Emergence of Social Conventions in Complex Networks

Jordi Delgado [a,1]

[a] *Departament de Llenguatges i Sistemes Informatics, Universitat Politècnica de Catalunya, Campus Nord, Mòdul C6, 08034 Barcelona, Spain*

Abstract

The emergence of social conventions in multi-agent systems has been analyzed mainly in settings where every agent may interact either with every other agent or with nearest neighbours, according to some regular underlying topology. In this note we argue that these topologies are too simple if we take into account recent discoveries on real networks. These networks, one of the main examples being the Internet, are what is called *complex*, that is, either graphs with the *small-world* property or *scale-free* graphs. In this note we study the efficiency of the emergence of social conventions in complex networks, that is, how fast conventions are reached. Our main result is that complex graphs make the system much more efficient than regular graphs with the same average number of links per node. Furthermore, we find out that scale-free graphs make the system as efficient as fully connected graphs.

Key words: Conventions; Emergent Behavior; Coordination; Multi-Agent Systems

1 Introduction

The study of social conventions in human societies is more than forty years old [21,33]. Social conventions, according to [33], are a special type of norms related to coordination problems, that is, conventions are "those regularities of behavior which owe either their origin or their durability to their being solutions to recurrent (or continuous) co-ordination problems, and which, with time, turn normative" [33, p.96-97]. In contrast, proper social norms are usually meant as solutions to problems of cooperation and originate from conflicts. In this sense, Multi-Agent Systems (MAS) are by no means different, that is,

[1] Email: jdelgado@lsi.upc.es

MAS also may need to coordinate and/or cooperate in order to work as intended [15]. There are definitions of norms and conventions more suitable to MAS, see for example the formal, logic oriented definitions of [16,6], and the game-theoretic definitions of [31] (see section 3), where norms and conventions may be viewed as either obligations, ends/goals or constraints on behavior [34, Chap.2]). However, in this work we will deal only with coordination problems, hence we will focus on social conventions.

Two ways of introducing conventions in MAS have been explored: the off-line design, where every agent has the conventions "hard-wired" from the beginning, and the on-line design, also known as emergent design, where the collective of agents decides, through interaction, which are the most suitable conventions given the current state of the system. The former design is clearly unsuitable in dynamical and changing environments, where one cannot know *a priori* what are the conditions under which the system will operate (this has been argued in [31,35]). In this case, the dynamical nature of on-line conventions appears to be most appropriate. It is not difficult to think of other situations where these non-fixed conventions may be an advantage, for example in the case of agents with changing goals.

In the simplest multi-agent system every agent may interact with every other agent. This means that the underlying topology is a graph with an all-to-all connectivity pattern. However, this is not very realistic. It is far more accurate to assume some restrictions in the pattern of interactions an agent may have. We can think of different possibilities: Regular graphs, lattices, etc. This has already been (partially) analysed, since emergence of conventions in MAS with topological restrictions has been studied in regular graphs (functional and product hierarchies, contract nets, decentralized and centralized markets, see [32] for definitions of these different types of graphs in the context of organization theory) and lattices [19,20,32,35]. This work is quite interesting, since it shows that the underlying MAS topology is important in the efficiency of the emergence of conventions; however, regular topologies are not very realistic either. If we pay attention to the topology of *real* networks, we will find out that most of them have a very particular topology: they are *complex* networks [2,4,25,7,11,14,36] with non-trivial wiring schemes. Notice that one of the possible environments for a MAS, the Internet, is among the most prominent complex networks found in the real world. This fact is clearly relevant also to Organization Theory, where the notion of network plays a predominant role [12,32]. Complex networks are well characterized by some special properties, such as the connectivity distribution (either exponential or power-law) or the *small-world* property [25,37].

In this note we will study the efficiency of the emergence of social conventions in MAS with a complex underlying topology. We will follow the conceptual framework introduced by Shoham & Tennenholtz [29–32] and our measure of efficiency will be one of those introduced in the work of Kittock [19]: the time it takes to reach a 90% of the agents in the system to use the same convention. This will be detailed in section 3. In section 2 we will give a detailed description of the graph models of complex networks we have used with their most relevant properties. We have used two different action update rules (they will be defined in section 3): a generalized version of the Simple Majority rule (GSM) and the well known Highest Current Reward rule (HCR). There is a theorem on the emergence of conventions in MAS using the HCR [31], but, as far as we know, to Simple Majority rule has not been used yet. Here we will provide some analytic evidence of the convergence of the GSM in graphs fulfilling some special properties. Our results on both systems will be described in section 4. Finally, we will discuss our results in section 5.

2 Graph Models

There are several models of graphs we are going to use as the underlying topology of our MAS. First, we define the following model of regular graphs (these are the regular graphs underlying the systems explored in [19]):

Definition 1 ([19]) *$C_{N,K}$ is the graph on N nodes such that node i is adjacent to nodes $(i+j) \mod N$ and $(i-j) \mod N$ for $1 \leq j \leq K$.*
K_N is the complete N-nodes graph (every node is adjacent to all the $N-1$ other nodes).

The $C_{N,K}$ graphs are called *contract nets with communication radius K* in [32].

As we pointed out in section 1, recent discoveries on real networks lead us to think that regular graphs are not the most realistic environment for MAS. Lots of real networks have been studied [4,5,7,9,14,25,36] though the most interesting result for us is that the Internet is a *complex* network, a scale-free graph with small-world properties [2,4]. Since the Internet is a quite reasonable environment for a MAS, and since the underlying topology is important for the efficiency of the emergence of conventions (as shown by [19], see subsection 3.2), it is quite clear that the study of the efficiency of the emergence of conventions in complex networks will provide more realistic results.

The graphs we will use as models of complex networks are:

- *Small-world* graphs W_N: These are highly clustered graphs (like regular lattices) with small characteristic path lengths (like random graphs) [37,36]. This is the small-world property. We will choose the Watts-Strogatz model as model of small-world graphs.
- *Scale-free* graphs S_N^γ: These are graphs with a connectivity distribution $P(k)$

(the probability that a node has k adjacent nodes) of the form $P(k) \propto k^{-\gamma}$. We will choose the Albert-Barabási extended model as model of scale-free graph.

Hence, we use the term "complex" only in reference to graphs, meaning either "scale-free" or "small-world". In the case of the Internet, the probability that a certain web document points to k documents follows a distribution of the form $P(k) \propto k^{-2.45}$ [4,8] and the exponent of the connectivity distribution of Internet Service Providers is $\gamma = 2.5$ [11].

Albert, Barabási and Jeong have recently proposed a set of different models for scale-free graphs, based on the growing process of the Internet and other real complex networks. We have used the Albert-Barabási [3] *extended* model as model of scale-free graphs, since it gives us some control over the exponent γ of the graph. The underlying idea is that of growth with preferential connectivity, where the most "popular" nodes get most of the links. This model was built on a simpler one [7,8], able to generate graphs with exponent $\gamma = 2.9 \pm 0.1$ (by setting $p = q = 0$ in the algorithm detailed below we recover this previous model). We will define precisely these graphs by giving an algorithm to build them.

The algorithm depends on 4 parameters: m_0 (initial number of nodes), m (number of links added and/or rewired at every step of the algorithm), p (probability of adding links) and q (probability of edge rewiring). The procedure is: Start the algorithm with m_0 isolated nodes, and perform at every step one of these three actions:

1 With probability p add m ($\leq m_0$) new links. We pick two nodes randomly. The starting point of the link is chosen uniformly and the end point of the new link will be chosen according to the following probability distribution:

$$\Pi_i = \frac{k_i + 1}{\sum_j (k_j + 1)}$$

where Π_i is the probability of selecting the i-th node, and k_i is the number of edges of node i. This process is repeated m times.

2 With probability q, m edges are rewired. That is, we repeat m times: Choose (uniformly) at random one node i and a link l_{ij}. Delete this link. Choose another (different) node k with probability $\{\Pi_l\}_{l=1\ldots N}$ and add the new link l_{ik}.

3 With probability $1 - p - q$ add a new node with m links. These new links will connect the new node to m other nodes chosen according to $\{\Pi_l\}_{l=1\ldots N}$.

Once we get the desired number N of nodes, we stop the algorithm. The graphs generated with this algorithm are scale-free *random* graphs, that is, there are no correlations among edges [27]. It can be shown [3] that in the limit of

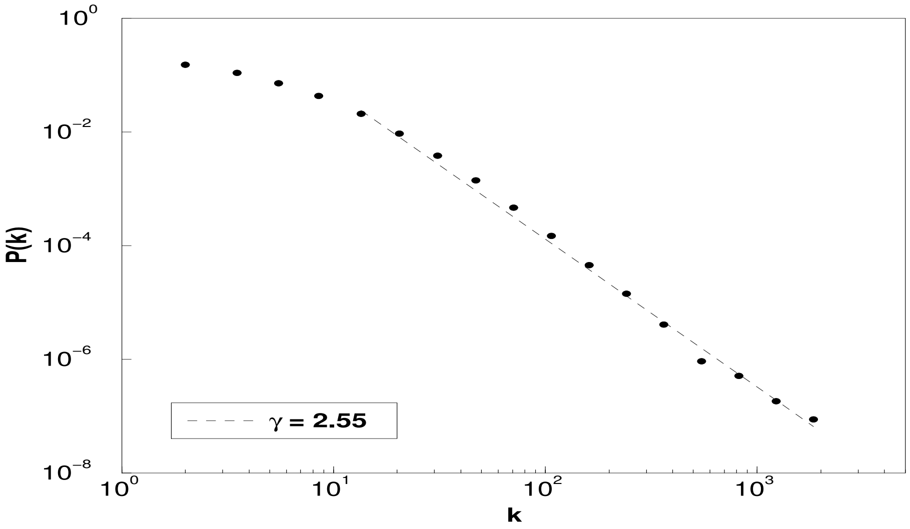

Fig. 1. This figure shows the connectivity distribution of a graph generated with the Albert-Barabási extended model (see text). Parameters are $N = 5{\times}10^4$, $p = q = 0.4$, $m_0 = 4$ and $m = 2$, so the exponent should be $\gamma = 2.3$. As we see in the plot, the real exponent is $\gamma \sim 2.5$. The data were logarithmically binned.

large N, when $p = q$, this algorithm ends up with a graph with connectivity distribution

$$P(k) \propto (k+1)^{-\left(\frac{2m(1-p)+1-2p}{m}+1\right)}$$

that can be approximated, when $k >> 1$, by $P(k) \propto k^{\gamma}$ where $\gamma = \frac{2m(1-p)+1-2p}{m} + 1$. The graphs in our experiments are no larger than $N = 10^5$, therefore the theoretical exponent suffers from finite-size effects and must be computed numerically. (see figure 1).

Albert-Barabási's scale-free graphs have not the small-world property. So we have chosen another graph model to work with: The Watts-Strogatz model. It depends on two parameters, connectivity (K) and randomness (P), given the size of the graph (N). This model starts with a $C_{N,K}$ graph and then every link is rewired at random with probability P, that is, for every link l_{ij} we decide whether we change the "destination" node with probability P; if this is the case, we choose a new node k uniformly at random (no self-links allowed) and add the link l_{ik} while erasing link l_{ij}. In fact, for $P = 0$ we have $W_N = C_{N,K}$ and for $P = 1$ we have a completely random graph (but not scale-free). For intermediate values of P there is the "small-world" region, where the graph is highly clustered (which means it is *not* random) but with a small characteristic path length (a property shared with random graphs). The Watts-Strogatz model does not generate scale-free graphs, since the distribution $P(k)$ associated to these graphs is exponential [8].

3 Social Conventions in MAS

First we will describe some general properties of the systems we are going to study. Details will require separate subsections. The MAS we deal with are extremely simple, but also *necessarily* simple if we want to get to any conclusion about its dynamics. The use of these simple settings in MAS theory has been largely discussed in [31,35], to which we refer for more information.

Our MAS will consist of N agents on a graph, where every agent will be located on a node of the graph. Its adjacent agents will be called its *neighbors*. Every agent will be in one out of two states (or actions), called A and B. The system will evolve in time, and at each time step one agent will be selected at random, for state updating [2]. Different rules to update agent's state will define different systems. In this note we will study two different rules: the *generalized simple majority* rule (detailed in subsection 3.1) and the *highest current reward* rule (detailed in subsection 3.2).

We define a *social convention* as in [31]

Definition 2 ([31]) *A social law is a restriction on the set of actions available to agents. A social law that restricts the agents' behavior to one particular action is called a social convention.*

In our case a social convention will be reached if all the N agents are either in state A or in state B.

From [19] we will get the performance measure we use to evaluate how fast conventions arise in our systems, it is the *convergence time* T_c: the convergence time for a given level of convergence c is the earliest time at which $C_t \geq c$, where C_t is the convergence of a system at time t, that is, the fraction of agents using the majority action (either A or B). In this note we will focus on the study of the average time to a fixed convergence (we set c to 90%, following [19]).

[2] The dynamics we use is *asynchronous*, following previous work [19,20,31]. We will depart from Walker & Wooldridge formalization [35] because the dynamics they use (their function r, used to define a *run*) imposes a *synchronous* dynamics, where all agents interact at once. This is, at least, problematic. It is well known that some "emergent" properties of synchronous systems are not due to the system itself, but to global correlations introduced by this synchronous update [17,22].

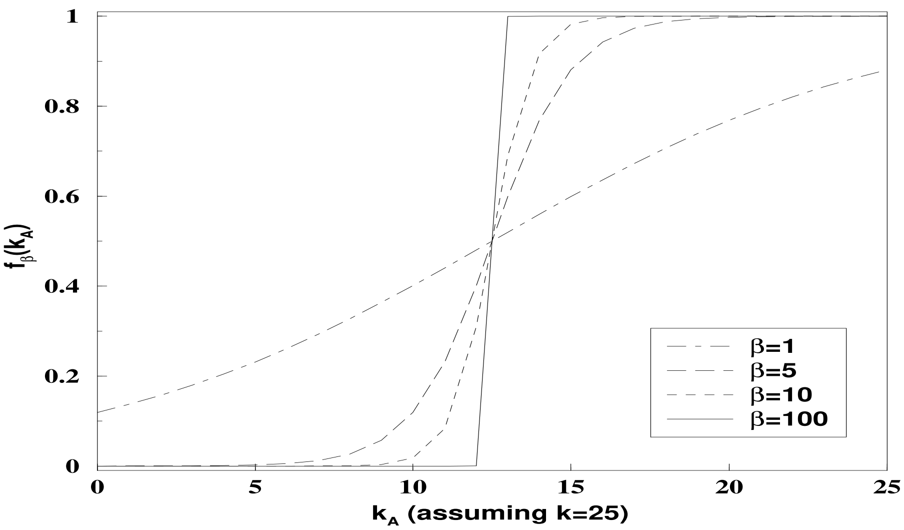

Fig. 2. Generalized simple majority rule: If an agent chosen for updating is in state B and has 25 neighbors, this figure shows how the probability of changing state varies as a function of the number of neighbors in state A, for different values of β.

3.1 Generalized Simple Majority: Definition

We generalize herein the simple majority rule, as was defined in [35]. We have N agents on a graph, so we have a well defined neighborhood for every agent. The initial state of the system is a random state (either A or B) for every agent. Now, at every time step one agent, say the j-th, is chosen randomly. Let us suppose that agent j has k neighbors and that k_A neighbors are in state A (so there are $k - k_A$ neighbors in state B). If agent j is in state S, let $\bar{S}$ be the complementary state. Thus, agent j will change to state $\bar{S}$ with probability

$$f_\beta(k_{\bar{S}}) = \frac{1}{1 + e^{2\beta(2k_{\bar{S}}/k - 1)}}$$

This rule generalizes simple majority since for $\beta \to \infty$ we recover the change of state only when more than $k/2$ neighbors are in state $\bar{S}$ (see figure 2). Notice that Walker & Wooldridge's simple majority rule in a system with two-state agents and any type of graph as underlying topology is a deterministic version of what was called a *convention evolution setting* in [32]. There is no theorem assuring convergence in the emergence of conventions in the system defined with the GSM rule, but we can provide some analytical evidence that this is the case. We use what in physics is called a *mean-field* argument [26]. Let $N_A(t)$ be the number of agents in state A at time t and $\rho(t) = N_A(t)/N$ be the density of agents in state A. We will assume the following *homogeneity* condition: for every agent with k neighbors, the number of neighbors in state A is $k_A(t) \simeq k\rho(t)$. This condition is completely fulfilled for K_N graphs (obviously), and approximately fulfilled for S_N^γ graphs and W_N graphs with $P \to 1$, since these are random graphs. In this case, the probability of change will depend on $\rho(t)$.

Let us assume an agent is in state B and has k neighbors. The agent will change with probability

$$f_\beta(k_A(t)) = \frac{1}{1+e^{2\beta(2k_A(t)/k-1)}} \simeq \frac{1}{1+e^{2\beta(2k\rho(t)/k-1)}} = \frac{1}{1+e^{2\beta(2\rho(t)-1)}} = f_\beta(\rho(t))$$

By the same argument, if the agent is in state A, it will change with probability $f_\beta(k_B(t)) = f_\beta(1-\rho(t))$.

Now, we can write an equation for the evolution of $\rho(t)$. First, notice that the variation of $\rho(t)$ after a small time interval Δt is proportional to Δt, that is, $\rho(t+\Delta t) = \rho(t) + \partial_t \rho(t)\Delta t + O(\Delta t^2)$. Then, we can neglect the $O(\Delta t^2)$ term (since we want to perform a continuum approximation $\Delta t \to 0$) and compute the variation of $\rho(t)$ as the balance between the agents switching from state B to state A and the agents switching from state A to state B. On one hand, the fraction of agents in state B (that is, $1-\rho(t)$) that change to state A in a time interval Δt is the product $(1-\rho(t))f_\beta(\rho(t))\Delta t$, provided Δt is small enough; on the other hand, the fraction of agents that switch form state A to state B in Δt is $\rho(t)f_\beta(1-\rho(t))\Delta t$, also for small Δt. That is,

$$\rho(t+\Delta t) - \rho(t) = \left[(1-\rho(t))f_\beta(\rho(t)) - \rho(t)f_\beta(1-\rho(t))\right]\Delta t$$

Thus, after $\Delta t \to 0$, the mean-field equation for $\rho(t)$ can be written as

$$\partial_t \rho(t) = (1-\rho(t))f_\beta(\rho(t)) - \rho(t)f_\beta(1-\rho(t))$$

After substitution of $f_\beta(\rho(t))$, this equation reads

$$\partial_t \rho = -\rho + \frac{1+e^{2\beta(2\rho-1)}}{2+e^{-2\beta(2\rho-1)}+e^{2\beta(2\rho-1)}}$$

finally, with the change $x(t) = 2\beta(2\rho(t)-1)$ we get to

$$\partial_t x = -x + 2\beta\frac{e^x - e^{-x}}{2+e^x+e^{-x}}$$

We want to study the stable fixed-points of $x(t)$, since these will give us information on the final state of the system. Thus, we must find the solutions of $\partial_t x(t) = 0$. It can be shown that, for $\beta > 1$ the only stable fixed-points of this equation are $x_1 = -2\beta$ and $x_2 = 2\beta$, that is, $\rho_1 = 0$ (state B is the reached convention) and $\rho_2 = 1$ (state A is the final state of all the agents). Let us point out that this result implies the convergence to a social convention in systems using the simple majority rule, as defined in [35]. Initial conditions will break the symmetry of the solutions, that is, an initial fraction of agents in state A slightly larger than the initial fraction of agents in state B will get the system to a $\rho = 1$ stationary state, and vice-versa (see figure 3). In this note we will not study the effect of β, setting $\beta = 10$.

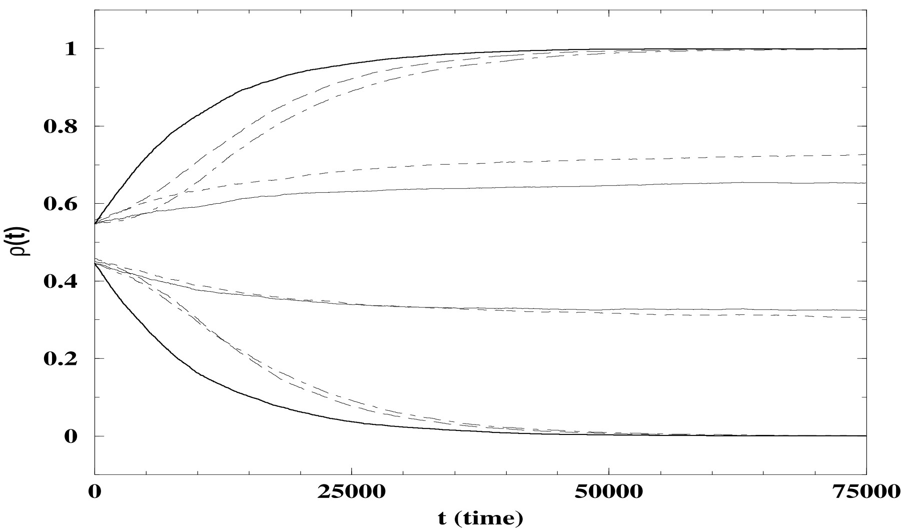

Fig. 3. Evolution in time of the density of agents in state A for the system defined in subsection 3.1. Several graphs have been used ($N = 10^4$): K_N (thick solid line), $C_{N,12}$ (solid line), $S_N^{2.5}$ (dot-dashed line, $m_0 = 4$, $m = 2$, $p = q = 0.4$), S_N^3 (long-dashed line, $m_0 = 7$, $m = 6$, $p = q = 0$) and W_N (dashed line, $P = 0.05$ and $K = 12$, inside the small-world region). We observe a fast convergence in the graphs that fulfill the homogeneity condition, that is, the scale-free graphs and the K_N graphs.

3.2 Highest Current Reward: Definition

The framework in which we will work here was introduced by Shoham & Tennenholtz [29–32] some time ago, though it is in frequent use nowadays (see [10,13,23] for example). In this note we will adapt from [31] the definitions and theorems we need, not dwelling on justifications of this formal framework (it was eloquently done in [31]). We will focus on *coordination* games [21,30]

Definition 3 ([31]) *A payoff matrix G 2×2 defines a 2-person 2-choice symmetric coordination game if G has the form*

$$\begin{pmatrix} x & u \\ v & y \end{pmatrix}$$

where $x > v$ and $y > u$

Essentially the idea is that every player has two available actions, say A and B. If both players play A, both players receive a payoff of x. If they play B they receive a payoff of y. When the players do not agree, for example, player 1 plays A and player 2 plays B, the former receives a payoff of u and the latter a payoff of v; the remaining situation is symmetric. The condition on the entries of G makes clear that to play the same action is the best choice.

Specifically, we will use the *pure coordination game* [21] G, where $x = y = +1$ and $u = v = -1$.

Now, once defined the game we need to define the players. Our MAS will be composed of N agents (every agent is a player) that will interact with other agents, playing the game G once per interaction. What we are interested in is whether the dynamics of this system makes all the agents reach a social convention. In our particular setting, this means that we want to know whether all the agents will end up playing one of the two possible actions of the game G, say A and B.

Following [19], every agent, say the k-th, will be characterized by a *memory* M_k of size M (same size for all the agents) and an action a_k (to play the next time agent k is selected, so the value of a_k is either A or B). The memory M_k will record some information on the M last plays of the agent k: The value of the position i of the memory M_k will be a tuple $< a_k^i, p_k^i, t^i >$ where t^i is the time the i-th play took place, a_k^i is the action played by agent k and p_k^i is the payoff received ($1 \leq i \leq M$). However, in this work we will not study the effect of memory, setting $M = 1$.

Now we must define the dynamics of the system (a variant of $n - k - g$ stochastic social games [31] where we will take into account the underlying topology). At every time step t, a pair of agents will be selected to play the game G, where one of them will be randomly chosen and the other will be one of its neighbors, according to the underlying graph. They will receive a payoff (either $+1$ or -1) depending on their actions. Let us assume that at time t, agents k (with memory M_k and action a_k) and l (with memory M_l and action a_l) are chosen to play. Every agent will receive a certain payoff, say p_k and p_l. Now, agent k must decide which action it is going to play next time it is chosen, as a function of its memory M_k, the action a_k played and the payoff received p_k. It uses the *Highest Current Reward* rule. Agent k will compute the payoff received for using action A in the last M plays in which it has been involved: $P_A^k = \sum_{i:a_k^i=A} p_k^i$, where P_B^k is defined in the same way. Agent k will add p_k to either P_A^k or P_B^k, depending on a_k. Now, agent k can decide: Next time it is chosen to play, the action chosen by the agent k will be either A if $P_A^k > P_B^k$, B if $P_B^k > P_A^k$ or a_k otherwise. Finally agent k updates its memory, deleting the oldest entry and adding the tuple $< a_k, p_k, t >$ (agent l will do the same thing, the rest of the system will do nothing).

Shoham & Tennenholtz [30,31] provide a general theorem that guarantees the convergence of our system to a stable social convention. With the system as defined above, an immediate consequence of Shoham & Tennenholtz result is:

Theorem 1 ([31], theorem 12) *Given a 2-person 2-choice symmetric coordination game, with the dynamics as defined and using the HCR action* *selection rule:*

- $\forall\epsilon > 0$ *there exists a bounded number* Γ*, such that if the system runs for* Γ *iterations then the probability that a social convention will be reached is greater than* $1-\epsilon$.
- *Once the convention is reached, it will never be left.*

Kittock [19] studied numerically the efficiency of the emergence of conventions in regular graphs $C_{N,K}$ and K_N. His main result was that the underlying topology has a profound effect on the efficiency with which conventions emerge, and he conjectured that this efficiency depends essentially on the *diameter* of the graph.

Shoham & Tennenholtz showed [31] that the update rule known as *External Majority* (EM) (equivalent to the HCR rule, but taking into account only the majority of the states seen in the last M interactions, not the payoff received) was equivalent to the HCR update rule in 2-person 2-choice coordination games. However, notice that their definition of EM is different from Walker & Wooldridge definition [35] of the simple majority rule, in which the simultaneous state of neighbours is used to update agents' state. So, the GSM rule (based, as defined above, on Walker & Wooldridge's rule) is different from Shoham & Tennenholtz's EM rule and their result cannot be extended to our GSM rule without further consideration.

4 Convergence time of social conventions in complex networks.

Once we know that social conventions will emerge in the systems we are interested in, we would like to know how fast these conventions will be reached.

The results of our experiments can be seen in figs 4, 5 and 6. First we performed experiments analogous to those of Kittock [19]. We measured $T_{90\%}$ vs. N in systems using the HCR rule and four different underlying graphs (fig 4): regular graphs K_N and $C_{N,12}$, scale-free graphs (generated with the Albert-Barabási algorithm detailed in section 2, with parameters $m_0 = 4$, $m = 2$ and $p = q = 0.4$ which provide graphs with exponents $\gamma \simeq 2.5$ and average connectivity $\simeq 12$) and small world graphs (generated with the Watts-Strogatz algorithm detailed in section 2, with parameters $P = 0.1$ and $K = 12$ so that the graphs have the small world property). We have chosen graphs with the same (average) connectivity per node in order to perform a reasonable comparison, and the exponent of the scale-free graphs in such a way that it be similar to the exponent of the Internet. For every N, we ran 25 simulations of the system initialized randomly (agents with random initial state, either A or B with probability 0.5), averaging the results (see table).

Fig 4	Graph Parameters	Size$\times 10^3$ (N)	Samples
K_N	None needed	1, 2.5, 5, 7.5, 10, 25, 50	25
$C_{N,12}$	None needed	0.1, 0.25, 0.5, 0.75, 1	25
$S_N^{2.5}$	$m_0 = 4$, $m = 2$, $p = q = 0.4$	1, 2.5, 5, 7.5, 10, 25, 50	25
W_N	$P = 0.1$, $K = 12$	1, 2.5, 5, 7.5, 10	25

An identical experiment was performed with systems using the GSM rule (fig 5), though in this case we used, besides the above mentioned graphs, scale-free graphs with a different exponent (parameters $m_0 = 7$, $m = 6$ and $p = q = 0$ which provide graphs with exponent $\gamma \simeq 3$ and an average connectivity $\simeq 12$). See table.

Fig 5	Graph Parameters	Size$\times 10^3$ (N)	Samples
K_N	None needed	1, 2.5, 5, 7.5, 10, 25, 50, 75, 100	25
$C_{N,12}$	None needed	0.1, 0.25, 0.5, 0.75	25
$S_N^{2.5}$	$m_0 = 4$, $m = 2$, $p = q = 0.4$	1, 2.5, 5, 7.5, 10, 25, 50, 75, 100	25
S_N^{3}	$m_0 = 7$, $m = 6$, $p = q = 0$	1, 2.5, 5, 7.5, 10, 25, 50, 75, 100	25
W_N	$P = 0.1$, $K = 12$	1, 2.5, 5, 7.5, 10, 25, 50	25

For reasons explained below, a third experiment was necessary (fig 6) in which we measured $T_{90\%}$ vs. P for systems with different update rules (HCR and GSM) and underlying Watts-Strogatz graphs of size 10^4 and connectivity $K = 12$. We ran 25 simulations (each with random initial conditions) for every P ($P = 0.05$ and then from $P = 0.09$ to $P = 0.9$ with an increment of $\Delta P = 0.09$) and averaged the results (see table).

Fig 6	N	K	P	Samples
HCR rule	10^4	12	$P = 0.05$ and $P = 0.09 \ldots 0.9$ ($\Delta P = 0.09$)	25
GSM rule	10^4	12	$P = 0.05$ and $P = 0.09 \ldots 0.9$ ($\Delta P = 0.09$)	25

From our numerical work (see figures 4 and 5, these figures are representative of results obtained with different sets of parameters) we may conjecture that $T_{90\%} = O(N^3)$ for $C_{N,K}$ graphs (which was already observed by Kittock [19]) and $T_{90\%} = O(N \log N)$ for complex graphs and K_N graphs (this is the lower bound predicted analytically in [31,32]) for the HCR rule. Results for the GSM rule are $T_{90\%} = O(N^3)$ for $C_{N,K}$ graphs and $T_{90\%} = O(N)$ for complex graphs and K_N graphs. Besides, we observe that an underlying small-world graph makes the system less efficient than an underlying scale-free graph, despite they have the same behavior.

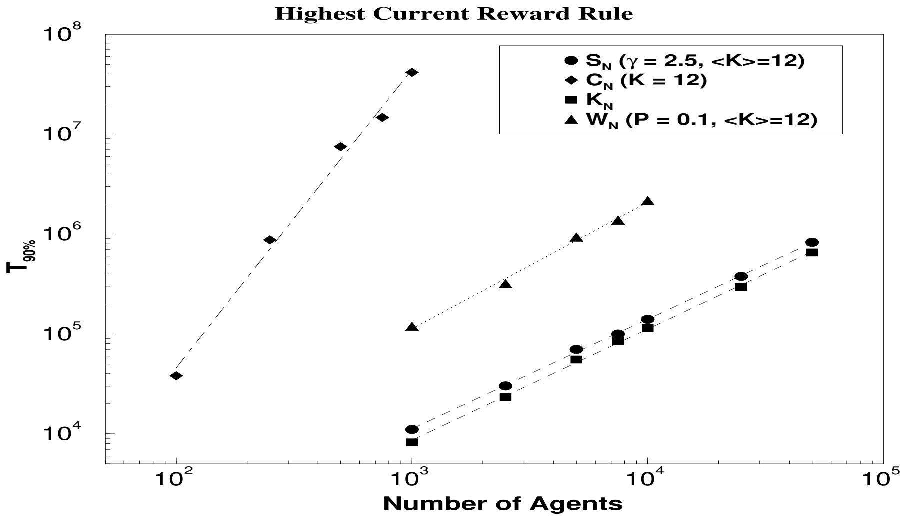

Fig. 4. HCR rule: $T_{90\%}$ vs. N, averaged over 25 samples for each N. Several graphs have been used: K_N, $C_{N,12}$, $S_N^{2.5}$ ($m_0 = 4$, $m = 2$, $p = q = 0.4$) and W_N ($P = 0.1$ and $K = 12$, inside the small-world region). All the graphs have the same average connectivity per node (except K_N, for obvious reasons).

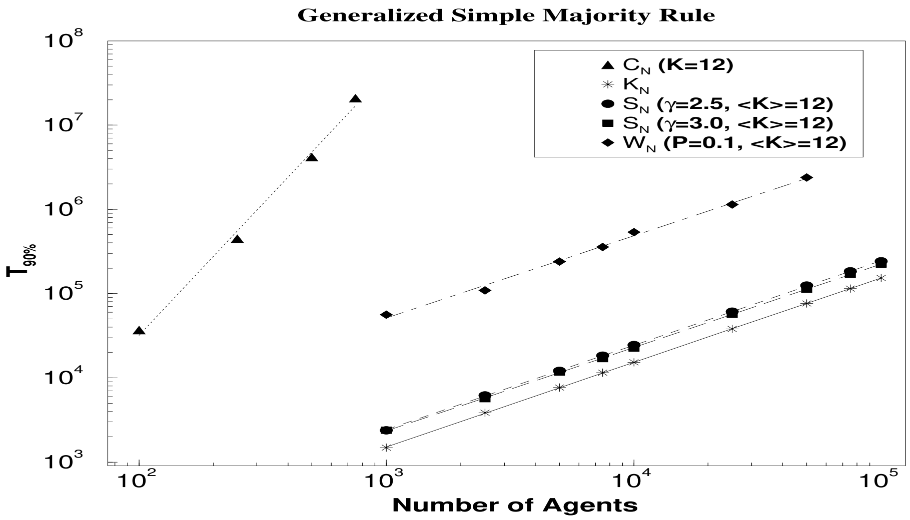

Fig. 5. GSM rule: $T_{90\%}$ vs. N, averaged over 25 samples for each N. Several graphs have been used: K_N, $C_{N,12}$, $S_N^{2.5}$ ($m_0 = 4$, $m = 2$, $p = q = 0.4$), S_N^3 ($m_0 = 7$, $m = 6$, $p = q = 0$) and W_N ($P = 0.1$ and $K = 12$, inside the small-world region). All the graphs have the same average connectivity per node (except K_N, for obvious reasons).

Kittock's conjecture provides us with a partial explanation of the observed behavior. According to [19], the efficiency of the emergence of social conventions depends on the diameter of the graph. The diameter of $C_{N,K}$ grows linearly with N [36] but the diameter of complex graphs grows *logarithmically* with N [25], hence the difference between the growth of $T_{90\%}$ in regular graphs and complex graphs. However, the precise relation between the linear growth of the diameter in regular graphs and the $O(N^3)$ behavior of $T_{90\%}$ for both rules, and between the logarithmic growth of the diameter in complex graphs and the $O(N \log N)$ behavior of $T_{90\%}$ for the HCR rule, $O(N)$ behavior for the GSM rule, remains to be fully justified by means of analytical arguments.

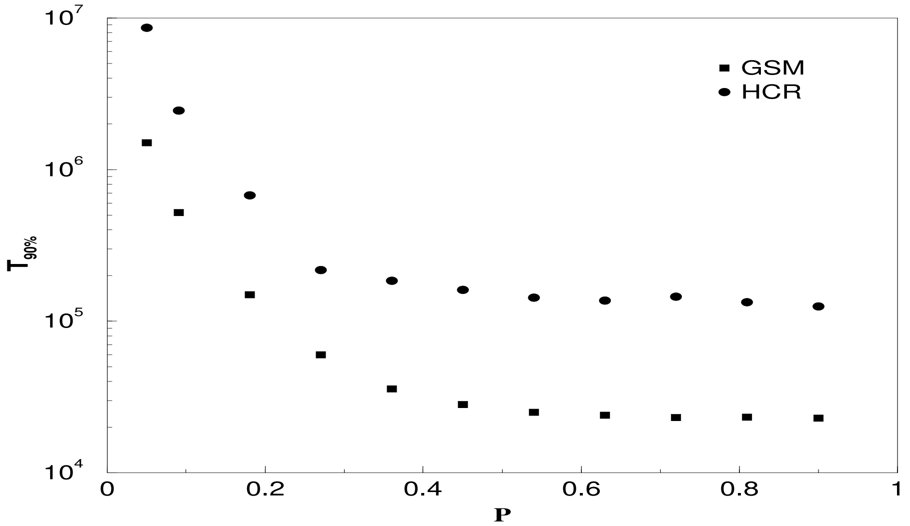

Fig. 6. $T_{90\%}$ vs. P for systems using the two different action update rules studied in this note, with underlying graphs generated with the Watts-Strogatz model. Parameters are $N = 10^4$ and $K = 12$

Now, the efficiency of the emergence of social conventions in systems with underlying scale-free graphs is almost as good as with K_N graphs, despite having a constant (with respect to N) average connectivity. Notice that K_N graphs are optimal with respect to $T_{90\%}$, provided Kittock's conjecture is correct, since this quantity depends on graph diameter and this equals 1 for K_N graphs. Besides, underlying small-world graphs are less efficient than scale-free graphs, despite they have same behavior with respect to N. This is so because what is important here is the randomness (in the sense mentioned above) of the scale-free graphs we have used, since randomness reduces the graph diameter. However, the small-world property seems to have no effect on $T_{90\%}$. We may perform some experiments to test this hypothesis. The Watts-Strogatz model allows us, by means of the parameter P, to go from small-world graphs (small P) to random graphs ($P \to 1$) with an exponential $P(k)$. Thus, measuring $T_{90\%}$ on the Watts-Strogatz model with varying P will make clear the importance of randomness. We see in figure 6 that $T_{90\%}$ decreases with P, without noticing the small-world zone for small P: The graph becomes more and more random and the system becomes more and more efficient. This fact makes clear, again, that the diameter seems to be the important factor in the efficiency with which conventions are reached. Thus, our results are fully consistent with Kittock's results.

5 Summary and Prospects

In this note we have introduced the analysis of MAS with underlying complex topologies. We have defined simple MAS with which to study the efficiency of the emergence of social conventions in complex networks. On the one hand we have defined MAS with the action update rule called the generalized simple majority rule, providing analytical evidence of convergence to a social convention, and, on the other hand, we have studied the well-known MAS with the highest current reward rule as action update rule. On both systems we have performed a numerical study of $T_{90\%}$ as a function of N and, in graphs defined according to the Watts-Strogatz model, of P. Our results on both systems are consistent with the hypothesis that the diameter of the graph underlying the MAS is of essential importance in the efficiency with which conventions are reached [19]. We have found a topology that makes the system as efficient as the K_N graph but at a lower cost, where the cost is the average number of links per node.

Some questions are still open: It remains to be analytically justified the precise relation we have found among the growth of the diameter for different classes of graphs and the behavior of $T_{90\%}$.

There are many ways to extend the work introduced in this note. We may perform a similar study of the effect of either memory (M in systems using the HCR rule) or randomness in state switching (β in systems using the GSM rule). The same study may be repeated with cooperative games, since these games were also considered by Shoham & Tennenholtz [29–31] and Kittock [19]. Furthermore, preliminary results indicate that the behavior of cooperative games in complex networks is far from trivial [1,36]. Also, it would be interesting to study MAS playing stag-hunt games [18,28] in complex networks, since these games are frequently used in the study of emergent conventions in game theory. These complex topologies could also be introduced in organization theory, since, quoting [12], “work on organizational design suggests that different architectures influence performance and there is no one right organizational design for all tasks”.

Finally, it has been shown that systems with mobile elements have also some of the properties of complex networks [24], so a study of coordination and cooperation games in mobile agents should also be relevant to MAS theory.

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