs of vertices depicted in Fig. 5.2, any of the pairs joined by the edges with this second color will not coincide with any of the pairs joined according to the first color, because any two vertices ξ_i, ξ_j only share one edge e_{ij}. An example of the disposition of the edges across the vertices belonging to a possible second color is given in Fig. 5.3. The relative disposition of the

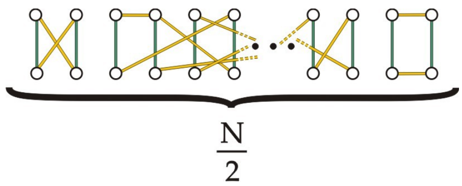


Figure 5.3: Addition to Fig. 5.2 of the $\frac{N}{2}$ edges of G belonging to a second color.

edges belonging to the second color with reference to the edges belonging to the first color can be any possible combination such as the one in Fig. 5.3.

On the other hand, as asserted in *Corollary 1*, it is clear that it is possible to divide the edges of the first color, depicted in Fig. 5.2, in sets of Q_c disjoint edges, if $\frac{N}{2}$ is a multiple of Q_c. An example of this is shown in Fig. 5.4. But what *Lemma 3* asserts is that it

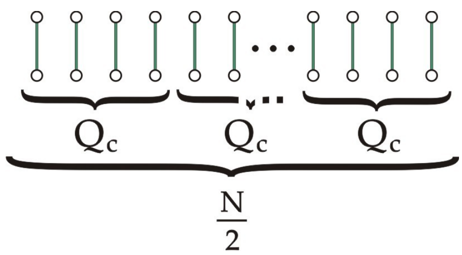


Figure 5.4: Example of Q_c being submultiple of $\frac{N}{2}$.

is possible to make sets of Q_c disjoint edges, even if Q_c is not a submultiple of $\frac{N}{2}$, where

$1 \leq Q_c \leq \frac{N}{2}$.[1] Let ϱ be

$$\varrho \triangleq \left(\frac{N}{2} \mod Q_c \right) . \tag{5.13}$$

Then, a situation such as the one depicted in Fig. 5.5 will occur.

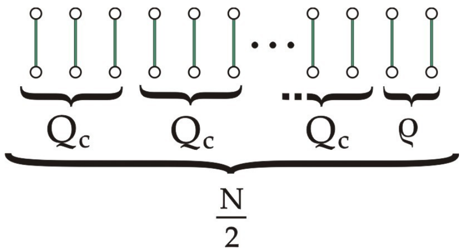


Figure 5.5: Example of Q_c not being submultiple of $\frac{N}{2}$.

Since, as shown in Fig. 5.3, each of the two vertices of every of the ϱ edges belonging to the first color may be each connected to a different (in general) edge belonging to the second color, there will be at most 2ϱ edges belonging to the second color that are not disjoint to the ϱ remaining edges belonging to the first color. Thus, there will be $\frac{N}{2} - 2\varrho$ edges belonging to the second color that are disjoint to the remaining edges belonging to the first color. On the other hand, given ϱ, the largest possible size of Q_c is $\frac{N}{2} - \varrho$ (which corresponds to just one whole set of edges in the fisrt color). Recall that the goal is to build up a set of Q_c disjoint edges merging the ϱ remaining edges belonging to the first color with some edges belonging to the second color, all of which have to be disjoint with that ones belonging to the first color. And this is possible because, as said, the number of edges belonging to the second color which are not adjacent to the remaining edges belonging to the first color is, at least, $\frac{N}{2} - 2\varrho$, and the number of necessary disjoint edges belonging to the second color for completing a set of Q_c edges together with the ϱ remaining edges is (for this worst case of Q_c being $\frac{N}{2} - \varrho$) $\frac{N}{2} - 2\varrho$.

The same reasoning can be applied recursively to any two sets of the elements of two consecutive colors. □

Lemma 4: Considering an edge coloring to a complete graph G with an odd number N of vertices with L different colors and with the same number of edges Q_c to each color, the not used color by the adjacent edges of each vertex is different for each vertex, so that, each color is used by $\frac{N-1}{2}$ edges.

Proof: Suppose that the negation of *Lemma 4* is true. Then, as a consequence, there are $\mu > 1$ vertices whose adjacent edges use the same $N-1$ colors. Therefore, $N-1-(N-\mu-1) = \mu$ colors amongst these $N-1$ colors have to be perforce used once in all the rest of the vertices

[1] Recall that the case $N > 2M$ is not of interest in this work and note, on the other hand, that $Q_c = N - M > \frac{N}{2} \Rightarrow N > 2M$.

(which are $N-\mu$). Consequently, there are μ colors that have to be used once by the adjacent edges to each of the N vertices. And this is impossible because it is not possible to create disjoint pairs of vertices from an odd number of vertices, and the number of vertices is odd by hypotesis. $\square$

Lemma 5: It is possible to make an edge coloring to a complete graph G with an odd number N of vertices with L different colors and with the same number of edges Q_c to each color, for any $Q_c \in \mathbb{N}, 1 \leq Q_c < \frac{N}{2}$, for Q multiple of Q_c.

Proof: Each vertex of G is joined to $N-1$ edges. All these edges are adjacent because they all share this vertex. Therefore, the $N-1$ edges joined by a vertex have each a different color, given an edge coloring of G. If this coloring uses the minimum possible number of different colors $\kappa = N$, for the odd case, then, each vertex will not be adjacent to one of the N used colors.

From here on, *Lemma 5* can be proved in the same way than *Lemma 3*. $\square$

Proposition 1: It does exist a partition of $\mathcal{C}$ into L subsets $\mathcal{C}_s$ of $Q_c = N - M$ elements $\mathcal{C}_{ij}$, where $M < N \leq 2M,\ M \in \mathbb{M},\ N \in \mathbb{N}$, which satisfies

$$\bigcap_{C_{ij} \in \mathcal{C}_s} \mathcal{C}_{ij} = \varnothing \,, \qquad \forall\, \mathcal{C}_s \,,$$

for Q multiple of Q_c.

Proof: It follows from *Lemma 3* if N is even and from *Lemma 5* if N is odd.[2]$\square$

Proposition 1 implies that the solution of the problem formulated in the first paragraph of this section exists. So, the proposed fairness-oriented scheme is feasible.

5.3 Proposed method

According to what have been exposed in 5.2, in order to design the hopping sequeces, a method for creating the L sets of Q_c pairs of colliding nodes should follow the same building pattern than it has been done in the proof of *Lemma 3*. This means that, given the matrix $\mathcal{N}$ defined in 5.2, the sets of pairs of colliding nodes $\mathcal{C}_s$ have to be created always with the elements of only one column of $\mathcal{N}$, if possible. When a subset has been created, all the elements ξ_i, ξ_j that build up the pairs $\mathcal{C}_{ij}$ of $\mathcal{C}_s$ have to be removed from $\mathcal{N}$. When the remaining elements in a column of $\mathcal{N}$ are less than N_c, then, the rest of elements to build up the set have to be all taken from another column of $\mathcal{N}$, the same column for all. And the next set have to be build up also with elements from that column, and so on.

[2]Guidance: each edge of the complete graph represents one pair out of the Q different pairs of colliding nodes; since all Q_c edges belonging to the same color are disjoint among them, in a partition of $\mathcal{C}$ into L subsets of Q_c pairs each, any of the pairs in a subset share any node.

5.4 Preliminary results

In this section the FOS is compared to the RS in terms of probability of collision, as an indicator of the better performance of the FOS, which is studied in detail in section 6.1. With the FOS, the probability of one node ξ_i to collide to some other node in a randomly chosen hop is given by

$$\mathbf{P}_{FOS}\left\{\text{one collision}\right\} = \begin{cases} 0 & \text{if } 1 < N \leq M \\ \\ \frac{2Q_c}{N} = \frac{2(N-M)}{N} & \text{if } M < N \leq 2M \end{cases} , \tag{5.14}$$

whereas the same[3] probability for the RS is equal to

$$\begin{aligned} \mathbf{P}_{RS}\left\{\text{at least one collision}\right\} &= 1 - \mathbf{P}_{RS}\left\{\text{no collisions}\right\} \\ &= 1 - \left(1 - \mathbf{P}_{RS}\left\{\text{one transmitter collide with } \xi_i\right\}\right)^{N-1} \\ &= 1 - \left(1 - \frac{1}{M}\right)^{N-1} , \qquad 1 < N < \infty . \end{aligned} \tag{5.15}$$

(5.14) and (5.15) can be written in terms of the ratio $\gamma = \frac{N}{M}$ as

$$\mathbf{P}_{FOS}\left\{\text{one collision}\right\} = \begin{cases} 0 & \text{if } 0 < \gamma \leq 1 \\ \\ 2\left(1 - \frac{1}{\gamma}\right) & \text{if } 1 < \gamma \leq 2 \end{cases} \tag{5.16}$$

and

$$\mathbf{P}_{RS}\left\{\text{at least one collision}\right\} = 1 - \left(1 - \frac{\gamma}{N}\right)^{N-1} = 1 - \left(1 - \frac{1}{M}\right)^{\gamma M-1} , \qquad 0 < \gamma < \infty , \tag{5.17}$$

respectively. In light of these expressions, whereas for the FOS the probability of one user to have at least one collision in a randomly chosen hop only depends on γ, for the RS this probability also depends on the absolute value of M and N. Both probabilities are compared with respect to γ in Fig. 5.6. The FOS outperforms considerably the random scheme in terms of probability of collision until a certain value of γ which is close to $\gamma = 1,7$.

[3] Note that for the FOS, since each node will experience at most one collision per hop, the probability of having one collision is the same than the probability of having at least one collision.

The value of γ for which both curves take the same value is denoted by γ_c. Above γ_c, the FOS has a higher probability of collision. Equaling (5.16) and the last member of (5.17) evinces that γ_c depends on M. However, neither the value of γ_c nor the curve of the RS change significantly if M changes. For M tending to infinity and γ remaining constant, (5.17) yields

$$\lim_{M\to\infty} \mathbf{P}_{RS}\,\{\text{at least one collision}\} = \lim_{M\to\infty} 1-\left(1-\frac{1}{M}\right)^{\gamma M-1} = 1-e^{-\gamma}\,, \qquad 0<\gamma<\infty\,. \tag{5.18}$$

In Fig. 5.6, the curves corresponding to (5.17), with $M = 5$ and with $M = 10$, and to (5.18) are displayed. It is clear that the three curves are similar, and their difference decreases for

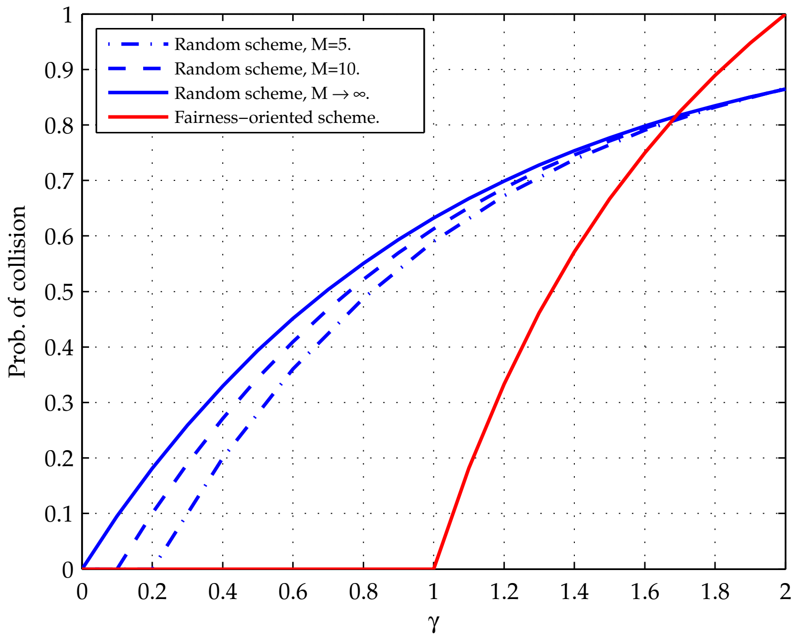


Figure 5.6: Probability of collision for the RS for $M = 5$, $M = 10$ and $M \to \infty$, and for the FOS.

higher values of γ. Hence, γ_c does not change considerably with respect to M and neither the whole curve of the probability of one user to experience at least one collision in one hop, for the RS, changes significantly.

Note that the probability of collision for the RS when $M = 10$ is equal to 0 when $\gamma = 0,1$. This is because when $\gamma = 0,1$, for $M = 10$, $N = 1$. Therefore, in this case there is only one transmitter and, consequently, it will not be interfered by any other transmitter; so, the probability of collision is equal to zero.

The probability of one user to have at least one collision in one hop is a good measure to evaluate the performance of the studied schemes. However, an analysis which only consideres this metric would be incomplete since important insights remaing hidden to this indicator. The probability of one user to be have at least one collision in one hop just takes into account two possible cases: having no collisions or colliding least with one transmitter. But it does not consider how many transmitters are colliding to one user in the case of having at least one collision. Therefore, the fact that the FOS has a higher probability of collision for $\gamma_c < \gamma \leq 2$ than the RS does not necessarily imply that the RS has a better performance in this range. If in this range one user is interfered with lower probability for the RS than for the FOS but, on the other hand, when this user is interfered it is interfered by more users with the RS than with the FOS in average terms, then the performance of the FOS may be still better. For this reason, as it will be seen in section 6.1.1, one important performance measure is the interference power. This measure will play a central role for this work.

6 Performance Evaluation

6.1 Analysis

In this section the three schemes are analysed for the cases of static and dynamic slow hopping and fast hopping. At the same time, for every scheme the two limiting cases where a hypothetical receiver is placed in the center of the disk or on its border are regarded. All other possible cases will be bounded by these two limiting cases. In the following subsections the evaluation criteria and metrics are first presented and the analysis is next developed.

6.1.1 Metrics

The overall interference power, the probability of colliding at least once per hop and the BER

For comparing the performance of the considered schemes to the new scheme in terms of interference balancing, the use of some appropriate metrics is necessary. As asserted in [1] with regard to distributed random access ad hoc networks, "The outage probability is the most meaningful and pertinent measurement of the performance. It is directly related to the classical network transport capacity and to the transmission capacity." So, the *outage probability* (OP) is a key parameter to take into account when evaluating the behaviour of distributed random access ad hoc networks, such as the ones represented by the model chosen in the present work. The OP is defined as the probability that the instantaneous signal to interference plus noise ratio (SINR) seen by a reference receiver is lower than a given threshold β. Below β the reception is considered not successful. Thus, the outage probability depends directly on the SINR, which is next defined. Suppose a node ξ_i belonging to the ad hoc network which during a certain period of time aims to transmit information to ξ_j, a node which does not belong to the ad hoc network but which has exactly the same features than all the nodes in the ad hoc network when they act as receivers. Henceforth, ξ_j will be named the *intended transmitter*, and denoted by $\xi_\Gamma \equiv \Gamma \triangleq \xi_i$, and ξ_j the *virtual reference receiver*, denoted by $\Theta \triangleq \xi_j$. Then, all nodes that during the time of transmission of Γ are transmitting in the same frequency channel than Γ will be denoted from here on as *interferers*. While Γ is a node selected from the N transmitters in the ad hoc network, Θ does not belong to this group, but it is just a hypothetical receiver placed somewhere in the disk which acts as a reference receiver in order to measure the interference situation in the network. In fact, Θ is considered to belong to the $N_T - N$ nodes that do not transmit.

In the described situation, the average *signal to interference plus noise ratio* (SINR) seen by Θ is given by

$$SINR = \frac{P_{R_\Gamma}}{P_N + P_I} \ , \tag{6.1}$$

where P_{R_Γ} is the received power from the intended transmitter Γ, P_N is the noise power in the receiver of an *additive white gaussian noise* (AWGN) signal $n(t)$ with zero mean and *power spectral density* (PSD) $\frac{N_0}{2}$, and P_I is the overall power of all interferers. It is assumed that the signals of all interferers are uncorrelated and that the interferent signals have zero mean. As a consequence, the overall interference received power P_I is equal to the sum of the individual received powers from every interferer.

Therefore,the SINR depends basicly on the *overall interference power* P_I, and also on P_{R_Γ} and on the average thermal noise power P_N. However, as asserted in [28], the noise contribution to the SINR can be neglected for a number of interferers N great enough. In this work it will be assumed that this condition is satisfied and, so, instead of the SINR, the considered parameter will be SIR. Likewise, P_{R_Γ} is not very significant comparing with P_I when $N \gg 1$. For this reason and moreover because the considered schemes differ only in P_I, and P_{R_Γ} is the same in the three cases, in this thesis the main chosen metric is P_I. The other two chosen metrics are the probability of one transmitter to experience at least one collision in a randomly chosen hop, $\mathbf{P}\{\text{at least one collision}\}$, and the *bit error rate* (BER). Whereas P_I will be used in all the considered cases, $\mathbf{P}\{\text{at least one collision}\}$ and the BER will be only used in some specific cases. The BER is directly related to the SIR and to the outage probability, and consequently, it is also a highly suitable metric for this thesis purposes. Nevertheless, the BER depends also on the structure and type of modulation of every particular receiver, so, there is not a closed-form expression for it.

Besides, due to the randomness of the position of the nodes according to the network model described in 3.1 (and in the case of the RS, also due to its inherent randomness), P_I is a random variable. Therefore a statistical approach is required in both analysis and simulations. The expectation $\mathbf{E}\{\cdot\}$ of these three parameters reflects the average interference effect across users of every scheme. Likewise, the variance $\mathbf{Var}\{\cdot\}$ reflects the variability of this interference across all users in the network. Thus, whereas the expectation will be used for evaluating the efficiency of every scheme from the perspective of interference avoidance, the variance will be used as an indicator of the equity and the fairness of the interference distribution across the users. In the discussion of the results, the variances of P_I for the three schemes will be denoted as $\mathbf{Var}_{FOS}\{P_I\}$, $\mathbf{Var}_{RS}\{P_I\}$ and $\mathbf{Var}_{CS}\{P_I\}$. An important metric will be the ratio between the variances of two schemes. The ratio between the variance of P_I for the scheme A and the variance of P_I for the scheme B, where A and B can be F, for the FOS, R, for the RS, and C, for the CS, is

$$\Delta_{VarAB} \triangleq \frac{\mathbf{Var}_A\{P_I\}}{\mathbf{Var}_B\{P_I\}} \ . \tag{6.2}$$

The two boundary cases

The virtual reference receiver Θ has to represent any of the nodes of the ad hoc network when they act as receivers, so, it can be placed in any point within the circle of radius R where the ad hoc network is placed. It can bee easily checked that the upper and lower bounds for the expected interference power $\mathbf{E}\{P_I\}$ are, respectively, the case when Θ is placed in the center of W and the case when Θ is placed in some point on the border of W. Note that, since the nodes positions are uniformly distributed, there is central simetry in W. Thus, the interference situation depends only on the distance to the center of W.

For calculating the statistical moments of the overall interference power P_I, it is necessary to calculate the statistical moments also for P_{Ri}, the received power by the reference receiver Θ from an interferer ξ_i. According to the definition of the power decay in section 3.8, P_{Ri} will be

$$P_{Ri}(r_{i\Theta}) = \frac{1}{(r_{i\Theta}+1)^{\alpha}} \ , \tag{6.3}$$

where $r_{i\Theta}$ is the distance between ξ_i and Θ. Thus, P_{Ri} depends, on the one hand, on the path loss exponent α and on the other hand on $r_{i\Theta}$, which is a random variable. Therefore, P_{Ri} is also a random variable. The distribution of $r_{i\Theta}$ depends in turn on the position of both ξ_i and Θ because the distribution of $r_{i\Theta}$ is the distribution of the position of ξ_i in W with respect to Θ. Then, the CDF of $r_{i\Theta}$ is obtained as follows: since the interferers are placed uniformly in the disk, the probability of one transmitter to be at distance equal or smaller than r is given by a quotient of two areas, as done in [1]. For the case of Θ being in the center, the derived expression for the CDF is

$$F_D(r) = \mathbf{P}\left\{r_{i\Theta} \le r\right\} = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2} \tag{6.4}$$

where the divided areas are the two circles depicted in Fig. 6.1.

Thus, its PDF is

$$f_D(r) = \frac{\partial}{\partial r}(F_D(r)) = \frac{\partial}{\partial r}\frac{r^2}{R^2} = \frac{2r}{R^2}$$

For the case of Θ being placed on the border of the disk, the CDF is obtained similarly, as depicted in Fig. 6.3. It can be shown the area A of the intersection between two circles with radii R and r whose centers are separated a distance d has the expression

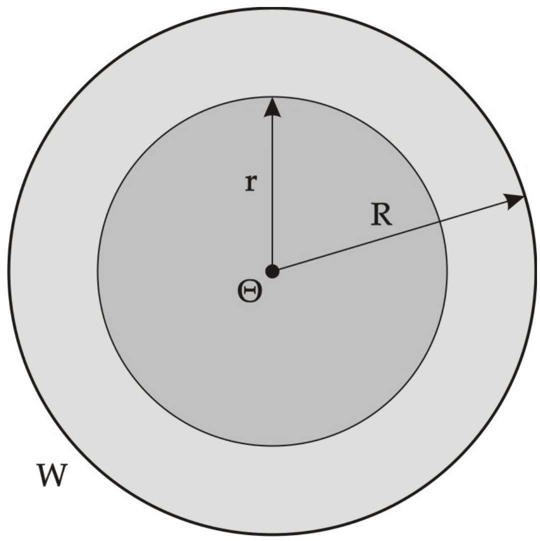


Figure 6.1: Illustration of the divided areas in 6.4.

$$A = r^2 \arccos\left(\frac{d^2 + r^2 - R^2}{2dr}\right) + R^2 \arccos\left(\frac{d^2 + R^2 - r^2}{2dR}\right)$$
$$- \frac{1}{2}\sqrt{(-d + R + r)(d + r - R)(d - r + R)(d + r + R)}\ . \tag{6.5}$$

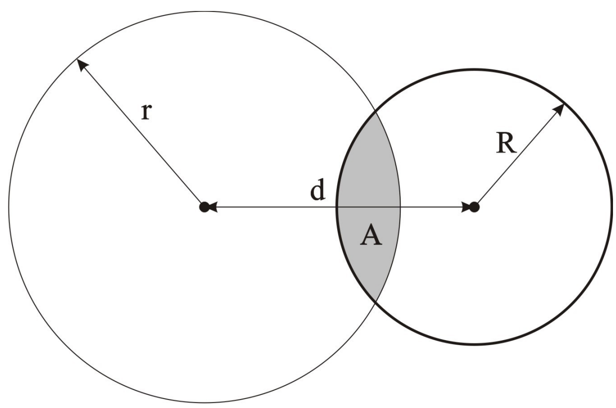


Figure 6.2: Intersection of two circles with radii R and r whose centers are separated by d.

When Θ is placed on the border of the circle, the distance between the two centers has to be $d = R$, as depicted in Fig. 6.3. Consequently, 6.5 turns into

$$A = r^2 \arccos\left(\frac{r}{2R}\right) + R^2 \arccos\left(\frac{2R^2 - r^2}{2R^2}\right) - \frac{1}{2} r\sqrt{4R^2 - r^2}\ . \tag{6.6}$$

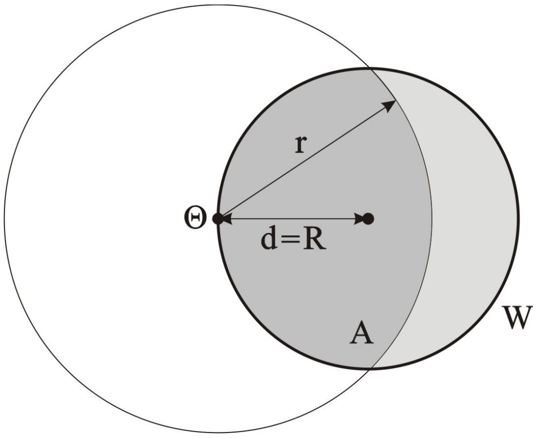


Figure 6.3: Intersection of two circles with radii R and r whose centers are separated a distance d

Thus, the CDF of the distance between any interferer and Θ is

$$F_D(r) = \mathbf{P}\left\{r_{i\Theta} \leq r\right\} = \frac{A(r)}{\pi R^2} = \frac{r^2}{\pi R^2}\arccos\left(\frac{r}{2R}\right) + \frac{1}{\pi}\arccos\left(\frac{2R^2 - r^2}{2R^2}\right) - \frac{r}{2\pi R^2}\sqrt{4R^2 - r^2}\,. \tag{6.7}$$

Hence, its PDF is

$$f_D(r) = \frac{\partial}{\partial r}(F_D(r)) = \frac{1}{\pi R^2}\left[2r\arccos\left(\frac{r}{2R}\right) - \frac{1}{2R}\frac{r^2}{\sqrt{1-\left(\frac{r}{2R}\right)^2}} + \frac{r}{\sqrt{1-\left(\frac{2R^2-r^2}{2R^2}\right)^2}} - \frac{1}{2}\sqrt{4R^2-r^2} + \frac{r^2}{2\sqrt{4R^2-r^2}}\right]. \tag{6.8}$$

6.1.2 Fast hopping analysis

The FOS with the virtual receiver placed in the center

First of all it has to be clarified that all the given interference power expressions in the analysis section either for the FOS or for the CS are always valid for the case $M < N \leq 2M$; otherwise, the interference power is zero for both schemes, as explained above. If, as said, P_{Ri} is the power received by Θ from an interferer ξ_i, the average received energy from this interferer during one hop period is $E_{Ri} = P_{Ri}T_h$. Thus, since with the FOS each transmitter will collide with each other once per hopping sequence, the average interference power received at Θ in a whole hopping sequence, is

$$\widetilde{P_I} = \frac{1}{L\,T_h} \sum_{i \neq \Gamma}^{N} P_{Ri}\, T_h = \frac{1}{L} \sum_{i \neq \Gamma}^{N} P_{Ri}\ . \tag{6.9}$$

As hopping sequences are repeated periodically, with period LT_h, the average interference power received by Θ after X sequences, i.e., after the time interval $X(LT_h)$, is

$$\widehat{P_I} = \frac{1}{X} \sum_{j=1}^{X} \widetilde{P_I} \tag{6.10}$$

In fast hopping, sequences will be repeated many times before the network situation changes.[1] Therefore, since sequences are periodic, the limit is taken. So, the average interference power received by Θ is

$$P_I = \lim_{X\to\infty} \widehat{P_I} = \lim_{X\to\infty} \frac{1}{X} \sum_{j=1}^{X} \widetilde{P_I} = \widetilde{P_I}. \tag{6.11}$$

Then, according to (6.11), to (6.9) and to (6.3), the expectation of the overall interference power measured by Θ, for the FOS, is

$$\begin{aligned}
\mathbf{E}\{P_I\} &= \mathbf{E}\left\{ \frac{1}{L} \sum_{i\neq\Gamma}^{N} P_{Ri} \right\} = \mathbf{E}\left\{ \frac{1}{L} \sum_{i\neq\Gamma}^{N} \frac{1}{(r_{i\Theta}+1)^{\alpha}} \right\} = \frac{1}{L} \sum_{i\neq\Gamma}^{N} \mathbf{E}\left\{ \frac{1}{(r_{i\Theta}+1)^{\alpha}} \right\} \\
&= \frac{1}{L} \sum_{i\neq\Gamma}^{N} \int_0^R \frac{1}{(r+1)^{\alpha}} f_D(r)\, dr = \frac{N-1}{L} \int_0^R \frac{1}{(r+1)^{\alpha}} \frac{2r}{R^2}\, dr \\
&= \frac{2(N-1)}{L\,R^2} \int_0^R \frac{r}{(r+1)^{\alpha}}\, dr = \frac{2(N-1)}{L\,R^2} \frac{R^2}{2}\ {}_2F_1(\alpha, 2; 3; -R) \\
&= \frac{N-1}{L}\ {}_2F_1(\alpha, 2; 3; -R)\ ,
\end{aligned} \tag{6.12}$$

where ${}_2F_1(\alpha, 2; 3; -R)$ is the *Gauss hypergeometric function*, which can be generally represented by

[1] For the case of fast hopping, considering an arbitrarily large number of consecutive equal hopping secuences transmissions is equivalent to consider the average packet length arbitrarily large, and, therefore, it is equivalent to consider the packet transmission time T_p arbitrarily long.

$$F(\alpha, \beta; \gamma; z) = \frac{1}{B(\beta, \gamma - \beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}\, dt \ ,$$

where $\Re\{\gamma\} > \Re\{\beta\} > 0$, and where $B(x, y)$ is the *Beta function*

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt \ ;$$

see [36]. Since the hopping sequence length for the FOS is

$$L = \frac{N(N-1)}{2(N-M)} \ ,$$

(6.12) yields

$$\mathbf{E}\{P_I\} = \frac{2(N-M)}{N}\ {}_2F_1(\alpha, 2; 3; -R) \ . \tag{6.13}$$

The variance of P_I can be expressed as

$$\mathbf{Var}\{P_I\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\{P_I\}\right]^2 . \tag{6.14}$$

The second moment of P_I is calculated as

$$\begin{aligned}\mathbf{E}\left\{P_I^2\right\} &= \mathbf{E}\left\{\left(\frac{1}{L}\sum_{i\neq\Gamma}^{N}\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right)^2\right\} \\ &= \frac{1}{L^2}\left[(N-1)(N-2)\mathbf{E}\ \underbrace{\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}}_{i\neq j \Rightarrow \frac{1}{(r_{i\Theta}+1)^{\alpha}}\ ,\ \frac{1}{(r_{j\Theta}+1)^{\alpha}}\ \text{indep.}} + (N-1)\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\right] ,\end{aligned}$$

$\forall i, j = 1, \ldots, N;\ i, j \neq \Gamma$. Then,

$$
\begin{aligned}
\mathbf{E}\left\{P_I^2\right\} &= \frac{1}{L^2}\left[(N-1)(N-2)\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\} + (N-1)\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\}\right] \\
&= \frac{1}{L^2}\left[(N-1)(N-2)\left(\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\right)^2 + (N-1)\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\}\right] \\
&= \frac{1}{L^2}\left[(N-1)(N-2)\left(\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\right)^2 + \frac{2(N-1)}{R^2}\int_0^R \frac{r}{(r+1)^{2\alpha}}\,dr\right] \\
&= \frac{N-1}{L^2}\left[(N-2)\left({}_2F_1(\alpha,2;3;-R)\right)^2 + {}_2F_1(2\alpha,2;3;-R)\right] . \qquad (6.15)
\end{aligned}
$$

Thus, the variance of P_I results in

$$
\begin{aligned}
\mathbf{Var}\left\{P_I\right\} &= \frac{N-1}{L^2}\left[(N-2)\left({}_2F_1(\alpha,2;3;-R)\right)^2 + {}_2F_1(2\alpha,2;3;-R) - (N-1)\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] \\
&= \frac{N-1}{L^2}\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right] . \qquad (6.16)
\end{aligned}
$$

Replacing L by (5.2), (6.16) yields

$$
\mathbf{Var}\left\{P_I\right\} = \frac{4(N-M)^2}{N^2(N-1)}\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right] . \qquad (6.17)
$$

The FOS with the virtual receiver placed at the border

In this case, the definition of P_I is the same than in the previous case, given by (6.11) and (6.9), with the only difference of the PDF of $r_{i\Theta}$, which now is the one given by (6.8). Thus, the expectation of P_{Ri} is

$$\mathbf{E}\left\{P_{Ri}\right\} = \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} = \int_0^{2R} \frac{1}{(r+1)^{\alpha}} f_D(r)\,dr$$

$$= \frac{1}{\pi R^2}\left[2\int_0^{2R} \frac{1}{(r+1)^{\alpha}} r \arccos\left(\frac{r}{2R}\right)\,dr - \frac{1}{2R}\int_0^{2R} \frac{1}{(r+1)^{\alpha}} \frac{r^2}{\sqrt{1-\left(\frac{r}{2R}\right)^2}}\,dr\right.$$

$$+ \int_0^{2R} \frac{1}{(r+1)^{\alpha}} \frac{r}{\sqrt{1-\left(\frac{2R^2-r^2}{2R^2}\right)^2}}\,dr - \frac{1}{2}\int_0^{2R} \frac{1}{(r+1)^{\alpha}} \sqrt{4R^2-r^2}\,dr$$

$$\left. + \frac{1}{2}\int_0^{2R} \frac{1}{(r+1)^{\alpha}} \frac{r^2}{\sqrt{4R^2-r^2}}\,dr\right]. \tag{6.18}$$

The second moment of P_{Ri} is similarly obtained as

$$\mathbf{E}\left\{P_{Ri}^2\right\} = \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\} = \int_0^{2R} \frac{1}{(r+1)^{2\alpha}} f_D(r)\,dr$$

$$= \frac{1}{\pi R^2}\left[2\int_0^{2R} \frac{1}{(r+1)^{2\alpha}} r \arccos\left(\frac{r}{2R}\right)\,dr - \frac{1}{2R}\int_0^{2R} \frac{1}{(r+1)^{2\alpha}} \frac{r^2}{\sqrt{1-\left(\frac{r}{2R}\right)^2}}\,dr\right.$$

$$+ \int_0^{2R} \frac{1}{(r+1)^{2\alpha}} \frac{r}{\sqrt{1-\left(\frac{2R^2-r^2}{2R^2}\right)^2}}\,dr - \frac{1}{2}\int_0^{2R} \frac{1}{(r+1)^{2\alpha}} \sqrt{4R^2-r^2}\,dr$$

$$\left. + \frac{1}{2}\int_0^{2R} \frac{1}{(r+1)^{2\alpha}} \frac{r^2}{\sqrt{4R^2-r^2}}\,dr\right]. \tag{6.19}$$

The integrals in (6.18) and (6.19) cannot be solved analytically. Consequently, their values have to be approximate by means of numerical methods. As in the previous case, the result depends on the two parameters α and R, the decay coefficient and the radius of the disk, respectively. From here on, the expresions in (6.18) and (6.19) will be denoted as $F_{border}(\alpha, R) \triangleq \mathbf{E}\left\{P_{Ri}\right\}$ and $F_{border}(2\alpha, R) \triangleq \mathbf{E}\left\{P_{Ri}^2\right\}$.

Therefore, the expectation of the overall interference power seen by the receiver Θ placed at the border of the disk while the intended transmitter Γ is transmitting to, for the FOS, is

$$\mathbf{E}\left\{P_I\right\} = \mathbf{E}\left\{\frac{1}{L}\sum_{i\neq\Gamma}^{N} P_{Ri}\right\} = \mathbf{E}\left\{\frac{1}{L}\sum_{i\neq\Gamma}^{N}\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} = \frac{1}{L}\sum_{i\neq\Gamma}^{N}\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}$$

$$= \frac{1}{L}\sum_{i\neq\Gamma}^{N} F_{border}(\alpha,R) = \frac{N-1}{L}F_{border}(\alpha,R) = \frac{2(N-M)}{N}F_{border}(\alpha,R)\ . \quad (6.20)$$

It is easy to derive the variance of P_I for this case and it can be obtained directly from (6.16) resulting in

$$\mathbf{Var}\left\{P_I\right\} = \frac{N-1}{L^2}\left[F_{border}(2\alpha,R) - \left(F_{border}(\alpha,R)\right)^2\right]$$

$$= \frac{4(N-M)^2}{N^2(N-1)}\left[F_{border}(2\alpha,R) - \left(F_{border}(\alpha,R)\right)^2\right]\ . \quad (6.21)$$

The RS with the virtual receiver placed in the center

For the RS the overall interference power measured at Θ while the intended receiver Γ is transmitting, when Θ is placed in the center of the disk, is

$$P_I = \lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}\sum_{j\neq\Gamma}^{N} I_{ij}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\ . \quad (6.22)$$

In (6.22), the sequence length has been supposed to be infinite. Thus, hopping sequences are assumed to be random sequences instead of pseudorandom sequences. This supposition also implies thet the packet length has been considered arbitrarly large. In section 6.3.1, it is seen that there is no big differences on the results between considering L infinite or considering $L = M$. On the other hand, in (6.22), I_{ij} is a Bernoulli random variable which is equal to 1 if the user ξ_j is transmitting in the same frequency channel than the intended transmitter Γ in the i-th hop. Otherwise, I_{ij} is equal to 0. Thus, the expectation of P_I is

$$\mathbf{E}\left\{P_I\right\} = \lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}\sum_{j\neq\Gamma}^{N}\mathbf{E}\left\{I_{ij}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}$$

$$= \lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}\sum_{j\neq\Gamma}^{N}\mathbf{E}\left\{I_{ij}\right\}\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}$$

$$= \sum_{j\neq\Gamma}^{N}\mathbf{E}\left\{I_{ij}\right\}{}_2F_1(\alpha,2;3;-R)\;. \tag{6.23}$$

The expectation of I_{ij} is given by

$$\mathbf{E}\left\{I_{ij}\right\} = 0\cdot\mathbf{P}\left\{I_{ij}=0\right\} + 1\cdot\mathbf{P}\left\{I_{ij}=1\right\}$$

$$= 0\cdot\frac{M-1}{M} + 1\cdot\frac{1}{M}$$

$$= \frac{1}{M} \tag{6.24}$$

Thus,

$$\mathbf{E}\left\{P_I\right\} = \sum_{j\neq\Gamma}^{N}\frac{1}{M}\,{}_2F_1(\alpha,2;3;-R) = {}_2F_1(\alpha,2;3;-R)\sum_{j\neq\Gamma}^{N}\frac{1}{M}$$

$$= \frac{(N-1)}{M}\,{}_2F_1(\alpha,2;3;-R)\;. \tag{6.25}$$

The variance of the interference power will be calculated, as in (6.1.2), using

$$\mathbf{Var}\left\{P_I\right\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\left\{P_I\right\}\right]^2, \tag{6.26}$$

where the first term is

$$\mathbf{E}\left\{P_I^2\right\} = \mathbf{E}\left\{\left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}\sum_{j\neq\Gamma}^{N} I_{ij}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right)^2\right\} = \mathbf{E}\left\{\left(\lim_{L\to\infty}\frac{1}{L}\sum_{j\neq\Gamma}^{N}\sum_{i=1}^{L} I_{ij}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right)^2\right\}$$

$$= \mathbf{E}\left\{\left(\lim_{L\to\infty}\frac{1}{L}\sum_{j\neq\Gamma}^{N}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\sum_{i=1}^{L} I_{ij}\right)^2\right\} = (N-1)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{2\alpha}}\left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right)^2\right\}$$

$$+ (N-1)(N-2)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\frac{1}{(r_{k\Theta}+1)^{\alpha}}\left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right)\left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ik}\right)\right\}, \tag{6.27}$$

where $j,k \in \{1,\ldots,N\}$; $j,k \neq \Gamma$. Furthermore, since $j \neq k$, $\frac{1}{(r_{j\Theta}+1)^{\alpha}}$ and $\frac{1}{(r_{k\Theta}+1)^{\alpha}}$ are statistically independent. Likewise, I_{ij} and I_{ik} are also independent for any value of i. And also $(r_{x\Theta}+1)^{\alpha}$ and I_{iy} are independent, for $x = j,k$ and $y = j,k$. Hence[2],

$$\mathbf{E}\left\{P_I^2\right\} = (N-1)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{2\alpha}}\right\}\mathbf{E}\underbrace{\left\{\left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right)^2\right\}}_{(\lim_{L\to\infty} f(L))^2=\lim_{L\to\infty}(f(L))^2}$$

$$+ (N-1)(N-2)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}\mathbf{E}\left\{\frac{1}{(r_{k\Theta}+1)^{\alpha}}\right\}\mathbf{E}\left\{\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right\}\mathbf{E}\left\{\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ik}\right\}$$

$$= (N-1)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{2\alpha}}\right\}\mathbf{E}\left\{\lim_{L\to\infty}\left(\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right)^2\right\}$$

$$+ (N-1)(N-2)\left(\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}\right)^2\left(\mathbf{E}\left\{\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right\}\right)^2$$

$$= (N-1)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{2\alpha}}\right\}\lim_{L\to\infty}\frac{1}{L^2}\left[L\mathbf{E}\left\{I_{ij}^2\right\} + L(L-1)\mathbf{E}\left\{I_{ij}I_{pj}\right\}\right]$$

$$+ (N-1)(N-2)\left(\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}\right)^2\left(\mathbf{E}\left\{\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L} I_{ij}\right\}\right)^2, \tag{6.28}$$

[2]For the identity $(\lim_{L\to\infty} f(L))^2 = \lim_{L\to\infty}(f(L))^2$, used in this page, see [31].

where $i, p \in \{1, \ldots, L\}$. As $i \neq p$, for the same user j, I_{ij} and I_{pj} correspond to different hops; so, I_{ij} and I_{pj} are independent. Thus,

$$\begin{aligned}
\mathbf{E}\left\{P_I^2\right\} &= (N-1)\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{2\alpha}}\right\} \lim_{L\to\infty}\left[\frac{1}{L}\mathbf{E}\left\{I_{ij}^2\right\} + \left(1-\frac{1}{L}\right)\left(\mathbf{E}\left\{I_{ij}\right\}\right)^2\right] \\
&+ (N-1)(N-2)\left(\mathbf{E}\left\{\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\}\right)^2 \left(\lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}\mathbf{E}\left\{I_{ij}\right\}\right)^2 \\
&= (N-1)\,{}_2F_1(2\alpha,2;3;-R)\frac{1}{M^2} + (N-1)(N-2)\left({}_2F_1(\alpha,2;3;-R)\right)^2\frac{1}{M^2} \\
&= \frac{N-1}{M^2}\left[\,{}_2F_1(2\alpha,2;3;-R) + (N-2)\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] . \qquad (6.29)
\end{aligned}$$

Consequently, the variance of the interference power yields

$$\begin{aligned}
\mathbf{Var}\left\{P_I\right\} &= \frac{N-1}{M^2}\left[\,{}_2F_1(2\alpha,2;3;-R) + (N-2)\left({}_2F_1(\alpha,2;3;-R)\right)^2 - (N-1)\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] \\
&= \frac{N-1}{M^2}\left[\,{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]. \qquad (6.30)
\end{aligned}$$

The RS with the virtual receiver placed at the border

As done for the FOS, the derived expressions for the expectation and for the variance of P_I for the case when the virtual reference receiver Θ is placed on the border of the disk come directly from the ones obtained with the virtual receiver placed in the center as follows

$$\mathbf{E}\left\{P_I\right\} = \frac{(N-1)}{M}\,F_{border}(\alpha, R) \qquad (6.31)$$

and

$$\mathbf{Var}\left\{P_I\right\} = \frac{N-1}{M^2}\left[\,F_{border}(2\alpha, R) - \left(F_{border}(\alpha, R)\right)^2\right] . \qquad (6.32)$$

The CS with the virtual receiver placed in the center

In this scheme, the average interference power that Θ will see during a whole hopping sequence transmission (of length $L = M$) from the i-th interferer is given by

$$\widetilde{P_I} = I \frac{1}{L\,T_h} \sum_{j=1}^{L} \frac{1}{(r_{i\Theta}+1)^{\alpha}}\, T_h = I \frac{1}{(r_{i\Theta}+1)^{\alpha}} , \qquad i \in \{1,\ldots,N\},\ i \neq \Gamma , \tag{6.33}$$

where I is a Bernoulli random variable which equals 0 when any of the interferer transmitters is not colliding with the intended transmitter, Γ, and 1 otherwise. The PMF of I has the following values:

$$f_I(x=1) = \mathbf{P}\{I=1\} = \frac{2(N-M)}{N}$$

$$f_I(x=0) = \mathbf{P}\{I=0\} = 1 - \mathbf{P}\{I=1\} = 1 - \frac{2(N-M)}{N} = \frac{2M-N}{N}$$

It follows from (6.33) that after an arbitrarily large number X of transmitted hopping sequences, in which the interferer is always the same user ξ_i, the overall interference power that Θ will see has the same expression than (6.10), where $\widetilde{P_I}$ is now the one given in (6.33). Taking X to the limit (the packet length is considered to be arbitrarily large), the expression for P_I is the same than in (6.11). Thus, the expectation of P_I is given by

$$\mathbf{E}\{P_I\} = \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\mathbf{E}\{I\} = {}_2F_1(\alpha,2;3;-R)\left(0\cdot\mathbf{P}\{I=0\} + 1\cdot\mathbf{P}\{I=1\}\right)$$

$$= \frac{2(N-M)}{N}\, {}_2F_1(\alpha,2;3;-R) . \tag{6.34}$$

The variance of the interference power will be calculated using

$$\mathbf{Var}\{P_I\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\{P_I\}\right]^2 , \tag{6.35}$$

where the second moment is

$$\mathbf{E}\left\{P_I^2\right\} = \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\}\mathbf{E}\left\{I^2\right\} . \tag{6.36}$$

In this case,

$$\mathbf{E}\left\{I^2\right\} = \mathbf{E}\left\{I\right\} = \frac{2(N-M)}{N} \;, \tag{6.37}$$

yielding

$$\mathbf{E}\left\{P_I^2\right\} = \frac{2(N-M)}{N} \; {}_2F_1(2\alpha, 2; 3; -R) \;. \tag{6.38}$$

Thus,

$$\begin{aligned}\mathbf{Var}\left\{P_I\right\} &= \frac{2(N-M)}{N} \; {}_2F_1(2\alpha, 2; 3; -R) - \frac{4(N-M)^2}{N^2} \left({}_2F_1(\alpha, 2; 3; -R)\right)^2 \\ &= \frac{2(N-M)}{N} \left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{2(N-M)}{N} \left({}_2F_1(\alpha, 2; 3; -R)\right)^2 \right] \;. \end{aligned} \tag{6.39}$$

The CS with the virtual receiver placed at the border

As for the previous two schemes, the derivation of the expressions for the expectation and the variance of P_I for the case where Θ is placed at the border is analogous to the case when Θ is placed in the center. Therefore, in this case the resulting expression for the expectation of P_I is

$$\mathbf{E}\left\{P_I\right\} = \frac{2(N-M)}{N} \; F_{border}(\alpha, R) \tag{6.40}$$

and the result for the variance of P_I is

$$\mathbf{Var}\left\{P_I\right\} = \frac{2(N-M)}{N} \left[F_{border}(2\alpha, R) - \frac{2(N-M)}{N} \left(F_{border}(\alpha, R)\right)^2 \right] \;. \tag{6.41}$$

6.1.3 Static slow hopping analysis

As described in section 3.4, in the slow hopping case, the hop period is equal to the packet duration. Hence, since the reference framework according to which the interference balancing in each scheme is measured is one packet transmission, the performance evaluation in the case of slow hopping will be done from the perspective of one hop separately. This is valid for both static and dynamic slow hopping analysis.

As already said in section 3.4, in a slow hopping scenario some of the users will act as transmitters and some of them not during a hopping sequence transmission. For the case of static slow hopping it is not known *a priori* which nodes out of the N_T total nodes in the network have the role of transmitters in a randomly chosen hop. Therefore, it is assumed that every node is a transmitter with probability p. Thus, the expected number of nodes that are transmitting is equal to $N = N_T\, p$. Next, the three studied schemes will be analysed for the static slow hopping case.

The FOS with the virtual receiver placed in the center

The overall interference power seen by Θ during one hop period T_h when Θ is placed in the center, for the FOS, is given by

$$P_I = I \frac{1}{(r_{i\Theta} + 1)^\alpha} , \tag{6.42}$$

where I is a Bernoulli random variable equal to one when one node transmits on the same channel channel than Γ, and also if this user is a transmitter in this hop. Otherwise I is equal to zero. The probabilities for both cases are

$$\begin{aligned} \mathbf{P}\{I = 1\} &= \mathbf{P}\{\text{user } i \text{ is a transmitter}\}\, \mathbf{P}\{\text{user } i \text{ same channel than } \Gamma | \text{user } i \text{ is a transmitter}\} \\ &= \mathbf{P}\{\text{user } i \text{ is a transmitter}\}\, \mathbf{P}\{\text{user } i \text{ same channel than } \Gamma\} \ , \end{aligned} \tag{6.43}$$

where the second equation follows from the independence of the two events. The first factor is simply

$$\mathbf{P}\{\text{user } i \text{ is a transmitter}\} = p \ , \tag{6.44}$$

while the second is given by

$$\mathbf{P}\{\text{user } i \text{ same channel than } \Gamma\} = \frac{N_T - 1}{L} \ . \tag{6.45}$$

Therefore,

$$\mathbf{P}\{I = 1\} = \frac{N_T - 1}{L} p = \frac{2(N_T - M)}{N_T} p \tag{6.46}$$

and

$$\mathbf{P}\{I = 0\} = 1 - \mathbf{P}\{I = 1\} = 1 - \frac{N_T - 1}{L} p = \frac{N_T(1 - 2p) + 2Mp}{N_T} \ . \tag{6.47}$$

Thus,

$$\mathbf{E}\{I\} = 0 \cdot \mathbf{P}\{I=0\} + 1 \cdot \mathbf{P}\{I=1\} = \frac{2(N_T-M)p}{N_T} \; . \tag{6.48}$$

Consequently, the expectation of P_I is

$$\begin{aligned}\mathbf{E}\{P_I\} = \mathbf{E}\{I\}\,\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} &= \frac{2(N_T-M)p}{N_T}\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} \\ &= \frac{2(N_T-M)p}{N_T}\ {}_2F_1(\alpha,2;3;-R) \; . \end{aligned} \tag{6.49}$$

Likewise, the variance of P_I can be calculated using

$$\mathbf{Var}\{P_I\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\{P_I\}\right]^2 \; , \tag{6.50}$$

where

$$\mathbf{E}\left\{P_I^2\right\} = \mathbf{E}\left\{I^2\right\}\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\} = \frac{2(N_T-M)p}{N_T}\ {}_2F_1(2\alpha,2;3;-R) \; . \tag{6.51}$$

Hence,

$$\begin{aligned}\mathbf{Var}\{P_I\} &= \frac{2(N_T-M)p}{N_T}\ {}_2F_1(2\alpha,2;3;-R) - \left(\frac{2(N_T-M)p}{N_T}\right)^2 \left({}_2F_1(\alpha,2;3;-R)\right)^2 \\ &= \frac{2(N_T-M)p}{N_T}\left[{}_2F_1(2\alpha,2;3;-R) - \frac{2(N_T-M)p}{N_T}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] \; . \end{aligned} \tag{6.52}$$

The FOS with the virtual receiver placed at the border

With the same arguments than in the previous calculation, the expectation of P_I in this case yields

$$\mathbf{E}\{P_I\} = \frac{2(N_T-M)p}{N_T}\ F_{border}(\alpha,R) \; , \tag{6.53}$$

and its variance yields

$$\mathbf{Var}\{P_I\} = \frac{2(N_T-M)p}{N_T}\left[F_{border}(2\alpha,R) - \frac{2(N_T-M)p}{N_T}\left(F_{border}(\alpha,R)\right)^2\right] \; . \tag{6.54}$$

The RS with the virtual receiver placed in the center

In this case P_I is given by[3]

$$P_I = \sum_{i \neq \Gamma, \Theta}^{N_T} I_i \frac{1}{(r_{i\Theta} + 1)^{\alpha}} , \tag{6.55}$$

where I_i is a Bernoulli random variable with the same possible values than I in (6.42). So, equations (6.43) and (6.44) are the same in this case. But (6.45), in this case is

$$\mathbf{P}\left\{\text{user } i \text{ same channel than } \Gamma\right\} = \frac{1}{M} .$$

Thus,

$$\mathbf{E}\left\{I_i\right\} = 0 \cdot \mathbf{P}\left\{I_i = 0\right\} + 1 \cdot \mathbf{P}\left\{I_i = 1\right\} = \frac{p}{M}$$

Hence, the expectation of P_I is given by

$$\mathbf{E}\left\{P_I\right\} = \sum_{i \neq \Gamma, \Theta}^{N_T} \mathbf{E}\left\{I_i\right\} \mathbf{E}\left\{\frac{1}{(r_{i\Theta} + 1)^{\alpha}}\right\} = \frac{(N_T - 2)p}{M} \, {}_2F_1(\alpha, 2; 3; -R) . \tag{6.56}$$

Note that the second member in (6.56) can be expressed as

$$\mathbf{E}\left\{\frac{1}{(r_{i\Theta} + 1)^{\alpha}}\right\} \mathbf{E}\left\{\sum_{i \neq \Gamma, \Theta}^{N_T} I_i\right\} \tag{6.57}$$

because all random variables $r_{i\Theta}$ have the same distribution. Note, then, that $\sum_{i \neq \Gamma, \Theta}^{N_T} I_i$ is a binomial random variable with expectation

$$\mathbf{E}\left\{\sum_{i \neq \Gamma, \Theta}^{N_T} I_i\right\} = (N_T - 2)\frac{p}{M} , \tag{6.58}$$

which leads to te same result than the third member of (6.56).On the other hand, the variance of P_I is

$$\mathbf{Var}\left\{P_I\right\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\left\{P_I\right\}\right]^2 , \tag{6.59}$$

[3]Note that since Θ is considered to belong to the set of N_T nodes in the network, in this case it has to be excluded from the sum, as well as Γ.

where

$$\mathbf{E}\left\{P_I^2\right\} = \mathbf{E}\left\{\left(\sum_{i\neq\Gamma,\Theta}^{N_T} I_i \frac{1}{(r_{i\Theta}+1)^{\alpha}}\right)^2\right\}$$

$$= (N_T-2)\mathbf{E}\left\{I_i^2\left(\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right)^2\right\} + (N_T-2)(N_T-3)\mathbf{E}\left\{I_i I_j \frac{1}{(r_{i\Theta}+1)^{\alpha}}\frac{1}{(r_{j\Theta}+1)^{\alpha}}\right\} , \tag{6.60}$$

where $j \neq i$. Therefore,

$$\mathbf{E}\left\{P_I^2\right\} = (N_T-2)\mathbf{E}\left\{I_i^2\right\}\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\} + (N_T-2)(N_T-3)\left(\mathbf{E}\left\{I_i\right\}\right)^2\left(\mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\}\right)^2$$

$$= (N_T-2)\frac{p}{M}\,{}_2F_1(2\alpha,2;3;-R) + (N_T-2)(N_T-3)\frac{p^2}{M^2}\left({}_2F_1(\alpha,2;3;-R)\right)^2$$

$$= \frac{(N_T-2)p}{M}\left[{}_2F_1(2\alpha,2;3;-R) + \frac{(N_T-3)p}{M}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] . \tag{6.61}$$

Thus,

$$\mathbf{Var}\left\{P_I\right\} = \frac{(N_T-2)p}{M}\left[{}_2F_1(2\alpha,2;3;-R) + \frac{(N_T-3)p}{M}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]$$

$$- \left(\frac{(N_T-2)p}{M}\right)^2\left({}_2F_1(\alpha,2;3;-R)\right)^2$$

$$= \frac{(N_T-2)p}{M}\left[{}_2F_1(2\alpha,2;3;-R) - \frac{p}{M}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right] . \tag{6.62}$$

The RS with the virtual receiver placed at the border

Similarly to (6.56) and (6.62), the expectation and the variance of P_I for this case can be calculated as

$$\mathbf{E}\left\{P_I\right\} = \frac{(N_T-2)p}{M}\,F_{border}(\alpha,R) \tag{6.63}$$

and

$$\mathbf{Var}\left\{P_I\right\} = \frac{(N_T-2)p}{M}\left[F_{border}(2\alpha,R) - \frac{p}{M}\left(F_{border}(\alpha,R)\right)^2\right] . \tag{6.64}$$

The CS with the virtual receiver placed in the center

For this scheme, the overall interference power seen by Θ is given by

$$P_I = I\frac{1}{(r_{i\Theta}+1)^{\alpha}} , \tag{6.65}$$

where I is a Bernoulli random variable equal to one if the intended transmitter Γ belongs to the group of $2(N_T - M)$ users with a repeated hop sequence during the whole transmission and, moreover, if at the same time the user with the same sequence than Γ is a transmitter. Otherwise I is equal to zero. Therefore, the probabilities for the two values of this random variable are obtained as

$$\mathbf{P}\left\{I=1\right\} = \mathbf{P}\left\{\text{user } i \text{ is a transmitter}\right\}\mathbf{P}\left\{\text{user } i \text{ same channel than } \Gamma | \text{user } i \text{ is a transmitter}\right\} ,$$

and, since the two events are independent, it yields

$$\mathbf{P}\left\{I=1\right\} = \mathbf{P}\left\{\text{user } i \text{ is a transmitter}\right\}\mathbf{P}\left\{\text{user } i \text{ same channel than } \Gamma\right\} .$$

These probabilities are

$$\mathbf{P}\left\{\text{user } i \text{ is a transmitter}\right\} = p$$

and

$$\mathbf{P}\left\{\text{user } i \text{ same channel than } \Gamma\right\} = \frac{2(N_T-M)}{N_T} .$$

Therefore,

$$\mathbf{P}\left\{I=1\right\} = \frac{2(N_T-M)p}{N_T} \tag{6.66}$$

and

$$\mathbf{P}\left\{I=0\right\} = 1 - \mathbf{P}\left\{I=1\right\} = 1 - \frac{2(N_T-M)}{N_T} = \frac{N_T(1-2p)+Mp}{N_T} . \tag{6.67}$$

Thus,

$$\mathbf{E}\{I\} = 0 \cdot \mathbf{P}\{I=0\} + 1 \cdot \mathbf{P}\{I=1\} = \frac{2(N_T - M)p}{N_T} \; . \tag{6.68}$$

Consequently, the expectation of P_I is equal to

$$\begin{aligned} \mathbf{E}\{P_I\} &= \mathbf{E}\{I\}\, \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} = \frac{2(N_T - M)p}{N_T}\, \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{\alpha}}\right\} \\ &= \frac{2(N_T - M)p}{N_T}\; {}_2F_1(\alpha, 2; 3; -R) \; . \end{aligned} \tag{6.69}$$

On the other hand, the variance of P_I is calculated from

$$\mathbf{Var}\{P_I\} = \mathbf{E}\left\{P_I^2\right\} - \left[\mathbf{E}\{P_{interf}\}\right]^2 \; , \tag{6.70}$$

where

$$\mathbf{E}\left\{P_I^2\right\} = \mathbf{E}\left\{I^2\right\} \mathbf{E}\left\{\frac{1}{(r_{i\Theta}+1)^{2\alpha}}\right\} = \frac{2(N_T - M)p}{N_T}\; {}_2F_1(2\alpha, 2; 3; -R) \; . \tag{6.71}$$

Thus,

$$\begin{aligned} \mathbf{Var}\{P_I\} &= \frac{2(N_T - M)p}{N_T}\; {}_2F_1(2\alpha, 2; 3; -R) - \left(\frac{2(N_T - M)p}{N_T}\right)^2 \left({}_2F_1(\alpha, 2; 3; -R)\right)^2 \\ &= \frac{2(N_T - M)p}{N_T}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{2(N_T - M)p}{N_T}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] \; . \end{aligned} \tag{6.72}$$

The CS with the virtual receiver placed at the border

When the virtual reference receiver Θ is placed at the border of the disk, the derived expressions for the expectation and the variance of P_I for the CS are, respectively,

$$\mathbf{E}\{P_I\} = \frac{2(N_T - M)p}{N_T}\; F_{border}(\alpha, R) \tag{6.73}$$

and

$$\mathbf{Var}\{P_I\} = \frac{2(N_T - M)p}{N_T}\left[F_{border}(2\alpha, R) - \frac{2(N_T - M)p}{N_T}\left(F_{border}(\alpha, R)\right)^2\right] \; . \tag{6.74}$$

6.1.4 Dynamic slow hopping analysis

As previously said, in the case of dynamic slow hopping, as happens in the case of fast hopping, it is *a priori* known which N users from among the N_T users will be transmitting during every transmission. This implies that, regarding the statistical analysis of the dynamic slow hopping case, the derived formulae are the same than that ones derived in the static slow hopping case if N_T is replaced by N and the probability p takes the value $p = 1$. Indeed, all of the N regarded transmitters in the case of dynamic slow hopping, are surely transmitters, so, p must be equal to one. The derived expressions for each scheme are listed in the following.

The FOS with the virtual receiver placed in the center

$$\mathbf{E}\{P_I\} = \frac{2(N-M)}{N}\ {}_2F_1(\alpha, 2; 3; -R) \tag{6.75}$$

$$\mathbf{Var}\{P_I\} = \frac{2(N-M)}{N}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{2(N-M)}{N}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] \tag{6.76}$$

The FOS with the virtual receiver placed at the border

$$\mathbf{E}\{P_I\} = \frac{2(N-M)}{N}\ F_{border}(\alpha, R) \tag{6.77}$$

$$\mathbf{Var}\{P_I\} = \frac{2(N-M)}{N}\left[F_{border}(2\alpha, R) - \frac{2(N-M)}{N}\left(F_{border}(\alpha, R)\right)^2\right] \tag{6.78}$$

The RS with the virtual receiver placed in the center

$$\mathbf{E}\{P_I\} = \frac{(N-1)}{M}\ {}_2F_1(\alpha, 2; 3; -R) \tag{6.79}$$

$$\mathbf{Var}\{P_I\} = \frac{(N-1)}{M}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{1}{M}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] \tag{6.80}$$

The RS with the virtual receiver placed at the border

$$\mathbf{E}\{P_I\} = \frac{(N-1)}{M}\ F_{border}(\alpha, R) \tag{6.81}$$

$$\mathbf{Var}\{P_I\} = \frac{(N-1)}{M}\left[F_{border}(2\alpha, R) - \frac{1}{M}\left(F_{border}(\alpha, R)\right)^2\right] \tag{6.82}$$

The CS with the virtual receiver placed in the center

$$\mathbf{E}\left\{P_I\right\} = \frac{2(N-M)}{N}\ {}_2F_1(\alpha, 2; 3; -R) \tag{6.83}$$

$$\mathbf{Var}\left\{P_I\right\} = \frac{2(N-M)}{N}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{2(N-M)}{N}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] \tag{6.84}$$

The CS with the virtual receiver placed at the border

$$\mathbf{E}\left\{P_I\right\} = \frac{2(N-M)}{N}\ F_{border}(\alpha, R) \tag{6.85}$$

$$\mathbf{Var}\left\{P_I\right\} = \frac{2(N-M)}{N}\left[F_{border}(2\alpha, R) - \frac{2(N-M)}{N}\left(F_{border}(\alpha, R)\right)^2\right] \tag{6.86}$$

Probability of collision and BER

For the case of dynamic slow hopping, these two metrics will also be used. In this section it is regarded the scenario where hopping sequences have already been assigned and, therefore, they are known. The probability that one user experiences at least one collision in a randomly chosen hop with the FOS has the expression given in (5.16). The same probability for the RS is given in (5.17) and the limit case is given in (5.18). For the CS, in contrast, once the hopping sequences are known, it does not matter which is the randomly chosen hop because in all hops the same collisions take place. Thus, for the CS, the probability of colliding in a randomly chosen hop is 1 for $2(N-M)$ users and 0 for the rest.

Regarding the BER, a BPSK modulation has been chosen. Accordingly, for an AWGN channel without interference the BER is calculated as

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \tag{6.87}$$

where E_b is the energy per bit, $\frac{N_0}{2}$ is the PSD of the noise and $Q(\cdot)$ is the *tail probability* of the *standard gaussian distribution* (see [30]). Assume that in the case of study in this work the interference has a flat power spectral density in the bandwidth $2B$ with level $\frac{N_0}{2}$, i.e., the interference can be regarded as an AWGN. Then, the expected received interference power is equal to

$$\mathbf{E}\left\{P_I\right\} = \frac{N_0}{2}2B = \frac{N_0}{T}. \tag{6.88}$$

Then, equaling (6.88) to (6.75), N_0 for the FOS yields

$$N_0 = T\,\frac{2(N-M)}{N}\;{}_2F_1(\alpha, 2; 3; -R)\;. \tag{6.89}$$

On the other hand, for the signal received from the intended transmitter, since the transmitted power is equal to 1, $E_b = 1 \cdot T_b$, where T_b is the bit duration, which is equal to the symbol duraton T for a BPSK modulation. Hence,

$$P_{b_{FOS}} = Q\left(\sqrt{\frac{2\;T\;{}_2F_1(\alpha, 2; 3; -R)}{T\;\frac{2(N-M)}{N}\;{}_2F_1(\alpha, 2; 3; -R)}}\right) = Q\left(\sqrt{\frac{1}{1-\frac{1}{\gamma}}}\right)\;. \tag{6.90}$$

Proceeding similarly, the BER for the RS is

$$P_{b_{RS}} = Q\left(\sqrt{\frac{2}{\gamma - \frac{1}{M}}}\right)\;, \tag{6.91}$$

and for the CS, for the users that it is already known that will collide, the BER is

$$P_{b_{RS}} = Q\left(\sqrt{2}\right)\;, \tag{6.92}$$

while for the rest of the users $P_{b_{RS}} = 0$, where $P_{b_{FOS}}$, $P_{b_{RS}}$ and $P_{b_{RS}}$ denote the BER for the FOS, the RS and the CS, respectively.

6.2 Simulations

P_I depends on random factors such as the position of the interferers and who will interfere Γ during a hopping sequence transmission. Therefore, after a statistical analysis from the theoretical perspective, simulating the system behaviour is an appropriate manner to validate the results derived analytically.

Since the histogram of a random variable would characterise its PDF for an infinite number of realisations, simulating the statistical moments such as the expectation and the variance, as it is done here, in fact consists on recreating the experiment that defines the random variable by means of a model, repeating the experiment a number k of times and estimate the expectation and the variance from the obtained results. The greater the number of iterations k, the lower the error between the estimated and the true parameters. For all the simulations carried out in this thesis, the chosen number of iterations is $k = 10^6$.

For the simulations, the system model (see section 3) has been implemented as follows: in every iteration, a fixed number of N points is placed throughout a circle with radius R according to a binomial point process. That is, all N points are distributed uniformly throughout the disk. Thus, for simulating the binomial point process, points have been generated according to the following two PDFs:

$$f_R(r) = \frac{2r}{R^2} , \qquad 0 \leq r \leq R \tag{6.93}$$

and

$$f_\Theta(\theta) = \frac{1}{2\pi} , \qquad 0 \leq \theta < 2\pi . \tag{6.94}$$

An example of one realisation of the implemented point process points is depicted in Fig. 6.4.

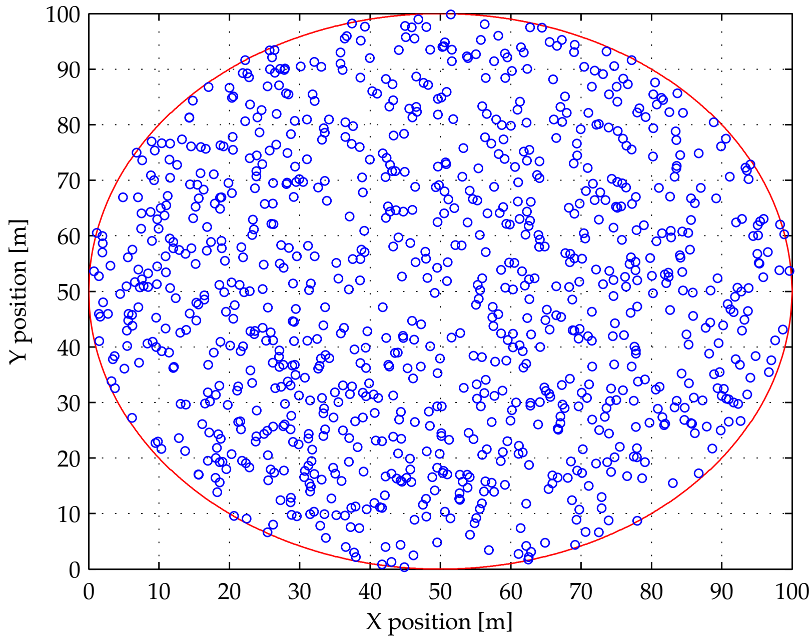


Figure 6.4: One realisation of a binomial point process for $N = 1000$, on a disk with radius $R = 50$ m.

For implementing the three schemes for the fast hopping case, it has been decided to make correspond each iteration of the simulation to a whole hopping sequence for every scheme. Hence, the obtained value of P_I for each scheme at the end of each iteration is the power that every transmitter experiences. The positions of the transmitters are changed in each iteration. Thus, for the FOS the implemented sequence length is $L = \frac{N(N-1)}{2(N-M)}$, for the CS it is $L = M$, and for the RS there are two possibilities: if the sequence length is considered to be infinite, as in section 6.1.2, the chosen value for the simulation is $L = 1000$; the other possibility is to fix the sequence length to $L = M$ (see the corresponding subsection in section 6.3).[4]

On the other hand, as set in section 6.1.1, for both fast and slow hopping, Θ has to be placed in the center and at the border of the circle, depicted in Fig. 6.4 for $R = 50$ m. For simulating these scenarios, what has been done is simply to calculate the decay of the average transmitted power between each interferer and the center of the disk or one point on the border of the disk, and multiply it by the transmitted power (fixed to 1 W for all users) for obtaining P_{Ri}.

In the case of fast hopping, for the random scheme, once the intended transmitter is randomly chosen amongst the N transmitters, the power from all the $N-1$ interfering transmitters is summed up and divided by the sequence length. This is done in each iteration. For the RS, in every hop, a Bernoulli random variable is implemented for deciding whether each of the $N-1$ possible interferers is interfering the intened transmitter with probability $\frac{1}{M}$ or not. And since it is done for all the $N-1$ transmitters, the binomial random variable described in section 6.1.2 is implemented in each hop. This is repeated for L hops and averaged with respect to L. All this steps correspond to one iteration. Finally, for the CS, in each iteration only one interferer among the $N-1$ remaining transmitters is randomly chosen and its interference power during M hops is calculated and averaged by M. Then, in each iteration, a Bernoulli random variable is implemented to decide wheter the intended transmitter belongs to the set of interfered nodes, with probability $\frac{2(N-M)}{N}$, or not.

In the case of slow hopping, the FOS and the CS have almost the same algorithm for the implementation but with different realisations of the Bernoulli random variable which decide in every hop whether Γ is interfered, with probability $\frac{2(N-M)}{N}$ or not. For the RS, the implementation is the same than for the fast hopping case but only for one hop instead of being for L hops.

6.3 Discussion

In this section the results derived both analitically and by means of simulations in the previous sections will be evaluated and discussed. The performance evaluation of each scheme is analysed in most of the graphics according to the expectation and to the variance of the interference power P_I with respect to a key parameter, γ, the ratio between the number of transmitters and the number of frequency channels. With regrad to the notation, recall that the variances of P_I for the FOS, the RS and the CS will be respectively denoted in some parts of this section as $\mathbf{Var}_{FOS}\{P_I\}$, $\mathbf{Var}_{RS}\{P_I\}$ and $\mathbf{Var}_{CS}\{P_I\}$.

[4] It is convenient bearing in mind that, in most of the provided graphics in section 6.3, the chosen value for M is $M = 10$, so that $L = 1000$ is, in relative terms, great enough to well approximate an infinite sequence length. This can be checked in Fig. 6.6, where the curve of the variance of P_I with the RS obtained by simulation with $L = 1000$ is roughly identical to the curve obtained analitically.

6.3.1 Fast frequency hopping

With the virtual receiver placed in the center

As shown in Fig. 6.5, for any value of $0 < \gamma \leq 2$, the expected interference power seen by a user with the RS is much higher than for the FOS and the CS, which have the same expression for the expectation of P_I. This figure also illustrates that, in the interval $0 < \gamma \leq 1$, for the FOS and the CS interference power results in zero because the hopping sequences with these schemes are orthogonal. With the RS, in contrast, there is interference for any γ, with $N > 1$ (when there is only one user in the ad hoc network, obviously, there is no interference). As previously stated, the FOS, as well as the CS, is optimal in terms of the total number of collisions per hop, i.e., with these schems the total number of collisions that take place in each hop is the minimum possible. Thus, the expected interference power is also the minimum possible with these schemes. On the other hand, in Fig. 6.5, the linear increase of the expectation of P_I for the RS can also be observed.

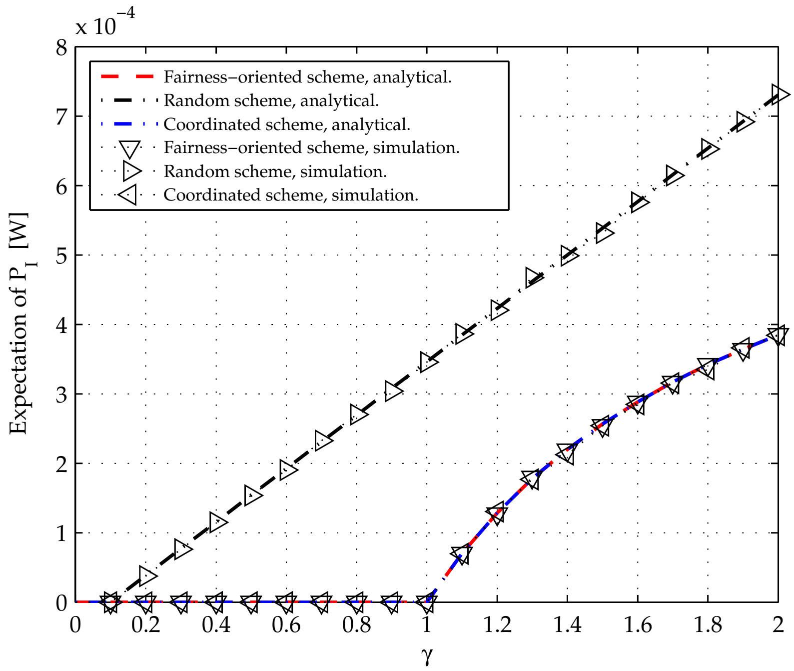


Figure 6.5: Fast hopping: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$ and $R = 50$ m.

With regard to the variance of P_I for the three studied schemes, it is clear that the FOS outperforms the other two schemes many times. Recall that the variance of the interference power seen by a user with each scheme indicates how equitably interference is balanced among all users and, therefore, it is directly related to the main goal of this thesis. Thus, from the point of view of fairness, in the interval $1 < \gamma \leq 2$, since the FOS has the lowest variance, this scheme is the one with the most equitable balancing of P_I among users. The scheme with worst performance in terms of fairness in the fast hopping case is the CS. On the other hand, in the interval $0 < \gamma \leq 1$ the FOS and the CS have zero variance, since P_I is equal to zero. In contrast, with the RS there is also variance among users in this interval.

As a remark, note that in Fig. 6.6, for the case of the CS, there is a higher difference between the analytical curve and the one obtained by simulation than for the other two schemes. This is due to the fact that the estimation of the variance (inherent to the simulation) for the CS has more variance than the estimation for the other two schemes, for the same number of iterations in the simulation. The CS has more variance because, whereas for this scheme in each iteration (and one iteration is one realization of P_I) the same interferer is interfering one interfered user, for the other two schemes one realization of P_I involves many users. Therefore, with the same number of iterations, the estimated variance of P_I is higher because of the randomness of the position of the users on the disk.

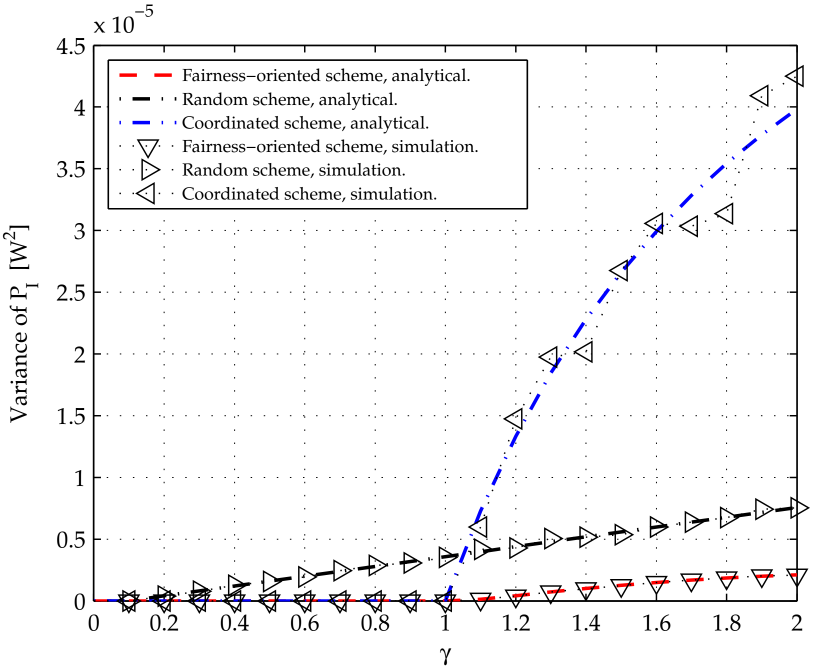


Figure 6.6: Fast hopping with Θ in the center of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

The variation of $\mathbf{Var}\{P_I\}$ with respect to γ is not linear neither for teh FOS nor for the CS (see equations (6.17), (6.30) and (6.39)). Furthermore, apart from depending on γ, these expressions also depend on N or M in absolute terms. To better compare the behaviour of each scheme with respect to the parameter γ, the ratios of the variances are next analysed. Recall that

$$\Delta_{VarAB} = \frac{\mathbf{Var}_A\{P_I\}}{\mathbf{Var}_B\{P_I\}} \tag{6.95}$$

is the ratio between the variance of P_I for the scheme A and the variance of P_I for the scheme B, where A and B can be F, for the FOS, R, for the RS, and C, for the CS. Thus, for the fast hopping case, with the virtual receiver Θ placed in the center of the disk,

$$\Delta_{VarRF} = \frac{\frac{N-1}{M^2}\left[\,{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}{\frac{4(N-M)^2}{N^2(N-1)}\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}$$

$$= \frac{N^2(N-1)^2}{4M^2(N-M)^2} = \frac{1}{4}\gamma^2\left(\frac{\gamma-\frac{1}{M}}{\gamma-1}\right)^2 , \qquad 1 < \gamma \leq 2 \, , \tag{6.96}$$

and

$$\Delta_{VarCF} = \frac{\frac{2(N-M)}{N}\left[{}_2F_1(2\alpha,2;3;-R) - \frac{2(N-M)}{N}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}{\frac{4(N-M)^2}{N^2(N-1)}\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}$$

$$= \frac{N(N-1)}{2(N-M)}\frac{\left[{}_2F_1(2\alpha,2;3;-R) - \frac{2(N-M)}{N}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}{\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}$$

$$= \frac{\gamma}{2}\frac{\gamma M-1}{\gamma-1}\frac{\left[{}_2F_1(2\alpha,2;3;-R) - 2\left(1-\frac{1}{\gamma}\right)\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}{\left[{}_2F_1(2\alpha,2;3;-R) - \left({}_2F_1(\alpha,2;3;-R)\right)^2\right]} , \qquad 1 < \gamma \leq 2 \, . \tag{6.97}$$

It is clear, then, that both ratios depend on γ and M, aside from R and α. Fig. 6.7 displays $\Delta_{VarRF}(\gamma, M)$ and $\Delta_{VarCF}(\gamma, M)$ with respect to γ for the cases $M = 10$ and $M = 20$. In this figure it can be seen that for values of γ close to 1, Δ_{VarRF} and Δ_{VarCF} take higher values. In fact, as (6.96) and (6.97) display, Δ_{VarRF} and Δ_{VarCF} tend to infinity when γ tends to one because the variance for the FOS tends faster to zero than the variance for the CS. Thus, the performance of the FOS compared to the other two studied schemes is more beneficial in terms of fairness for $1 < \gamma \ll 2$. However, in reality, γ will not be arbitrarily close to 1 because neither the number of transmitters N in the ad hoc network nor the number of available frequency channels M will be arbitrarily high.

For $M = 10$, the variance P_I with the RS is at least roughly 4 times greater than the one seen with the FOS (this is for $\gamma = 2$). In contrast, Δ_{VarCF} is roughly 19 for $\gamma = 2$. Δ_{VarRF} and Δ_{VarCF} become higher when M increases, as Fig. 6.96 shows. However, the difference between the ratio $\Delta_{VarRF}(\gamma, M)$ for the cases $M = 10$ and $M = 20$ is very low, whereas

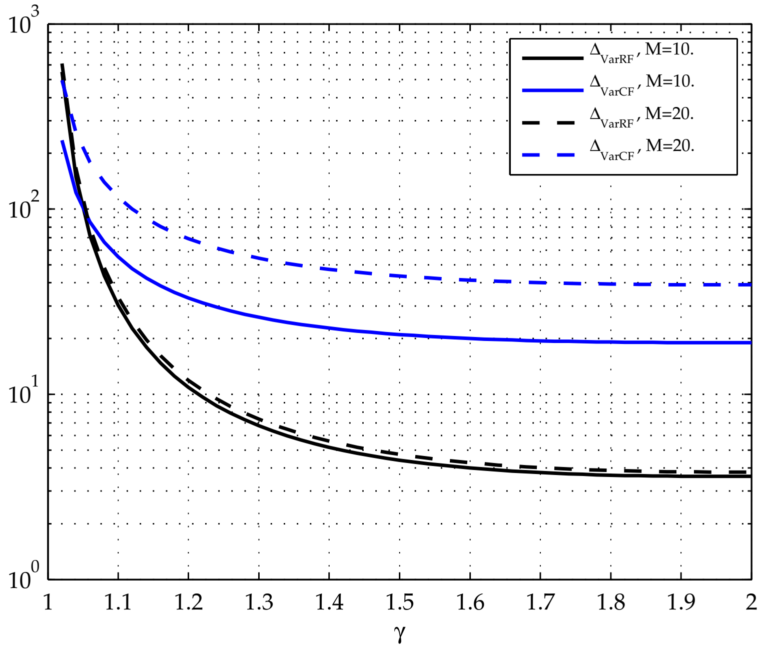


Figure 6.7: Fast hopping with Θ in the center of W: $\Delta_{VarRF}(\gamma, M)$ and $\Delta_{VarCF}(\gamma, M)$, for $M = 10$ and $M = 20$, with $R = 50$ and $\alpha = 3$.

$\Delta_{VarCF}(\gamma, M)$ for $M = 20$ takes values at least roughly 39 times higher $\Delta_{VarCF}(\gamma, M)$ for $M = 10$.

Hence, the FOS has a much more equitable interference balancing across all users in the ad hoc network because of the smaller variance. Besides, the FOS performance in terms of equity is much better compared to the other schemes for values close to 1 and for high values of M. This last fact is more significant when comparing the CS with the FOS (note that $\Delta_{VarCF}(\gamma, M)$ depends linearly on M) than when comparing the RS the FOS, where the dependence with M is weaker. Regarding the expected interference power level for each user, the FOS achieves the lowest possible value for all $0 < \gamma \leq 2M$, together with the CS; whereas the RS has a higher interference level. So, the FOS outperforms the other two studied schemes in both metrics, interference level and interference balancing across users. And the CS outperforms the RS in terms of interference level per user but the RS is better than the CS in terms of interference balancing for $1 < \gamma \leq 2$.

However, one should bear in mind that the RS in practice exhibits a higher variance of P_I because L is typically finite. The definition of P_I given by 6.22 implies making the assumption that the RS uses purely random hopping sequences with $L \to \infty$ and, hence, with no periodicity. In fact, in a more realistic scenario, the receiver has to be able to reproduce the sequence pattern generated by the transciever and, consequently, pseudorandom sequences with finite length are used. Hence, now $L = M$ is assumed. So then, in this case the definition for the average interference power is

$$P_I = \frac{1}{L}\sum_{i=1}^{L}\sum_{j\neq\Gamma}^{N} I_{ij}\frac{1}{(r_{j\Theta}+1)^{\alpha}} \tag{6.98}$$

Thus, according to this definition of P_I and proceeding similarly as in section 6.1.2, the obtained expectation of P_I is given by

$$\mathbf{E}\left\{P_I\right\} = \frac{N-1}{M}\ {}_2F_1(\alpha, 2; 3; -R)\ , \tag{6.99}$$

which is in fact the same expression than the one obtained in section 6.1.2. Regarding the variance of P_I, the derivation is also analogous to that one done in section 6.1.2 and it results in

$$\mathbf{Var}\left\{P_I\right\} = \frac{N-1}{M}\left[\left(\frac{1}{L}+\left(1-\frac{1}{L}\right)\frac{1}{M}\right)\ {}_2F_1(2\alpha, 2; 3; -R) - \frac{1}{M}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]\ . \tag{6.100}$$

And giving to the sequence length the value $L = M$, it yields

$$\mathbf{Var}\left\{P_I\right\} = \frac{N-1}{M^2}\left[\frac{2M-1}{M}\ {}_2F_1(2\alpha, 2; 3; -R) - \left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]\ . \tag{6.101}$$

The analytical variances for the three schemes are displayed in Fig. 6.8, this time according to the definition of P_I given in (6.98) for the RS. Comparing the curves corresponding to the RS in Fig. 6.8 and in Fig. 6.6 reveals that with the definition of P_I given in (6.98), its variance is roughly two times the variance obtained with the previous definition of P_I with $L \to \infty$. This ratio tends to 2 when M tends to infinity, as the factor $\frac{2M-1}{M}$ manifests in (6.101). Thus, the RS, in terms of equitable interference balancing, is two times worse compared to the Fos with the new definition of P_I.

Apart from this, the factor $\frac{N-1}{M^2}$ can be expressed in terms of γ as

$$\frac{\gamma}{M} - \frac{1}{M^2}\ , \tag{6.102}$$

and, therefore,

$$\lim_{M\to\infty}\left(\frac{\gamma}{M} - \frac{1}{M^2}\right) = 0\ . \tag{6.103}$$

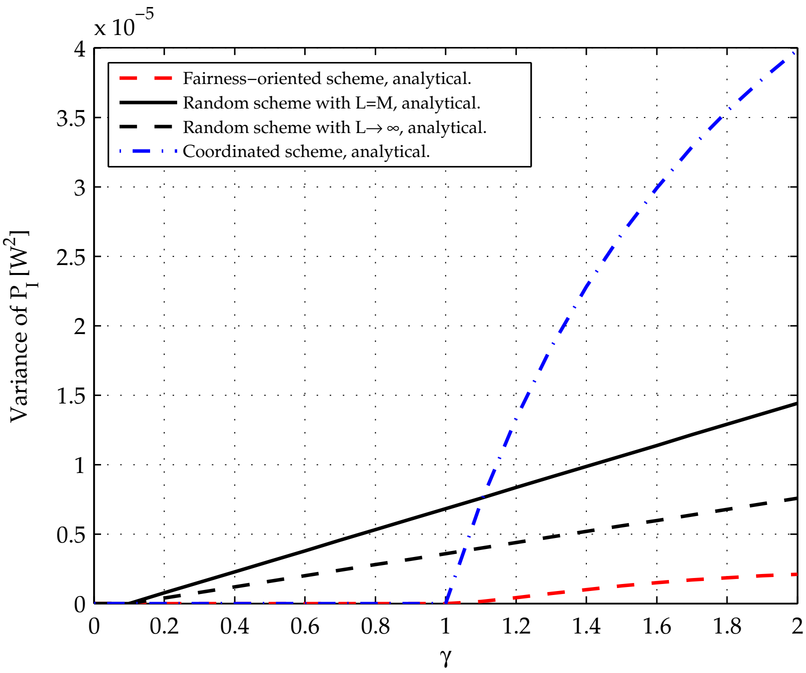


Figure 6.8: Fast hopping with Θ in the center of W: $\mathbf{Var}\{P_I\}$, for the three schemes, where the RS has $L = M$. Analytical results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

And for the RS with the definition of P_I for an infinite sequence length, it also happens that

$$\lim_{M\to\infty} \mathbf{Var}_{RS}\left\{P_I\right\} = \lim_{M\to\infty} \left(\frac{\gamma}{M} - \frac{1}{M^2}\right) \left[\,{}_2F_1(2\alpha, 2; 3; -R) - \left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] = 0\,. \tag{6.104}$$

In contrast, for the CS it does not happen:

$$\begin{aligned}\lim_{M\to\infty} \mathbf{Var}_{CS}\left\{P_I\right\} &= \lim_{M\to\infty} 2\left(1-\frac{1}{\gamma}\right)\left[{}_2F_1(2\alpha, 2; 3; -R) - 2\left(1-\frac{1}{\gamma}\right)\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] \\ &= 2\left(1-\frac{1}{\gamma}\right)\left[{}_2F_1(2\alpha, 2; 3; -R) - 2\left(1-\frac{1}{\gamma}\right)\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]\,.\end{aligned} \tag{6.105}$$

Note that in the three above given limits M and N tend to infinity but γ remains constant. So then, for a given γ between 1 and 2, if the number of frequency channels tends to infinity, with the RS the variance of P_I tends to zero, whereas for the CS the variance does not depend on M, once γ is fixed. This is because with the CS if γ is fixed, the probability of one user to belong to the set of colliding users during one hopping sequence does not change if M increases. Therefore, the PDF of P_I does not change.

On the other hand, despite of the increase of the variance of P_I for the second definition, given in (6.98), for the RS, the variance for the CS is still much higher, as shown in Fig. 6.8. Hence, the CS is still the one with the worst performance in terms variance. This is because with the RS in every hop the interferers are different users in general and, furthermore, being interfered in one hop does not imply being interfered in the whole sequence, in contrast to the CS. Regarding the FOS, it has the lowest variance of P_I of the three schemes because, with this cheme, the randomness of P_I is just due to the random position of the interferer transmitters, not due to which are the transmitters that will interfere one transmitter because it is *a priori* known that each transmitter will collide with all the rest in each hopping sequence. In other words, if all transmitters where placed in a deterministic fixed position, all at the same distance d to the reference receiver Θ (that is, all placed on a circumference of radius d centered in Θ), compared to the other two schemes, the FOS would be completely equitable because all transmitters would see exactly the same average interference power. Hence, the FOS is the fairest of the three schemes. However, since the position of the $N-1$ transmitters that will collide with a transmitter during a hopping sequence is a random variable, the interference power that Θ will see when this user will be the intended transmitter is not exactly the same than with another intended transmitter. Because when a user ξ_i is the intended transmitter, there are $N-1$ interferers (the rest of the transmitters), and when another user ξ_j is the intended transmitter, it has $N-2$ common interferers with ξ_i, which are placed in the same place in both cases, but ξ_i will also be an interferer to ξ_j and vice versa. I.e., if nodes were distributed according to a Binomial point process uniformly in W just as well as it has been assumed in this thesis but, after this, their positions were known (so, they were deterministic), the transmitters would not exactly see the same interference with the FOS. However, with this scheme nodes would see the lowest interference level than with the other two schemes, statistically, as seen above. For this reason, the interference power that nodes see with the FOS, in general, is not the same for all nodes. Thus, the variance that the FOS presents, supposing again the position of the interferers to be a random variable, is due partially to the fact that the position of the interferers is random, but it is also due to the fact that each transmitter is not interferred exactly by the same transmitters and, therefore, transmitters do not bear exactly the same interference power. The difference of the interference power borne by two transmitter becomes negligible for a great enough N. In the limit case, when $N \to \infty$, the FOS would be absolutely fair because all nodes would bear exactly the same average interference power. Here the expression for the variance of P_I for the FOS given by (6.17) is again written:

$$\mathbf{Var}\left\{P_I\right\} = \frac{4(N-M)^2}{N^2(N-1)} \left[{}_2F_1(2\alpha, 2; 3; -R) - \left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right] . \tag{6.106}$$

It is clear that, as above said, one part of the expression, $\left[{}_2F_1(2\alpha, 2; 3; -R) - \left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]$, depends on the randomness of the position of the nodes, and the other part, $\frac{4(N-M)^2}{N^2(N-1)}$, depends on the design of the scheme, that is, it depends on how collisions have been distributed by means of the design of the hopping sequences. This last expression in terms of γ can be expressed as

$$\frac{4(N-M)^2}{N^2(N-1)} = \frac{4(\gamma-1)^2}{\gamma^2(N-1)} \ , \tag{6.107}$$

and, as just said,

$$\lim_{N\to\infty} \frac{4(\gamma-1)^2}{\gamma^2(N-1)} = 0 \ . \tag{6.108}$$

So, the FOS tends to be absolutely fair when N tends to infinity.

With the virtual receiver placed at the border

As the expressions of the expectations and variances of P_I for the three schemes[5] show in section 6.1.2, the only difference between the expectations and variances corresponding to the case of Θ being placed in the center of the disk and Θ being placed at the border of the disk is the replacement of the factors ${}_2F_1(\alpha, 2; 3; -R)$ and ${}_2F_1(2\alpha, 2; 3; -R)$ by the factors $F_{border}(\alpha, R)$ and $F_{border}(2\alpha, R)$, respectively. I.e., only the spatial distribution of the nodes with respect to the reference receiver Θ has changed but not the way in which nodes collide among them, because the hopping sequences are designed in the same way in the case of Θ being at the border of the disk.

As seen in Fig. 6.9, in this case Θ sees less interference power than when it is placed in the center of the disk, because in average terms, when Θ is placed at the border, it is more faraway from a randomly chosen transmitter placed somewhere in the disk. The curves of the expectations only change with respect to the case with Θ in the center simply by a scale factor $\frac{F_{border}(\alpha,R)}{{}_2F_1(\alpha,2;3;-R)} < 1$. Therefore, the same conclusions regarding the expectations obtained in the case of Θ in the center are also valid in this case.

As regards the variances, comparing the expressions in section 6.1.2, one notices that for the cases of the FOS and the RS the factor

$$\left[{}_2F_1(2\alpha, 2; 3; -R) - ({}_2F_1(\alpha, 2; 3; -R))^2\right]$$

or, equivalently,

$$\left[F_{border}(2\alpha, R) - (F_{border}(\alpha, R))^2\right] \ ,$$

[5]In this section, the definition of P_I for the Random scheme is again the one given in section 6.1.2.

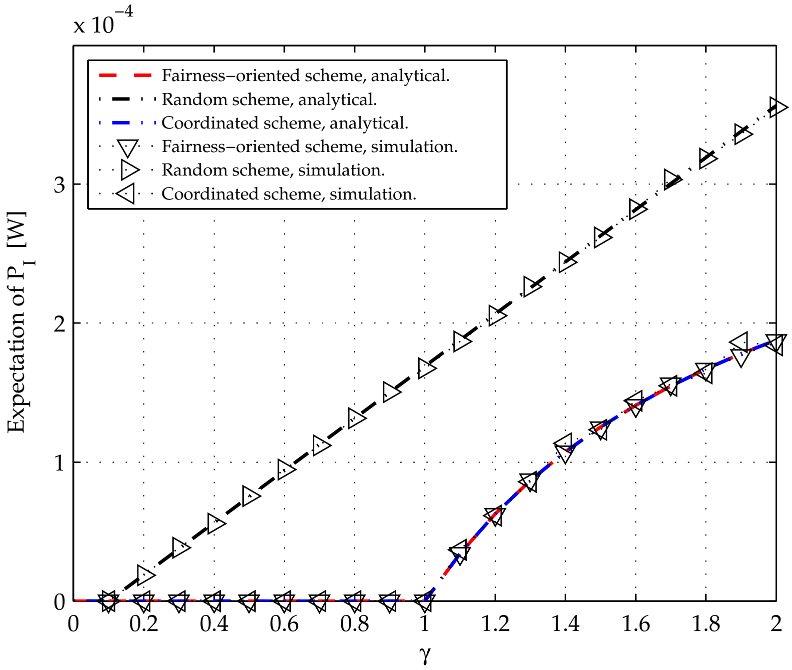


Figure 6.9: Fast hopping with Θ at the border of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$ and $R = 50$ m.

appears multiplying the rest of the expression of the variance. For the CS, in contrast, it does not happen. This means that, whereas for the FOS and the RS, the ratio between their respective variances for the case of Θ being placed in the center or at the border does not depend on γ, for the CS, this ratio of variances depends on γ. However, as displayed in Fig. 6.10, the curves of the variances of the FOS and the RS have been reduced in $0 < \gamma \leq 2$ roughly a factor 2 but, for the specific values of $\alpha = 3$ and $R = 50$ m, for the CS the variance has also been reduced roughly a factor 2 with respect to the case of Θ being in the center of the disk.

So, in this case, the conclusions regarding the three variances would be the same than for the case of Θ being in the center. Nevertheless, for other values of R and α, the ratio between $\mathbf{Var}_C\{P_I\}$ for Θ being placed in the center and at the border has a higher variation with respect to γ. This means that, whereas in this case Δ_{VarRF} is the same than in the case of Θ being placed in the center, it does not exactly happen with Δ_{VarCF}, specially for low values of R and α. The ratio between the variance of P_I for the CS and the variance of P_I for the FOS in this case is

$$\Delta_{VarCF} = \frac{\gamma}{2}\frac{\gamma M - 1}{\gamma - 1}\frac{\left[F_{border}(2\alpha, R) - 2\left(1 - \frac{1}{\gamma}\right)\left(F_{border}(\alpha, R)\right)^2\right]}{\left[F_{border}(2\alpha, R) - \left(F_{border}(\alpha, R)\right)^2\right]}\,, \qquad 1 < \gamma \leq 2\,. \tag{6.109}$$

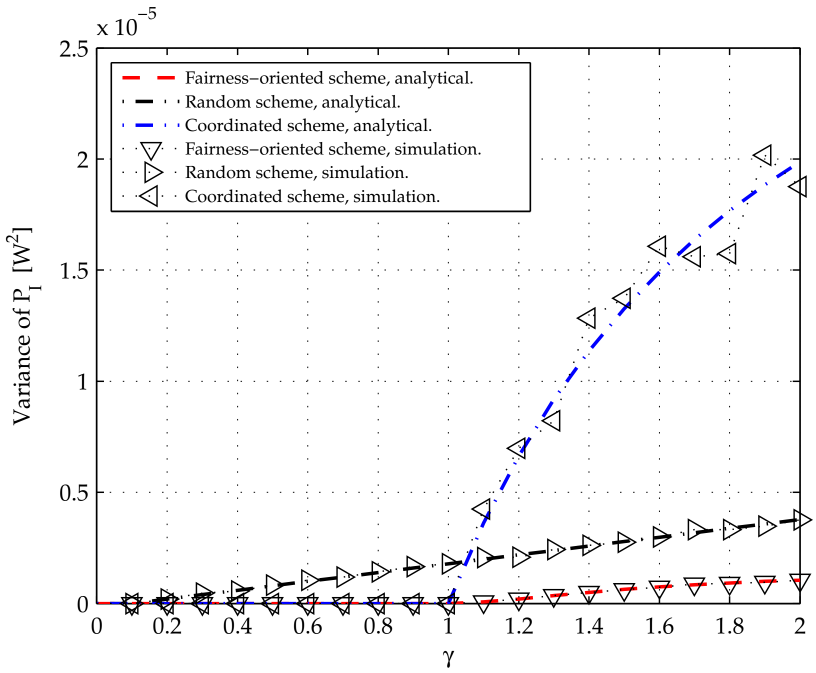


Figure 6.10: Fast hopping with Θ at the border of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

This expression is depicted in Fig. 6.11,as well as the same ratio Δ_{VarCF} for the case of Θ being placed in the center of the disk. The fact that the curve for the case of Θ being placed in the border of the disk is below the one for the other case means that the varience for the FOS is more times lower than the variance for the CS when Θ is placed in the center of the disk than when it is placed at the border.

Likewise, it is manifest that the difference between the ratios in Fig. 6.11 is greater as γ is closer to 1. Therefore, comparing the Fairness-oriented scheme to the Coordinated scheme, the performance of the Fairness-oriented scheme, in terms of equity in interference balancing, is many times better than the Coordinated scheme. However, for values of γ close to 1 (and above it), in the case of Θ being placed on the border of the disk, the performance of the Fairness-oriented scheme is less times better than in the case of Θ being placed in the center of the disk. The difference between both cases, nevertheless, decreases when γ increases and it becomes negligible for $1 \ll \gamma < 2$. So, for $1 \ll \gamma < 2$, the performance of the Fairness-oriented scheme in comparison with the other schemes is almost the same independently of where the virtual reference receiver Θ is placed. For Θ being placed in an in-between point, between the border and the center, the Fairness-oriented scheme will also display a performance, in comparison to the other schemes, which will be in between its performances for the two extreme cases. It has also to be recalled that the performance of the Fairness-oriented scheme, relatively to the performance of the Random scheme, is the same independently of the position of Θ, and, consequently, the same conclusions regarding it stated in the previous section are likewise valid in this section.

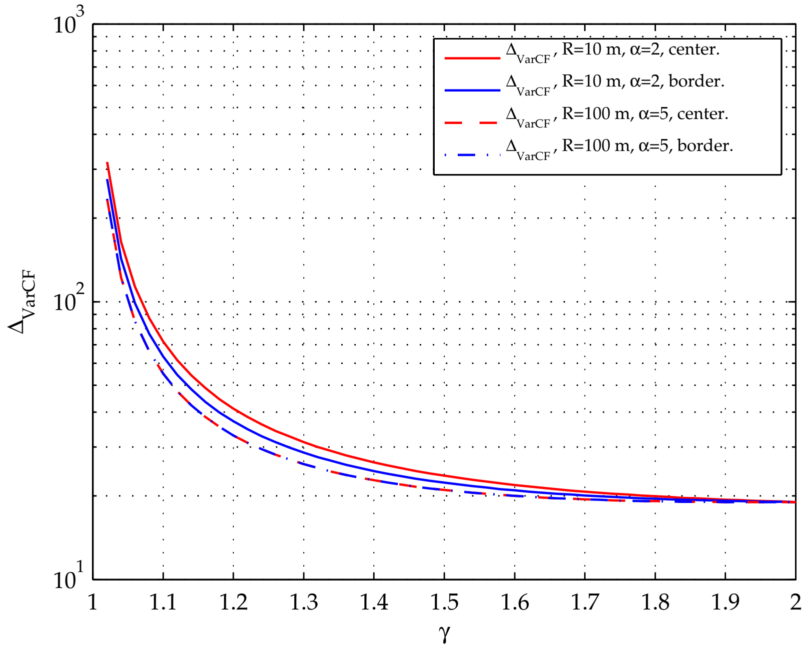


Figure 6.11: Fast hopping: Δ_{VarCF} for Θ in the center and in the border of W; each case, in turn, divided into the cases $R = 10$ and $\alpha = 2$, and $R = 100$ and $\alpha = 5$. $M = 10$ for all cases.

6.3.2 Static slow frequency hopping

As stated in 6.1.3, in static slow hopping, whereas the total number of users in the network, N_T, is a known parameter, the number of users which are transmitting during at least one hopping sequence (and which are these users) is a random variable with an expectation equal to $N = N_T p$. p is the probability of one user to be a transmitter. From here on, it will be assumed that $p = \frac{1}{2}$, therefore, $N = \frac{N_T}{2}$. Since N is not known and it may change after each hopping sequence transmission, the hopping sequences are assigned to all the N_T users. Besides, since in slow hopping the interference power is calculated from the perpective of only one hop separately, the derived expressions for the expectation and the variance of P_I with the FOS are indentical for the CS. This fact is explained in more detail in the following subsection.

With the virtual receiver placed in the center

Since in the case of slow frequency hopping it is assumed that the hop period is equal to the average time of one packet transmission, the expressions derived in section 6.1.3 only analyse the performance of the three schemes in each hop, independently of the other hops. For this reason, the FOS exhibits the same performance than the CS in terms of both expectation and variance of P_I, as depicted in Fig. 6.12 and in Fig. 6.13. Therefore, if the FOS is studied only taking into account one hop separately, the resulting behaviour is considerably worse than if it is analysed in a whole hopping sequence, as done for the case of fast frequency hopping. In fact, from the point of view of just one single hop, the FOS is identical to the CS. They are the same scheme from this point of view. The difference between both is how transmitters collide among them in a whole hopping sequence. And it is after a whole hopping sequence, or after an integer number of hopping sequences, when the FOS works properly and achieves a fair balancing of the interference across all transmitters. Thus, supposing that one transmitter transmits more than one packet consecutively, then, analysing the schemes regarding all these consecutive hops, as done for the case of fast hopping, the obtained performance of the FOS would improve, tending to that one obtained in the fast hopping case as the number of analysed hops tends to L. As seen, for the case of fast hopping, the FOS and the CS have the same expectation of P_I, but the FOS presents a many times lower variance of P_I. Therefore, analysing both schemes from the perspective of one hop, as in section 6.1.3, or from the perspective of one hopping sequence, as done in section 6.1.2, constitute two extreme cases. Thus, in the case of static slow hopping, for an average number of consecutive transmitted packets, the FOS will present a curve for the varaince between the ones displayed for these two extreme cases. Hence, the bigger is the number of consecutive transmitted packets, the better is the performance of the FOS, also compared to the performance of the CS, which always presents the same variance, no matter the number of consecutive transmitted packets. This is due to the fact that with the CS each of the collided transmitters is colliding with the same transmitter in a whole sequence.

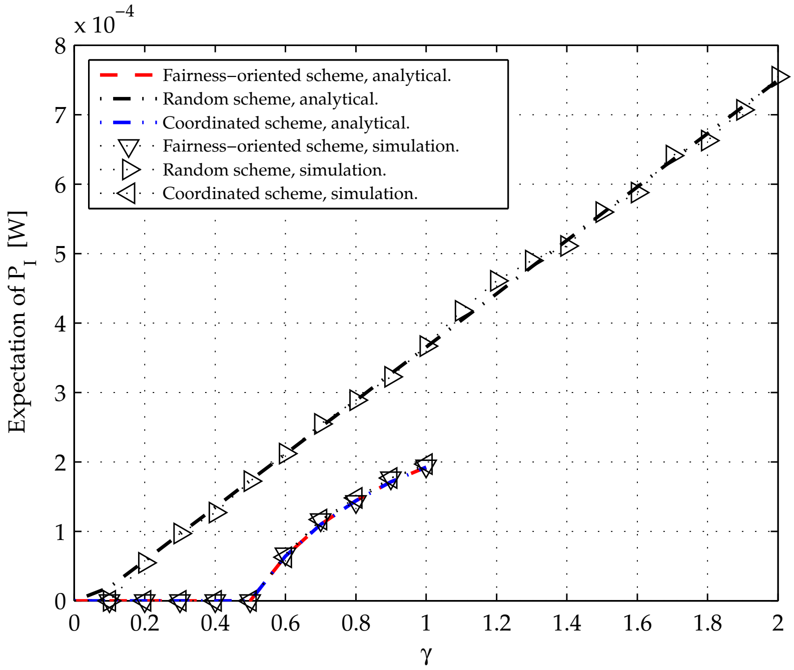


Figure 6.12: Static slow hopping with Θ in the center of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$ and $R = 50$ m and $p = \frac{1}{2}$

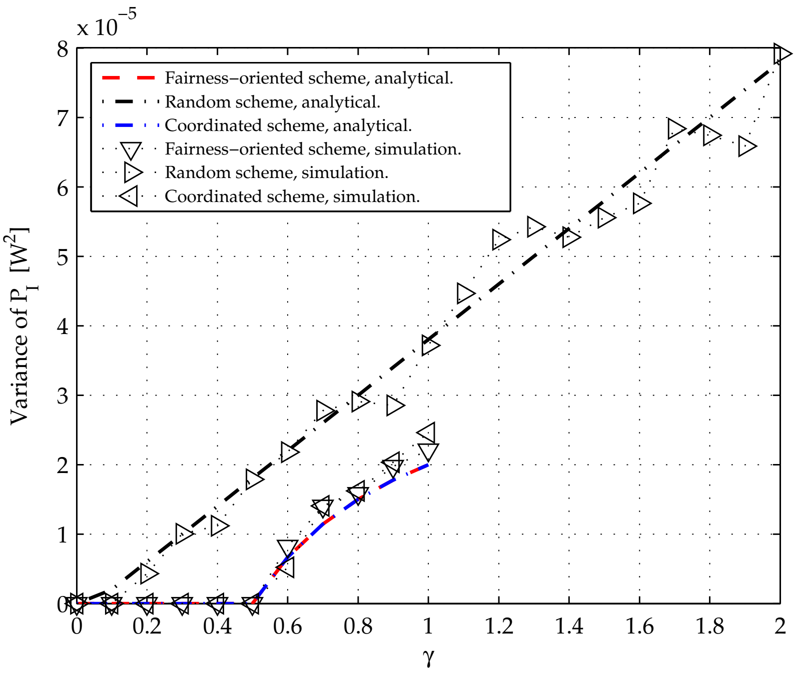


Figure 6.13: Static slow hopping with Θ in the center of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$, $R = 50$ m and $p = \frac{1}{2}$.

From the point of view of one hop, the RS displays a worse behaviour than the other two schemes both in terms of expected interference power per user and in terms of the variance of the interference power as Fig. 6.12 and Fig. 6.13 show. As seen, in the fast hopping case, the RS outperforms the CS in terms of variance but in the slow hopping case it occurs inversely. This is because in the case of fast hopping, whereas the RS profits from measuring the interference power received during a whole sequence for reducing the variance by averaging several realisations of the interference power in every hop, the CS does not profit from averaging the interference power of several hops because the interferer is in all hops the same transmitter. The ratio between the variance of P_I with the RS and with the FOS for the static slow hopping case with Θ placed in the center of the disk is

$$\Delta_{VarRF} = \frac{\frac{N_T-2}{M}p\left[{}_2F_1(2\alpha,2;3;-R) - \frac{p}{M}\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}{\frac{2N_T-2M}{N_T}p\left[{}_2F_1(2\alpha,2;3;-R) - \frac{2N_T-2M}{N_T}p\left({}_2F_1(\alpha,2;3;-R)\right)^2\right]}\ , \qquad M < N_T \leq 2M\ . \tag{6.110}$$

And, as said, in the static slow hopping case, the expected number of transmitters in each hop, N, has been assumed to be $N = \frac{N_T}{2}$, where N_T is the total number of users in the network, which is constant during T_{valid}. In other words, the probability of one user to be a transmitter during a certain hop has been assumed to be $p = \frac{1}{2}$. And, on the other hand, as also said obove, in this case the parameter γ has been defined as $\gamma \triangleq \frac{N}{M}$ (and not as

$\gamma \triangleq \frac{N_T}{M}$). So, (6.110) yields

$$\Delta_{VarRF} = 2\gamma\frac{\gamma - \frac{1}{M}}{2\gamma - 1}\frac{\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{1}{2M}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}{\left[{}_2F_1(2\alpha, 2; 3; -R) - \left(1 - \frac{1}{2\gamma}\right)\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}\ , \qquad \frac{1}{2} < \gamma \leq 1\ . \tag{6.111}$$

Δ_{VarRF} is displayed in Fig. 6.14. As it can be deduced from 6.111, $\Delta_{VarRF} \to \infty$ when $\gamma \to \frac{1}{2}^+$. Then, around $\gamma = 1$ Δ_{VarRF} reaches its absolute minimum. In contrast, as it happens for the fast hopping case around $\gamma = 1$, the FOS outperforms many times the RS, from the point of view of the variance of P_I, for values of γ close to $0,5$. As remarked in section 6.3.1, if the difference between the chosen value for γ and $\frac{1}{2}$ is lower, the number of times that the variance of P_I for the FOS is lower than for the RS increases assimptotically to infinity. However, in practice it will be limited by the values of M and N in absolute terms; they cannot be infinitely large, but they will be fixed by the circumstances. Nevertheless, where the FOS works optimally is, consequently, for values of γ close to $\frac{1}{2}$.

As displayed in Fig. 6.14, the performance of the FOS in comparison to the RS is better for fast hopping than for dynamic slow hopping, in terms of the variance of P_I. And for the range $\frac{1}{2} < \gamma \leq 1$, Δ_{VarRF} is not defined for the fast hopping case because, in that case, there is no interference in this range because sequences are orthogonal. This does not happen for the static slow hopping case in this range. On the other hand, as Fig. 6.14 depicts, although there is, in fact, a slightly greater difference between the curves for $M = 10$ and $M = 100$ in the case of fast hopping than in the case of slow hopping, in both cases this difference is not very significant. So, despite the fact that for a greater M the FOS achieves a greater equity in comparison to the RS, this enhancement with respect to M is not remarkable in relative terms and, furthermore, changing M does not change significantly the shape of the obtained curves.

As final remarks in this section, it has to be said, firstly, that the fact that the curves for the variance and the expectation of P_I, depicted in Fig. 6.12 and in Fig. 6.13 respectively, look so similar for these specific values for M, R and α is partially casual and partially due to a certain similarity between the expressions for both statistical moments in the case of slow hopping. Secondly, it is manifest, in the second of these two figures, that the fluctuation of the curves obtained by simulation around the analytical values is, in this case, similar for the three schemes, whereas, in the fast hopping case, as seen, the curve of the variance for the CS obtained by simulation displays a larger fluctuation than the others. This is again due to the fact that, since in this case the analysis has been done regarding one hop single-handedly, the estimators of the variance for the three schemes which are implicitly implemented in a simulation have similar variances. And, thirdly, an important issue to highlight is the fact that, in this case, the expectation and the variance either for the FOS or for the CS are not zero in $0 < \gamma \leq 1$ but in $0 < \gamma \leq 0,5$. This, on the one hand, is due to the above mentioned assumption that $p = \frac{1}{2}$ or, equivalently, $N = \frac{N_T}{2}$, and, on the other hand, it is due to the definition $\gamma \triangleq \frac{N}{M}$, instead of $\gamma \triangleq \frac{N_T}{M}$. Hence, when $\gamma = \frac{1}{2}$,

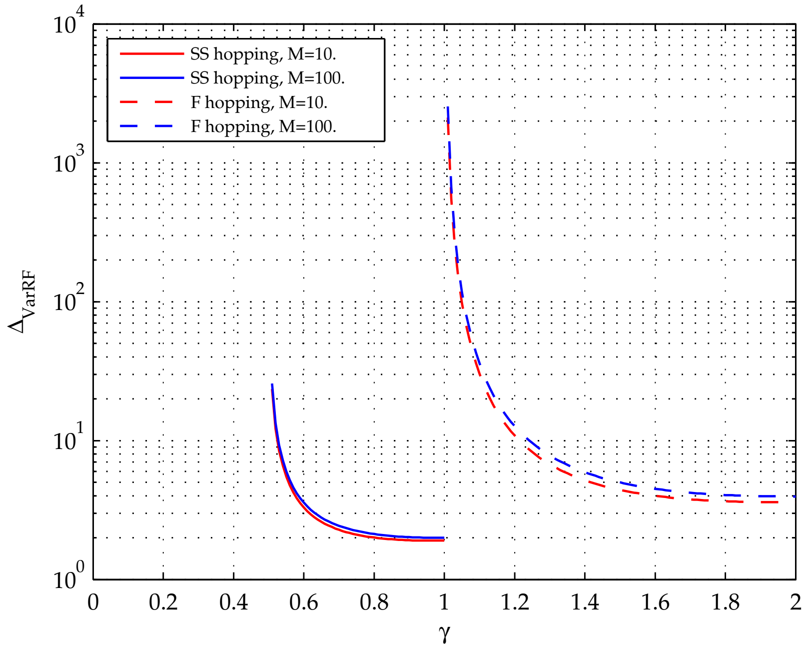


Figure 6.14: Fast and static slow hopping with Θ in the center of W: Δ_{VarRF}. Analytical results for $M = 10$ and $M = 100$, and for $\alpha = 3$, $R = 50$ m and $p = \frac{1}{2}$.

$N_T = 1$ and, so, above this value, sequences are not orthogonal anymore and, consequently, interference occurs. That is the reason why of the curves displayed in Fig. 6.12 and in Fig. 6.13 show this behaviour. This will be more carefully analysed and compared to the dynamic slow hopping case in the next section.

With the virtual receiver placed at the border

The behaviour of the three schemes when the reference virtual receiver Θ is placed on the border of the disk, as shown in Fig. 6.15 and Fig. 6.16, is very similar than for the case of Θ placed in the center of the disk. As it happens in the case of fast hopping, in the static slow hopping case, the curves either for the expectations or the variances corresponding to the three schemes have roughly the same shape except for a factor, which is approximately equal to 2.

So, as it happens in the fast hopping case, either the expected average interference level for each user and its variance are approximately 2 times lower when Θ is placed on the border of the disk than when it is placed in the center of the disk, for each of the three schemes and for any γ belonging to the range of interest.

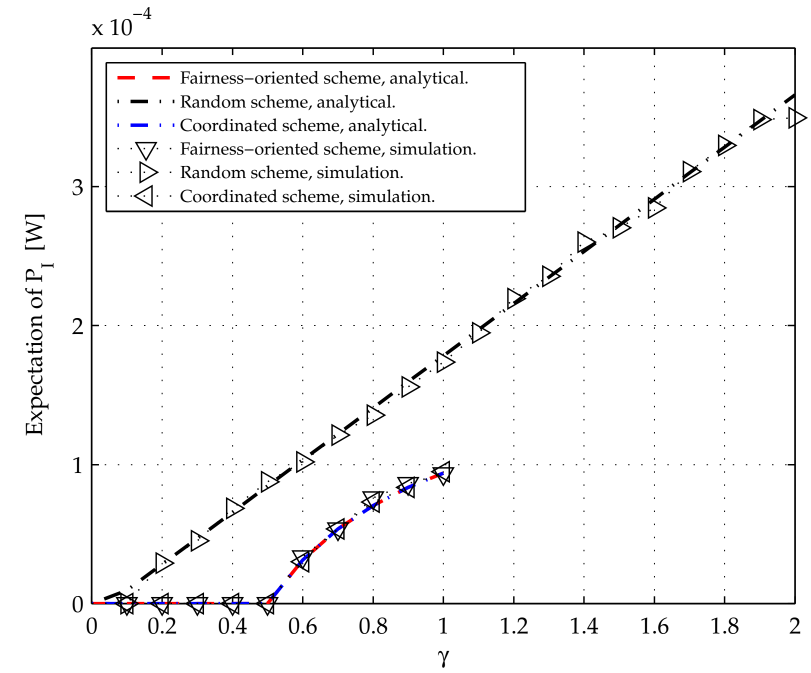


Figure 6.15: Static slow hopping with Θ in the border of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$, $R = 50$ m and $p = \frac{1}{2}$.

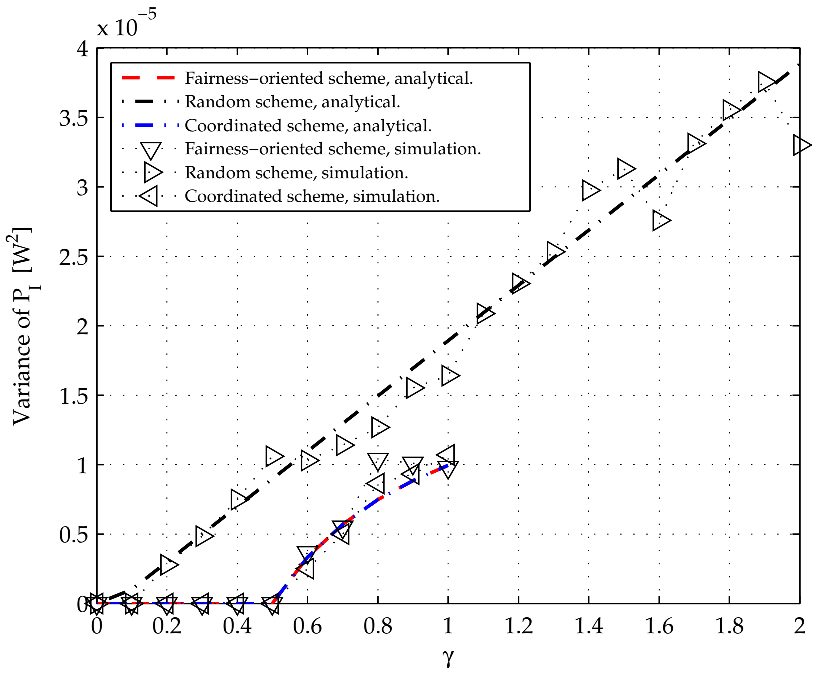


Figure 6.16: Static slow hopping with Θ in the border of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$, $R = 50$ m and $p = \frac{1}{2}$.

6.3.3 Dynamic slow frequency hopping

As said in the analysis section, whereas in the static slow hopping case the number of transmitters in every hop, N, is not known, but it is only known the total number of users in the network N_T, in the dynamic slow hopping case N is *a priori* known. Therefore, the assignment of the hopping sequences is done to the N transmitters, as in the fast hopping case, and not to all users independently of whether they are transmitting or not in a certain hop, as in the static slow hopping case.

With the virtual receiver placed in the center

In Fig. 6.17 and Fig. 6.17, the expectations and the variances of P_I for the three schemes are displayed. As happens in the static slow hopping case, the behaviour of the FOS is the same than the behaviour of the CS, if they are analysed from the point of view of only one hop separately. Hence, these two schemes display the same curves either for the expectation and the variance of P_I also for the case of dynamic slow hopping.

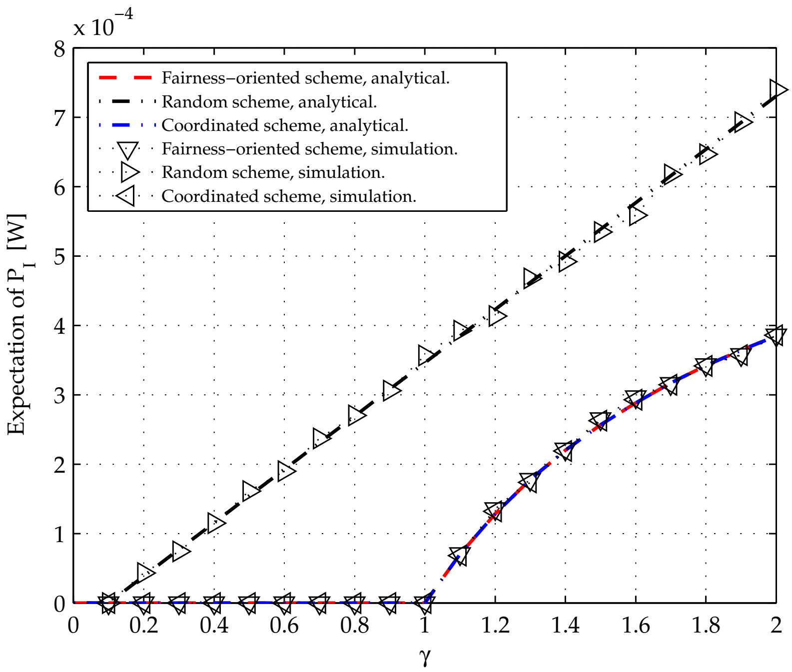


Figure 6.17: Dynamic slow hopping with Θ in the center of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$ and $R = 50$ m.

Both the FOS and the CS outperform the RS either in terms of the expected average interference power per user or in terms of its variance, and, therefore, in terms of equity in the interference allocation. The ratio between the variance of P_I obtained with the RS and the one obtained with the FOS in this case is

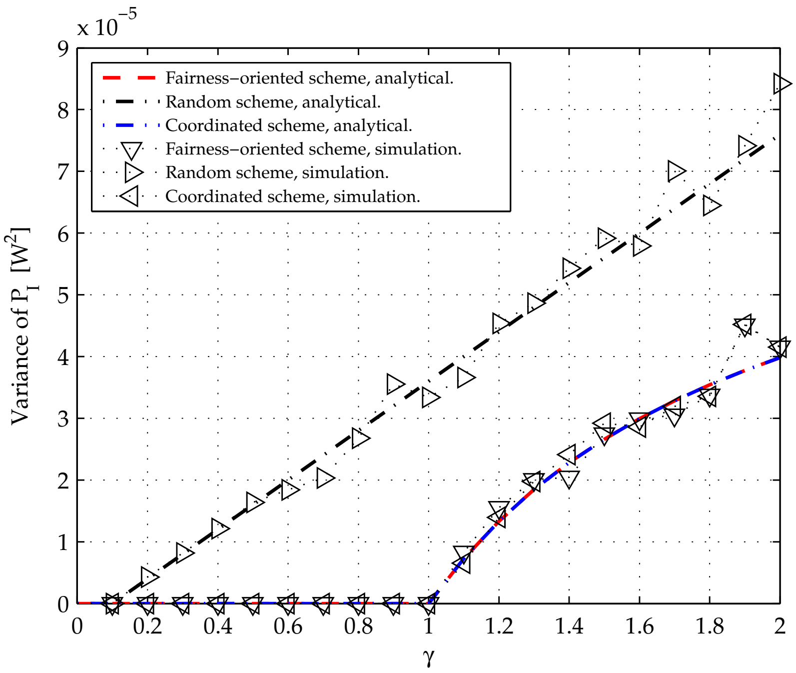


Figure 6.18: Dynamic slow hopping with Θ in the center of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

$$\Delta_{VarRF} = \frac{\frac{(N-1)}{M}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{1}{M}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}{\frac{2(N-M)}{N}\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{2(N-M)}{N}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}$$

$$= \frac{\gamma}{2}\frac{\gamma - \frac{1}{M}}{\gamma - 1}\frac{\left[{}_2F_1(2\alpha, 2; 3; -R) - \frac{1}{M}\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}{\left[{}_2F_1(2\alpha, 2; 3; -R) - 2\left(1 - \frac{1}{\gamma}\right)\left({}_2F_1(\alpha, 2; 3; -R)\right)^2\right]}, \qquad 1 < \gamma \leq 2\,. \tag{6.112}$$

This ratio, as well as the one corresponding to the same two schemes for the static slow hopping case and for the fast hopping case, is depicted in Fig. 6.19 for $M = 10$ and $M = 100$. As it can be seen, in the three cases Δ_{VarRF} tends to infinity when γ tends to the respective lower limits of the ranges where Δ_{VarRF} is defined in each case (i.e., 1 for the case of dynamic slow hopping and fast hopping and $\frac{1}{2}$ for static slow hopping). Thus, for values close to these limits, $\mathbf{Var}\{P_I\}$ with the FOS is very high number of times lower than $\mathbf{Var}\{P_I\}$ with the RS. Therefore, the FOS is many times better than the RS, in fast and in dynamic slow hopping $1 < \gamma \ll 2$, and in the static case for $\frac{1}{2} < \gamma \ll 1$. However, in fast and dynamic slow hopping, in the range $\frac{1}{2} < \gamma \leq 1$, the performance of the FOS is optimal because there is no interference power, whereas for the case of static slow hopping there is already interference power for this range. Note that Δ_{VarRF} is very similar for static and dynamic slow hopping at least for the values of M, R and α chosen in Fig. 6.19.

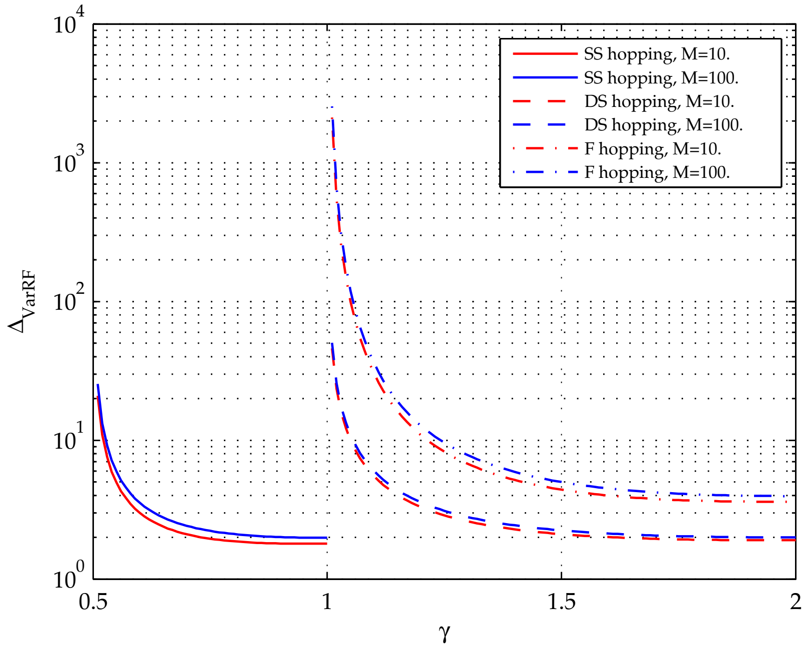


Figure 6.19: Fast, dynamic and static slow hopping with Θ in the center of W: Δ_{VarRF}. Analytical results for $M = 10$ and $M = 100$, and for $\alpha = 3$, $R = 50$ m and $p = \frac{1}{2}$.

Fig. 6.20 depicts the variances of P_I with the three schemes for the cases of static and dynamic slow hopping, for $M = 10$. In keeping with the conclusions extracted from Fig. 6.19, Fig. 6.20 confirms that both the FOS and the CS have a better performance in dynamic slow hopping, in the range $\frac{1}{2} < \gamma \leq 1$. Regarding the RS, it is easy to verify that the expectation and the variance of P_I with the CS have the same expression for static and dynamic slow hopping.

Regarding the expected average interference power, it can bee seen that with the FOS and the CS in dynamic slow hopping there is no interference in the range $\frac{1}{2} < \gamma \leq 1$, whereas in the static case these two schemes present interference in this range.

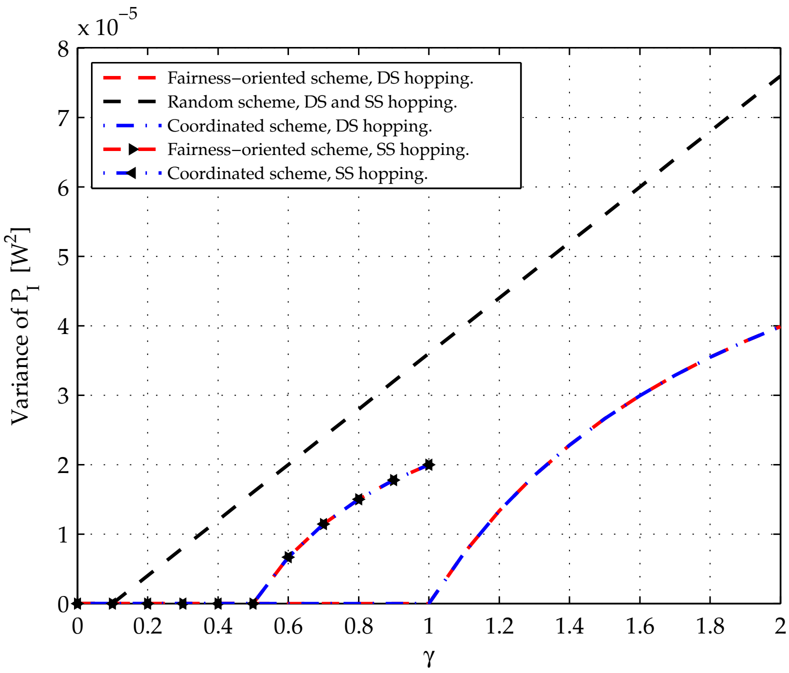


Figure 6.20: Dynamic and static slow hopping with Θ in the center of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

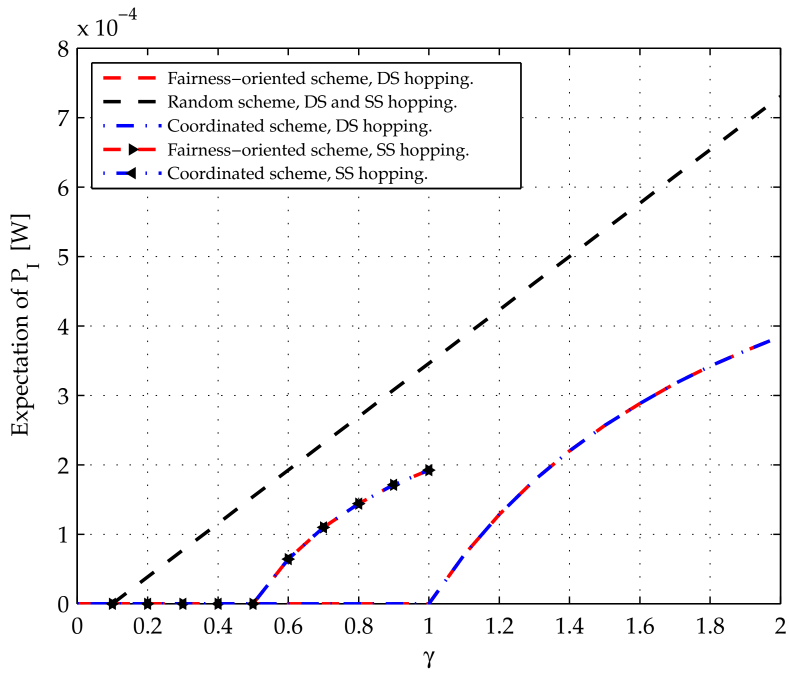


Figure 6.21: Dynamic and static slow hopping with Θ in the center of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical results for $\alpha = 3$ and $R = 50$ m.

With the virtual receiver placed at the border

For $M = 10$, $\alpha = 3$ and $R = 50$, the expectation and the variance of P_I when Θ is at the border are roughly one half of those corresponding to the case where Θ is in the center, as Fig. 6.22 and Fig. 6.23 illustrate.

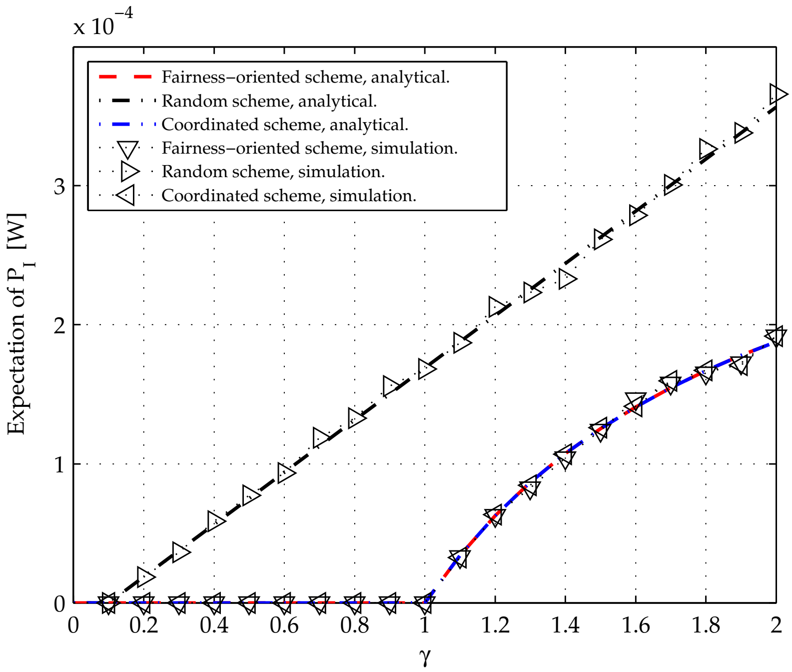


Figure 6.22: Dynamic slow hopping with Θ in the border of W: $\mathbf{E}\{P_I\}$, for the three schemes. Analytical and simulation results for $\alpha = 3$ and $R = 50$ m.

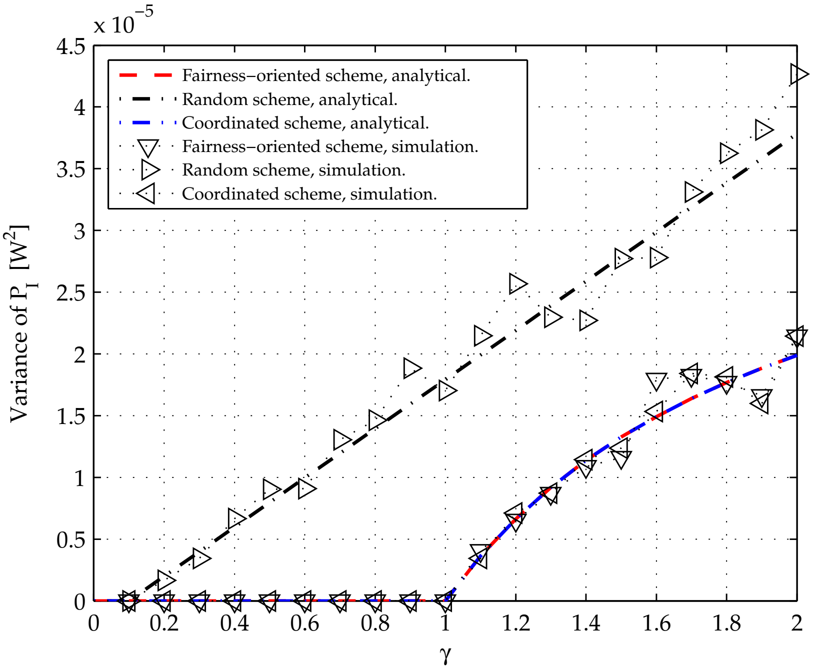


Figure 6.23: Dynamic slow hopping with Θ in the border of W: $\mathbf{Var}\{P_I\}$, for the three schemes. Analytical and simulation results for $M = 10$, $\alpha = 3$ and $R = 50$ m.

Probability of collision and BER for dynamic slow frequency hopping

It has been said that the behaviour of the FOS is the same than the behaviour of the CS, if they are analysed from the point of view of only one hop separately. But actually in most real cases one transmitter will transmitt more than one packet and therefore, the perspective of one hop, which penalizes the FOS compared to the CS, has to be extended to more than one hop. Hence, a good perspective to see the benefits of the FOS is to study the probability of collision of one user in a randomly chosen hop, once hopping sequences are already assigned. In this scenario, with the CS it is already known which nodes will collide in a randomly chosen hop because in all hops of a sequence the colliding nodes are the same. In contrast, with the FOS, it is not known which nodes will collide in a randomly chosen hop, once the hopping sequences are known.

Following section 6.1.4, for dynamic slow hopping, the probability of having at least one collision in a randomly chosen hop is depicted in Fig. 6.24. The curves for the FOS and for the RS, with $M \to \infty$, are the same than the ones provided in section 5.4. It is clear that the FOS outperfroms the RS in terms of probability of collision in most of the range $0 < \gamma \leq 2$, and although the probability of collision with the FOS is higher than with the RS for values of γ close to 2, it has well seen that in terms of P_I the FOS is much better than the RS also for values of γ close to 2. With the FOS and with the RS, the probability of collision is the same for all users. However, with the CS, the probability of collision in a randomly chosen hop is not the same for all users because it is already known which users are colliding in each hop. Thus, in the range $1 < \gamma \leq 2$, as depicted in Fig. 6.24, this probability is equal to 1 for $2(N-M)$ transmitters and equal to 0 for the rest. Therefore, the interference balancing across all users with the FOS is much more equitable than with the CS because all users have the same probability of collision.

This fact can also bee seen by means of the BER with a BPSK modulation at the output of the demodulator with each of the three schemes. See the formulae of the curves depicted in Fig. 6.25 in section 6.1.4. Observing Fig. 6.25, it is manifest that the FOS displays a much lower BER than the RS. On the other hand, with the CS, since $2(N-M)$ transmitters have $P_b = Q\left(\sqrt{2}\right)$ and the rest have $P_b = 0$, there is a high imbalance among users, whereas with the FOS all users have the same BER. So, the FOS is the most efficient scheme in terms of interference balancing.

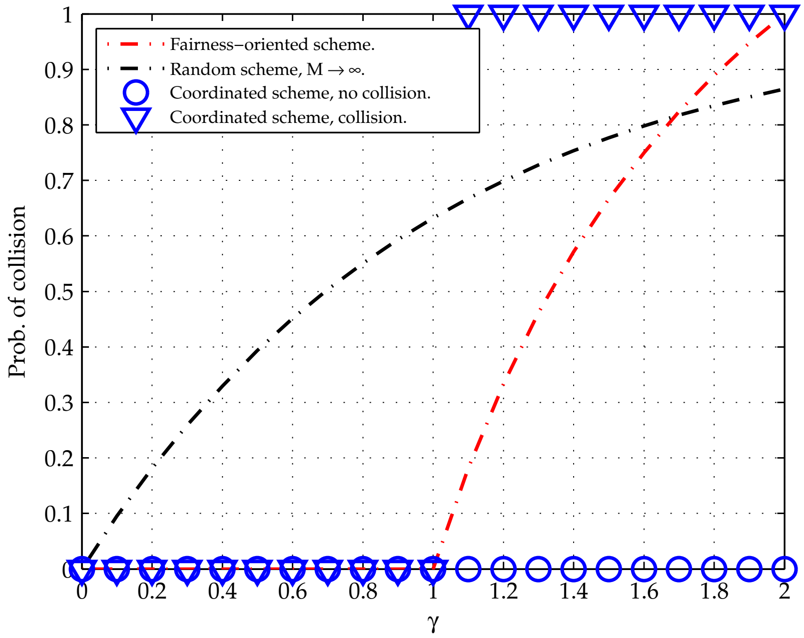


Figure 6.24: Dynamic slow hopping, probability of having at least one collision in a randomly chosen hop for the three schemes.

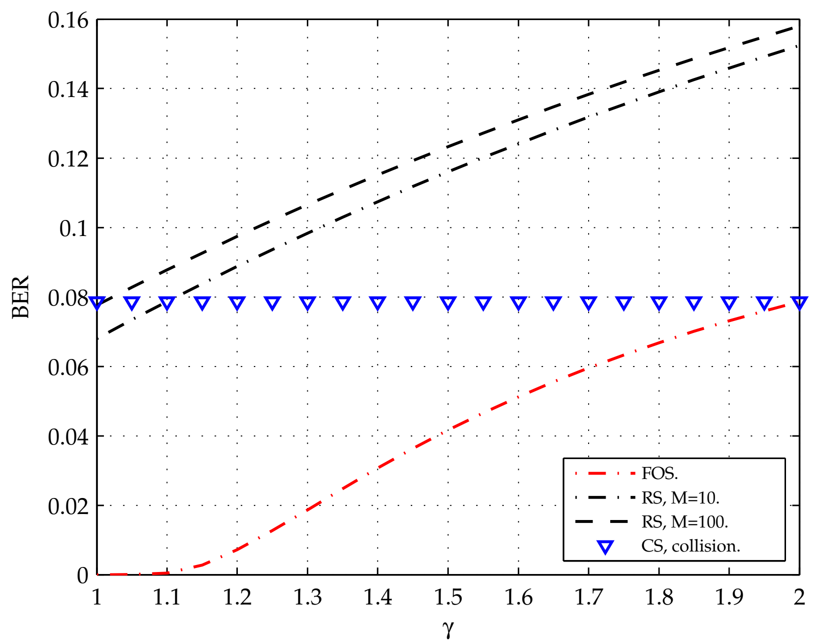


Figure 6.25: Dynamic slow hopping, BER for the three schemes.

7 Conclusions

The aim of this thesis was, on the one hand, designing a new cooperative scheme for balancing the interference across users in FHSS wireless ad hoc networks equitably and, on the other, evaluating its performance by comparing it to two traditional approaches, the RS and the CS. The fairness-oriented new scheme (FOS) has been designed in terms of collisions distribution, which is the basic step for designing the frequency hopping sequences. The FOS, as well as the RS and the CS, has been described and analysed for both static and fast frequency hopping, and the existence of the proposed method for interference-balancing of the FOS has been proved.

The FOS achieves an equitable interference-balancing among all users and, at the same time, it is optimal from the perspective of the interference power seen by each user. This is because the FOS has been designed to guarantee the minimum possible number of collisions per frequency hop while making all users experience the same number of collisions in a hopping sequence. In the performance evaluation chapter, for fast frequency hopping, it has been seen that the FOS outperforms the RS and the CS in terms of the variance of the interference power experienced by each user. Specifically, the FOS exhibits the best behaviour in comparison to the other two schemes for values of γ close to one, i.e., when N is not much higher than M, but it maintains a considerably better performance than the RS and the CS in the whole range $1 < \gamma \leq 2$. Comparing the FOS to the RS in particular, in the range $0 < \gamma \leq 1$, in contrast to the RS, with the FOS there is no interference because hopping sequences remain orthogonal. For $1 < \gamma \leq 2$ orthogonality is no longer preserved but, still, the variance of the interference power is much higher with the RS than with the FOS. So, the FOS is totally preferable than the RS. Comparing the FOS to the CS leads to a similar conclusion. With the CS, hopping sequences are also orthogonal for $\gamma \leq 1$, but for $\gamma > 1$ the variance of the interference power is many times higher than for the FOS. In fact, in the range $1 < \gamma \leq 2$ the CS is much worse than the RS in terms of variance. Yet, the CS is better than the RS concerning the number of collisions per hop, which is optimal for the CS, as well as for the FOS.

For slow frequency hopping, the FOS also displays a better performance in terms of the variance of the interference power than the RS, even though in this case the outcome of the FOS is not as much better than the outcome of the RS than in fast hopping. Likewise, in slow hopping, once the hopping sequences have been distributed, with the FOS all users have the same probability of collision in a randomly chosen hop, whereas with the CS some users collide with probability one and the rest do not collide. Thus, in this case the FOS also outperforms the CS in terms of fairness.

To conclude, the FOS stands as an efficient fairness-oriented approach to the problem of temporarily having more users that may transmit than available frequency channels in a FHSS wireles ad hoc network. From now on, further reasearch on this point would be of interest either to enhance the present scheme (see chapter 8) or to design suitable protocols to implement it.

8 Further Research

8.1 Fairness-oriented scheme extension

Evidently, the assumption made in section 3.8 that all frequency channels have the same attenuation affects the analysis results from the perspective of fairness of interference distribution: if different attenuation for different frequency channels is regarded, then, the amount of interference for every user has a larger variance because, apart from depending on the chosen scheme and on the position of the interferers, it also depends on whether collisions to closer interferes occur through hardly attenuated channels and collisions to more remote interferers occur through softly attenuated channels or vice versa. Whereas the RS and the CS have the same variance either if this assumption is made or not, our scheme has an added random factor. It should be studied and discussed. A possible solution for the FOS to be more equitable would be to make sequences M times longer (with a new sequence length ML) so as to every user collide to any of the other users in all possible frequency channels. This solution would be completely fair. Another solution (much more complex than the previous) would be to estimate the attenuation of every channel (channel estimation) and make the decision about in which channel make two given users collide according to the distance between them and the channel attenuation so as to get a similar amount of interference to all users in just one sequence length L.

8.2 An alternative scheme: OFDM shifted carriers

An alternative scheme to the FOS would be an approach based on OFDM. This scheme would consist in assigning in each hop an OFDM carrier to each user. Therefore, for the case when $N > M$, there would be N OFDM carriers in the same total bandwidth W than for the case $1 < N \leq M$. Thus, the OFDM carriers would be shifted and orthogonality would not be preserved in this case. For the case $M = N$, the bandwidth between carriers is $B = \frac{1}{T}$, where T is the symbol duration and, hence, they are orthogonal. If $N > M$, the bandwidth between carriers will be $\frac{W}{N+1} = \frac{(M+1)B}{N+1} = \frac{\left(1+\frac{1}{M}\right)B}{\gamma+\frac{1}{M}}$. Thereby, all carriers will be interfered by all the others at the same time but with different proportion. The interference power from closer channels will be higher than the one from remote channels. It would be of interest to check out whether with this approach the average interference level experienced by one transmitter is lower than with the FOS, which *a priori* seems to be likely to happen. Fairness would be achieved with this scheme by making all users hop into all N frequency channels in one hopping sequence. One suitable strategy to do it in a pseudorandom manner would be maybe the one proposed in [18].

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